Boosting

Sanjoy Dasgupta

University of California, San Diego

Topics we'll cover

1. Weak learners
2. The AdaBoost learning algorithm

Weak learners

It is often easy to come up with a **weak classifier**, one that is marginally better than random guessing:

$$\Pr(h(X) \neq Y) \leq \frac{1}{2} - \epsilon$$

A learning algorithm that can consistently generate such classifiers is called a **weak learner**.

Is it possible to systematically boost the quality of a weak learner?

The blueprint for boosting

Given: data set $(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})$.

- Initially give all points equal weight.
- Repeat for $t = 1, 2, \ldots$:
  - Feed weighted data set to the weak learner, get back a weak classifier h_t
  - Reweight data to put more emphasis on points that h_t gets wrong
- Combine all these h_t's linearly

AdaBoost

Data set $(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})$, labels $y^{(i)} \in \{-1, +1\}$.

1. Initialize $D_1(i) = 1/n$ for all $i = 1, 2, \ldots, n$
2. For $t = 1, 2, \ldots, T$:
   - Give D_t to weak learner, get back some $h_t : \mathcal{X} \to [-1, 1]$
   - Compute h_t's margin of correctness:

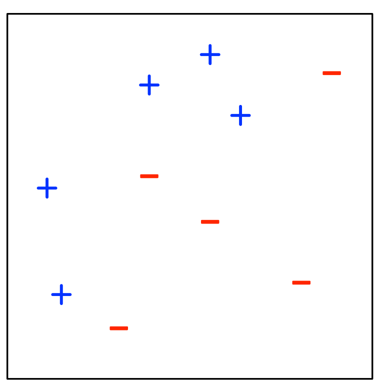
$$r_t = \sum_{i=1}^{n} D_t(i) y^{(i)} h_t(x^{(i)}) \in [-1, 1]$$

$$\alpha_t = \frac{1}{2} \ln \frac{1 + r_t}{1 - r_t}$$

   - Update weights: $D_{t+1}(i) \propto D_t(i) \exp\left(-\alpha_t y^{(i)} h_t(x^{(i)})\right)$
3. Final classifier: $H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$

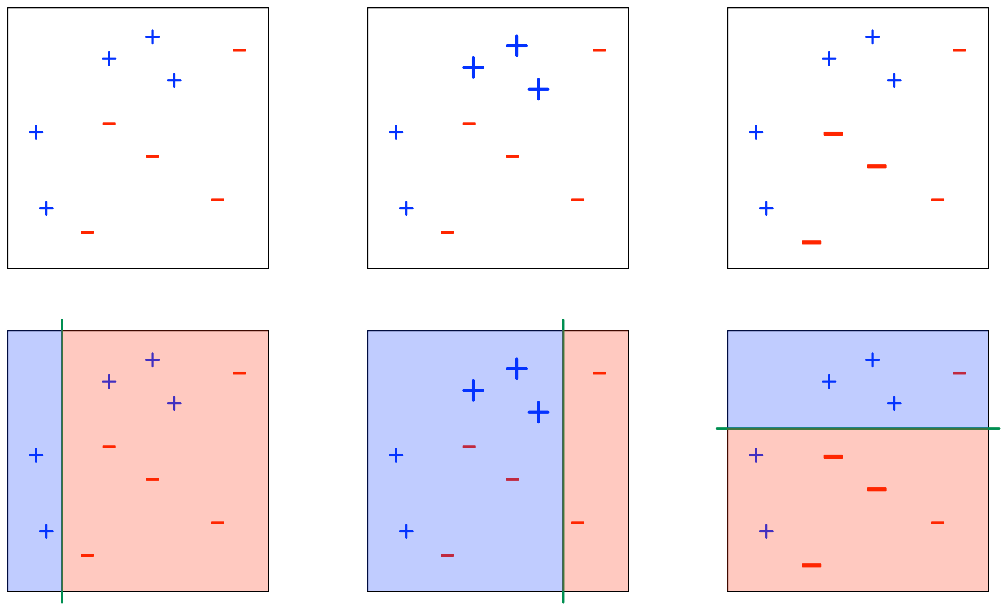
Example (Freund-Schapire)

Training set:



