Lecture 2: Intensity Surfaces and Gradients

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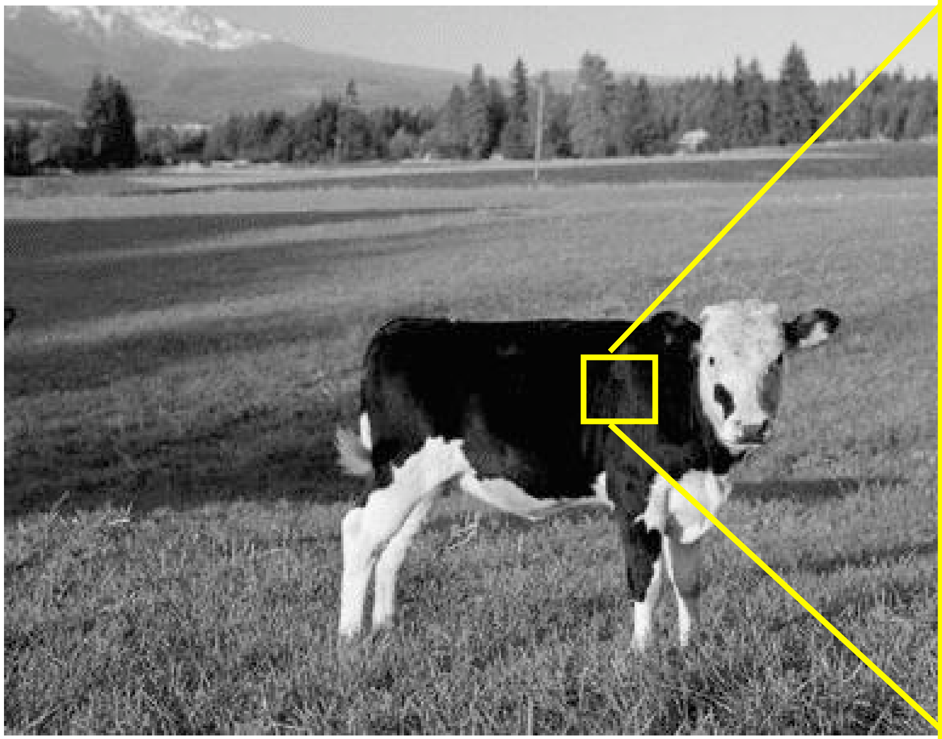
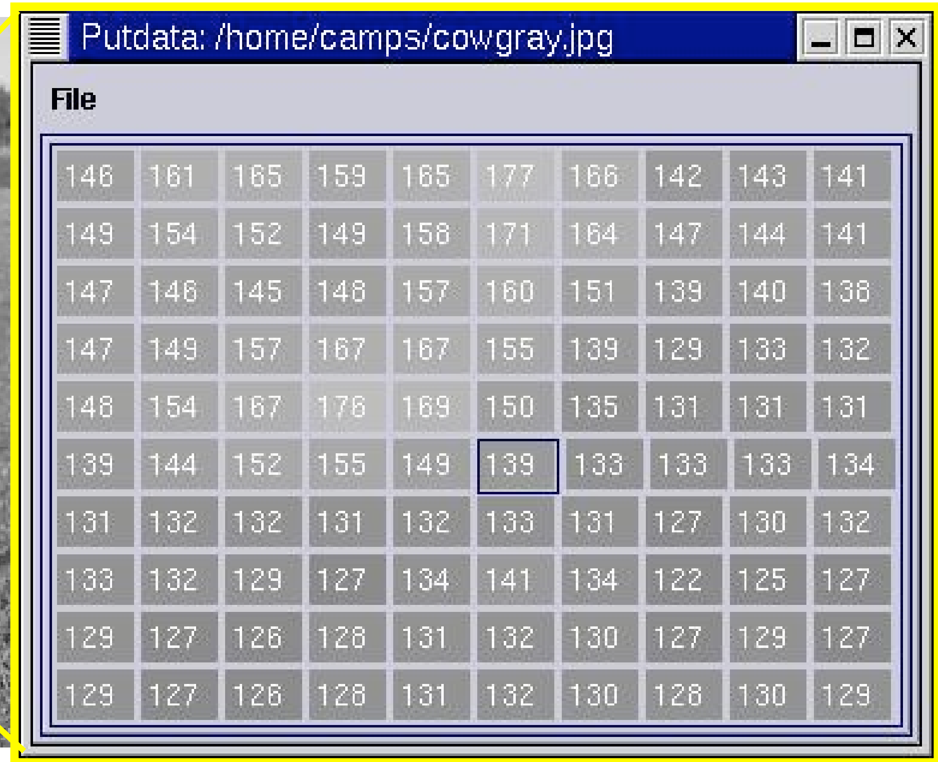
Visualizing Images

Recall two ways of visualizing an image

Intensity pattern

2d array of numbers





146	161	165	159	165	177	166	142	143	141
149	154	152	149	158	171	164	147	144	141
147	146	145	148	157	160	151	139	140	138
147	149	157	167	167	155	139	129	133	132
148	154	167	176	169	150	135	131	131	131
139	144	152	155	149	139	133	133	133	134
131	132	132	131	132	133	131	127	130	132
133	132	129	127	134	141	134	122	125	127
129	127	126	128	131	132	130	127	129	127
129	127	126	128	131	132	130	128	130	129

We "see it" at this level

Computer works at this level

Bridging the Gap

Motivation: we want to visualize images at a level high enough to retain human insight, but low enough to allow us to readily translate our insights into mathematical notation and, ultimately, computer algorithms that operate on arrays of numbers.

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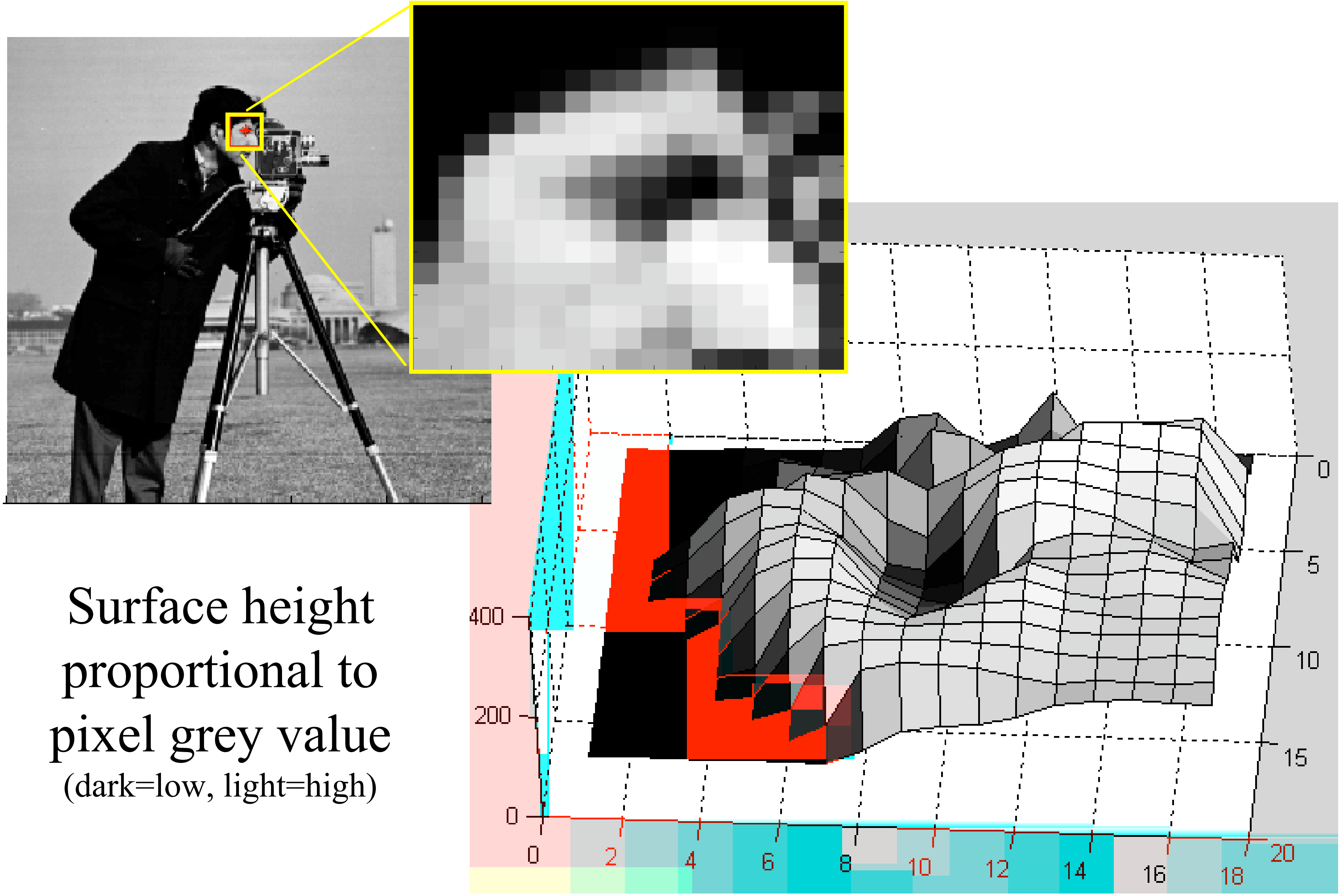
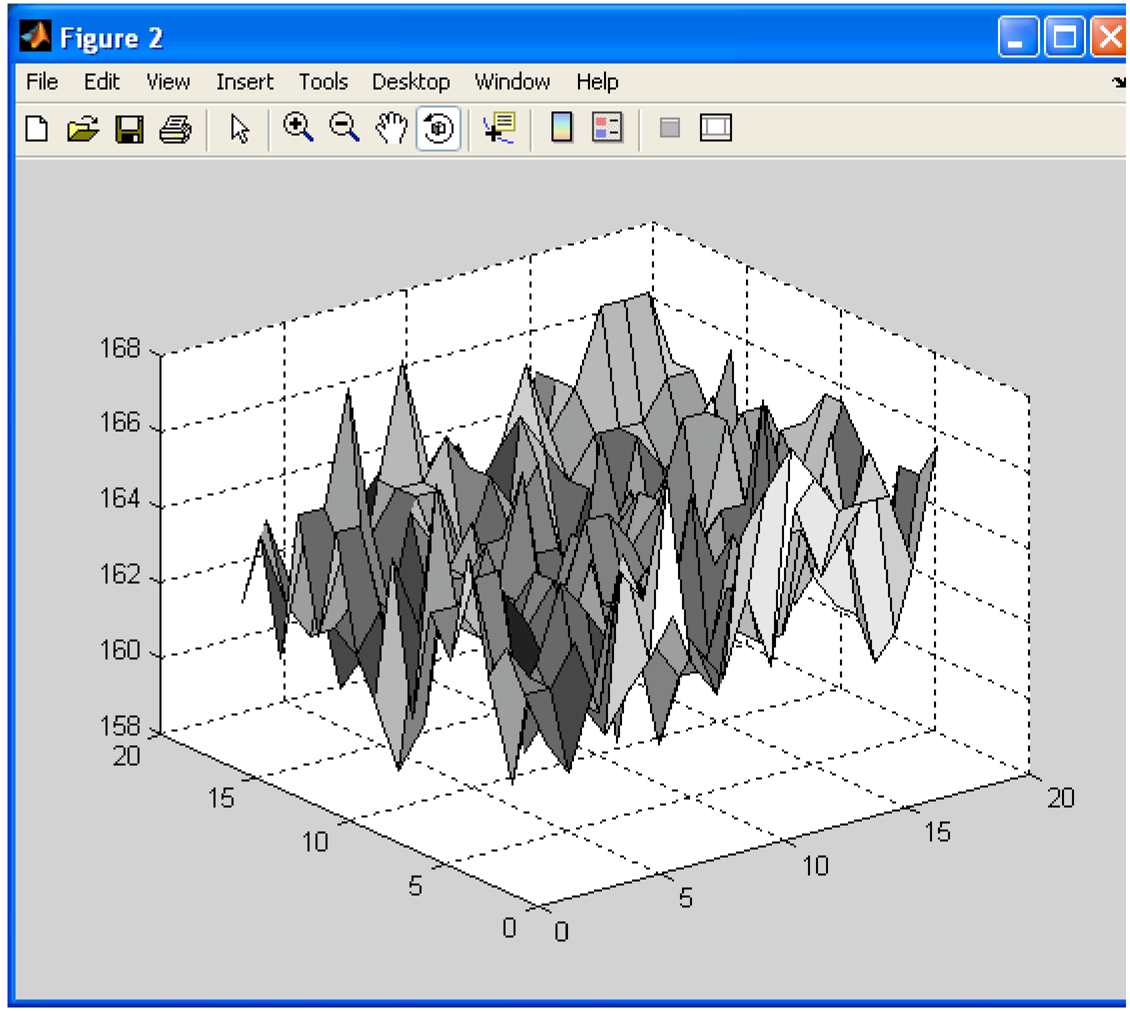
Images as Surfaces



Surface height proportional to pixel grey value

(dark=low, light=high)

Examples



Note: see demoImSurf.m in matlab examples directory on course web site if you want to generate plots like these.

