A simple linear classifier

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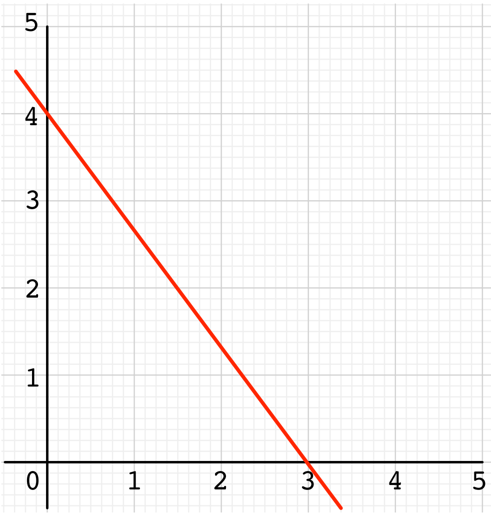
Topics we'll cover

1. Linear decision boundary for binary classification
2. A loss function for classification
3. The Perceptron algorithm

Linear decision boundary for classification: example



- What is the formula for this boundary?
- What label would we predict for a new point x?

Linear classifiers

Binary classification problem: data $x \in \mathbb{R}^d$ and labels $y \in \{-1, +1\}$

- Linear classifier:
  - Parameters: $w \in \mathbb{R}^d$ and $b \in \mathbb{R}$
  - Decision boundary $w \cdot x + b = 0$
  - On point x, predict label $\text{sign}(w \cdot x + b)$
- If the true label on point x is y:
  - Classifier correct if $y(w \cdot x + b) > 0$

A loss function for classification

What is the **loss** of our linear classifier (given by w, b) on a point (x, y)?

One idea for a loss function:

- If $y(w \cdot x + b) > 0$: correct, no loss
- If $y(w \cdot x + b) < 0$: loss $= -y(w \cdot x + b)$

A simple learning algorithm

Fit a linear classifier w, b to the training set using **stochastic gradient descent**.

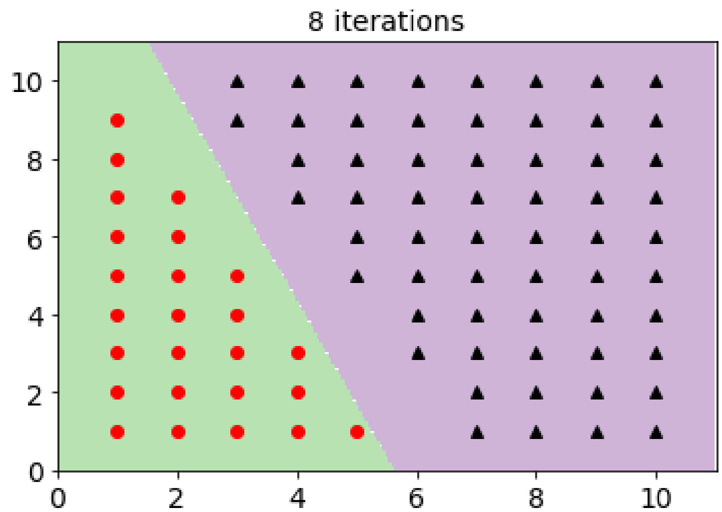
- Update w, b based on just one data point (x, y) at a time
- If $y(w \cdot x + b) > 0$: zero loss, no update
- If $y(w \cdot x + b) \leq 0$: loss is $-y(w \cdot x + b)$

The Perceptron algorithm

- Initialize $w = 0$ and $b = 0$
- Keep cycling through the training data (x, y):
  - If $y(w \cdot x + b) \leq 0$ (i.e. point misclassified):
    - $w = w + yx$
    - $b = b + y$

The Perceptron in action

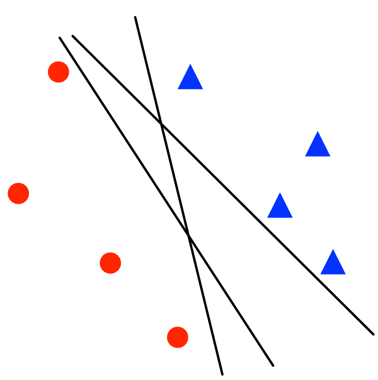
85 data points, linearly separable.



Perceptron: convergence

If the training data is linearly separable:

- The Perceptron algorithm will find a linear classifier with zero training error
- It will converge within a finite number of steps.



But is there a better, more systematic choice of separator?