Novel analytical solutions to a new formed model of the (2+1)-dimensional BKP equation using a novel expansion technique

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Submission Info

Communicated by Albert Luo Received 01-09-2021 Accepted 01-09-2021 Available online 01-09-2021

Keywords

Nonlinear PDEs BKP equation Exact solution Traveling wave solutions

Abstract

In this article, we present a comprehensive analytical study to obtain the exact traveling wave solutions to a new formed model of the (2+1)-dimensional BKP equation. We construct exact solutions of the considered model using a recently developed expansion technique. This current proposed technique has been successfully implemented to obtain a few exact solutions of a new formed (2+1)-dimensional BKP equation. In order to understand the physical interpretation of solutions effectively, the 2D and 3D graphs are plotted for each type of the solutions obtained for different particular values of the parameters. Furthermore, it is found that the obtained solutions are periodic and solitary wave solutions. We anticipate that the proposed method is reliable and can be applied for obtaining wave solutions of the other nonlinear evolution equations (NLEEs).

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1 Introduction

Nowadays the exact solution of nonlinear evolution equation is a very active research area. In the study of nonlinear physical phenomena, the exact analytical traveling wave solutions of the non–linear partial–differential equations (NPDEs) plays a very significant role. The non-linear physical wave phenomena takes place in several engineering and scientific fields. For example, in plasma physics, optical fibers, chemical physics and geochemistry, biology, solid state physics and fluid mechanics, the non-linear evolution equations are broadly used as a model to study the physical phenomena.

Various powerful effective techniques have been established to obtain the exact traveling wave solutions of various nonlinear evolution equations. Some of these powerful techniques are the sine-Gordon method [1–3], the Sardar sub-equation technique [4–6], the modified Sardar sub-equation method [7,8], the Riccati-Bernoulli sub-ODE method [9,10], the $\left(\frac{G'}{G}\right)$ -expansion method [11–16], the $\left(\frac{G'}{G},\frac{1}{G}\right)$ -expansion method [17–20], the modified exp-function method [21–24], the simple equation method (SEM) [25,26], the tanh–coth expansion method [27–29], $\left(\frac{1}{G'}\right)$ -expansion technique [20,30–32], Lie symmetry approach [33–36] and so on.

Recently, Ahmed et al. have found mixed lump, periodic-lump and breather soliton solutions to the (2+1)-dimensional KP equation with the help of symbolic computation using the Hirota bilinear

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method [37]. They have also observed some new type of characteristics of the periodic–lump solutions and kinky–breather solitons. Kayum et al. have established stable traveling wave solutions to the nonlinear Klein–Gordon equation in particle physics, condensed matter physics, solid state physics, the gas dynamics and nonlinear optics equation utilizing the sine-Gordon expansion technique [38]. They have established the familiar stable wave solutions covering an extensive range which related to free parameters. They have obtain various new solutions for the particular values of the parameters. In the article [39], a generalized (G'/G)-expansion technique is used for deriving the exact solitary wave solutions generated in the form of trigonometric, hyperbolic and rational functions for the the non-linear and regularized long wave (RLW) as well as for the Riemann–wave (RW) models. Shakeel et al. [23] have obtained various types of analytical exact wave solutions such as complex function and hyperbolic solutions of the strain-wave equation found in micro-structured solids, which is very important in the field of solid physics, utilizing the modified exp-function technique.

Tripathy et al. [40] and Khaliq et al. [41] have successfully applied the proposed expansion technique to obtain some novel exact traveling wave solutions of the ion sound Langmuir wave model and the (2 + 1)-dimensional Boussinesq equation respectively. They have demonstrated the physical interpretation of nonlinear processes and the efficiency of the proposed method.

In this current work, our main purpose is to obtain the new exact traveling wave solutions to a new formed equation of the (2+1)-dimensional BKP model. In this work, for the first time we have used $\left(\frac{G'}{G'+G+A}\right)$ -expansion technique [41–43] to obtain a few analytical solutions to a new formed equation of the (2+1)-dimensional BKP model. We have plotted the 3D and 2D graphics simulations of the newly obtained exact solutions. By observing the various nature of the waves and from numerical simulations one can find more useful information regarding the new reduced form of the (2+1)-dimensional generalized BKP equation.

This work is arranged as follows: In the Section 2, we have explained the algorithm of the $\left(\frac{G'}{G'+G+A}\right)$ -expansion technique. In Section 3, we have implemented the proposed method to obtain the exact traveling wave solutions to a new form of the (2+1)-dimensional B-type Kadomtsev-Petviashvili (BKP) equation. Results and discussions are given in Section 4. Lastly, Section 5 is devoted to the conclusions.

