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Trajectory Tracking for a Group of Mini Rotorcraft Flying in Formation

J.A. Guerrero * **P. Castillo** * **Y. Challal** *

* *Heudiasyc UMR 6599 Université de Technologie de Compiégne, Centre de Recherches de Royallieu, 60200 France. (e-mail: {jguerrer, castillo, ychallal}@hds.utc.fr)*

Abstract:
In this paper, we study the problem of flight formation control and trajectory tracking control for a group of mini rotorcraft. The goal is to achieve and maintain a stable formation for multiple mini rotorcraft while guaranteeing the tracking of a desired time-varying reference. A two-level controller is developed where a individual control strategy guarantees the stabilization of each mini rotorcraft and a collective control strategy ensures a geometric formation and trajectory tracking. A nonlinear controller based on separated saturations is used as a individual controller to stabilize every mini rotorcraft. In order to achieve a collective behavior, i.e. formation and trajectory tracking, we adopt a leader/follower structure of multiple robot systems. The resulting system allow us to use the centroid of the coordinated control subsystem for trajectory tracking purposes. After a initial transient in which every mini rotorcraft will lock himself to the desired geometric formation, then the center of mass which will converge to the desired trajectory while keeping a stable formation. The analytic results are supported by simulation tests.

Keywords: Trajectory tracking, nonlinear control, Flight formation, UAVs

1. INTRODUCTION

The new developments powered by the technological revolution of the last century have spurred a broad interest in Unmanned Aerial Vehicles (UAVs). The explosion in computation and wireless communication capabilities as well as the advent of miniaturization technologies have increased the interest in replacing single complex systems by interacting multi-agent systems with interconnected structure. In this framework, new problems needs to be addressed such as consensus, coordination, formation control and tracking. The recent literature reports a rich collection of results during the last decades Chen and Serrani [2006]-Beard et al. [2001]. Some of the existing approaches for multiple UAV flying in formation and coordination of multiple autonomous robot systems include the leader/follower, virtual structure and behavioral-based control.

In the leader/follower architecture, one or more agents are designated as leaders driving the group behavior by generating commands while the other agents are designated as followers which should track the commands generated by the leaders as in Chen and Serrani [2006] and Kristiansen et al. [2006]. The virtual structure approach considers every agent as an element of a larger structure Leonard and Fiorelli [2001] and Beard et al. [2001]. Finally, the behavioral control in Arrichiello et al. [2006] and Balch and Arkin [1998] is based on the decomposition of the main control goal into tasks or behaviors. This approach also deals with collision avoidance, flock centering, obstacle avoidance and barycenter.

In a multi-vehicle system, information exchange among vehicles is needed for coordination and cooperation. To model the communication between vehicles, directed or undirected graphs have been used in Beard et al. [2001], Tanner et al. [2003a], and Olfati [2006] where every node in a graph is considered as an agent or robot which can have information exchange with all or several agents. By using this technique, several control strategies have been developed, e.g. Olfati [2006], Ren [2007], Lee and Li [2003], Lee and Spong [2006] and Hokayem et al. [2007]. In Olfati [2006], the authors present several algorithms for consensus and obstacle avoidance for multiple-agent systems. Lee and Li [2003] and Lee and Spong [2006] presents a passive decomposition approach for consensus and formation control.

The work reported in the literature is by now quite vast and addresses different approaches for mini rotorcraft stabilization Castillo et al. [2005], Bouabdallah et al. [2004], Lara et al. [2010] and Guerrero et al. [2010a] among others. In Castillo et al. [2005] a nonlinear control based on nested saturations is presented. In this approach, the dynamics is decoupled into lateral and longitudinal dynamical subsystems. Thus, nested saturations control was used to stabilize each subsystem. In Lara et al. [2010], the authors propose a robust linear PD controller considering parametric interval uncertainty. There, the authors present a robust stability analysis and compute the robustness margin of the system with respect to the parameters uncertainty. Recent approaches for trajectory tracking control for multi-agent systems have been reported in the literature Ren [2007], Hokayem et al. [2007], Porfiri et al. [2007]-Sun et al. [2009]. In Ren [2007] an algorithm for trajectory tracking of a time varying reference for a single integrator multi-agent system has been presented. In Hokayem et al.

