# A New Variable Step-Size NLMS Adaptive Filtering Algorithm

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# A New Variable Step-Size NLMS Adaptive Filtering Algorithm

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***Abstract* - There is a contradiction in classical adaptive filtering algorithm that fast convergence speed comparing with low steady state error. In order to improve this contradiction, this paper presents a new non-parametric Variable Step-Size NLMS algorithm. The new algorithm uses gradient vector's features to achieve the step iteration and a certain approximation process was made for it. Software simulation and hardware implementation verify the effectiveness of the algorithm.**

***Keywords-* *adaptive filtering;NLMS algorithm;variable step-size;spatial filtering***

## I. INTRODUCTION

Least mean square algorithm (LMS) adaptive filtering is the most widely used typical algorithm in practice. The improved normalized LMS algorithm (NLMS) has the same structure as the LMS algorithm, but a lower computational complexity, faster convergence rate than the standard LMS algorithm, so it has been widely used [1]. The fixed step size parameter $\mu$ in LMS algorithm and NLMS algorithm controls algorithm's stability, convergence speed and steady state error. Generally, if the step size parameter is large, the algorithm has a faster convergence speed comparing with larger convergence error; however, if the step size parameter is small, the algorithm has a lower convergence speed but the convergence error is smaller. In order to overcome the contradiction above, people made a number of variable step size LMS algorithm [2]-[9]. The basic idea of these algorithms is using a larger step size in the initial stage making it has a fast convergence speed, with the convergence is deeper, reducing the step size for it has a smaller steady state error.

As the system's output error is gradually reduced, Kwong derived a variable step size LMS algorithm using instantaneous error energy [2]. Because the noise, iterative step cannot be carried out as expected iterative process, and in the steady state the noise can affect the result, so that the efficiency of the algorithm will be greatly reduced. In order to alleviate the uncorrelated noise's influence, Aboulnasr used adjacent errors cross-correlation function amount of smoothing instead of the error energy in Kwong's algorithm, which has a good convergence [3]. Pazaitis had made step iterated using the higher-order function of error [4]. Iterative step of this algorithm cannot be affected by Gaussian noise. The algorithms in [5] and [6] using the mean square error for selecting the appropriate parameters $\alpha$ and $\beta$ to the step iteration, which have good steady-state performance.

Another branch of variable size algorithm is using gradient vector to iterate step. Mader utilized the character of gradient vector to present a new algorithm. The character is that the gradient vector mode's value is large initially and smaller tends to zero in steady state and the gradient direction is consistent in the initial stage while change frequently after the system converged, which is good to meet the variable step size algorithm's requirement for step size. That algorithm has a fast convergence rate and a small steady state error [7]. Mader presents a theoretical optimal variable step size algorithm, which can makes mean square weight deviation in each iteration decrease to the maximum that can be the fastest speed of convergence[8]. However, this algorithm is not applicable in practice for it used the additional error statistic, which cannot be gotten in iterative process. Shin had approximated and simplified Mader's algorithm to make the algorithm more applicable in practice [9].

This paper presents a new NLMS algorithm based on gradient vector updating step size. This new algorithm uses a first-order filter to smooth the gradient vectors to reduce noise's impact and update steps using smoothed gradient vector's Euclidean norm square to improve the convergence rate. The algorithm achieved rapid convergence without reducing the steady state error. Software simulation and experimental results prove that the proposed algorithm has good fast convergence and steady state performance. The detailed algorithm description, simulation and experimental verification are below.