Mean = 164 Std = 1.8

Examples

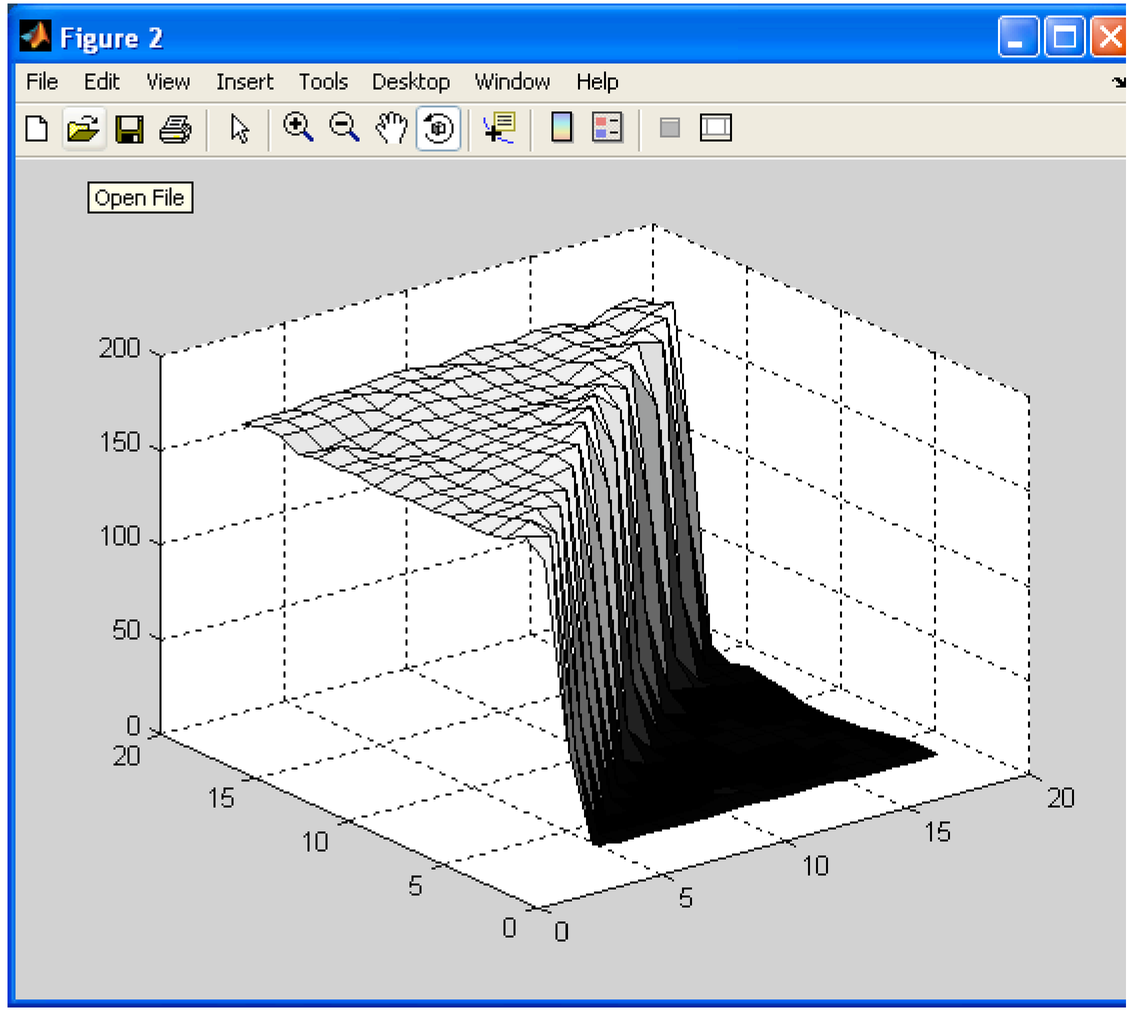


Figure 2
File Edit View Insert Tools Desktop Window Help
Open File
200
150
100
50
0
20
15
10
5
0
0
5
10
15
20

Examples

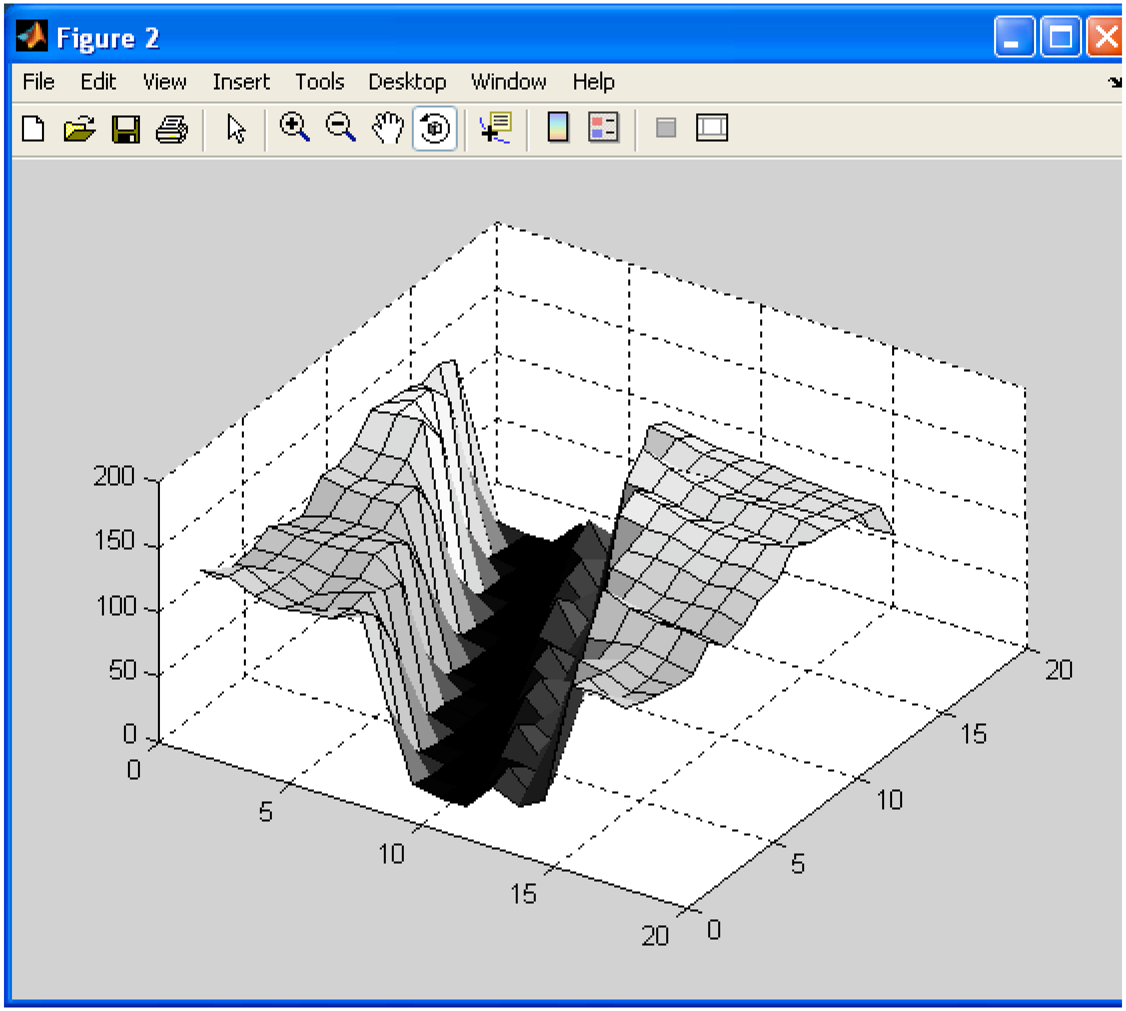


Figure 2
File Edit View Insert Tools Desktop Window Help
200
150
100
50
0
0
5
10
15
20
0
5
10
15
20

