Master Project on MCSS
Progress Report 1

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1 Introduction

We consider the collective action of a bee colony. Each bee in a colony could possibly sting after observing a threat in the surrounding environment, and warn other bees by releasing pheromone. By sensing the pheromone released in the environment, other bees in the colony may also sting. Since stinging leads to the termination of an individual bee, it reduces the total defense capability as well. We studies how the actions of a bee changes with regarding to its surrounding the environment.
In this progress report, the following points are being presented:

1. Description of the system
2. Probability distribution of the number of stinging bees
3. Markov chain formalization
4. Parameter inference.

2 Formal description

There are 3 assumptions on the system:

1. Each bee release an unit amount of pheromone immediately after stinging.

2. A bee dies after stinging and releasing pheromone. In the other words, no bee can sting more than once.

3. Stinging behaviour only depends on the concentration of pheromone in the environment.

Formally, the system we are modeling is a tuple (N, ϕ, A)

1. $N \in \mathbb{N}$ is the number of bees.

2. ϕ is the concentration of pheromone in the environment.

3. A is the aggressiveness of each individual bee.

4. is the stinging probability of each bee

Steady state
The question we are concerning about is that, given a system, how many bees would sting in the steady state. Formally Y is a discrete random variable indicates the number of dead bees at steady state, we want to find the probability distribution of Y
We consider *synchronicity*

1. Asynchronous

2. Semi-synchronous

Given n bees in an isolated box. After stinging, each bee release unit Δ amount of pheromone and then dies. We denote that

- $s(i)$ is the probability of a bee sting at a pheromone level $i\Delta(i \in \mathbb{N})$

2.1 Fully asynchronous experiment

The asynchronous update is to model the fact that there is almost improbable for two bees to sting at exactly the same time, and any bees release pheromone immediately after its death. By that observation we can assume that each bee sting at different level of pheromone.

Claim: In *asynchronous model*, the probability of seeing dead bees at the steady state is

$$P(Y = j) = \binom{n}{j} s(0)s(1)\ldots s(j-1)(1 - s(j-))^{n-j} \tag{1}$$

However, the claim does not hold under synchronous update. To disproof the claim under the asynchronous assumption, we first show a counter example. For simplicity we assume $n = 2, s(0) = 0.4, s(1) = 0.5$. By (1) it follows that

$$\begin{aligned}
P(Y = 0) &= \binom{2}{0}(1 - s(0))^2 = 0.36 \\
P(Y = 1) &= \binom{2}{1}s(0)(1 - s(1)) = 0.4 \\
P(Y = 2) &= \binom{2}{2}s(0)s(1)(1 - s(2))^0 = \binom{2}{2}s(0)s(1) = 0.2
\end{aligned}$$

We obtain that $P(Y = 0) + P(Y = 1) + P(Y = 2) = 0.96 \neq 1$, thus the claim is incorrect. It is due to the fact that $P(Y = 1)$ is incorrectly counted.

2.2 Fully synchronous experiment

Synchronous assumes that the pheromone diffuse almost immediately in among the bee colony, and each bee decide to sting or not to sting immediately after sensing the pheromone concentration. Thus The state of each bee.

3 System modeling using Markov Chain

3.0.1 Discrete time Markov Chain

A *Discrete Time Markov Chain* is a tuple, where

- a
- b

Markov property

3.0.2 State spaces

3.0.3 Transition probabilities

3.0.4

4 Parameterized modeling and Bayesian inference

4.1 Parameterized DTMC

4.2 Statistical parameter inference

4.2.1 Definition

Statistical method for parameter inference is presented in [1] Input: distribution on steady state of number of dead bees Output:

4.2.2 Method description

Observing

4.3 Bayesian parameter inference

4.3.1 Problem declaration

Prior distribution
Posterior distribution
Marginal distribution

4.3.2 Method description

References

[1] Matej Hajnal et al. “Data-Informed Parameter Synthesis for Population Markov Chains”. In: *International Workshop on Hybrid Systems Biology.* Springer. 2019, pp. 147–164.