Convexity II

Topics we'll cover

1. Second derivative test for convexity
2. Convexity examples

Second-derivative test for convexity

A function of several variables, $F(z)$, is convex if its second-derivative matrix $H(z)$ is positive semidefinite for all z.

More formally:
Suppose that for $f : \mathbb{R}^d \rightarrow \mathbb{R}$, the second partial derivatives exist everywhere and are continuous functions of z. Then:

1. $H(z)$ is a symmetric matrix
2. f is convex $\Leftrightarrow$ $H(z)$ is positive semidefinite for all $z \in \mathbb{R}^d$

Example

Is $f(x) = \|x\|^2$ convex?

Example

Fix any vector $u \in \mathbb{R}^d$. Is this function $f : \mathbb{R}^d \to \mathbb{R}$ convex?

$$f(z) = (u \cdot z)^2$$

Least-squares regression

Recall loss function: for data points $(x^{(i)}, y^{(i)}) \in \mathbb{R}^d \times \mathbb{R}$,

$$L(w) = \sum_{i=1}^{n} (y^{(i)} - (w \cdot x^{(i)}))^2$$