Comparison of adaptive lattice filters to LMS transversal filters for sinusoidal cancellation

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COMPARISON OF ADAPTIVE LATTICE FILTERS TO LMS TRANSVERSAL FILTERS FOR SINUSOIDAL CANCELLATION

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Abstract

This paper compares the performance of the recursive least squares lattice (RLSL) and the normalized step-size stochastic gradient lattice (SGL) algorithms to that of the LMS transversal algorithm for the cancellation of sinusoidal interferences. It is found that adaptive lattice filters possess a number of advantages over the LMS transversal filter making them the preferred ANC filter structure if their increased computational costs can be tolerated. These advantages include faster convergence, notch bandwidths which are independent of the input power, and the generation of less harmonic distortion. The notch bandwidth of each filter structure is related to its respective convergence time constant. Experimental results obtained with the 32-bit floating-point Lattice Development System (LDS) are presented to verify our analytical results.

I. Introduction

The adaptive noise canceller (ANC) is a well known technique for eliminating interference corrupting a desired signal [1]. It is a particularly useful technique if the interference is not known precisely or if it is nonstationary. The ANC illustrated in Fig. 1 eliminates interference from a desired signal by adaptively filtering a reference of the interference signal, x(n), and subtracting this from the primary, d(n). The primary is the received signal containing the corrupted desired signal. For sinusoidal interferences, the ANC filter forms narrowband notches centered at each of the interference frequencies [1].

The purpose of this paper is to compare the performance of the adaptive lattice ANC filters to that of the LMS transversal ANC filter. LMS transversal filters have been widely used in ANC applications and their filtering characteristics have been carefully analyzed in [1]-[5]. Adaptive lattice algorithms possess certain theoretical properties which promise superior performance over LMS transversal filters, especially for the cancellation of sinusoidal interferences [6]-[10]. In particular, it will be shown that adaptive lattice filters generate little or no harmonic distortion and have a convergence rate and notch bandwidth which are both independent of the statistics of the input signals.

It was established in [3] that the LMS weight vector consists of a Wiener component, a random misadjustment component, and a time-varying (periodic) non-Wiener component. The Wiener component is the optimal Wiener-Hopf solution and the random misadjustment component is generated by the gradient estimation noise [2]. The time-varying non-Wiener component is generated by the modulation between uncorrelated periodic components of the reference and primary inputs. It will be shown that the RLSL coefficient vector also has a time-varying non-Wiener component for exponential weighting factor W < 1. These time-varying non-Wiener components of both filters create notches of non-zero bandwidth. Physical insight is contributed by relating the notch bandwidth to the nonstationary weight or coefficient time constants for the respective filters.

Two adaptive lattice algorithms will be analyzed: the a posteriori, prewindowed RLSL and the normalized step-size SGL [6]. Section II briefly reviews the convergence rates of the LMS transversal and adaptive lattice filters. In Section III, expressions for the ANC notch bandwidth of both the LMS and the lattice algorithms are derived and found to obey an equivalent relationship. Section IV shows that the RLSL algorithm generates no harmonic distortion. Section V presents experimental results taken on the 32-bit LDS [11] supporting the theoretical developments.

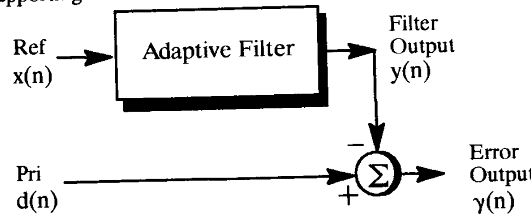

Fig. 1. Adaptive Noise Canceller (ANC).

II. Convergence Rate

The convergence of an N length LMS filter was shown in [2] to consists of N exponentially decaying modes whose individual time constants are inversely proportional to the eigenvalues of the correlation matrix formed by the tapped-delay inputs. These N convergence time constants for the mean value of the LMS weight vector are given by [2],

$$\tau_{LMS}^{i} \approx \frac{1}{2\mu\lambda_i}\frac{1}{f_s} \quad \text{(sec)} \quad i = 1,2,\ldots,N. \tag{1}$$

where μ is the LMS step-size, λ_i are the eigenvalues of the reference correlation matrix, and f_s is the sample frequency in Hz. The corresponding N time constants for the LMS mean-square error (MSE) are exactly one-half those of Eq. (1) [2]. The eigenvalues of a reference consisting of a single sinusoid in noise which has been lowpass filtered at B Hz can be written as [4],[12],