Examples

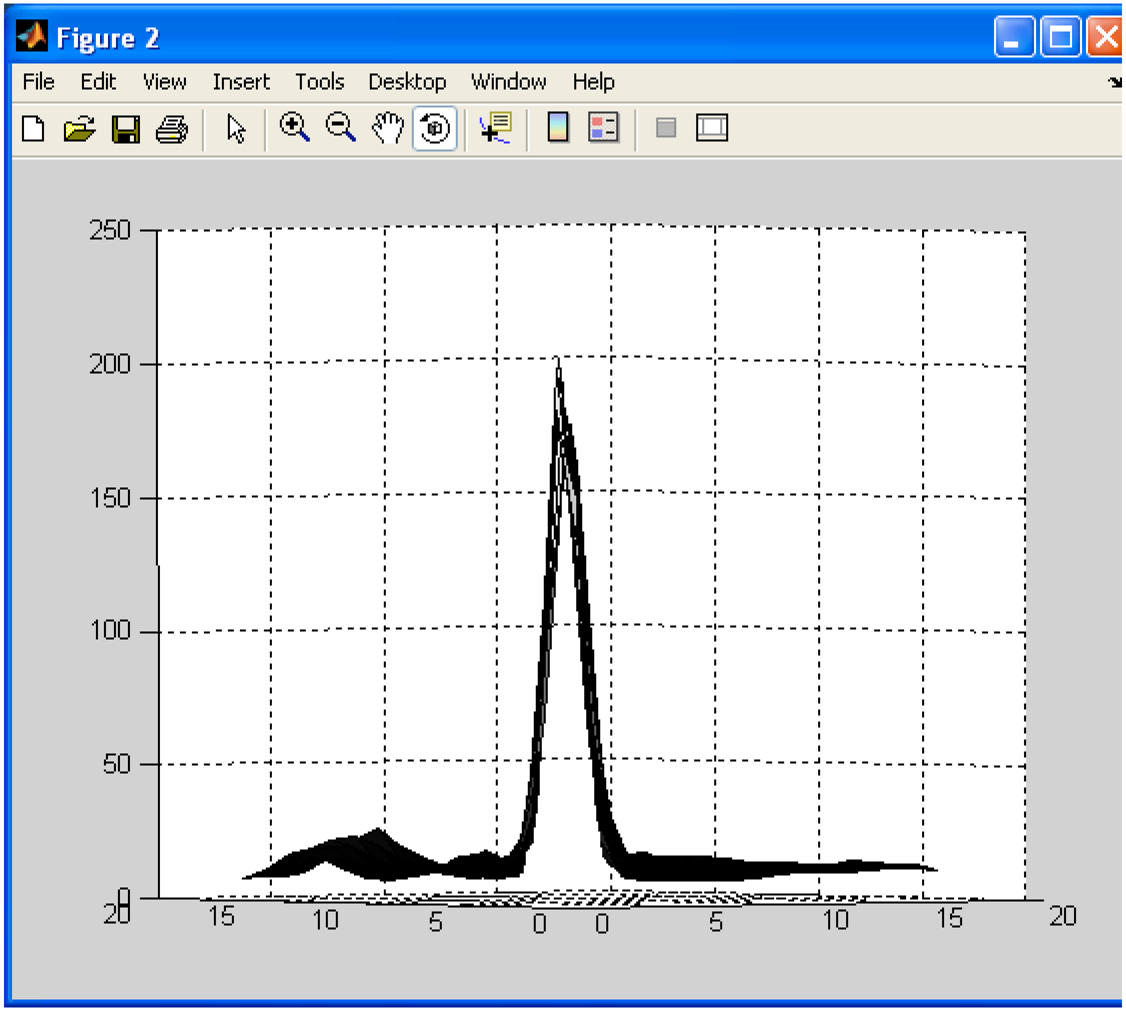


Figure 2
File Edit View Insert Tools Desktop Window Help
250
200
150
100
50
0
20
15
10
5
0
0
5
10
15
20

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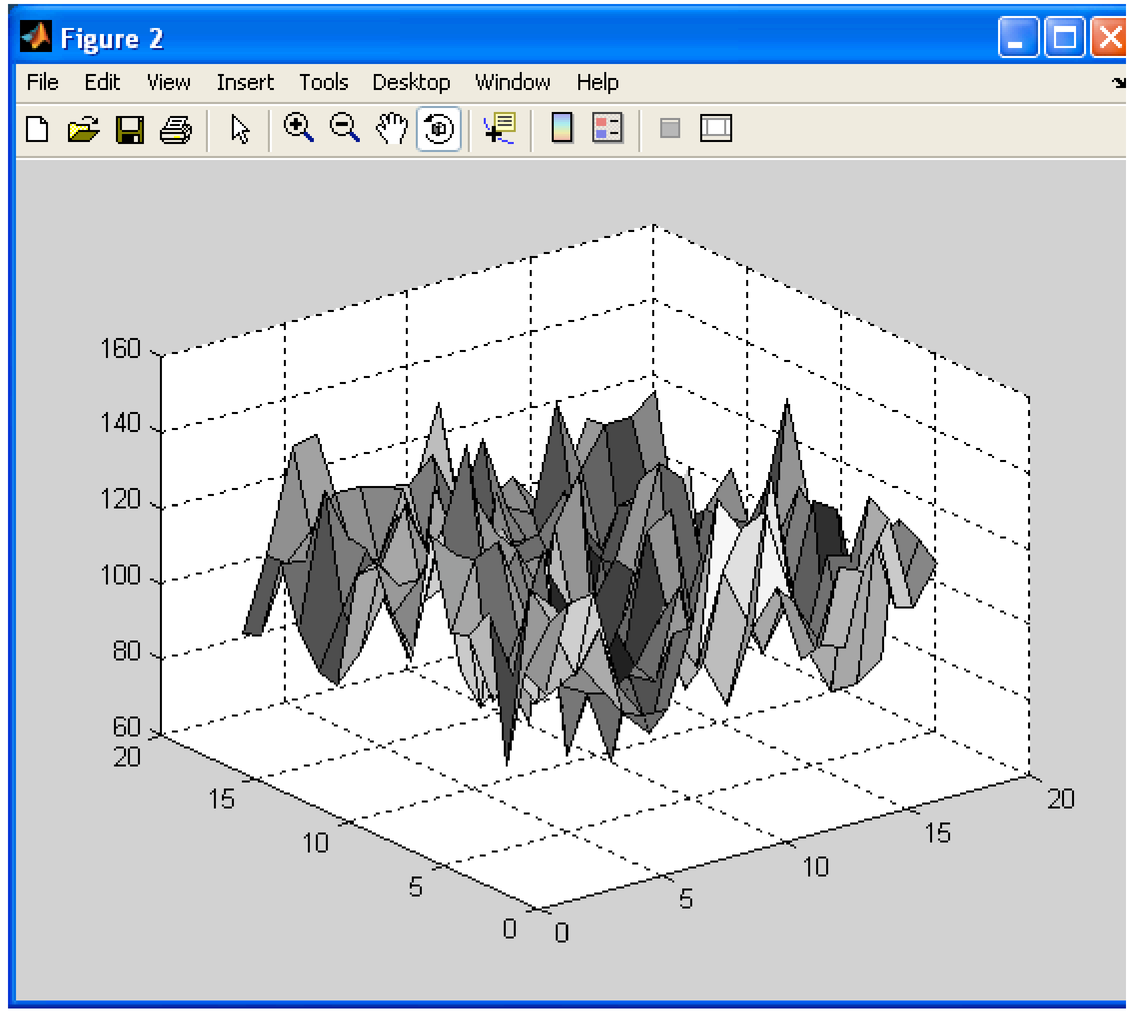
Examples



Mean = 111 Std = 15.4

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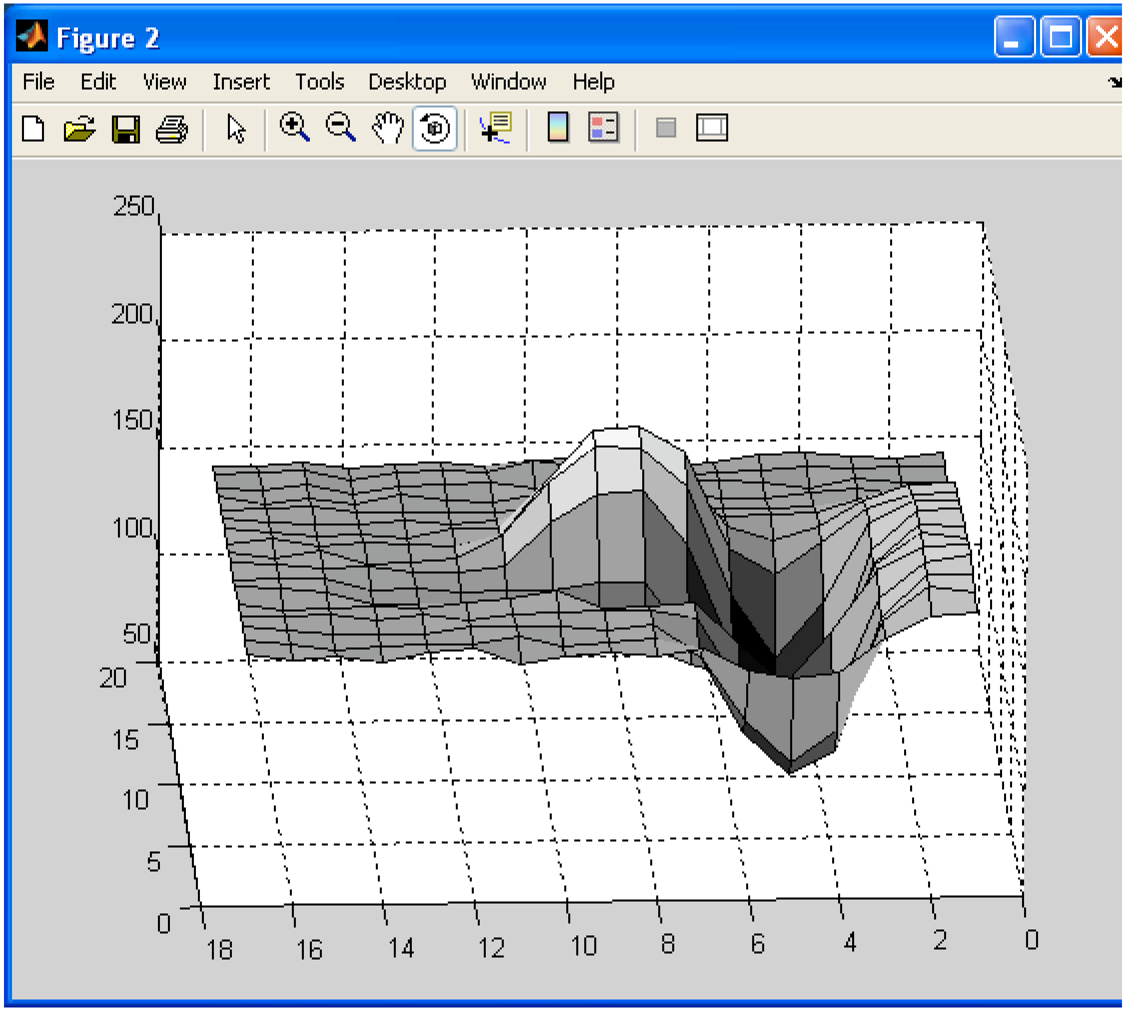
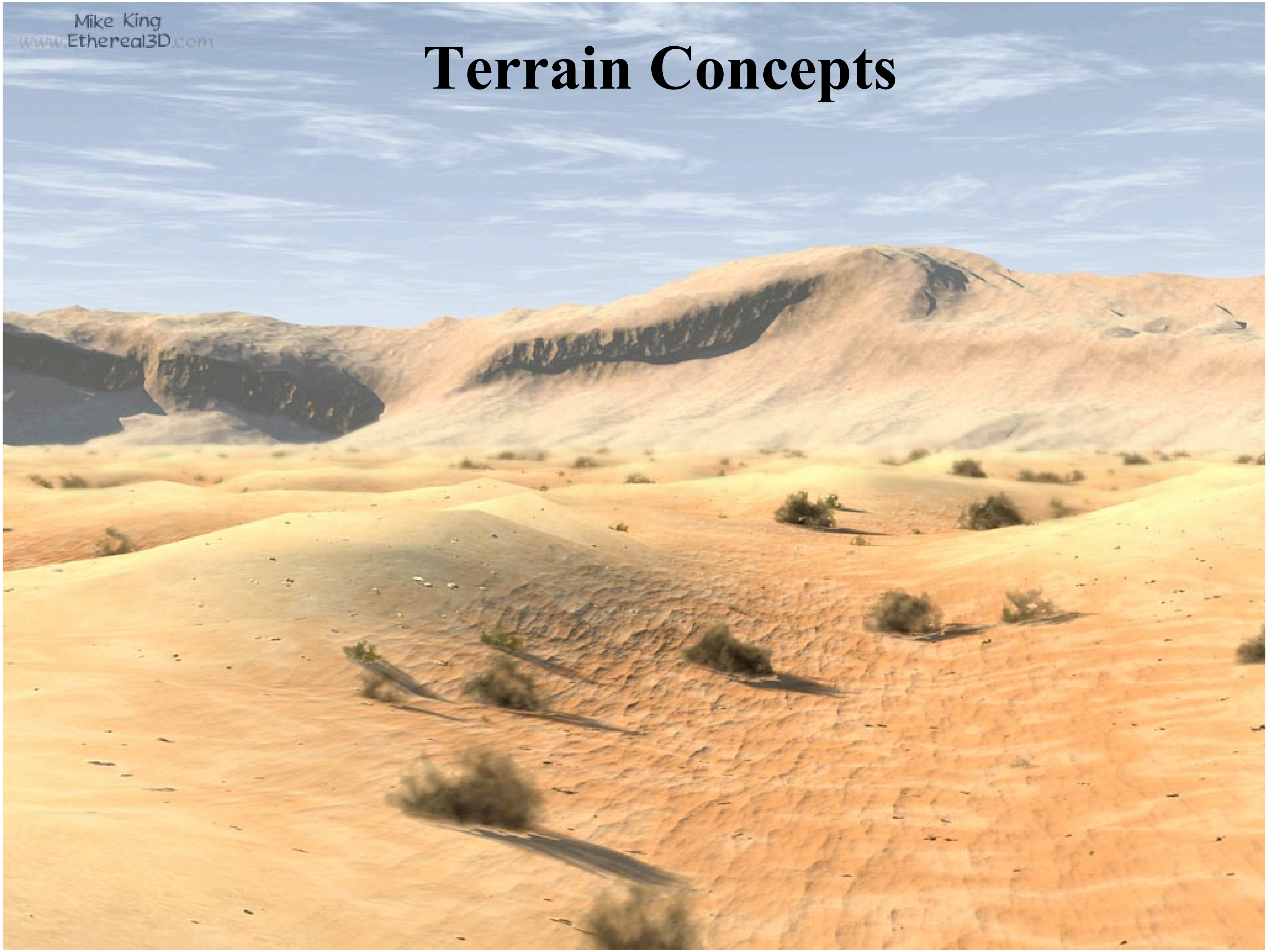
Examples



How does this visualization help us?