[2007], the authors present a bilateral teleoperation control approach for the multi-agent trajectory tracking problem. A flight formation control based on a four integrators coordination control can be found in Guerrero et al. [2010a]. This approach addresses the problem of formation and tracking of a constant reference for the center of mass. A second approach to flight formation control can be found in Guerrero and Lozano [2010b] which is based on a forced consensus algorithm to achieve a multiple mini rotorcraft flight formation and tracking of a constant reference.

This work addresses the problem of trajectory tracking for a group of mini rotorcraft flying in formation based on separated saturations and a coordination control strategy. In this approach every mini rotorcraft is considered as an agent to be coordinated and to follow a time-varying reference. The proposed control scheme is based on the idea that lateral and longitudinal subsystems are decoupled which enable us to implement a decoupled coordination of the lateral and longitudinal subsystems. In this way, the multiple mini rotorcraft platoon can hover and thus following a desired time-varying reference while maintaining a stable formation.

This paper is organized as follows: in Section 2 the dynamical model of the proposed architecture is analyzed. In section 3, the trajectory tracking control algorithm is obtained and its validation in simulation is presented in section 4. Finally, the conclusions and future work are discussed in section 5.

2. DYNAMIC MODEL AND VEHICLE STABILIZATION

Since the purpose of this work is to develop a trajectory tracking for a group of miniature quadrotor flying in formation, let us consider the dynamical model introduced in Castillo et al. [2005]

$$\ddot{x} = -F_T\ \sin(\theta) \tag{1}$$
$$\ddot{y} = F_T\ \cos(\theta)\sin(\phi) \tag{2}$$
$$\begin{aligned}\ddot{z} &= F_T\ \cos(\theta)\cos(\phi) - 1\\ \ddot{\phi} &= \tilde{\tau}_\phi\\ \ddot{\theta} &= \tilde{\tau}_\theta\\ \ddot{\psi} &= \tilde{\tau}_\psi\end{aligned}$$

where F_T is the thrust force vector int he body system.

Remark that, there are other ways to represent the orientation of rigid bodies, e.g. quaternion; however for the purpose of this work Euler angles represent a simple and practical solution to be adopted.

Vehicle Stabilization

In order to propose a coordination non linear control strategy, it is necessary in a first time to stabilize the rotorcraft in hover flight. Hence, the following nonlinear control strategies are designed

$$F_T \triangleq \frac{-\sigma_{z_2}(k_{z_1}\dot{z}) - \sigma_{z_1}(k_{z_2}(z - z^d)) + 1}{\cos(\phi)\cos(\theta)} \tag{3}$$
$$\tilde{\tau}_\psi \triangleq -\sigma_{\psi_2}(k_{y_1}\dot{\psi}) - \sigma_{\psi_1}(k_{y_2}(\psi - \psi^d)) \tag{4}$$

where F_T and $\tilde{\tau}_\psi$ represent the altitude and the heading control inputs, k_{z_1}, k_{z_2}, k_{y_1} and k_{y_2} are positive constant, z^d and ψ^d are the desired values, and $\sigma_{\{z_i,\psi_i\}}$ is a saturation function. The control laws are based in saturation functions and obtained using the Lyapunov analysis. In addition, the amplitudes of the saturation functions can be chosen in such a way that $\cos\phi\cos\theta \neq 0$.

Observe that the altitude and the heading systems in closed-loop represent two integrators in cascade. In the follows, its stability analysis will be presented and by the recurrence theorem this result is used later to stabilize a chain of integrators in cascade, more details see Sanahuja [2010].

Let us consider the system

$$\begin{aligned}\dot{\chi}_1 &= \chi_2\\ \dot{\chi}_2 &= u_\chi\end{aligned} \tag{5}$$

Propose the following control input

$$u_\chi = -\sigma_{b_2}(\bar{k}_2\chi_2) - \sigma_{b_1}(\bar{k}_1\chi_1)$$

where $\bar{k}_i > 0$ is a constant and $|\sigma_{b_i}(\cdot)| \leq M_{b_i}$ is a saturation function.

