Nearest neighbor classification

Topics we'll cover

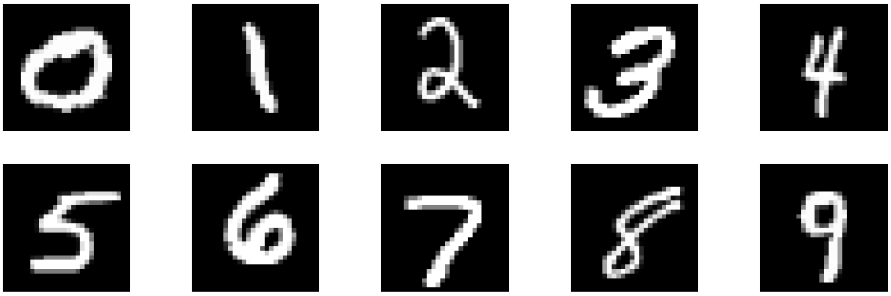
1. What is a classification problem?
2. The training set and test set
3. Representing data as vectors
4. Distance in Euclidean space
5. The 1-NN classifier
6. Training error versus test error
7. The error of a random classifier

The problem we'll solve today

Given an image of a handwritten digit, say which digit it is.

 $\Longrightarrow$ 3

Some more examples:



The machine learning approach

Assemble a data set:

The MNIST data set of handwritten digits:

- **Training set** of 60,000 images and their labels.
- **Test set** of 10,000 images and their labels.

And let the machine figure out the underlying patterns.

Nearest neighbor classification

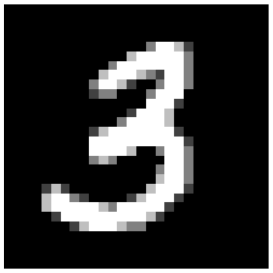
Training images $x^{(1)}, x^{(2)}, x^{(3)}, \ldots, x^{(60000)}$
Labels $y^{(1)}, y^{(2)}, y^{(3)}, \ldots, y^{(60000)}$ are numbers in the range $0 - 9$

How to **classify** a new image x?

- Find its nearest neighbor amongst the $x^{(i)}$
- Return $y^{(i)}$

The data space

How to measure the distance between images?



MNIST images:

- Size 28×28 (total: 784 pixels)
- Each pixel is grayscale: 0-255

Stretch each image into a vector with 784 coordinates:

- Data space $\mathcal{X} = \mathbb{R}^{784}$
- Label space $\mathcal{Y} = \{0, 1, \ldots, 9\}$

The distance function

Remember Euclidean distance in two dimensions?

$$z = (3, 5)$$

$$x = (1, 2)$$

Euclidean distance in higher dimension

Euclidean distance between 784-dimensional vectors x, z is

$$\|x - z\| = \sqrt{\sum_{i=1}^{784} (x_i - z_i)^2}$$

Here x_i is the ith coordinate of x.

Nearest neighbor classification

Training images $x^{(1)}, \ldots, x^{(60000)}$, labels $y^{(1)}, \ldots, y^{(60000)}$

To classify a new image x:

- Find its nearest neighbor amongst the $x^{(i)}$ **using Euclidean distance in** $\mathbb{R}^{784}$
- Return $y^{(i)}$

How accurate is this classifier?

Accuracy of nearest neighbor on MNIST

Training set of 60,000 points.

- What is the error rate on training points? **Zero.**
  In general, **training error** is an overly optimistic predictor of future performance.
- A better gauge: separate test set of 10,000 points.
  Test error = fraction of test points incorrectly classified.
- What test error would we expect for a *random classifier*?
  (One that picks a label $0 - 9$ at random?) **90%.**
- Test error of nearest neighbor: **3.09%.**

Examples of errors

Test set of 10,000 points:

- 309 are misclassified
- Error rate 3.09%

Examples of errors:

Query 

NN