## II. ADAPTIVE ALGORITHM

### *A. NLMS Algorithm*

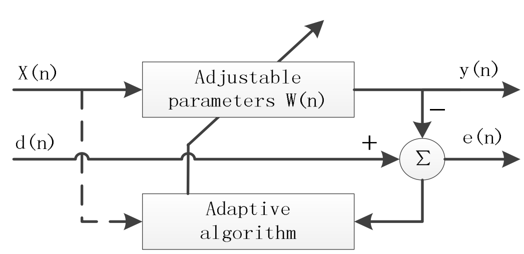


Figure 1. Adaptive filter schematic diagram

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Adaptive filter Schematic diagram is show as Fig. 1. In this figure, the $X(\mathrm{n})$, $W(\mathrm{n})$, $e(\mathrm{n})$, $d(\mathrm{n})$, $y(\mathrm{n})$ respectively stand for the input signal, the weight coefficient, the error signal, the desired output signal and the actual output signal. The classical NLMS algorithm updates the weights as follows:

$$e(n) = d(n) - \mathrm{y}(n) = d(n) - W^H(n)X(n) \tag{1}$$

$$\mu(\mathrm{n}) = \frac{\alpha}{X^T(n)X(n)} \tag{2}$$

$$W(n+1) = W(n) + 2\mu(n)e^*(n)X(n) \tag{3}$$

In the above recurrence formula, $\alpha$ is a constant ( $0<\alpha<2$ ).Usually, we have to plus a small positive constant in the denominator of formula (2) to avoid the result equals to 0. Therefore, updating iteration formula of NLMS algorithm as following:

$$W(n+1) = W(n) + 2\frac{\alpha}{\gamma + X^T(n)X(n)}e^*(n)X(n) \tag{4}$$

In it, $0<\gamma<1$

### B. Kwong-NLMS algorithm

Kwong-NLMS algorithm uses instantaneous error energy to update the step size and the expressing as following:

$$\mu(n) = \alpha\mu(n-1) + \gamma e^2(n) \tag{5}$$

In it, $\alpha$ is a constant ( $0<\alpha<1$ ) and $\gamma>0$ . From the formula (5), the step size will increase as the mean square error (MSE) increasing, and the Kwong-NLMS algorithm weight value iteration formula is as following:

$$W(n+1) = W(n) + 2\frac{\mu(\mathrm{n})}{\gamma + X^T(n)X(n)}e^*(n)X(n) \tag{6}$$

### C. The Proposed NLMS Algorithm

We need to design a variable step size algorithm which the main indicator is the deviation between the solution in adaptive process and the optimal solution. When the deviation is large, large step should be used to accelerate the convergence speed; when the deviation is small, small step should be used to reduce steady state error. This paper presents a new NLMS algorithm based on gradient vector updating step size. This new algorithm uses a first-order filter to smooth the gradient vectors to reduce noise's impact and update steps using smoothed gradient vector's Euclidean norm square. In this way, the deviation of mean square weight will be known to improve the convergence speed. The iterative weight formulas are shown below:

$$g(n) = \beta g(n) + (1-\beta)\frac{e^*(n)X(n)}{\gamma + \alpha X^T(n)X(n)} \tag{7}$$

$$\mu_g(n) = p\,\|\, g(n)\,\|^2 \tag{8}$$

$$W(n+1) = W(n) + \mu_g(\mathrm{n})\frac{e^*(n)X(n)}{\gamma + \mu_g(\mathrm{n})X^T(n)X(n)} \tag{9}$$

In the formulas above, $g(\mathrm{n})$ is the smooth of gradient vector, $\|\bullet\|^2$ is the Euclidean norm square, $\beta$ is close to 1, $\gamma$ has the same function in classical NLMS, $\alpha$ , $p$ are constants higher than 0, the selection of parameter $p$ will be given in the simulation graphs.

In the actual hardware implementation, both in the FPGA and DSP, the divide operation will take many clocks, which will seriously affect the system performance. In order to enhance the system's real-time; this paper gives the following approximation:

$$\gamma + \alpha X^T(n)X(n) \approx 2^m \quad (\mathrm{m}=0,1,2,3\cdots) \tag{10}$$

The function of above approximation is change the division operation to shift operation which can be completed within one clock. This will greatly enhance the real-time function for the hardware implementation of algorithm.