2 The algorithm of the $\left(\frac{G'}{G'+G+A}\right)$ - expansion technique

The steps of the $\left(\frac{G'}{G'+G+A}\right)$ -expansion technique to find the exact traveling wave solutions of the nonlinear evolution equations (NLEEs) are discussed in this section. Consider the general non-linear evolution equation of the following type

$$f(u, u_{v}, u_{x}, u_{t}, u_{xx}, u_{vv}, u_{vx}, u_{xt}, u_{xv}, \dots) = 0,$$
(1)

where the function $u \equiv u(x, y, t)$ is not known, f is a polynomial function involving u and its higher order derivatives together with nonlinear terms. Subscripts in this Eq. (1) indicate the various partial derivatives. The main steps of the $\left(\frac{G'}{G'+G+A}\right)$ - expansion technique are given below:

Step-I: The traveling wave variable is defined as $\xi = nx + my - pt$ by taking three independent variables x, y, and t into one variable ξ . Here n, m and p are constants. Let us assume that the solution of Eq. (1) in ξ as

$$u(x, y, t) = w(\xi). \tag{2}$$

Using the above transformation $\xi = nx + my - pt$ and $u(x, y, t) = w(\xi)$ in the Eq. (1), we get the following ordinary differential equation (ODE):

$$g(w, w', w'', w''', \dots) = 0,$$
 (3)

where $w' = \frac{dw}{d\xi}$, $w'' = \frac{d^2w}{d\xi^2}$, $w''' = \frac{d^3w}{d\xi^3}$

Step-II: We suppose that the exact analytical solutions of Eq. (3) can be written in the following form:

$$w(\xi) = \sum_{k=0}^{N} a_i \left(\frac{G'}{G' + G + A} \right)^k, \tag{4}$$

where $G \equiv G(\xi)$ satisfies the following auxiliary second order ODE:

$$G'' + BG' + CG + AC = 0. (5)$$

Here the prime indicates for ordinary derivative with respect to ξ . In the above equation A, B, C are real constants and $a_k (k = 0, 1, 2, ..., N)$ are arbitrary constants.

Step-III: By the homogeneous balance principle and balancing the highest order derivative of w with the highest order nonlinear term in Eq. (3), determine the positive integer N. Furthermore, the coefficients a_k (k = 0, 1, 2, ... N) can be found by solving a system of linear algebraic equations which will come from suggested method. Then using the values of A, B and C and Eq. (5), the function $G(\xi)$ will be evaluated.

Step-IV: Lastly, on substitution of Eq. (4) in the Eq. (3), we can obtain the polynomial of $\left(\frac{G'}{G'+G+A}\right)$. Then by equating the coefficients of like powers of $\left(\frac{G'}{G'+G+A}\right)$ from both sides, we will get a system of algebraic equations for η , a_k (k = 0, 1, 2, ...N), A, B, and C.

3 Implementation of the proposed technique

In this current section, we have presented the implementation of the proposed $\left(\frac{G'}{G'+G+A}\right)$ -expansion method to derive the exact analytical wave solutions to a new reduced form of the (2+1)-dimensional BKP model. The reduced new form (2+1)-dimensional BKP model could be obtained from the (3+1)-dimensional generalized BKP model by setting the spatial variable z=x [44]. The new formed equation of the (2+1)-dimensional BKP model is presented below [44, 45]:

$$u_{xxxy} + \alpha (u_y u_x)_x + (u_y + 2u_x)_t - (u_{yy} + 2u_{xx}) = 0,$$
(6)

where α is a constant. Recently Kara et al. [45] applied the $\left(\frac{G'}{G}, \frac{1}{G}\right)$ -expansion method to the new form of the (2+1)-dimensional BKP equation and obtained exact solutions in the form of hyperbolic, trigonometric and rational functions. Utilizing the wave transformation $u(x, y, t) = w(\xi)$ where $\xi = nx + my - pt$, the Eq. (6) reduces to the following ODE

$$n^{3}mw^{iv} + 2\alpha n^{2}mw'w'' - (2n^{2} + m^{2} + 2np + mp)w'' = 0.$$
(7)

By assuming $w'(\xi) = U(\xi)$ and $\eta = -(2n^2 + m^2 + 2np + mp)$, the Eq. (7) reduces to

$$n^{3}mU''' + 2\alpha n^{2}mUU' + \eta U' = 0.$$
(8)

Integrating Eq. (8) with respect to ξ and assuming the integration constant equal to zero, we obtain

$$n^{3}mU'' + \alpha n^{2}mU^{2} + \eta U = 0.$$
 (9)