Terrain Concepts



Terrain Concepts

www.pianoladynancy.com/ wallpapers/1003/wp86.jpg

Terrain Concepts

Basic notions:
- Uphill / downhill
- Contour lines (curves of constant elevation)
- Steepness of slope
- Peaks/Valleys (local extrema)

More mathematical notions:
- Tangent Plane
- Normal vectors
- Curvature

Gradient vectors (vectors of partial derivatives) will help us define/compute all of these.

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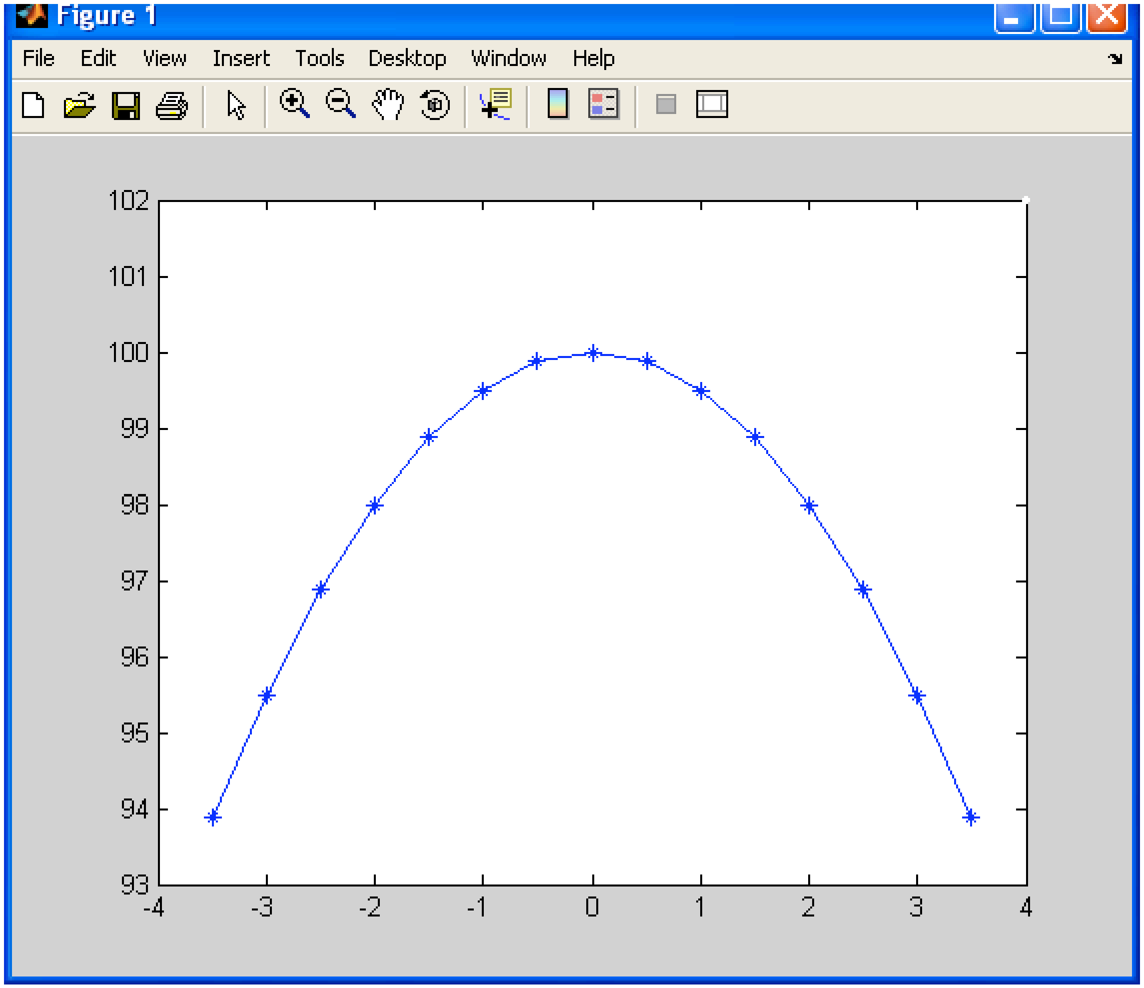
Math Example : 1D Gradient

Consider function f(x) = 100 - 0.5 * x^2



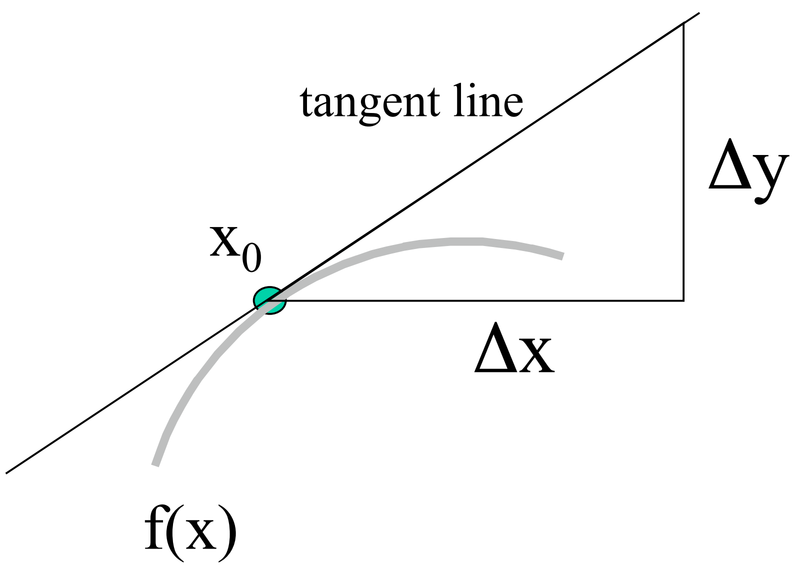
Math Example : 1D Gradient

Consider function f(x) = 100 - 0.5 * x^2

Gradient is df(x)/dx = - 2 * 0.5 * x = - x

Geometric interpretation:

gradient at x_0 is slope of tangent line to curve at point x_0



slope = Δy / Δx

$= df(x)/dx \Big|_{x_0}$

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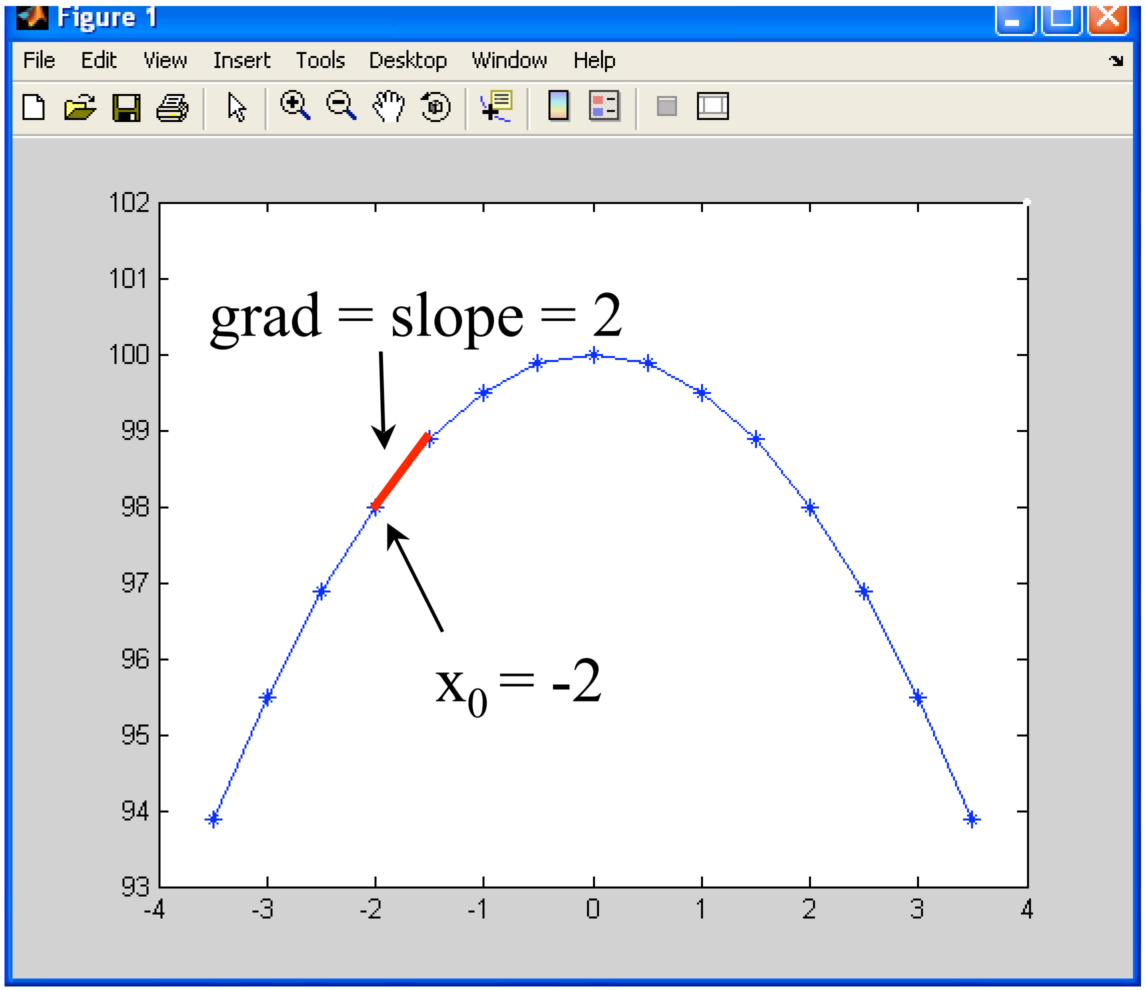
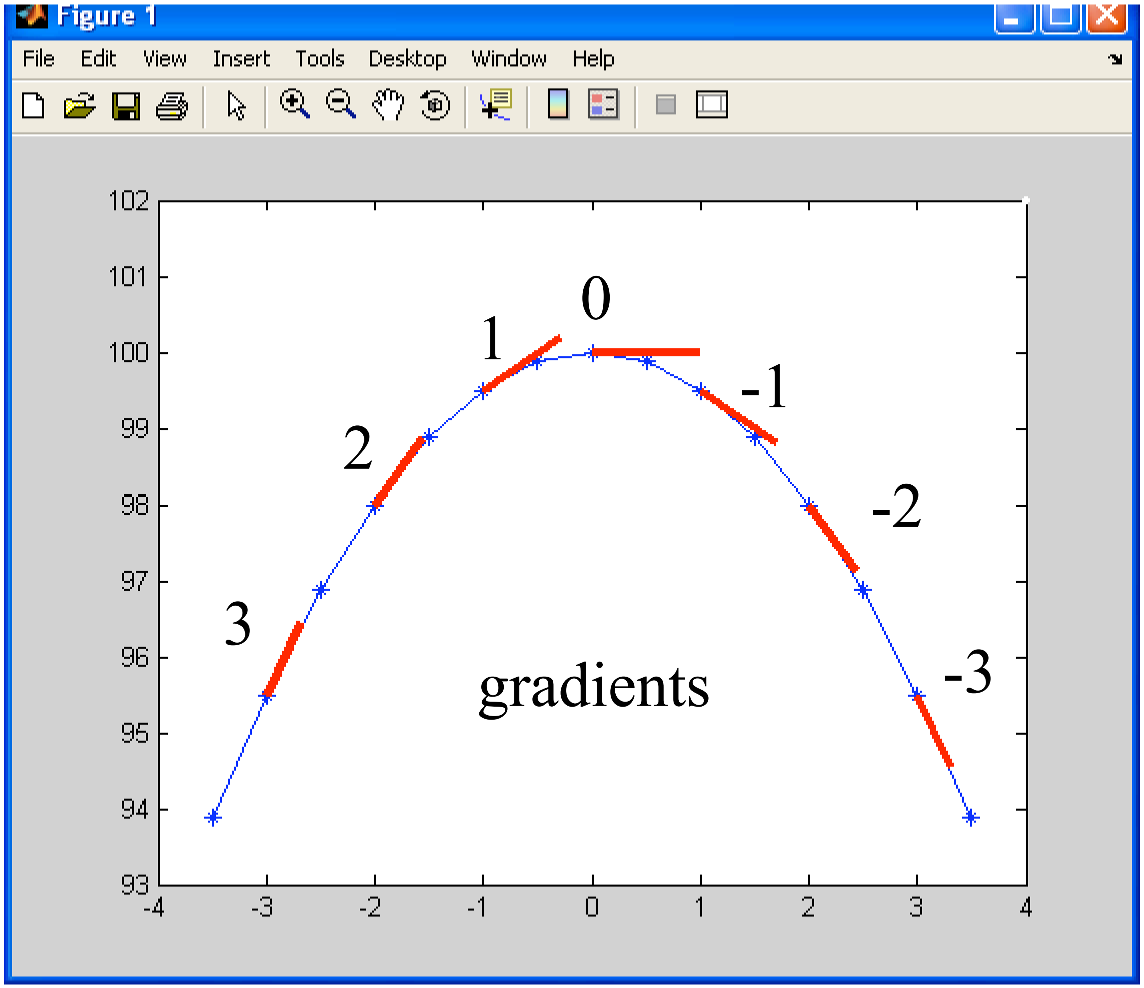
Math Example : 1D Gradient