$$\lambda_{1,2} \approx \frac{\sigma_r^2}{2}\left[N \pm \frac{sin(\omega_r N)}{sin(\omega_r)}\right] + \frac{f_s}{2B}\sigma_n^2 \tag{2}$$

$$\lambda_{3,\ldots,N} \approx \frac{f_s}{2B}\sigma_n^2 \tag{3}$$

where $\omega_r = 2\pi f_r/f_s$ is the normalized interference frequency, and σ_r^2 and σ_n^2 are the power of the interference sinusoid and noise in the reference. It is apparent from Eq. (1) that the smallest eigenvalue will determine the slowest converging time constant. The largest eigenvalue determines the bounds of the LMS step-size μ which guarantees convergence [2]. Accordingly, the performance of the LMS algorithm is strongly dependent on the condition number, $\kappa = \lambda_{max}/\lambda_{min}$, of the reference input signal.

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A comprehensive analysis of the convergence rate of adaptive lattice algorithms has yet to be published due in part to the highly non-linear dependence of the lattice internal parameters on the input signals. At least three cases exist for which the convergence rates must be considered;

1. initial convergence,
2. reconvergence from a deterministic input to a different deterministic input at time t_o.
3. reconvergence to a statistically different process at time t_o.

The convergence rate of the LMS transversal filter is described by Eq. (1) for all three cases. On the other hand, the RLSL filter exhibits essentially no time constant for the first and second cases [6]. It generates the minimum MSE for the given data at each instant in time since it is a deterministic algorithm. (The SGL algorithm only approximates the RLSL solution due to its statistical update algorithm.) However for the third case, the RLSL must forget old statistics and learn new statistics and thus exhibits a measurable time constant.

An approximate expression for the time constant of the mean of the adaptive lattice coefficients with statistically nonstationary inputs can be written as,

$$\tau_{lat} \approx \frac{1}{1-W}\,\frac{1}{f_s} \quad \text{(sec)} \tag{4}$$

Eq. (4) is based on the related works of [6],[8],[9],[10]. It was shown in [9] that the convergence time constant of the SGL MSE is exactly one-half of that of Eq. (4), analogous with the LMS case. It is important to recognize that τ_{lat} is completely independent of the statistics of the reference input, unlike τ_{LMS}. This can be attributed to the adaptive lattice algorithms near-orthogonalization of the reference signal.

III. Notch Bandwidth

The LMS ANC was shown in [3] to approximate a second ordered notch filter for a single noise-free interference sinusoid, assuming that linear time invariant conditions are satisfied and that the reference and primary inputs are uncorrelated. These results were extended in [4] for interferences in noise. Specifically, it was found in [3] that the transfer function, $H(z) = \Gamma(z)/D(z)$, of the LMS ANC can be written as,

$$H(z) = \frac{z^2 - [2\cos(\omega_r)]\,z + 1}{z^2 - 2\left\{1 - \frac{2\mu N (A_r)^2}{4}\right\}\cos(\omega_r)\,z + \left\{1 - \frac{2\mu N (A_r)^2}{2}\right\}} \tag{5}$$

where $\Gamma(z)$ and $D(z)$ are the Z-transforms of the error output, $\gamma(n)$, and the primary, $d(n)$ respectively. A close look at $H(z)$ in Eq. (5) reveals one of many important advantages of adaptive filtering over fixed filtering. In particular, the feedback inherent in the adaptive algorithm gives the all-zero transversal structure the ability to produce both *zeros and poles* in its transfer function. $H(z)$ consists of a pair of zeros on the unit circle exactly at the interference sinusoid's frequency, and a pair of poles just behind each zero lying approximately along the same radial. This allows very short filters to have extremely narrow transfer functions.

These results can be extended for M widely separated sinusoidal interferences. Under these assumptions, the reference input, $x(n) = \sum_{i=1}^{M} A_r^i \cos(\omega_r^i n + \theta_i)$, generates a multiple notch filter where the i^{th} notch bandwidth can be approximated as,

$$BW_{LMS}^i \approx \left\{2\mu N \frac{(A_r^i)^2}{2}\right\}\frac{f_s}{2\pi} \quad \text{(Hz)} \quad i = 1,2,\ldots,M. \tag{6}$$

Utilizing results from Eq. (6) and Eq. (1), it is seen that when the LMS algorithm is used to cancel multiple interferences with different powers, it generates a different notch bandwidth for each interference and each notch will converge with a different time constant.