## III. SIMULATION RESULT

In this section, the results of computer simulation and experimental verification about the proposed algorithm, the classical NLMS algorithm and the improved Kwong-NLMS algorithm are shown below:

### A. Time domain filter simulation

TABLE I. PARAMETERS OF ADAPTIVE ALGORITHM

| | |
|---|---|
| **Parameter** | Input signal sampling points: N=400 |
| | Filter order: k=128 |
| | Simulation times: Ng=100 |
| Algorithm | Parameter |
| NLMS | μ=0.99 |
| Kwong-NLMS | α=0.997;μ(0)=0.01;γ =0.018; |
| The Proposed NLMS | α=1/2^2;β=0.999;<br>γ=0.02;μ(0)=0.01;<br>p=1;g(0)=0; |

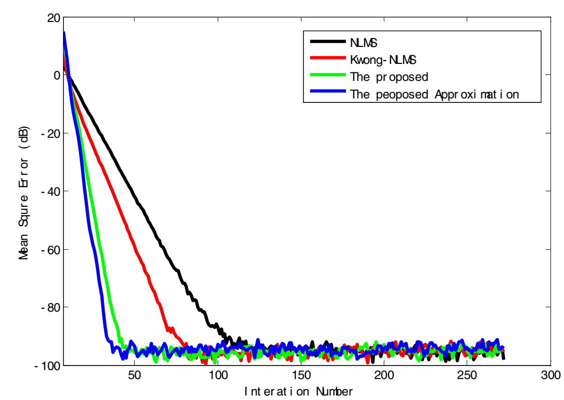



Figure 2. the MSE performance of all algorithms under the input is Gaussian white noise （SNR=10dB）

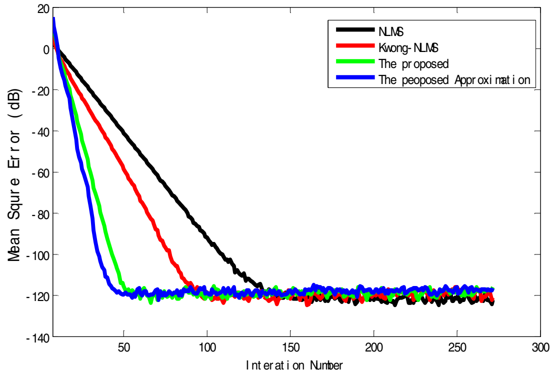



Figure 3. the MSE performance of all algorithms under the input is Gaussian white noise （SNR=20dB）

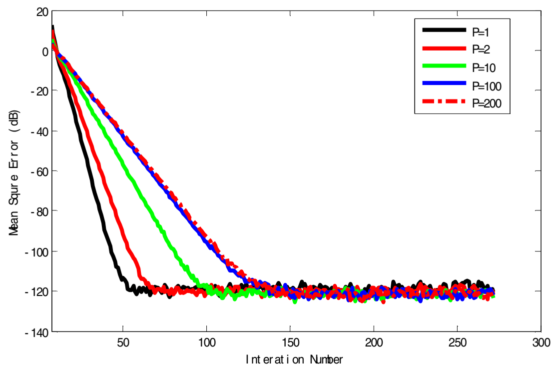
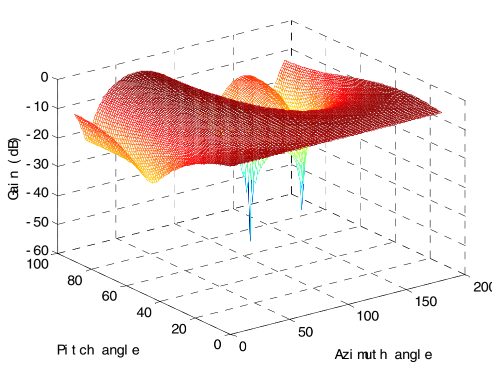


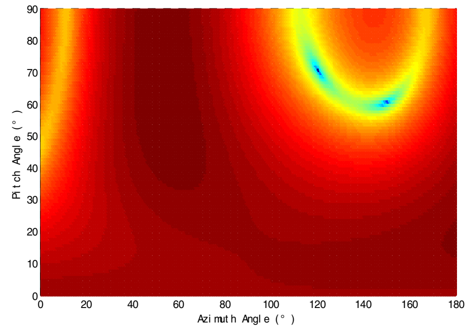

Figure 4. the MSE performance of all algorithms when input parameter $p$ is changed. （SNR=20dB）

