MACHINE LEARNING PHASE TRANSITIONS OF THE ONE-DIMENSIONAL ISING MODEL

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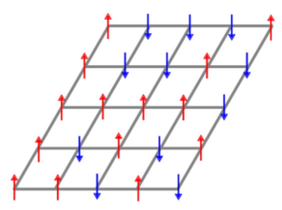
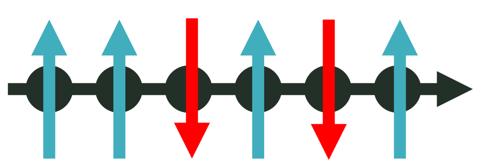
MOTIVATION

2D and 1D Ising Model



Two-Dimensional Ising Model

$T_c \neq 0$

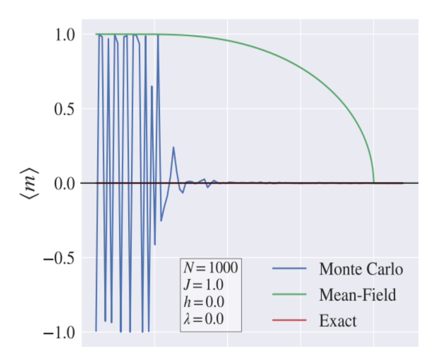


One-Dimensional Ising Model

$T_c = ?$

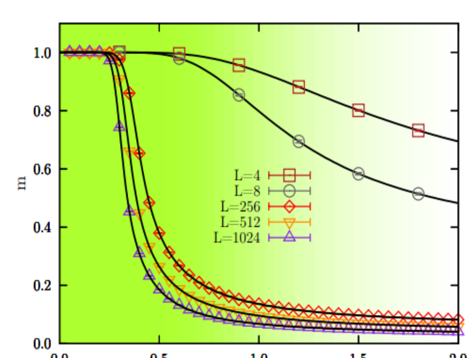
1D Ising Model

Alice Heiman thesis, 2021



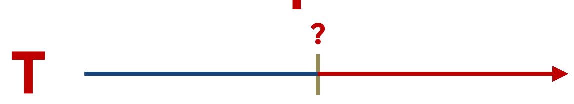
$T_c = 0$

Ferreira et al. 2021



$T_c \neq 0$

Open question: 1D Ising model with short-range exchange interactions exhibits a second-order phase transition at a finite temperature?



ACTIVITIES

METHOD: MONTE CARLO AND MACHINE LEARNING

Hamiltonian

$$\mathcal{H} = -J\sum_{i=1}^{N-1} s_i s_{i+1}$$

- J: Exchange interaction between ith and $(i+1)$th spin ($J = 1$).
- $s = \pm 1$ (spin up or down).
- $s_i s_{i+1}$: nearest neighbour.