Define the following positive function

$$V_1 = \frac{1}{2}\chi_2^2$$

then,

$$\dot{V}_1 = \chi_2\dot{\chi}_2 = -\chi_2\left[\sigma_{b_2}(\bar{k}_2\chi_2) + \sigma_{b_1}(\bar{k}_1\chi_1)\right]$$

Note that if $|\bar{k}_2\chi_2| > M_{b_1}$ then $\dot{V}_1 < 0$. This implies that $\exists T_1$ such that $\forall t > T_1$, $|\chi_2(t)| \leq M_{b_1}/\bar{k}_2$, choosing $M_{b_2} > M_{b_1}$, this yields

$$u_\chi = -\bar{k}_2\chi_2 - \sigma_{b_1}(\bar{k}_1\chi_1) \quad \forall\ t\ >\ T_1 \tag{6}$$

Define

$$\nu_\chi = \bar{k}_2\chi_1 + \chi_2$$

differentiating the above and using (6), it yields

$$\dot{\nu_\chi} = -\sigma_{b_1}\left[(\bar{k}_1/\bar{k}_2)\,(\nu_\chi - \chi_2)\right] \qquad \forall\ t > T_1$$

Define

$$V_2 = \frac{1}{2}\nu_\chi^2$$

thus,

$$\dot{V}_2 = \nu_\chi\dot{\nu_\chi} = -\nu_\chi\sigma_{b_1}\left[(\bar{k}_1/\bar{k}_2)\,(\nu_\chi - \chi_2)\right]$$

Note that, if $|\nu_\chi| > M_{b_1}/\bar{k}_2$, then $\dot{V}_2 < 0$. This implies that $\exists\ T_2 > T_1$ such that $\forall\ t > T_2$

$$|\nu_\chi| \leq M_{b_1}/\bar{k}_2 \tag{7}$$

If

$$\bar{k}_2^2 \geq 2\bar{k}_1 \tag{8}$$

then, $(\bar{k}_1/\bar{k}_2)\,|\nu_\chi - \chi_2| \leq M_{b_1}$.

Thus, from (6), (7) and (8) we obtain for $t > T_2$

$$u_\chi = -\bar{k}_1\chi_1 - \bar{k}_2\chi_2 = -K^T\bar{\chi} \tag{9}$$

where

$$K = \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \end{pmatrix} \quad \text{and} \quad \bar{\chi} = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

To prove the convergence to zero, we rewrite system (5) in the standard form

$$\dot{\bar{\chi}} = A\bar{\chi} + Bu_\chi \tag{10}$$

with

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Using (9) into (10), it follows that $\forall\, t > T_2$

$$\dot{\bar{\chi}} = (A - BK^T)\bar{\chi}.$$

Then, $\bar{k}_1$ and $\bar{k}_2$ need to be chosen such that the matrix $(A - BK^T)$ is stable and (8) is valid, more details see Sanahuja [2010].

Using the previous analysis, it follows that $\dot{z} \to 0$, $\dot{\psi} \to 0$, $z \to z^d$, and $\psi \to \psi^d$. Consequently, introducing (3) into (2) and (1) the lateral and longitudinal dynamics take

$$\begin{aligned} \ddot{y} &= \tan\phi \\ \ddot{\phi} &= \tilde{\tau}_\phi \\ \ddot{x} &= \frac{-\tan\theta}{\cos\phi} \\ \ddot{\theta} &= \tilde{\tau}_\theta \end{aligned}$$

It is assumed that, pitch and roll angle will be operated in a neighborhood of the origin, i.e., $|\phi, \theta| < \pi/10$. Moreover, the control approach will provide an upper bound for the attitude subsystem such that, $\tan\phi \approx \phi$. Then, the lateral dynamical system can be reduced to

$$\begin{aligned} \ddot{y} &\cong \phi \\ \ddot{\phi} &= \tilde{\tau}_\phi \end{aligned}$$

Notice that, the previous system represents four integrators in cascade. Using the previous control method and the recurrence theorem, the control strategy could be expressed as

$$\tilde{\tau}_\phi = \quad -\sigma_{\phi_4}(k_{r_4}\dot{\phi}) - \sigma_{\phi_3}(k_{r_3}\phi) - \sigma_{\phi_2}(k_{r_2}\dot{y}) - \sigma_{\phi_1}(k_{r_1}y) \tag{11}$$

Notice from the previous control analysis that, $\dot{\phi}, \phi, \dot{y}$ and $y \to 0$. Then, the longitudinal system could be also expressed like four integrators in cascade. Hence, we can propose

$$\tilde{\tau}_\theta = \quad -\sigma_{\theta_4}(k_{p_4}\dot{\theta}) - \sigma_{\theta_3}(k_{p_3}\theta) + \sigma_{\theta_2}(k_{p_2}\dot{x}) + \sigma_{\theta_1}(k_{p_1}x) \tag{12}$$

and also here, $\dot{\theta}, \theta, \dot{x}$ and $x \to 0$.