## B. *Spatial filter experiments*

In the experiment, the antenna array is a circular array and the element number is 4; the signal frequency is 1.575GHz and simulated by software; the center frequency is 1.58MHz; the SNR is -20dB; two broadband interference signals which bandwidth is 2MHz. Interference signal A's pitch angle is 70 °and azimuth is 120 °while interference signal B's pitch angle is 60 °and azimuth is 150 °. Sampling frequency is 6.2MHz and sampling time is 2ms. The Proposed NLMS algorithm simulation results were shown below.




(a)




(b)

Figure 5. Spatial the Proposed NLMS algorithm simulation figure:(a)Three-domain map(b)contour map

As shown above, the proposed NLMS algorithm is form two deep nulls in the two broadband, means effectively suppress two wideband interferences.

## C. *comparing results of GNSS spatial filtering*

One experiment is doing as the same requirements above. Figure 6 is the spatial filtering system output results under LMCV principle of the classical NLMS algorithm、the improved Kwong-NLMS algorithm and the proposed NLMS algorithm. The results are shown below:

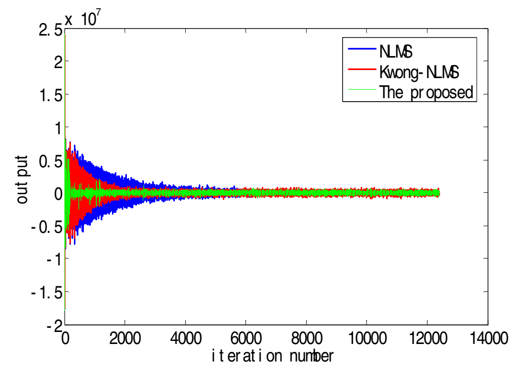



Figure 6. Spatial filtering system output results of the three algorithms

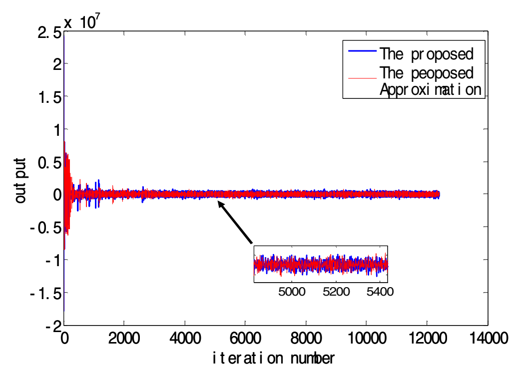



Figure 7. Spatial filtering system output results before and after The Proposed Algorithm approximation

Analysis of simulation results:

Simulation results Fig. 2 and Fig. 3 show that the new NLMS algorithm has a faster convergence speed comparing with the classical NLMS algorithm and Kwong-NLMS algorithm without losing the stead state error. From the Fig. 4, the new algorithm parameter $p$ will affect the speed of convergence. When the $p$ value increases, this effect decreases. In GNSS spatial filtering experiments, as in Fig. 5, the proposed algorithm can effectively suppress interference. As in Fig. 6, the new algorithm has a better performance in convergence rate comparing with classical algorithms, which is as same as software simulation results. Fig. 7 shows the system performance is nearly no change before and after the new algorithm approximating, which prove the effectiveness of approximation.

## IV. CONCLUSION

This paper proposes a new variable step size NLMS algorithm for solving the contradiction between the fast convergence speed and the low convergence error in classical NLMS algorithm. The computer simulation results and the application of spatial filtering in GNSS show that the proposed algorithm has a fast convergence when the output error performance is the same as in classical algorithms. For algorithms' approximation, the simulation results show it was effective to reduce the computational complexity and easy to hardware implementation, enhancing the real time performance.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