f(x) = 100 - 0.5 * x^2 df(x)/dx = - x



Math Example : 1D Gradient

$f(x) = 100 - 0.5 * x^2$ $\qquad$ $df(x)/dx = - x$



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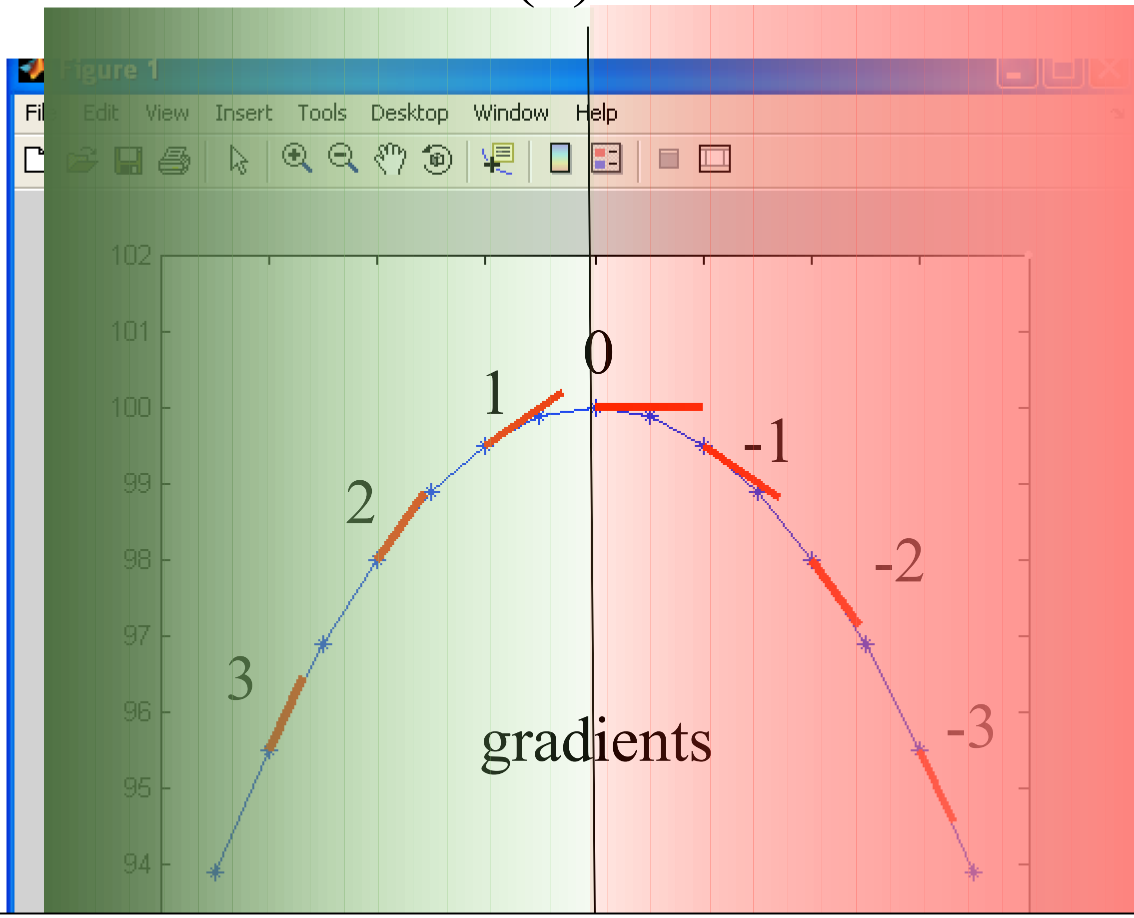
Math Example : 1D Gradient

f(x) = 100 - 0.5 * x^2   df(x)/dx = - x

Gradients on this side of peak are positive



Gradients on this side of peak are negative

Note: Sign of gradient at point tells you what direction to go to travel "uphill"

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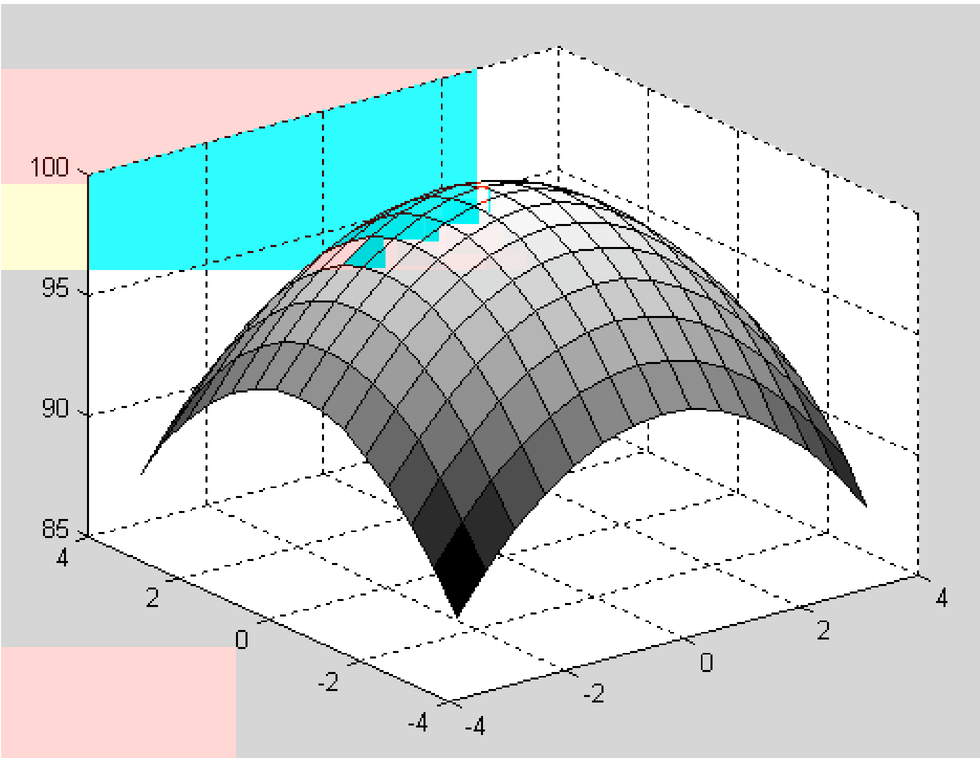
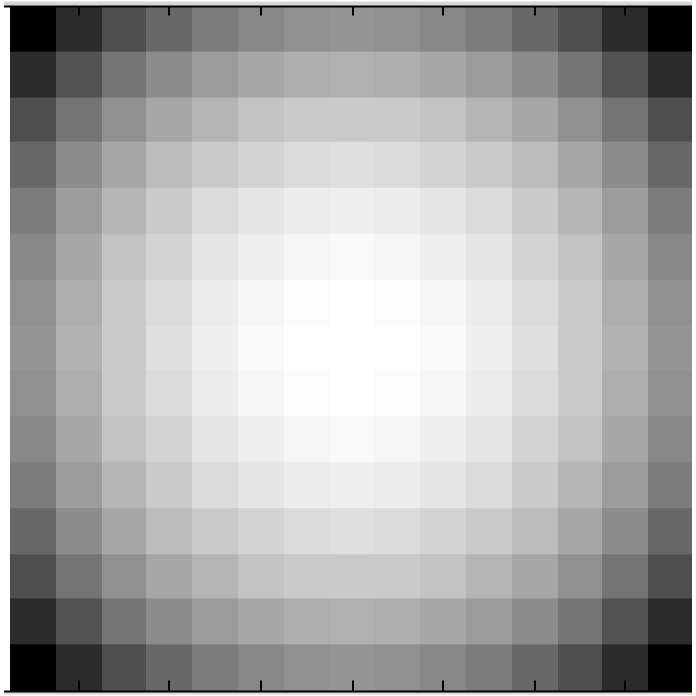
Math Example : 2D Gradient

f(x,y) = 100 - 0.5 * x^2 - 0.5 * y^2

df(x,y)/dx = - x   df(x,y)/dy = - y

Gradient = [df(x,y)/dx , df(x,y)/dy] = [- x , - y]