By the homogeneous balance principle, balancing between the terms U'' and U^2 in Eq. (9), gives N+2=2N which implies N=2. Hence from Eq. (4), the solution of Eq. (9) can be written as:

$$U(\xi) = \sum_{i=0}^{2} a_i \left(\frac{G'}{G' + G + A} \right)^i.$$
 (10)

Utilizing Eq. (10) into Eq. (9) and equating the coefficients of similar powers of $\left(\frac{G'}{G'+G+A}\right)$ of Eq. (9), we obtain a system of linear algebraic equations which is given below:

$$\begin{cases} a_{1}BCmn^{3} - 2a_{1}C^{2}mn^{3} + 2a_{2}C^{2}mn^{3} + a_{0}\eta + \alpha a_{0}^{2}mn^{2} = 0, \\ a_{1}B^{2}mn^{3} - 6a_{1}BCmn^{3} + 6a_{2}BCmn^{3} + 6a_{1}C^{2}mn^{3} \\ - 12a_{2}C^{2}mn^{3} + 2a_{1}Cmn^{3} + a_{1}\eta + 2\alpha a_{0}a_{1}mn^{2} = 0, \\ -3a_{1}B^{2}mn^{3} + 4a_{2}B^{2}mn^{3} + 9a_{1}BCmn^{3} - 24a_{2}BCmn^{3} \\ + 3a_{1}Bmn^{3} - 6a_{1}C^{2}mn^{3} + 24a_{2}C^{2}mn^{3} - 6a_{1}Cmn^{3} \\ + 8a_{2}Cmn^{3} + a_{2}\eta + \alpha a_{1}^{2}mn^{2} + 2\alpha a_{0}a_{2}mn^{2} = 0, \end{cases}$$

$$2a_{1}B^{2}mn^{3} - 10a_{2}B^{2}mn^{3} - 4a_{1}BCmn^{3} + 30a_{2}BCmn^{3} \\ - 4a_{1}Bmn^{3} + 10a_{2}Bmn^{3} + 2a_{1}C^{2}mn^{3} - 20a_{2}C^{2}mn^{3} \\ + 4a_{1}Cmn^{3} - 20a_{2}Cmn^{3} + 2a_{1}mn^{3} + 2\alpha a_{1}a_{2}mn^{2} = 0, \end{cases}$$

$$6a_{2}B^{2}mn^{3} - 12a_{2}BCmn^{3} - 12a_{2}Bmn^{3} + 6a_{2}C^{2}mn^{3} \\ + 12a_{2}Cmn^{3} + 6a_{2}mn^{3} + \alpha a_{2}^{2}mn^{2} = 0.$$

By solving the above system in Eq. (11), we obtain two sets of solutions as given below: SET-1:

$$\begin{split} \eta &= 4Cmn^3 - B^2mn^3, \ a_0 = -\frac{6Cn(-B+C+1)}{\alpha}, \\ a_1 &= \frac{6\left(B^2n - 3BCn - Bn + 2C^2n + 2Cn\right)}{\alpha}, \ a_2 = -\frac{6n(B-C-1)^2}{\alpha} \end{split}$$

SET-2:

$$\begin{split} \eta &= mn^3 \left(B^2 - 4C \right), \ a_0 = \frac{-B^2 n + 6BCn - 6C^2 n - 2Cn}{\alpha}, \\ a_1 &= \frac{6 \left(B^2 n - 3BCn - Bn + 2C^2 n + 2Cn \right)}{\alpha}, \ a_2 = -\frac{6n(B - C - 1)^2}{\alpha} \end{split}$$

For SET-1, we obtain the following exact traveling wave solutions: Case-I: When $\Lambda = B^2 - 4C > 0$

$$U_{11}(x,y,t) = -\frac{6nC(C-B+1)}{\alpha} + \frac{6\left(B^{2}n - 3BCn - Bn + 2C^{2}n + 2Cn\right)}{\alpha} \left[\frac{C_{1}\left(B + \sqrt{\Lambda}\right) + C_{2}\left(B - \sqrt{\Lambda}\right)e^{\sqrt{\Lambda}\xi}}{C_{1}\left(B + \sqrt{\Lambda} - 2\right) + C_{2}\left(B - \sqrt{\Lambda} - 2\right)e^{\sqrt{\Lambda}\xi}} \right] - \frac{6n(B-C-1)^{2}}{\alpha} \left[\frac{C_{1}\left(B + \sqrt{\Lambda}\right) + C_{2}\left(B - \sqrt{\Lambda}\right)e^{\sqrt{\Lambda}\xi}}{C_{1}\left(B + \sqrt{\Lambda} - 2\right) + C_{2}\left(B - \sqrt{\Lambda} - 2\right)e^{\sqrt{\Lambda}\xi}} \right]^{2}.$$