3. NONLINEAR CONTROL SCHEME

3.1 Consensus Agreement

One of the problems of working with multiple autonomous vehicles is the collision avoidance. A coordination strategy to ensure the formation and collision avoidance at the same time is here proposed. It should be noticed that a multi-agent approach ensures the flock centering as well as the collision avoidance among multi-agents. To develop this approach, we will start by analyzing the longitudinal kinematic model for the multi-quadrotor system which is given by

$$\dot{\mathbf{x}} = -\mathcal{L}\mathbf{x} \tag{13}$$

where $\mathcal{L}$ is the Laplacian matrix of the information exchange graph.

It is worth to mention that, dynamics (13) can also be written as $\dot{x}_i = \bar{u}_i \ \forall i = 1 : n$; with multiple agent consensus achieved using the following forced consensus algorithm $\bar{u}_i = -\sum_{j\in\mathcal{N}_i}(x_i - x_j)$ where $\mathcal{N}_i$ is the set of vehicles transmitting their information to the vehicle i. Notice that the above analysis is also used to the lateral dynamic.

Observe that, when using (11) and (12) all the states goes to the origin. And since, the control objective is to force the consensus of a set of quad-rotor vehicles to a desired position on the x and y-axis, we propose the following change of variables

$$x \triangleq \Sigma_{j\in\mathcal{N}_i}(x_j - x_i) \tag{14}$$

$$y \triangleq \Sigma_{j\in\mathcal{N}_i}(y_j - y_i) \tag{15}$$

where x_i, y_i, x_j and y_j represent the position of the i-th quad-rotor and the j-th quad-rotor to be coordinated.

Remark 1. On one hand a multiple mini rotorcraft consensus can be achieved by means of a single integrator consensus algorithm, then, (14) - (15) provide a simple way to solve the coordination problem. On the other hand, we may think of the neighbors position of a mini rotorcraft as the position reference and thus the stability of every mini rotorcraft is guarantied using the nonlinear control based on saturations.

From the previous control analysis, we have that x and $y \to 0$, and from (14) and (15), this implies that all $x_j \to x_i$, and similarly, $y_j \to y_i$. Therefore, the control laws $\tilde{\tau}_\theta$ and $\tilde{\tau}_\phi$ for the longitudinal and lateral subsystems of the i^{th}-minirotorcraft becomes

$$\begin{aligned} \tilde{\tau}_{\theta_i} =& -\sigma_{M_{\theta_4}}(k_{p_4}\dot{\theta}_i) - \sigma_{M_{\theta_3}}(k_{p_3}\theta_i) + \sigma_{M_{\theta_2}}(k_{p_2}\dot{x}_i) \\ &+\sigma_{M_{\theta_1}}(k_{p_1}\Sigma_{j\in\mathcal{N}_i}(x_j - x_i)) \end{aligned} \tag{16}$$

$$\begin{aligned} \tilde{\tau}_{\phi_i} =& -\sigma_{M_{\phi_4}}(k_{r_4}\dot{\phi}_i) - \sigma_{M_{\phi_3}}(k_{r_3}\phi_i) - \sigma_{M_{\phi_2}}(k_{r_2}\dot{y}_i) \\ &-\sigma_{M_{\phi_1}}(k_{r_1}\Sigma_{j\in\mathcal{N}_i}(y_j - y_i)) \end{aligned} \tag{17}$$

Notice that (16) and (17) cannot be used in practice due to the fact that a coordination to a fixed position implies that every mini rotorcraft will converge to the same position in the 3D space producing the collision of all mini rotorcrafts. In order to solve this problem, a simple leader relative position control is developed in the next section.

