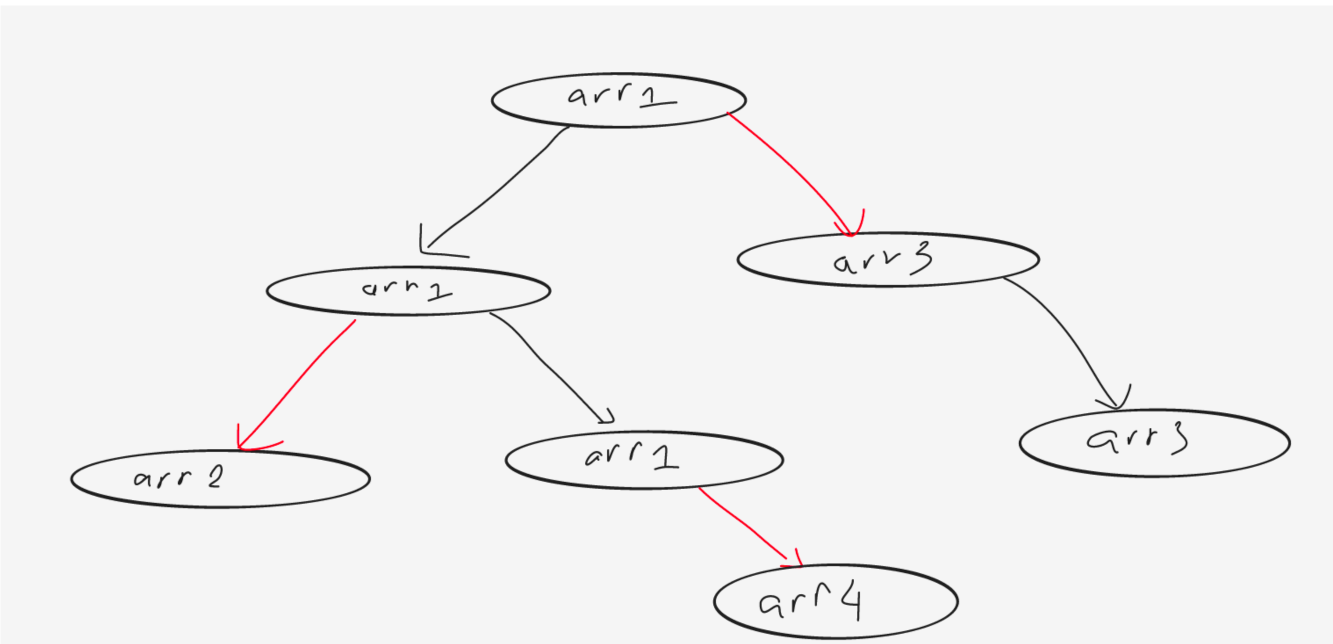
Abstract

You have to maintain a list of arrays. Initially, you have a single array $a_1, a_2, \ldots, a_n$ (array 1). You have to process three types of queries:

1. Change the value at index p of array k to x.
2. Return the sum in range $[a, b]$ of array k.
3. Create a copy of array k and append as the last array.

Tutorial



Lets check out the image above which is a illustrated tree of *applying queries* in this problem. Where

- Each node illustrate a version of a particular array.
- Each direct edge denotes as a query (red for query 3 and black for the others).

Therefore, for each queries we have to create new version of array. So data structure `persitent` is what we need to solve this problem.

For more meanings, `persistent` is a data structure that always preverse version itself when it modified (in this problem, it's applied query).

So how should we apply `persistent` in this problem?

Let's define a array has a number of version. The array 1 will have version 1. So when applying a query, a new version is created. And we will update the version corresponding to the array. More details,

- If the query is 1 or 2, the last version is the version of the current array k.
- Else, the last version is the version of new array.

Conclusion, we have to store the latest version of each array and for a query we need to create new version from the version of corresponding considered array.

The pseudo code as follow:

```
cur_ver = 2 # version 1 corresponds to array 1
ver = []
latest_ver = []
for q in Q:
    array_k, update_info = q
    update(q, latest_ver[array_k], cur_ver)
    latest_ver[array_k] = cur_ver
    cur_ver += 1
```

The complexity of above alogrithm is $O(np)$ which p the complexity of updating. We can use a popular data structure `Segment Tree` to implement the update function in $O(logn)$. But in this problem, it's a bit difficult that we have to combine 2 data structures above together.

Written by Tuan Huynh (11 - 9 - 2021)