$$(12)$$

Case-II: When $\Lambda = B^2 - 4C < 0$

$$U_{12}(x,y,t) = -\frac{6Cn(-B+C+1)}{\alpha} + \frac{6(nB^2 - 3nBC - nB + 2nC^2 + 2nC)}{\alpha} \times$$
(13)

$$\begin{split} &\left[\frac{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_2+C_1\sqrt{-\Lambda}\right)+\cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_1-C_2\sqrt{-\Lambda}\right)}{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B-2)C_2+C_1\sqrt{-\Lambda}\right)+\cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B-2)C_1-C_2\sqrt{-\Lambda}\right)}\right] \\ &-\frac{6n(B-C-1)^2}{\alpha}\times \\ &\left[\frac{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_2+C_1\sqrt{-\Lambda}\right)+\cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_1-C_2\sqrt{-\Lambda}\right)}{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B-2)C_2+C_1\sqrt{-\Lambda}\right)+\cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B-2)C_1-C_2\sqrt{-\Lambda}\right)}\right]^2. \end{split}$$

For SET-2, we get the following exact traveling wave solutions: Case-I: When $\Lambda = B^2 - 4C > 0$

$$U_{21}(x,y,t) = \frac{-B^{2}n + 6BCn - 6C^{2}n - 2Cn}{\alpha} + \frac{6\left(B^{2}n - 3BCn - Bn + 2C^{2}n + 2Cn\right)}{\alpha} \left[\frac{C_{1}\left(B + \sqrt{\Lambda}\right) + C_{2}\left(B - \sqrt{\Lambda}\right)e^{\sqrt{\Lambda}\xi}}{C_{1}\left(B + \sqrt{\Lambda} - 2\right) + C_{2}\left(B - \sqrt{\Lambda} - 2\right)e^{\sqrt{\Lambda}\xi}} \right] - \frac{6n(B - C - 1)^{2}}{\alpha} \left[\frac{C_{1}\left(B + \sqrt{\Lambda}\right) + C_{2}\left(B - \sqrt{\Lambda}\right)e^{\sqrt{\Lambda}\xi}}{C_{1}\left(B + \sqrt{\Lambda} - 2\right) + C_{2}\left(B - \sqrt{\Lambda} - 2\right)e^{\sqrt{\Lambda}\xi}} \right]^{2}.$$

Case-II: When $\Lambda = B^2 - 4C < 0$

$$U_{22}(x,y,t) = \frac{-B^{2}n + 6BCn - 6C^{2}n - 2Cn}{\alpha} + \frac{6\left(B^{2}n - 3BCn - Bn + 2C^{2}n + 2Cn\right)}{\alpha} \times$$

$$\left[\frac{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_{2} + C_{1}\sqrt{-\Lambda}\right) + \cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_{1} - C_{2}\sqrt{-\Lambda}\right)}{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B - 2)C_{2} + C_{1}\sqrt{-\Lambda}\right) + \cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B - 2)C_{1} - C_{2}\sqrt{-\Lambda}\right)}\right]$$

$$-\frac{6n(B - C - 1)^{2}}{\alpha} \left[\frac{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_{2} + C_{1}\sqrt{-\Lambda}\right) + \cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left(BC_{1} - C_{2}\sqrt{-\Lambda}\right)}{\sin\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B - 2)C_{2} + C_{1}\sqrt{-\Lambda}\right) + \cos\left(\frac{\sqrt{-\Lambda}}{2}\xi\right)\left((B - 2)C_{1} - C_{2}\sqrt{-\Lambda}\right)}\right]^{2}.$$
(14)

4 Results and discussion

In this section, we outline a variety of traveling wave solutions representations of the obtained results for the different values of the associated parameters. Integrating U_{11} , U_{12} , U_{21} and U_{22} given in Eqs. (12)-(14) with respect to ξ , we get the required solutions and have shown the 2D and 3D plots of four different solutions in different cases (see Figs. 1-4). The kink shape soliton solution for SET-1 (Case-1) is presented in Fig. 1 for the associated parameter values "t=1; $\alpha=1$; B=1; C=0.1; $C_1=1$, $C_2=1$, n=1; m=1; p=-0.8". In this figure, (I) represents the 3D plot of Eq. (12) and curve (II) represents the corresponding 2D plot of Eq. (12). Fig. 2 represents the 3D and 2D plots of the singular periodic shape wave solution at the parameter values "t=1, $\alpha=1$, B=1, C=1.1, $C_1=1$, $C_2=1$, n=1, m=1, p=-2.13333" for SET-1 (Case-2) of Eq. (13). One soliton solution of Eq. (14) for SET-2 (Case-1) at the parameter values "t=1, $\alpha=1$, B=1, C=0.15, $C_1=1$, $C_2=1$, n=1, m=1, p=-1.13333" is depicted in Fig. 3 in which (I) represents the 3D plot and curve (II) represents the corresponding 2D plot of Eq. (14). In Fig. 4, we have shown the 3D and 2D graphical representations of the singular periodic wave solution of Eq. (14) for SET-2 (Case-2) for the parameter values "t=1, $\alpha=1$, B=1, C=1.1, $C_1=0$, $C_2=1$, $C_1=1$, $C_$

