

Probability review I: Probability spaces, events, and conditioning

Topics we'll cover

- ① How to define the **probability space** for an experiment in which outcomes are random.
- ② How to formulate an **event** of interest.
- ③ The probability that two events both occur.
- ④ The **conditional probability** that an event occurs, given that some other event has occurred.
- ⑤ **Bayes' rule**.

Probability spaces

You roll two dice.

What is the probability they add to 10?

The **probability space** has two components:

- ① **Sample space** (space of outcomes).

- ② **Probabilities of outcomes**, summing to 1.

Events

Probability space:

- Outcomes: $\Omega = \{\text{all possible pairs of dice rolls}\}$
- Every pair $z = (z_1, z_2) \in \Omega$ has probability $1/36$.

Event of interest: the two dice add up to 10.

Multiple events

You have ten coins. Nine are fair, but one is a bad coin that always comes up tails.

- You close your eyes and pick a coin at random.
- You toss it four times, and it comes up tails every time.

What is the probability you picked the bad coin?

- Ten coins: nine are fair, one is a bad coin that always comes up tails.
- You pick a coin at random, toss it four times, and it's tails every time.

Conditioning

For two events A , B , **conditional probability**

$\Pr(B|A)$ = probability that B occurs, given that A occurs

Conditioning formula: $\Pr(A \cap B) = \Pr(A) \Pr(B|A)$

In our example:

- A : the bad coin is chosen
- B : all four tosses are tails

Want $\Pr(A|B)$

- Ten coins: nine are fair, one is a bad coin that always comes up tails.
- You pick a coin at random, toss it four times, and it's tails every time.

Event A : the bad coin is chosen. Event B : all tails

Bayes' rule

Two events A, B

- We are interested in A
- We can observe B

If we find out B occurred, how does it alter the probability of A ?

$$\text{Bayes' rule: } \Pr(A|B) = \Pr(A) \times \frac{\Pr(B|A)}{\Pr(B)}$$