Kernel methods I
Basis expansion

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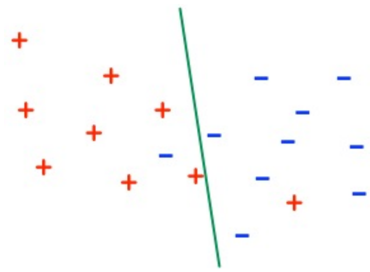
Topics we'll cover

1. Two deviations from linear separability
2. Learning quadratic boundaries using basis expansion

Deviations from linear separability

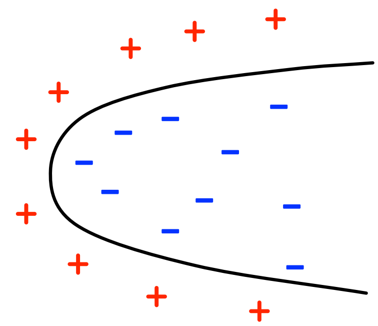
Noise



Find a separator that minimizes a convex loss function related to the number of mistakes.

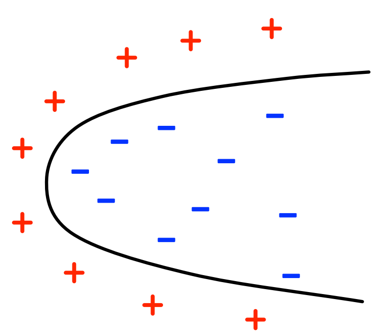
e.g. SVM, logistic regression.

Systematic deviation



What to do with this?

Adding new features



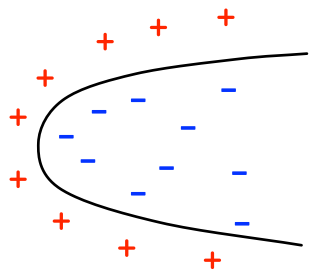
Actual boundary is something like $x_1 = x_2^2 + 5$.

- This is quadratic in $x = (x_1, x_2)$
- But it is linear in $\Phi(x) = (x_1, x_2, x_1^2, x_2^2, x_1 x_2)$

Basis expansion: embed data in a higher-dimensional feature space. Then we can use a linear classifier!

Basis expansion for quadratic boundaries

How to deal with a **quadratic** boundary?



Idea: augment the regular features $x = (x_1, x_2, \ldots, x_d)$ with

$$x_1^2,\ x_2^2,\ \ldots,\ x_d^2$$
$$x_1x_2,\ x_1x_3,\ \ldots,\ x_{d-1}x_d$$

Enhanced data vectors of the form:

$$\Phi(x) = (x_1, \ldots, x_d, x_1^2, \ldots, x_d^2, x_1x_2, \ldots, x_{d-1}x_d)$$

Quick question

Suppose $x = (x_1, x_2, x_3)$. What is the dimension of $\Phi(x)$?

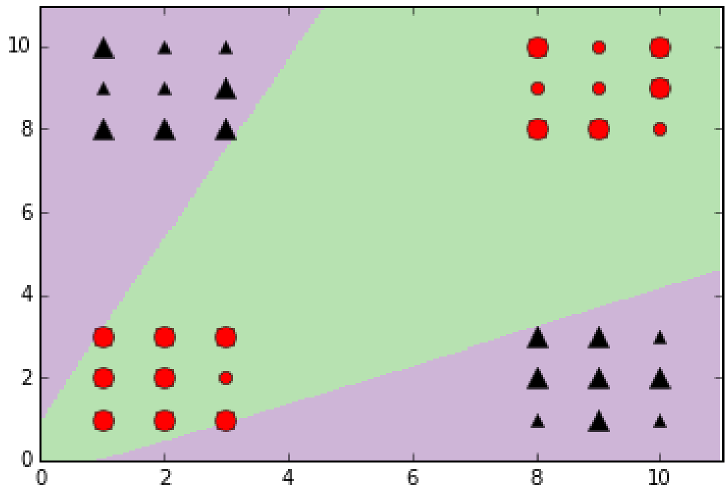
Suppose $x = (x_1, \ldots, x_d)$. What is the dimension of $\Phi(x)$?

Perceptron revisited

Learning in the higher-dimensional feature space:

- $w = 0$ and $b = 0$
- while some $y(w \cdot \Phi(x) + b) \leq 0$:
  - $w = w + y\,\Phi(x)$
  - $b = b + y$

Perceptron with basis expansion: examples



Perceptron with basis expansion: examples