An equivalent description of the notch effect lies with the time-varying non-Wiener component of the LMS weight vector. This time-varying solution "modulates" the reference sinusoid and heterodynes it into the primary, thereby creating the notch [3],[4]. Considering the time-varying weight vector, Eq. (6) can be rewritten using Eqs. (1) & (2) for a high interference-to-noise ratio (INR) reference as,

$$BW_{LMS}^i \approx \frac{1}{\pi\,(\tau_{LMS}^{\min})^i} \quad (Hz) \quad i = 1,2,\ldots,M. \tag{7}$$

where $(\tau_{LMS}^{\min})^i$ is the minimum time constant of the i^{th} interference for the LMS weight vector. Eq. (7) is a useful form since it clearly shows that infinitely narrow notch bandwidths can be realized as the convergence times approach infinity.

In [7] it was found, for a single sinusoidal interference, the normalized SGL ANC transfer function can be approximated by Eq. (5) with N = 2 and a normalized step-size:

$$H(z) = \frac{z^2 - [2\cos(\omega_r)]\,z + 1}{z^2 - [2W\cos(\omega_r)]\,z + [2W - 1]} \tag{8}$$

Eq. (8) places two zeros on the unit circle exactly at the interference's frequency while the location of the two poles is a function of W. Fig. 2 shows the pole movement as a function of W. As W approaches unity, the poles approach the zeros, forming an infinitely narrow notch at the interference frequency.

As with the LMS algorithm, the adaptive lattice notch effect can be equivalently described by the time-varying non-Wiener components in the lattice coefficients. To demonstrate this, we apply the reference $x(n) = A_r\cos(\omega_r n + \theta)$, and the primary $d(n) = A_p\cos(\omega_p n) + C A_r\cos(\omega_r n + \eta_r)$ to the RLSL and derive expressions for its internal variables. The parameter C determines the amount of interference corrupting the primary input. It is found in [13] that the closed form solution to the 0^{th} ordered RLSL joint process coefficient is given by,

$$\begin{aligned} K^{\gamma}(n|0) = {} & C\cos(\theta - \eta_r) \\ & + C(1-W) < \cos(2\omega_r n + \theta + \eta_r) >_W \\ & - C(1-W)\cos(\theta - \eta_r) < \cos(2\omega_r n + 2\theta) >_W \\ & + \frac{A_p(1-W)}{A_r} < \cos[(\omega_p - \omega_r)n - \theta] >_W \\ & + \frac{A_p(1-W)}{A_r} < \cos[(\omega_p + \omega_r)n + \theta] >_W \\ & + O\big((1-W)^2\big) \end{aligned} \tag{9}$$

where $< x(n) >_W = \sum_{i=0}^{n} W^{n-i}\,x(n)$. From Eq. (9) it is apparent that the misadjustment portion of the coefficient ((1–W) terms) is a time-varying function whose movement is described by Eq. (4). These terms are non-Wiener and modulate the reference sinusoid creating the notch effect.

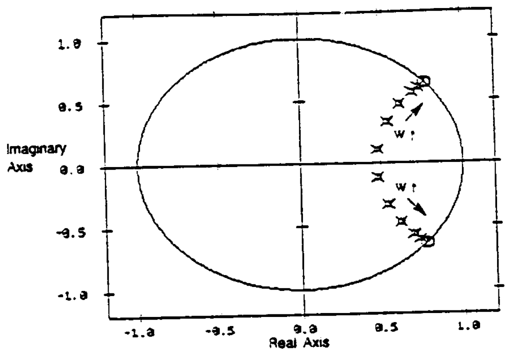

Fig. 2. Adaptive Lattice H(z) pole-zero plot for W= 0.62, 0.7, 0.8, 0.9, 0.95 (pri = noise; A_r = 0.146, f_r = 110Hz).

As with the LMS transversal filter, we find the same relationship between the notch bandwidth and the time constant for the adaptive lattice filter:

$$BW^i_{latt} \approx \frac{1}{\pi\, \tau_{RLS}} \quad (Hz) \quad i = 1, 2, \ldots, M \qquad (10)$$

$$\approx 2(1-W)\frac{f_s}{2\pi} \quad (\text{Hz})\,. \qquad (11)$$

Eq. (11) is independent of both the power in the interferences and the filter length. By equating Eq. (10) with Eq. (7), we find that the LMS notch bandwidth will equal the adaptive lattice notch bandwidth when their respective time constants are equal, ie., $1 - W = 2\mu\lambda_{max}$.