Gradient is vector of partial derivs wrt x and y axes

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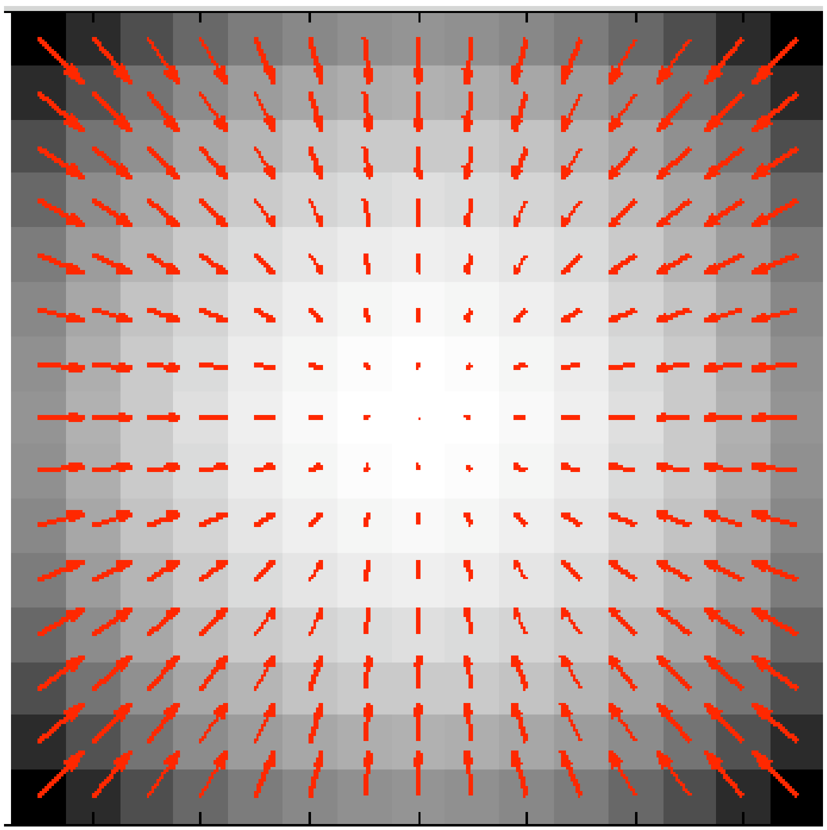
Math Example : 2D Gradient

$$f(x,y) = 100 - 0.5 * x^2 - 0.5 * y^2$$

$$\text{Gradient} = [df(x,y)/dx\ ,\ df(x,y)/dy] = [-x\ ,\ -y]$$



Plotted as a vector field, the gradient vector at each pixel points "uphill"

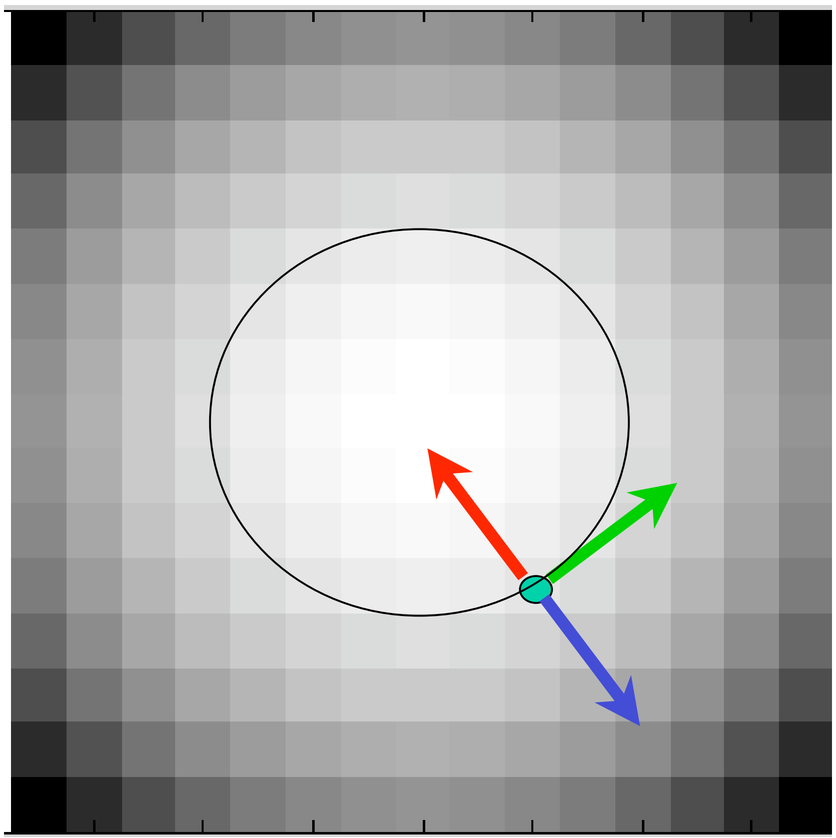
The gradient indicates the direction of steepest ascent.

The gradient is 0 at the peak
(also at any flat spots, and local minima,…but there are none of those for this function)

Math Example : 2D Gradient

f(x,y) = 100 - 0.5 * x^2 - 0.5 * y^2

Gradient = [df(x,y)/dx , df(x,y)/dy] = [- x , - y]



Let $g=[g_x,g_y]$ be the gradient vector at point/pixel (x_0,y_0)

Vector g points uphill

(direction of steepest ascent)

Vector - g points downhill

(direction of steepest descent)

Vector $[g_y, -g_x]$ is perpendicular, and denotes direction of constant elevation. i.e. normal to contour line passing through point (x_0,y_0)

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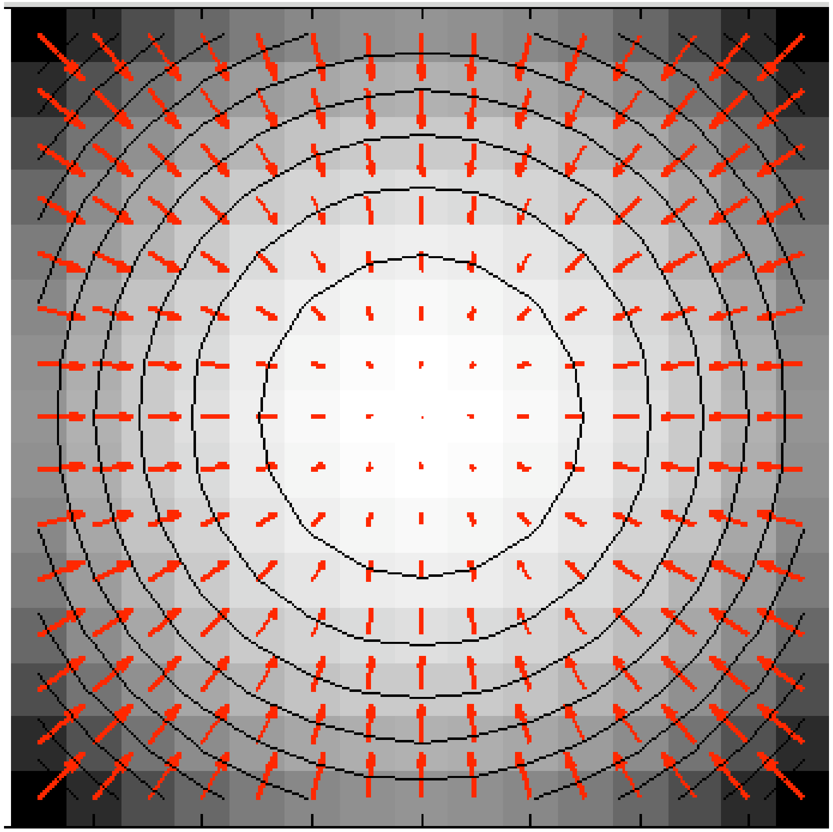
Math Example : 2D Gradient

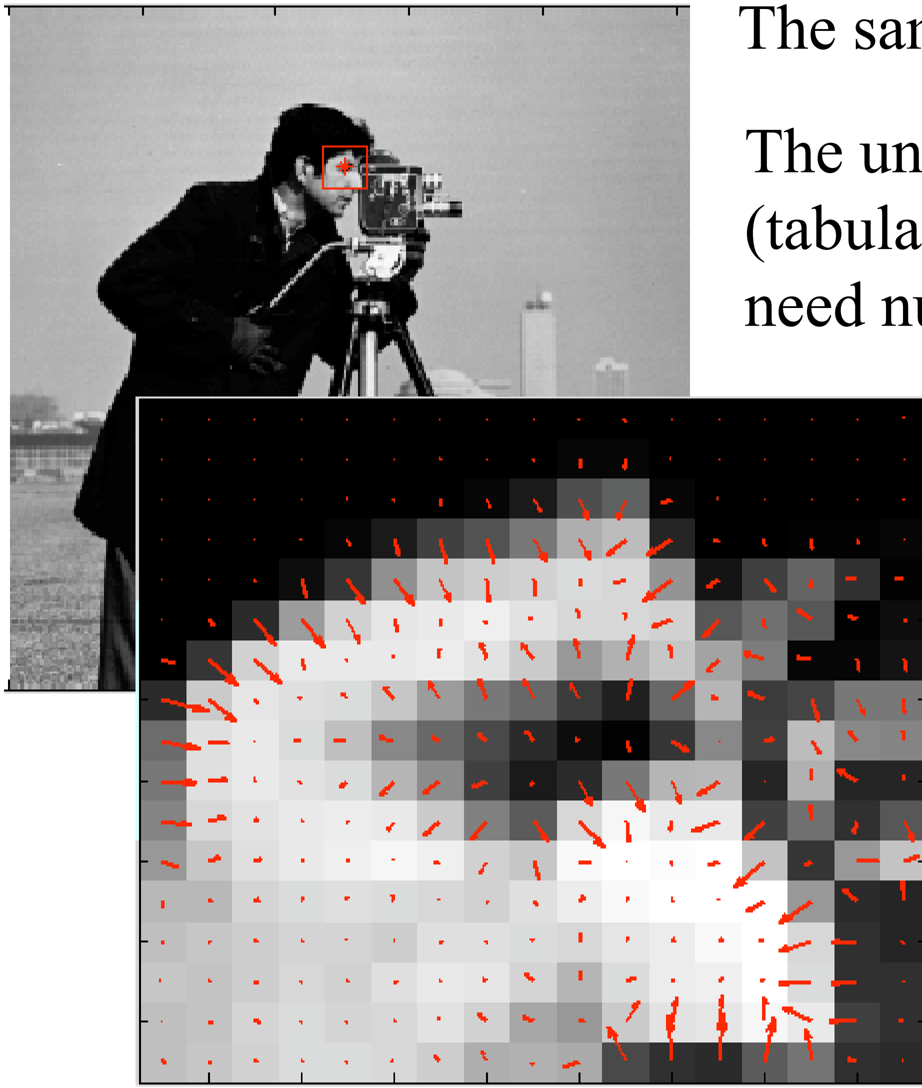
f(x,y) = 100 - 0.5 * x^2 - 0.5 * y^2

Gradient = [df(x,y)/dx , df(x,y)/dy] = [- x , - y]



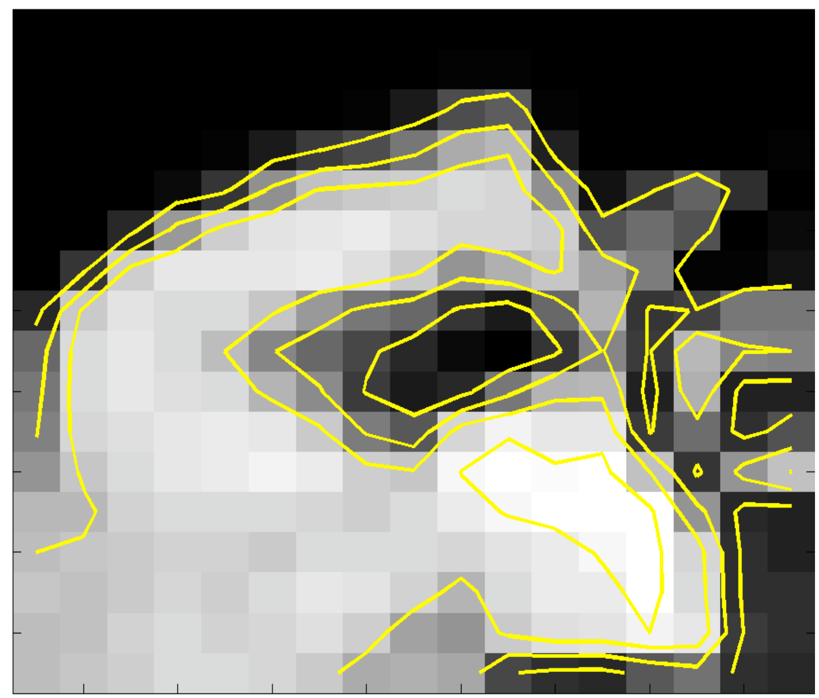
And so on for all points

Image Gradient



The same is true of 2D image gradients.