3.2 Formation Control

In this section, we propose a leader-relative position consensus (UAV formation) for the multi quadrotor system, i.e. the quadrotor vehicles will converge to a position with respect to the leader of the group. In this case, the following geometric formations are proposed, see Figure 1.

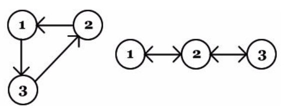

Fig. 1. Cyclic and Chain topologies of information exchange

Triangular Formation A triangular formation around a circle of radius r for the team of three quadrotor vehicles is proposed, see Figure 2. Assuming a cyclic information

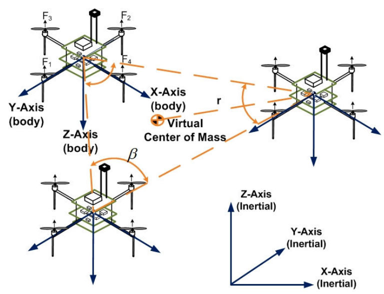

Fig. 2. Multiple mini rotorcraft flying in formation

exchange topology, the relative position is given by

$$x_1 - x_2 = r\cos(\pi/6); \quad y_1 - y_2 = r\sin(\pi/6) \qquad (18)$$
$$x_3 - x_1 = -r\cos(\pi/6); \quad y_3 - y_1 = -r\sin(\pi/6) \qquad (19)$$
$$x_2 - x_3 = r\cos(\pi/2); \quad y_2 - y_3 = 2r\sin(\pi/6) \qquad (20)$$

Assuming a chain information exchange topology, the relative position is given by

$$x_1 - x_2 = \cos(\pi/6); \quad y_1 - y_2 = \sin(\pi/6) \qquad (21)$$
$$x_2 - x_3 = \cos(\pi/2); \quad y_2 - y_3 = 2\sin(\pi/6) \qquad (22)$$

Therefore, we can use either (18)-(20) or (21)-(22) as a relative position reference x_i^d and y_i^d with respect to each other.

Using a relative position reference for the flight formation of multiple mini rotorcraft, equations (16) and (17) are rewritten as

$$\begin{aligned}\tilde{\tau}_{\theta_i} &= -\sigma_{M_{\theta_4}}(k_{p_4}\dot{\theta}_i) - \sigma_{M_{\theta_3}}(k_{p_3}\theta_i) + \sigma_{M_{\theta_2}}(k_{p_2}\dot{x}_i)\\ &\quad + \sigma_{M_{\theta_1}}\left(k_{p_1}\left(\Sigma_{j\in\mathcal{N}_i}(x_j - x_i) - x_i^d\right)\right) \qquad (23)\\ \tilde{\tau}_{\phi_i} &= -\sigma_{M_{\phi_4}}(k_{r_4}\dot{\phi}_i) - \sigma_{M_{\phi_3}}(k_{r_3}\phi_i) - \sigma_{M_{\phi_2}}(k_{r_2}\dot{y}_i)\\ &\quad - \sigma_{M_{\phi_1}}\left(k_{r_1}\left(\Sigma_{j\in\mathcal{N}_i}(y_j - y_i) - y_i^d\right)\right) \qquad (24)\end{aligned}$$

where x_i^d and y_i^d are the desired geometrical position reference with respect to the leader as shown in previous section. Thus, (23) and (24) are such that, the geometric flight formation of the multiple mini rotorcraft system is guarantied.

3.3 X4 Trajectory Tracking Control

Now, we will consider the case of trajectory tracking of a multiple vehicle system. It is assumed that, the leader of the group is always vehicle 1, see Figure 2. Then, (13) can be rewritten as

$$\dot{\mathbf{x}} = -\mathcal{L}\mathbf{x} + \mathbf{b}u_{1_x} \qquad (25)$$

where $\mathbf{b}^T = [1\ 0\ \ldots\ 0]$ and u_{1_x} is the input given to the leader. Define,

$$x_{CM} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

where N is the number of agents in the formation. Let x_{CM}^d be the desired value for x_{CM}. Thus, u_{1_x} can be stated as