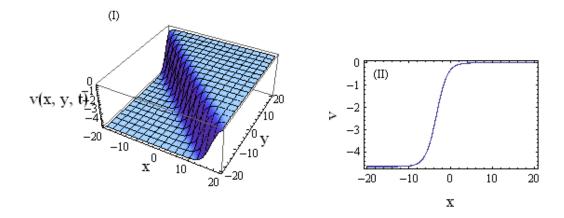


Fig. 1 The king shape soliton solution for $t = 1; B = 1; \alpha = 1; C = 0.1; C_1 = 1, C_2 = 1$: (I) represents the 3D plot and (II) represents the 2D plot of Eq. (12).

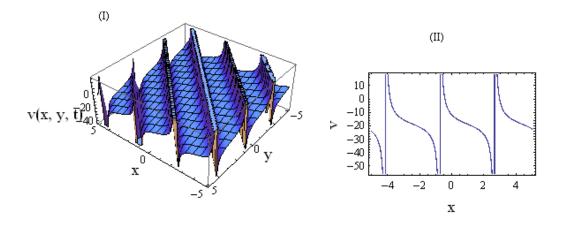


Fig. 2 The singular periodic wave solution for t = 1; $\alpha = 1$; B = 1; C = 1.1; $C_1 = 1$, $C_2 = 1$: (I) represents the 3D plot and (II) represents the 2D plot of Eq. (13).

5 Conclusions

The relatively new $\left(\frac{G'}{G'+G+A}\right)$ -expansion technique has been successfully utilized to derive the new exact traveling wave solutions to a new reduced form of the (2+1)-dimensional BKP equation. We have used the symbolic mathematical computation program (Mathematica), to show the graphical representation of the solutions. The exact traveling wave solutions are presented in the form of 2-D and 3-D figures. It is noticed that all the solutions obtained are in general form and involving a few parameters. The reported solutions are represented by equations (12), (13), (14) and (14). By assigning the particular values to the parameters, one kink shape soliton solution, two singular periodic shape wave solutions and one soliton solution are presented in Figs. 1-4.

Exploring the new exact analytical solutions of the new reduced form of the (2+1)-dimensional generalized BKP equation, we anticipate that the aforementioned technique could be applied to many other NPDES model arising in mathematical physics and engineering problems to obtain new exact wave solutions.

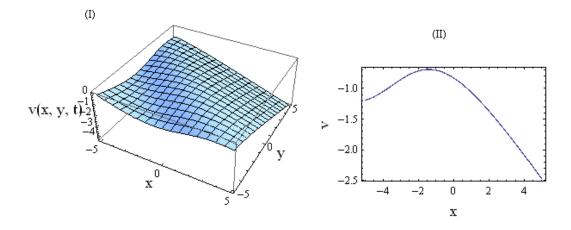


Fig. 3 One soliton wave solution for $t = 1, B = 1, \alpha = 1, C = 0.15, C_1 = 1, C_2 = 1$: (I) represents the 3D plot and (II) represents the 2D plot of Eq. (14)

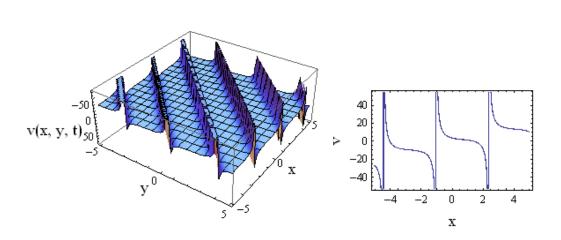


Fig. 4 The singular periodic wave solution for $t = 1, B = 1, C = 1.1, \alpha = 1, C_1 = 0, C_2 = 1$: (I) represents the 3D plot and (II) represents the 2D plot of Eq. (14)

Acknowledgments

The author would like to thank the anonymous referees and editor for all the apt suggestions and comments which improved both the quality and the clarity of the paper.

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