IV. Harmonic Distortion

Glover [3] has shown that the general solution to the LMS ANC filter for noise-free sinusoidal cancellation is composed of both time-invariant and time-variant components. The time-variant components generate unwanted sum and difference harmonics dependent on the interference and desired sinusoids. It was further shown in [3] that the time-variant components in the LMS error output are proportional to $\sin(\omega_r N)/\sin(\omega_r)$. Thus, a judicious choice of frequency and filter length, N, can minimize the harmonic distortion. However, this can be difficult in practical applications. The situation is further complicated by multiple interference sinusoids, especially if separate interference references are not available for a multiple reference channel ANC.

The harmonic distortion generated by adaptive lattice ANCs has been investigated by considering noise-free sinusoidal inputs. Simulations have shown [7] that the SGL generates less harmonic distortion than the LMS transversal ANC. Assuming the same reference and primary inputs as in the previous section, the RLSL ANC error output for a 2-stage RLSL can be written as [13].

$$\begin{aligned} \gamma(n|1) &\approx A_p \cos(\omega_p n) \\ &- \frac{A_p\,(1-W)}{1 + W^2 - 2W\cos(\omega_p - \omega_r)}\left[\cos(\omega_p n) - W\cos[\omega_p n + (\omega_p - \omega_r)]\right] \\ &- \frac{A_p\,(1-W)}{1 + W^2 - 2W\cos(\omega_p + \omega_r)}\left[\cos(\omega_p n) - W\cos[\omega_p n + (\omega_p + \omega_r)]\right] \\ &+ O\left((1-W)^2\right) \end{aligned} \qquad (12)$$

Note that Eq. (12) is completely free of any harmonic distortion. The (1-W) terms generate the notch effect as discussed in Section III.

V. Experimental Results

This section summarizes experimental results for single and multiple sinusoidal cancellation measured in real time on the LDS. The LDS is a 32-bit floating point machine custom designed to implement both adaptive transversal and adaptive lattice filters. A complete description of its design and operation can be found in [11].

A. Single Sinusoidal Cancellation

Figs. 3 & 4 demonstrate the reconvergence properties of the LMS and the RLSL ANC filters for the cancellation of a single interference sinusoid. In Fig. 3, the frequency of the noise-free interference is changed at time t = 0 from 140Hz to 130Hz. The LMS time constants were found by fitting an exponential decaying function to the ensemble averaged MSE. Excellent agreement is found with theory presented in Section 2. The RLSL was found to have no measurable time constant for values of W ranging from 0.9 to 0.99999. The RLSL did exhibit a time constant when the interference additive noise was suddenly changed. The ensemble averaged MSE results are plotted in Fig. 4 for a 30dB change. The 1/(1-W) dependence is clearly illustrated.

Fig. 5 presents harmonic distortion measurements for the LMS and both adaptive lattice ANCs. Harmonic distortion measurements where conducted with noise-free sinusoidal inputs. Plotted on the vertical axis is the magnitude of the FFT bin power for the largest harmonic signal. From Fig. 6 it is apparent that the RLSL does not generate any harmonics while the SGL harmonic generation becomes measurable for W < 0.98. The largest LMS harmonic showed a strong dependence on both the step-size and the filter length. By appropriately adjusting the filter length, a minimum harmonic distortion was found for N = 30, and was approximately equal to the SGL. Otherwise, the LMS algorithm generates more harmonic distortion than the SGL.

B. Multiple Sinusoidal Cancellation

ANC 3dB notch bandwidths for the cancellation of two interference sinusoids with unequal powers are presented in Figs. 6 & 7 for the LMS and RLSL algorithms respectively. The interference sinusoids at f_r^1 = 110Hz and f_r^2 = 145Hz differ in power by a factor of two. As predicted by theory, the LMS notch bandwidths differ by a factor of two, while the RLSL notch bandwidths are identical. Similar results have been obtained with the SGL.

VI. Conclusion

The analysis and experimental results presented demonstrate the superior performance of adaptive lattice filters with sinusoidal interferences in two key aspects: (1) they generate little or no harmonic distortion, and (2) their performance is not influenced by the reference power. However, each can generate notches with non-zero bandwidth created by modulation of the reference by time-varying non-Wiener components in their respective filter coefficients.