$$u_{1_x} = Nk\ \sigma_{M_{CM}}(x_{CM}^d - x_{CM}) \qquad (26)$$

where $\sigma_{M_{CM}}(\cdot)$ represents the saturation function and k is a positive gain. Note that, x_{CM} may not be directly measurable for the leader (vehicle 1). Notice that, for a cyclic topology of information exchange, the system is observable from the input and output of the leader. The state can therefore be observed from the input and output of vehicle 1. Introducing (26) into (25), it is follows that

$$\begin{aligned}\dot{x}_{CM} &= k\ \sigma_{M_{CM}}\left(x_{CM}^d - x_{CM}\right)\\ v_i^T\dot{\mathbf{x}} &= -\lambda_i(v_i^T\mathbf{x}) + v_i^T\mathbf{b}u_{1_x} \qquad \forall\ i = 2:N\end{aligned}$$

All the modes in the above equation are stable. When $u_{1_x} = 0$, these modes converge to zero which means that, $(x_i - x_j) \to 0$ for $i \neq j$. This property is obtained by using the coordinating control algorithm that leads the position dynamics to (25). These modes are uncontrollable when $v_i^T\mathbf{b} = 0$. In addition, there is a trade-off in the choice of gain k in (26). For smaller values of k, the speed of convergence of x_{CM} is slower, but the transient in the errors $(x_i - x_j)$ for $i \neq j$, will be smaller, see Lozano [2008]. For the lateral dynamic the previous analysis is also used.

Then, the trajectory tracking control for the leader of the group is given by

$$\begin{aligned}\tilde{\tau}_{\theta_1} &= -\sigma_{M_{\theta_4}}(k_{p_4}\dot{\theta}_i) - \sigma_{M_{\theta_3}}(k_{p_3}\theta_i) + \sigma_{M_{\theta_2}}(k_{p_2}\dot{x}_i)\\ &\quad + \sigma_{M_{\theta_1}}\left(k_{p_1}\left(\Sigma_{j\in\mathcal{N}_i}(x_j - x_i) - x_i^d - u_{1_x}\right)\right) \qquad (27)\\ \tilde{\tau}_{\phi_1} &= -\sigma_{M_{\phi_4}}(k_{r_4}\dot{\phi}_i) - \sigma_{M_{\phi_3}}(k_{r_3}\phi_i) - \sigma_{M_{\phi_2}}(k_{r_2}\dot{y}_i)\\ &\quad - \sigma_{M_{\phi_1}}\left(k_{r_1}\left(\Sigma_{j\in\mathcal{N}_i}(y_j - y_i) - y_i^d - u_{1_y}\right)\right) \qquad (28)\end{aligned}$$

Remark 2. Once the geometric formation has been achieved, the proposed control (27) and (28) guarantees the collision avoidance among quadrotors.

3.4 Time-varying reference tracking

It has been shown in previous sections that control law (26) ensures the convergence of the center of mass of a multi-vehicle system to a constant reference given to the leader. However, when the reference is varying in time there is a small bias in agents coordination.

In this section we consider the case of multi-agent trajectory tracking of a time varying reference. We will prove that a multi-vehicle system converge to the position time varying reference given only to the leader. Again, we are interested in the chain and cyclic topologies of information exchange. Let us consider the double integrator multi-vehicle system of the form

$$\ddot{x}_i = u_i$$

We define a change of variable

$$\xi_i \triangleq \dot{x}_i + \kappa x_i \tag{29}$$
$$\xi^d_{CM} \triangleq \dot{x}^d_{CM} + \kappa x^d_{CM}$$

where κ is a positive constant. Control input u_i is defined as

$$u_i \triangleq \bar{u}_i - \kappa \dot{x}_i \tag{30}$$

Differentiating (29) and using control (30), we obtain

$$\dot{\xi}_i = \bar{u}_i$$

Define the coordinating control $\bar{u}_i$ as

$$\bar{u}_i \triangleq -\Sigma_{j\in\mathcal{N}_i}(\xi_i - \xi_j) + b_i\tilde{u}_i$$

Thus, the following multi-agent system is obtained

$$\dot{\xi} = -\mathcal{L}\xi + b\tilde{u} \tag{31}$$

where $\mathcal{L}$ is the Laplacian matrix and the control law

$$\tilde{u}_l \triangleq \tilde{u}_{CM} \tag{32}$$
$$\tilde{u}_i \triangleq \dot{\xi}_j \text{ for some } j \in \mathcal{N}_i \tag{33}$$

where u_l is the input given to the leader, u_i is the input given to the i^{th} vehicle and $\tilde{u}_{CM} = k_{CM}\sigma_{M_{CM}}(\xi_{CM} - \xi^d_{CM})$.