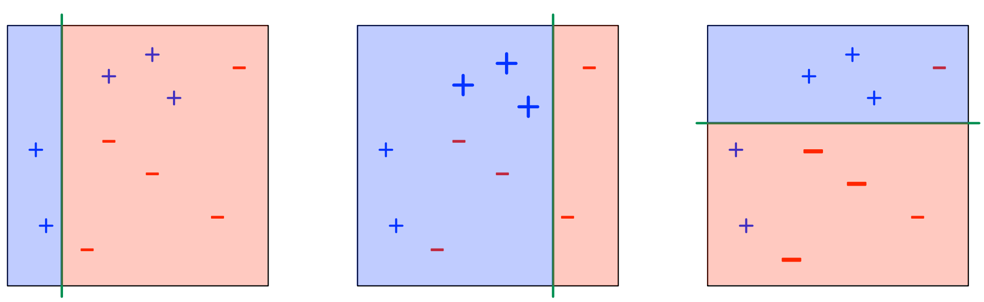
Use "decision stumps" (single-feature thresholds) as weak classifiers

D_1 D_2 D_3



h_1 h_2 h_3

$r_1 = 0.40, \alpha_1 = 0.42$ $r_2 = 0.58, \alpha_2 = 0.65$ $r_3 = 0.72, \alpha_3 = 0.92$

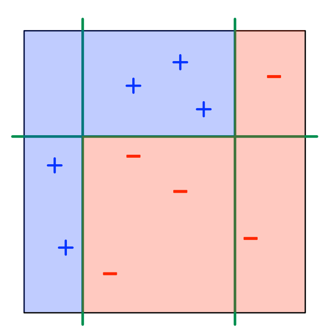


h_1 h_2 h_3

$\alpha_1 = 0.42$ $\alpha_2 = 0.65$ $\alpha_3 = 0.92$

Final classifier:

$\text{sign}\left(0.42h_1(x) + 0.65h_2(x) + 0.92h_3(x)\right)$



The surprising power of weak learning

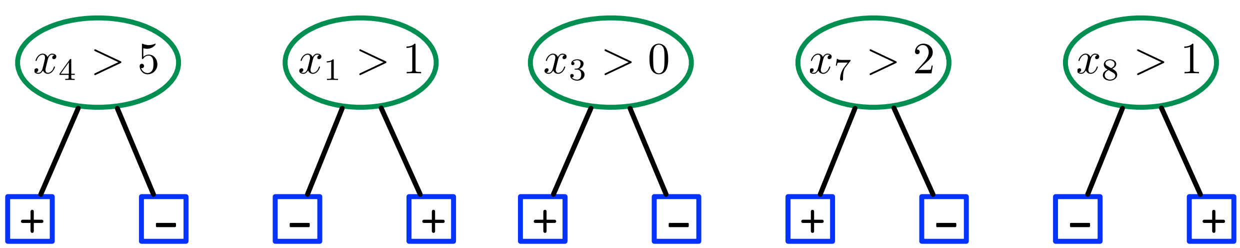
Suppose that on each round t, the weak learner returns a rule h_t whose error on the time-t weighted data distribution is $\leq 1/2 - \gamma$.

Then, after T rounds, the training error of the combined rule

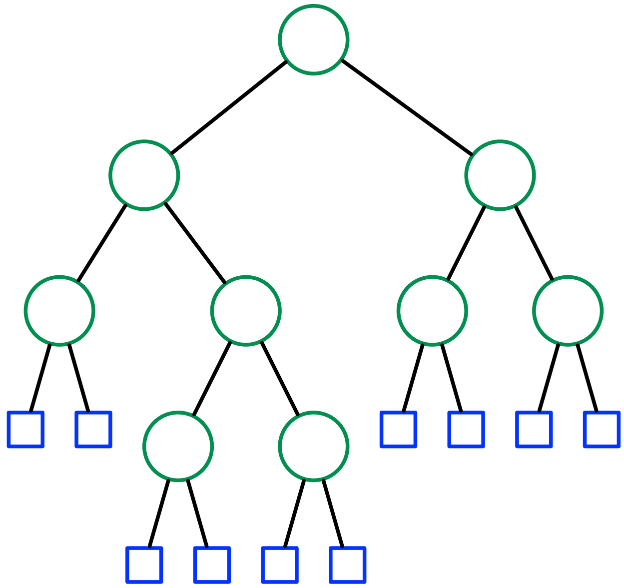
$$H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

is at most $e^{-\gamma^2 T/2}$.

Boosting decision stumps and trees



Boosting decision stumps and trees



Boosting decision stumps and trees