The underlying function is numerical (tabulated) rather than algebraic. So need numerical derivatives.



Numerical Derivatives

See also T&V, Appendix A.2

Taylor Series expansion

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + \frac{1}{3!}h^3 f'''(x) + O(h^4)$$

Manipulate:

$$f(x+h) - f(x) = hf'(x) + \frac{1}{2}h^2 f''(x) + O(h^3)$$

$$\frac{f(x+h) - f(x)}{h} = f'(x) + O(h)$$

Finite forward difference

Numerical Derivatives

See also T&V, Appendix A.2

Taylor Series expansion

$$f(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2 f''(x) - \frac{1}{3!}h^3 f'''(x) + O(h^4)$$

Manipulate:

$$f(x) - f(x-h) = hf'(x) - \frac{1}{2}h^2 f''(x) + O(h^3)$$

$$\frac{f(x) - f(x-h)}{h} = f'(x) + O(h)$$

Finite backward difference

Numerical Derivatives

See also T&V, Appendix A.2

Taylor Series expansion

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + \frac{1}{3!}h^3 f'''(x) + O(h^4)$$

subtract

$$-\left[f(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2 f''(x) - \frac{1}{3!}h^3 f'''(x) + O(h^4) \right]$$

$$f(x+h) - f(x-h) = 2hf'(x) + \frac{2}{3!}h^3 f'''(x) + O(h^4)$$

$$\frac{f(x+h) - f(x-h)}{2h} = f'(x) + O(h^2)$$

Finite central difference

Numerical Derivatives

See also T&V, Appendix A.2

Finite forward difference

$$\frac{f(x+h)-f(x)}{h} = f'(x) + O(h)$$

Finite backward difference

$$\frac{f(x)-f(x-h)}{h} = f'(x) + O(h)$$

Finite central difference

$$\frac{f(x+h)-f(x-h)}{2h} = f'(x) + O(h^2)$$

More accurate

Example: Temporal Gradient

A video is a sequence of image frames I(x,y,t).

Each frame has two spatial indices x, y and one temporal (time) index t.

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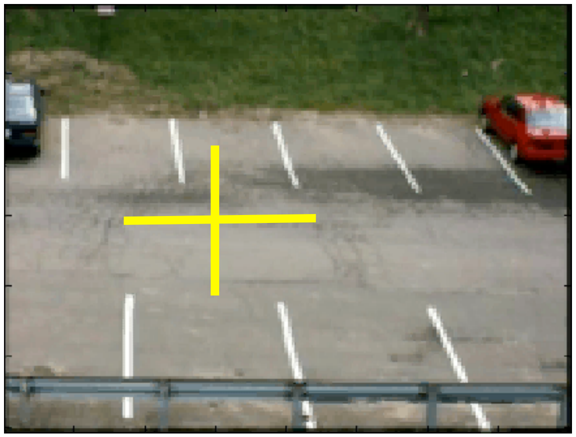
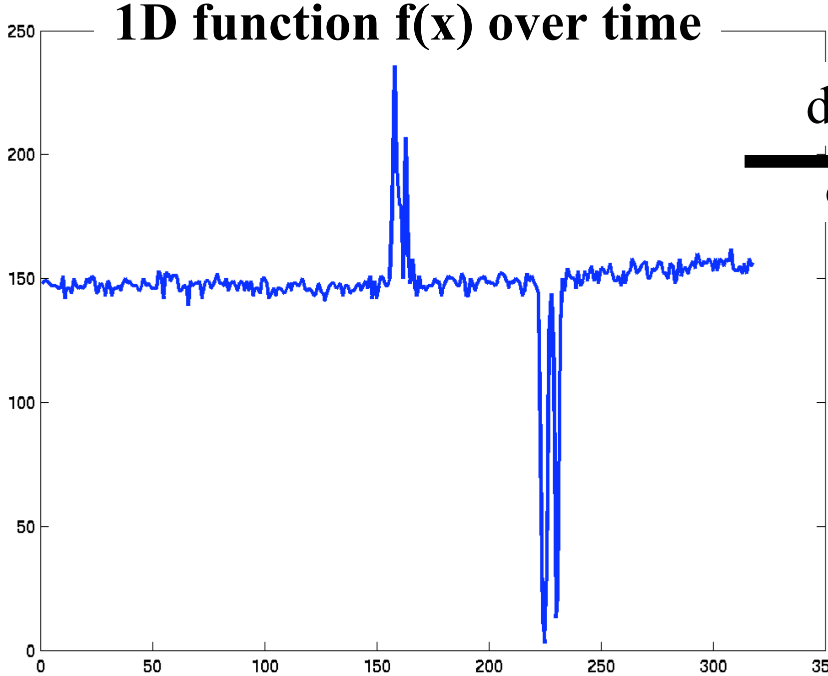
Example: Temporal Gradient



Consider the sequence of intensity values observed at a single pixel over time.

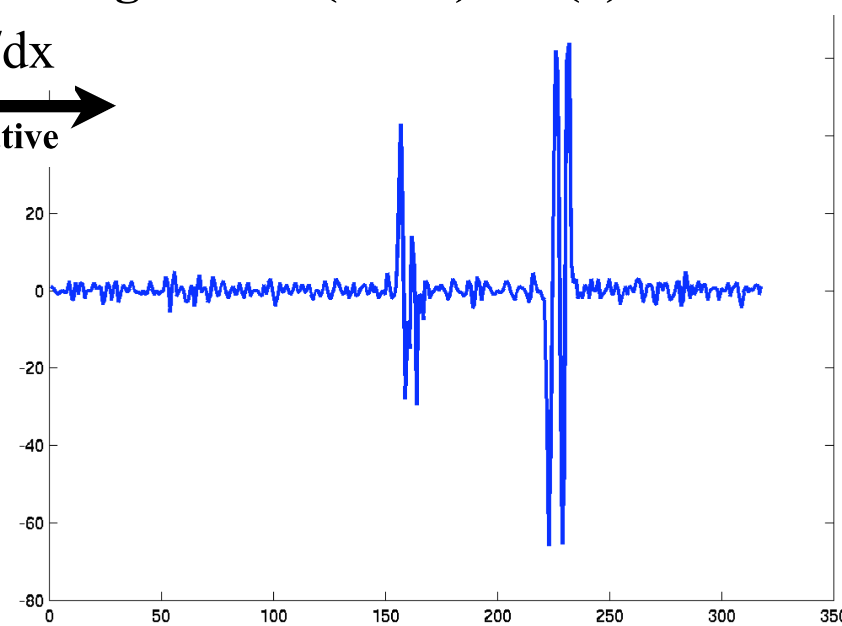
1D function f(x) over time



df(x)/dx

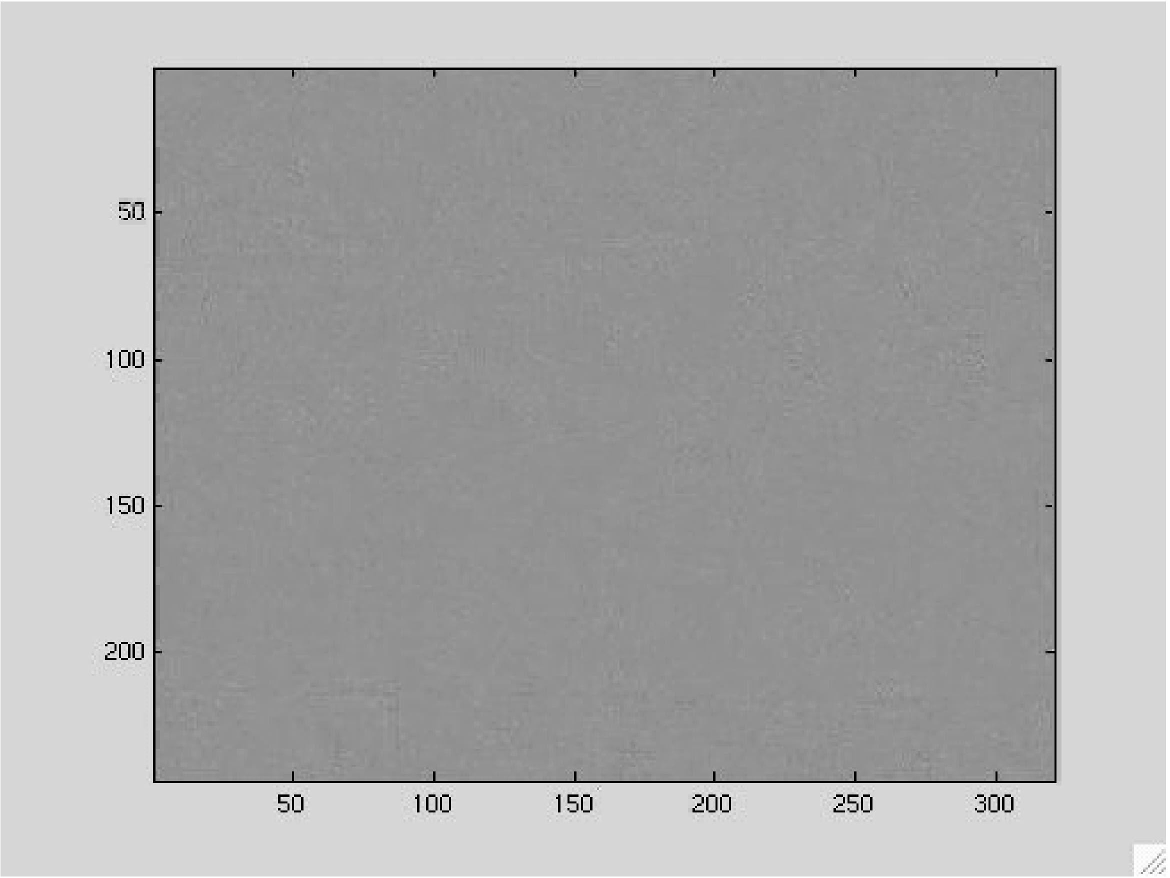
derivative

1D gradient (deriv) of f(x) over time



Temporal Gradient (cont)

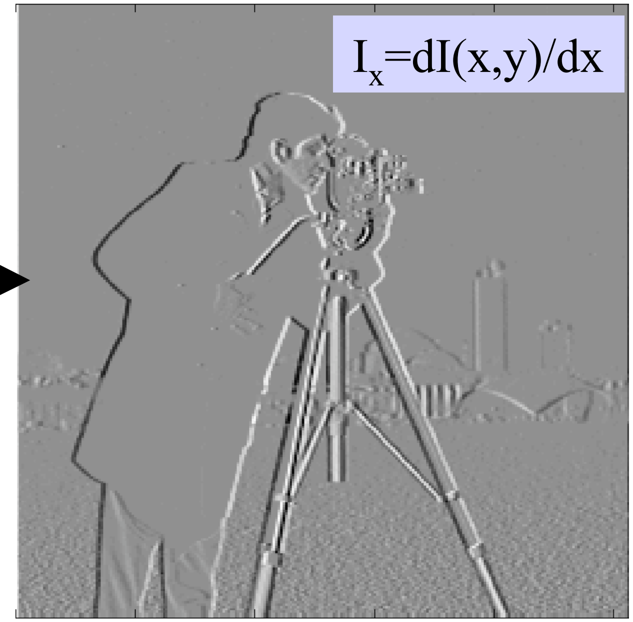
What does the temporal intensity gradient at each pixel look like over time?