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f_s = 1000*Hz* for all data

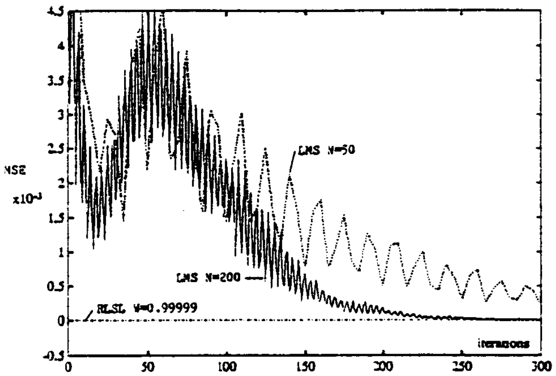

Fig. 3. Comparison of LMS and RLSL MSE time constants for interference frequency change at time t=0. (pri = noise + signal(150Hz); ref: σ_r^2 = 0.0093, f_r = 140Hz->130Hz).

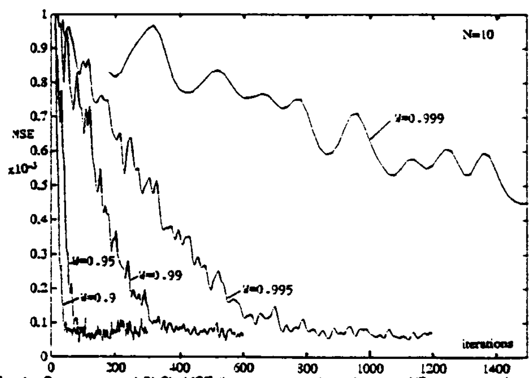

Fig. 4. Comparison of RLSL MSE time constants for reference INR change at time t=0. (pri = noise + int(163Hz); ref: σ_r^2 = 0.0023, INR = -13dB->+16.6dB).

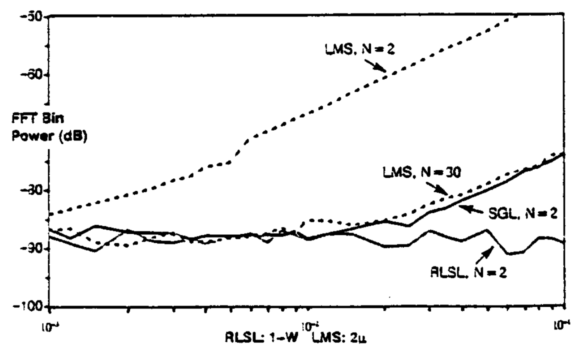

Fig. 5. ANC's Harmonic Freq = f_p - 2 f_r Bin Power plotted. (A_p = 0.046, f_p = 163Hz; A_r = 0.046, f_r = 167Hz; C=8).

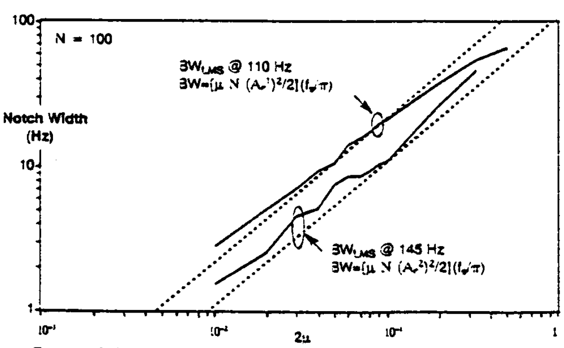

Fig. 6. LMS Transversal ANC multiple notch widths as a function of step size. (pri = noise; A_r^1 = 0.164, f_r^1 = 110Hz, A_r^2 = 0.116, f_r^2 = 145Hz).

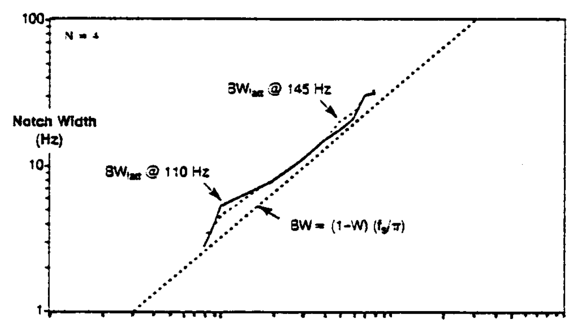

Fig. 7. RLSL ANC's notch width as a function of exponential weighting. (pri = noise; A_r^1 = 0.164, f_r^1 = 110Hz, A_r^2 = 0.116, f_r^2 = 145Hz).