Monte Carlo simulation

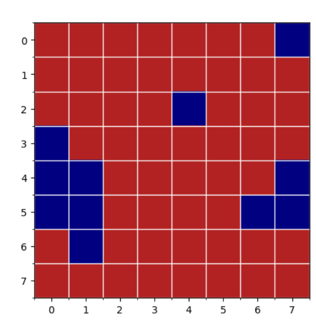
N lattice sites

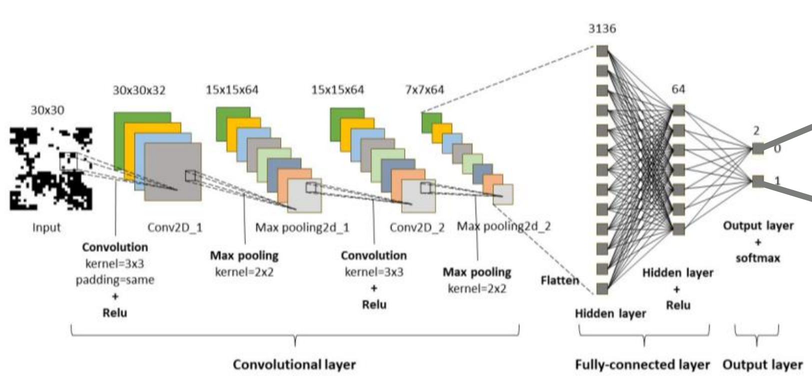
↓

The state space is of size 2^N

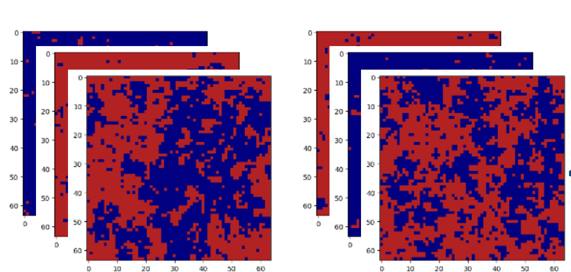
[-1 -1 -1 -1 -1 -1 -1 1]
[-1 -1 -1 -1 -1 -1 -1 -1]
[-1 -1 -1 -1 1 -1 -1 -1]
[1 -1 -1 -1 -1 -1 -1 -1]
[1 1 -1 -1 -1 -1 -1 1]
[1 1 -1 -1 -1 -1 1 1]
[-1 1 -1 -1 -1 -1 -1 -1]
[-1 -1 -1 -1 -1 -1 -1 -1]

Or



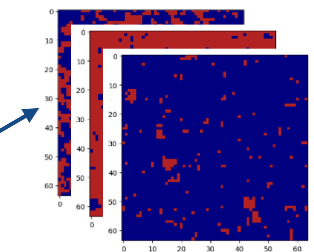


Convolutional Neural Network

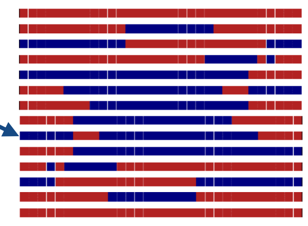


TRAIN 2D

VALID 2D



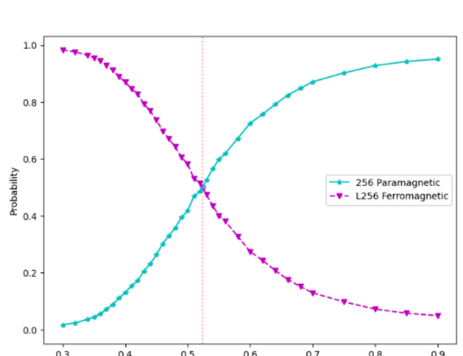
TEST ON 2D



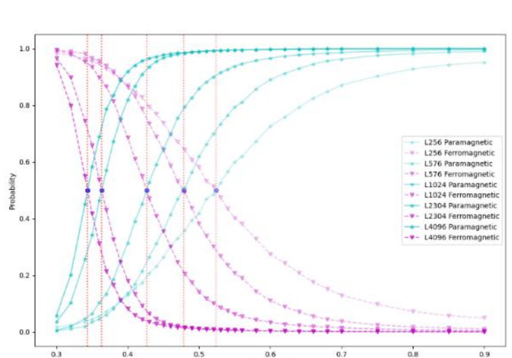
TEST ON 1D

RESULTS AND IMPACT

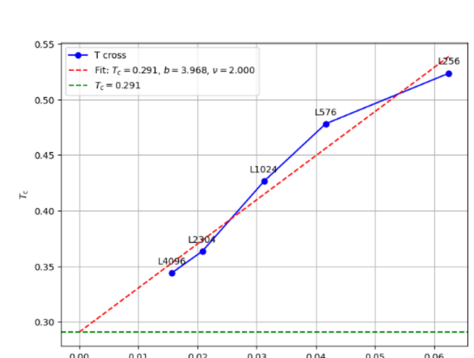
1D Ising Model



Output layer for L = 256



Output layer for L = 256, 576, 1024, 2304 and 4096



Finite-size scaling

Conclusion:
- 1D Ising Model exist a phase transition temperature
- $T_c = 0.291$ and $\nu = 2.0$

Reference: 1. Ariyadi M R N, Pribadi M R and Widiyanto E P 2023 2. Lucas Souza Ferreira *et al.* 2021

November 2024