Example: Spatial Image Gradients

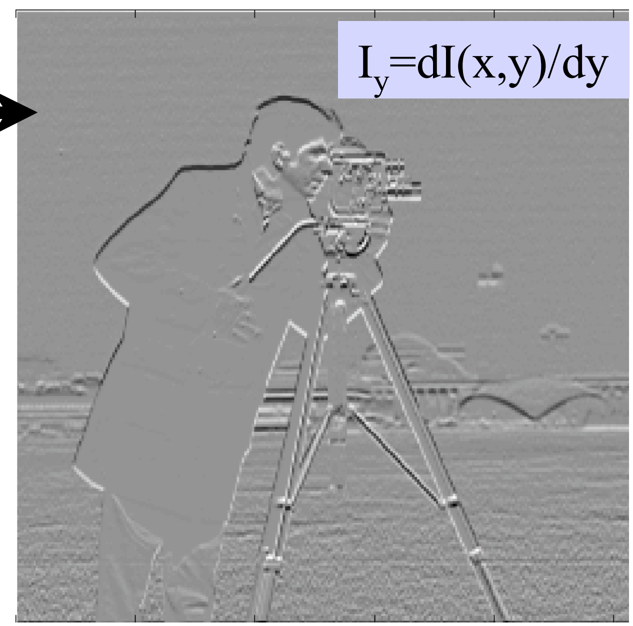
$$\frac{I(x+1,y) - I(x-1,y)}{2}$$

Partial derivative wrt x



$$\frac{I(x,y+1) - I(x,y-1)}{2}$$

Partial derivative wrt y



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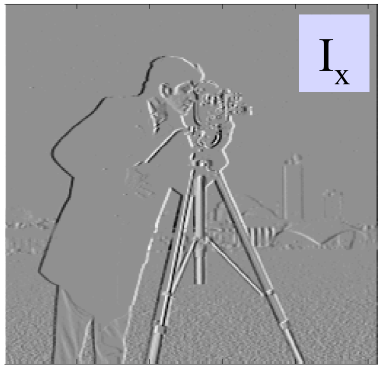
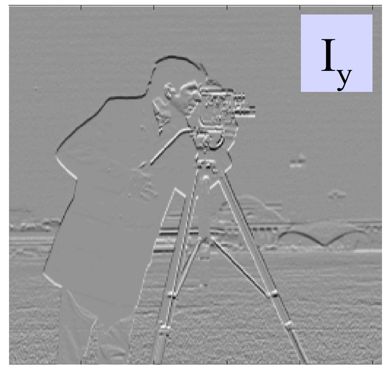
Functions of Gradients

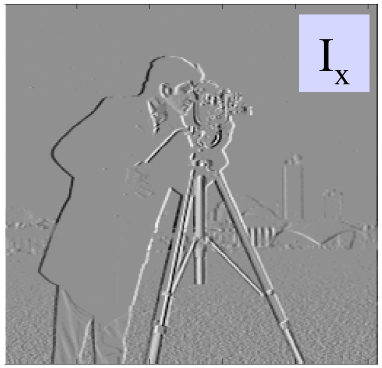


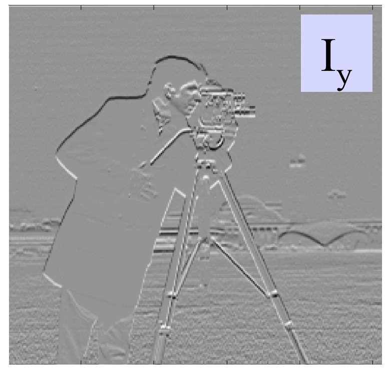


Magnitude of gradient
sqrt(Ix.^2 + Iy.^2)

Measures steepness of slope at each pixel

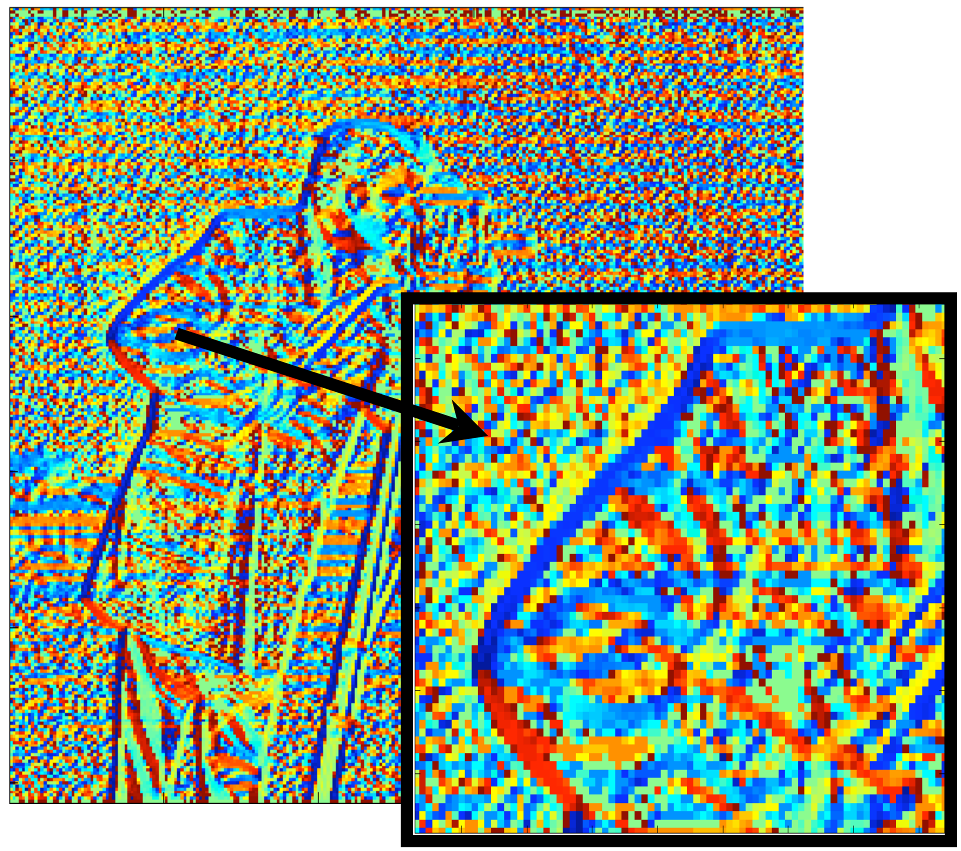
Functions of Gradients



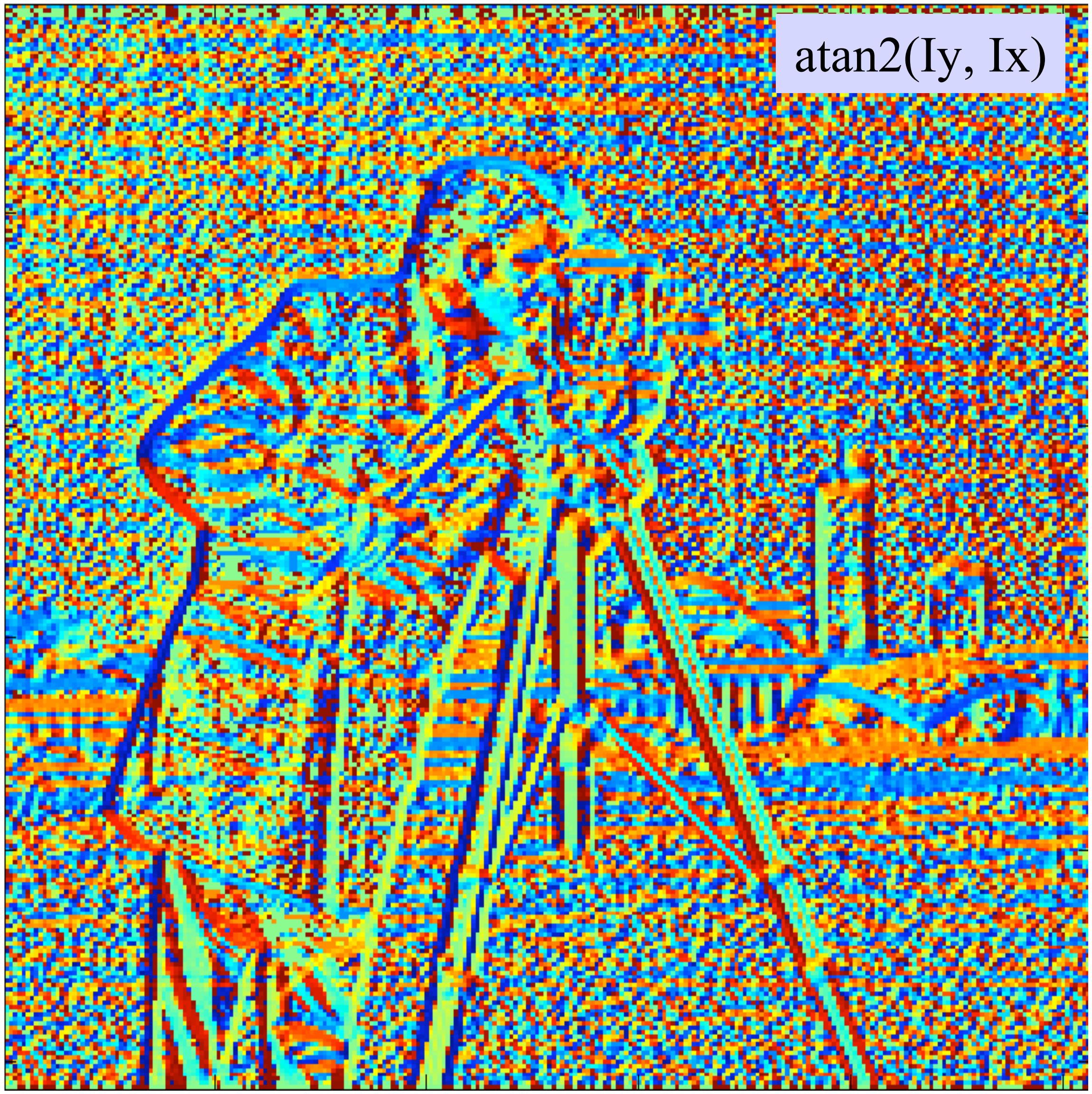


Angle of gradient
atan2(Iy, Ix)

Denotes similarity
of orientation of slope



Functions of Gradients



What else do we observe in this image?

Enhanced detail in low contrast areas (e.g. folds in coat; imaging artifacts in sky)

Next Time: Linear Operators

Gradients are an example of linear operators, i.e. value at a pixel is computed as a linear combination of values of neighboring pixels.