3.5 Chain Topology

Let us consider the case of three agents with chain topology of information exchange with agent 1 acting as the leader of the group

$$\dot{\xi}_1 = (\xi_2 - \xi_1) + \tilde{u}_{CM}$$
$$\dot{\xi}_2 = (\xi_1 - \xi_2) + (\xi_3 - \xi_2) + \dot{\xi}_1 \tag{34}$$
$$\dot{\xi}_3 = (\xi_2 - \xi_3) + \dot{\xi}_2 \tag{35}$$

this system can also be represented as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \tilde{u}_{CM} \\ \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} \tag{36}$$

Rewriting (35)

$$(\dot{\xi}_3 - \dot{\xi}_2) = -(\xi_3 - \xi_2)$$

then, from the above, this implies that $(\xi_3 - \xi_2) \to 0$.

From (34) it follows

$$(\dot{\xi}_2 - \dot{\xi}_1) = (\xi_1 - \xi_2) + (\xi_3 - \xi_2) \tag{37}$$

since $(\xi_3-\xi_2) \to 0$, then (37) reduces to $(\dot{\xi}_2-\dot{\xi}_1) = -(\xi_2 - \xi_1)$ which also implies that $(\xi_2 - \xi_1) \to 0$.

Premultiplying (36) by **1** eigenvector, we get

$$\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 = \tilde{u}_{CM} + \dot{\xi}_1 + \dot{\xi}_2$$
$$\dot{\xi}_3 = \tilde{u}_{CM} \tag{38}$$

Now, define

$$\tilde{u}_{CM} \triangleq -(\xi_{CM} - \xi^d_{CM}) + \dot{\xi}^d_{CM} \tag{39}$$

then, introducing (39) into (38), it follows that $(\dot{\xi}_3 - \dot{\xi}^d_{CM}) = -(\xi_{CM}-\xi^d_{CM})$ which implies that $(\xi_3-\xi^d_{CM}) \to 0$.

Assume that, $\xi^d_{CM} = x^d_{CM}$ and $\dot{x}^d_{CM} = 0$, then $(\xi_3 - x^d_{CM}) \to 0$ implies that $(x_3 - x^d_{CM}) \to 0$. Notice $(x_i - x_j) \to 0$ and $(x_3 - x^d_{CM}) \to 0$ then $(x_i - x^d_{CM}) \to 0$ for all $i = 1, 2, 3$.

Lemma 1. Consider a multi-agent system with the form (31) and control law (32)-(33), then, $\dot{x}_{CM} \to \dot{x}^d_{CM}$ as $t \to \infty$, moreover $(\dot{x}_i - \dot{x}^d_{CM}) \to 0$.

3.6 Cyclic Topology

Let us consider the following equations that represent the case of three agents with chain topology of information exchange and agent 1 acting as the leader of the group

$$\dot{\xi}_1 = (\xi_2 - \xi_1) + \tilde{u}_{CM}$$
$$\dot{\xi}_2 = (\xi_3 - \xi_2) + \dot{\xi}_3 \tag{40}$$
$$\dot{\xi}_3 = (\xi_1 - \xi_3) + \dot{\xi}_1 \tag{41}$$

or

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \tilde{u}_{CM} \\ \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} \tag{42}$$

From (41)

$$(\dot{\xi}_3 - \dot{\xi}_1) = -(\xi_3 - \xi_1)$$

which implies that $(\xi_3 - \xi_1) \to 0$.

Rewriting (40)

$$(\dot{\xi}_3 - \dot{\xi}_2) = -(\xi_3 - \xi_2)$$

then, $(\xi_3 - \xi_2) \to 0$.

