MATH 45 **REVIEW EXAM FINAL** **SPRING 2021**

Q1. Find the critical points , phase portrait of the given autonomous first order differential equation . Classify each critical point as asymptotically stable , unstable or semi stable . By hand sketch the typical solution curves in the region in the x y plane determined by the graphs of the equilibrium solutions :

$Y' = y (y - 1) (y - 3)$

Q2. Solve the linear differential equation and find if there are any transient terms in general solution :

a) $dy / dx - 3y = 6$ Ans : $y = -2 + c\, e^{3x}$,

b) $x y' + y = e^{x}$, $y(1) = 2$ Ans : $y = \frac{1}{x}e^{x} + \frac{2-e}{x}$

c) $(x+1) dy/dx + (x + 2) y = 2x\, e^{-x}$ Ans : $y = \frac{x^2}{x+1}\, e^{-x} + \frac{c}{x+1}e^{-x}$, entire solution is transient.

Q3. $Y = c_1 e^{x} + c_2 e^{-x}$ is a two parameter family of solutions of second order DE $y'' - y = 0$ .Find the solution of the second order IVP consisting of this differential equation and the given initial conditions.

a) $Y(1) = 0$, $y'(1) = e$ Ans : $y = \frac{e^x}{2} - \frac{e^{2-x}}{2}$

b) $Y(0) = 0$, $y'(0) = 0$ Ans : $y = 0$

Q4. Solve exact differential equations :

a) $(5x + 4y) dx + (4x - 8 y^3)dy = 0$ Ans : $c = \frac{5x^2}{2} + 4 xy - 2 y^4$

b) $(5y- 2x) y' -2y = 0$ Ans : $c = -2xy + \frac{5y^2}{2}$

c) $(2 y^2 + 3x) dx + 2xy\, dy = 0$ Ans : $x^2 y^2 + x^3 = c$

Q5. Solve the given differential equation by appropriate substitution :

a) $dy / dx = (x + 3y) / (3x + y)$ Ans : $(y - x)^2 = c(y + x)$

b) $dy/dx = \sin (x+y)$ Ans : $\tan (x+ y) - \sec (x + y) = x + c$

c) $dy/ dx = (1 - x - y) / (x + y)$ Ans : $(x+ y)^2 = 2x + c$

Q6.The indicated function $y_1 (x)$ is a solution of the given differential equation .Use the reduction of order to find the second solution .

a) $y'' + 2y' + y = 0$; $y_1 = x\, e^{-x}$ Ans : $y_2 = e^{-x}$

b) $y'' + 16 y = 0$; $y_1 = \cos 4x$ Ans : $y_2 = \sin 4x$

Q7.Find the general solution of the given differential equation:

a) $Y^{(4)}-4y'+y=0$, $y(1)=0$, $y'(1)=2$ Ans : $y = c_1e^x+c_{2x}e^x+c_3e^{-x}+c_4xe^{-x}$

b) $Y'' -4y'-5y = 0$, $y(1)=0$, $y'(1)=2$ Ans : $y=\frac{e^{1-t}}{3} + \frac{e^{5t-5}}{3}$

Q8. Solve by superposition approach $y'' + y = 4x + 10 \sin x$, $y(\pi) = 0$, , $y'(\pi) = 2$

Ans: $y = 9\pi \cos x + 7 \sin x + 4x - 5x \cos x$

Q9. Solve (using Annihilator approach) $y'' - 3y' = 8 e^{3x} + 4 \sin x$

Ans : $y = c_1 + c_2 e^{3x} + \frac{8}{3} x e^{3x} + \frac{6}{5} \cos x - \frac{2}{5} \sin x$

Q10. Solve (using Variation of Parameters) $y'' + y = \sin x$

Ans : $y = c_1 \cos x + c_2 \sin x - \frac{1}{2} x \cos x$

Q11 . Solve $y''+y = \sqrt{2} \sin\sqrt{2}t$, $y(0)=10$, $y'(0)=0$ Ans: $10\cos t+2\sin t-\sqrt{2}\sin\sqrt{2}t$

Q12. Find $L^{-1}\left[\frac{-2s+6}{s^2+4} \right]$ Ans : $-2\cos 2t + 3 \sin 2t$

Q13. Evaluate $L^{-1}\{\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\}$ Ans: $-\frac{16}{5} e^{t} + \frac{25}{6} e^{2t} + \frac{1}{30} e^{-4t}$

Q14.Solve the given differential equation by using Laplace transform:

$Y'' - 6y' + 13y = 0$ $y(0) = 0$, $y'(0) = -3$ Ans : $(-3 e^{3t} \sin 2t)/2$

Q15. Solve using Convolution theorem $L^{-1}\{ \frac{1}{s(s-a)^2} \}$ Ans : $(ate^{at} - e^{at} + 1)/a^2$

Q16. Evaluate the inverse transform

$L^{-1}\left[\frac{1}{s(s-1)}\right]$ Ans: e^t-1