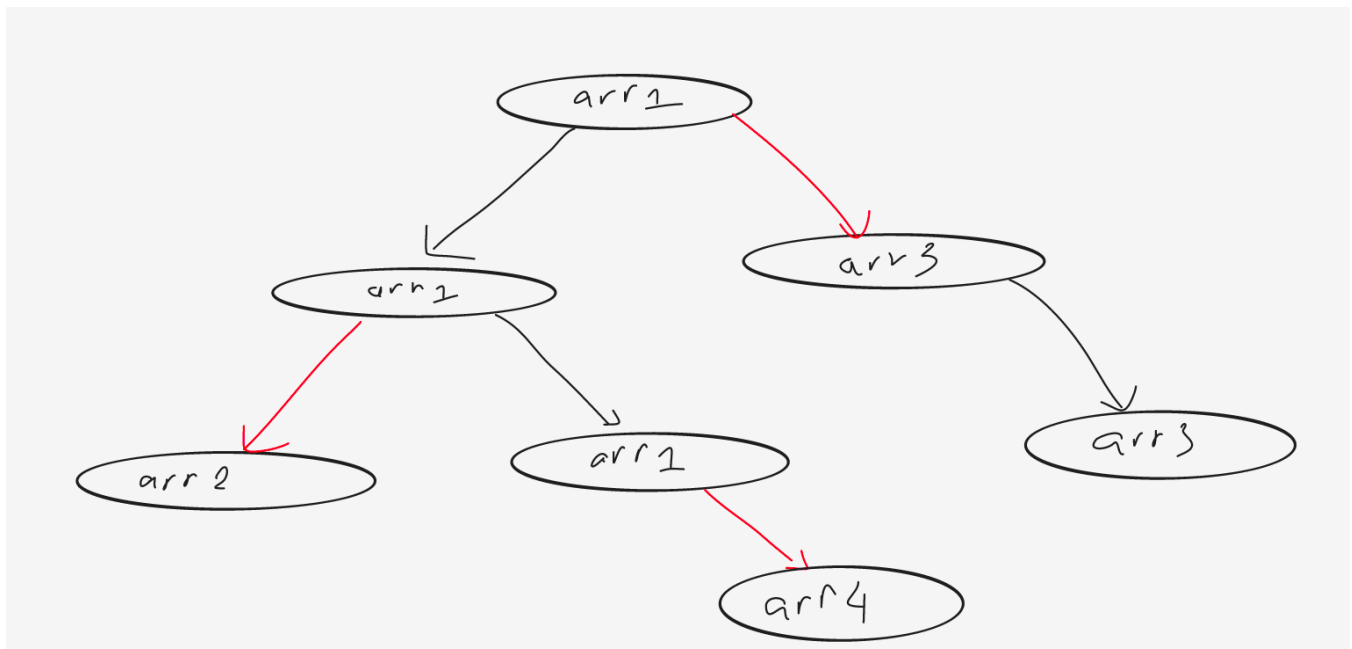


# Abstract

You have to maintain a list of arrays. Initially, you have a single array  $a_1, a_2, \dots, a_n$  (array 1). You have to process three types of queries:

1. Change the value at index  $p$  of array  $k$  to  $x$ .
2. Return the sum in range  $[a, b]$  of array  $k$ .
3. Create a copy of array  $k$  and append as the last array.

# Tutorial



Lets check out the image above which is a illustrated tree of *applying queries* in this problem. Where

- Each node illustrate a version of a particular array.
- Each direct edge denotes as a query (red for query 3 and black for the others).

Therefore, for each queries we have to create new version of array. So data structure **persistent** is what we need to solve this problem.

For more meanings, **persistent** is a data structure that always preverse version itself when it modified (in this problem, it's applied query).

So how should we apply `persistent` in this problem?

Let's define an array has a number of version. The array 1 will have version 1. So when applying a query, a new version is created. And we will update the version corresponding to the array. More details,

- If the query is 1 or 2, the last version is the version of the current array  $k$ .
- Else, the last version is the version of new array.

Conclusion, we have to store the latest version of each array and for a query we need to create new version from the version of corresponding considered array.

The pseudo code as follow:

```
cur_ver = 2 # version 1 corresponds to array 1
ver = []
latest_ver = []
for q in Q:
    array_k, update_info = q
    update(q, latest_ver[array_k], cur_ver)
    latest_ver[array_k] = cur_ver
    cur_ver += 1
```

The complexity of above algorithm is  $O(np)$  which  $p$  the complexity of the updating. We can use a popular data structure `Segment Tree` to implement the update function in  $O(\log n)$ . But in this problem, it's a bit difficult that we have to combine 2 data structure above together.

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