Premultiplying (42) by **1** eigenvector, we get

$$\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 = \tilde{u}_{CM} + \dot{\xi}_1 + \dot{\xi}_3$$
$$\dot{\xi}_2 = \tilde{u}_{CM} \tag{43}$$

Now, define

$$\tilde{u}_{CM} \triangleq -(\xi_{CM} - \xi^d_{CM}) + \dot{\xi}^d_{CM} \tag{44}$$

thus, introducing (44) into (43) it follows that

$$(\dot{\xi}_2 - \dot{\xi}^d_{CM}) = -(\xi_{CM} - \xi^d_{CM})$$

which implies that $(\xi_2 - \xi^d_{CM}) \to 0$. Assume that $\xi^d_{CM} = x^d_{CM}$ and $\dot{x}^d_{CM} = 0$, then $(\xi_2-x^d_{CM}) \to 0$ implies that $(x_2-x^d_{CM}) \to 0$. Due that $(x_i - x_j) \to 0$ and $(x_2 - x^d_{CM}) \to 0$ then $(x_i - x^d_{CM}) \to 0$ for all $i = 1, 2, 3$.

Lemma 2. Consider a multi-agent system of the form (31) with coordinating control law (32)-(33). Then $\dot{x}_{CM} \to \dot{x}^d_{CM}$ as $t \to \infty$, moreover $(\dot{x}_i - \dot{x}^d_{CM}) \to 0$.

4. SIMULATION RESULTS

To illustrate the proposed methodology, this section presents the simulation results concerning the tracking of a time-varying reference for a multiple mini quadrotor platton flying in formation. We consider three mini quadrotors evolving in the 3D space. Extensive simulations were run on a platoon of three rotorcrafts, with cyclic topology of information exchange, considering the 6-DOF nonlinear dynamical model. Initially, the platoon should converge to a triangular formation with center of mass at the $(0,0)$ position in the 2D plane. Once the platoon has converged to the triangular formation, the order to displace along the x-axis is given. After a short time, the constant reference is changed to a trigonometric function sinus while maintaining the heading in the same direction all the time. Finally, the reference is changed again to a constant reference. The initial conditions for inertial position and velocity are:

$$[x_1, y_1, z_1] = [0, 0.1, 0]\text{m}; \quad [\dot{x}_1, \dot{y}_1, \dot{z}_1] = [0, 0, 0]\text{m/s}$$
$$[x_2, y_2, z_2] = [-2, 0.5, 0]\text{m}; \quad [\dot{x}_2, \dot{y}_2, \dot{z}_2] = [0, 0, 0]\text{m/s}$$
$$[x_3, y_3, z_3] = [-2, -0.5, 0]\text{m}; \quad [\dot{x}_3, \dot{y}_3, \dot{z}_3] = [0, 0, 0]\text{m/s}$$

The simulation results show that the proposed nonlinear control strategy can be used to achieve a geometric formation as well as tracking a time-varying reference for a group of multiple mini rotorcrafts. Thus, using control inputs (23), (24), (3) and (4) on the mini rotorcraft acting as followers and (27), (28), (3) and (4) on the mini rotorcraft acting as leader, on the 6-DOF nonlinear dynamical model in simulation, we get the result shown in Figure 3.

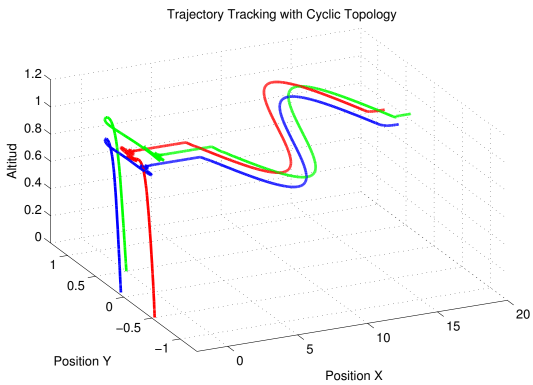

Fig. 3. Trajectory tracking (3D view)

5. CONCLUSIONS AND FUTURE WORK

A nonlinear control based on separated saturations and a forced consensus control to track a time-varying reference for flight formation of mini rotorcraft was developed. The x-position and the y-position of each mini rotorcraft were considered as dynamical agents with full information access. Trajectory tracking for the group of mini rotorcraft was achieved by using the virtual center of mass of the agents formation. Extensive simulations were run in order to show the performance of the developed control scheme. Future work in this area includes experimental tests on mini rotorcraft using real-time embedded control systems.

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