

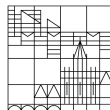
Bayesian parameter synthesis for markov population model.

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Introduction.

Preliminaries

Parameter synthesis

Katoen [1] definition

Definition (Discrete-time Markov Chain)

Parameter synthesis

Katoen [1] definition

Definition (PCTL)

Model checking PCTL

Katoen [1] definition

Definition (PCTL)

Model checking PCTL

Katoen [1] definition

Definition (PCTL)

Parameter synthesis

Katoen [1] definition

Definition (Parametric Discrete-time Markov Chain)

Parameter synthesis

Katoen [1] definition

Definition (Parameter synthesis)

Given a finite-state parametric Markov model, find the parameter values for which a given reachability property exceeds (or is below) a given threshold β .

Parameter synthesis and Parameter inference

What are the differences between parameter inference and parameter synthesis?

	Input	Ouput
Parameter inference	Model \mathcal{M}_θ Observed data D_{obs}	Parameter estimation $\hat{\theta}$
Parameter synthesis	Model \mathcal{M}_θ Reachability property Φ	$(\theta_1, \dots, \theta_N)$ $\forall \theta_i \in (\theta_1, \dots, \theta_N) : \mathcal{M}_{\theta_i} \models \Phi$

Synthesis: input: model \mathcal{M}_θ and property Φ , output is the set of satisfying parameters θ Inference: input: model \mathcal{M}_θ and observed data D_{obs} , output is the set of parameter estimation $\hat{\theta}$ This thesis: combines parameter inference and synthesis into data-informed, Bayesian parameter synthesis frameworks.

Problem description

Frameworks

Model checking step

A property Φ is

- ▶ a bounded reachability property and
- ▶ specified by PCTL.

Checking a model \mathcal{M}_θ against Φ

- ▶ Rational function evaluation
- ▶ Statistical model checking

Parameter synthesis step

Monte-Carlo Markov chain framework, basically modified Metropolis-Hasting algorithm.

Algorithm 1 Markov Chain Monte-Carlo with rational functions

```

1: procedure MCMC-RF
2:   Init  $\theta$  from prior distribution  $\pi(\theta)$ .
3:    $i \leftarrow 1$ 
4:   while  $i \leq N$  do
5:     Draw  $\theta_{cand}$  from  $Q(\theta|\theta_1, \dots, \theta_i)$ 
6:     if  $P(D_{obs}|\theta_{cand}) > P(D_{obs}|\theta_i)$  then
7:       Accept  $\theta_{cand}$  if  $\mathcal{M}_{\theta_{cand}} \models \Phi$ 
8:        $i \leftarrow i + 1$ 
9:     end if
10:  end while
11:  Return  $(\theta_1, \dots, \theta_N)$ 
12: end procedure
  
```


Parameter synthesis step

Algorithm 2 Sequential Monte-Carlo with rational functions

```

1: procedure SMC-RF
2:   Init  $\theta_1, \dots, \theta_N$  from prior distribution  $\pi(\theta)$ .
3:   Set  $w_i \leftarrow P(D_{obs}|\theta_i), 1 \leq i \leq N$ 
4:   for  $t \in (1, \dots, M)$  do
5:     Normalize  $w_1^t, \dots, w_N^t$ 
6:     Sample with replacement  $\theta_1^t, \dots, \theta_N^t$  from  $\theta_1^{t-1}, \dots, \theta_N^{t-1}$ 
7:     for  $\theta_i \in (\theta_1^t, \dots, \theta_N^t)$  do
8:        $\theta_i \leftarrow \text{MCMC-RF}(\theta_i), Q = F_i(\theta|\theta_1^{t-1}, \dots, \theta_N^{t-1})$ 
9:     end for
10:  end for
11:  Return  $(\theta_1, \dots, \theta_N)$ 
12: end procedure

```

Parameter synthesis step

Algorithm 3 Sequential Monte-Carlo with simulations

```

1: procedure RF-SMC
2:   Init  $\theta_1, \dots, \theta_N$  from prior distribution  $\pi(\theta)$ .
3:   for  $t \in (1, \dots, M)$  do
4:     Sample with replacement  $\theta_1^t, \dots, \theta_N^t$  from  $\theta_1^{t-1}, \dots, \theta_N^{t-1}$ 
5:     for  $\theta_i \in (\theta_1^t, \dots, \theta_N^t)$  do
6:       Draw  $\theta_{cand}$  from  $F_i(\theta | \theta_1^{t-1}, \dots, \theta_N^{t-1})$ 
7:       if Statistical Model Checking  $\mathcal{M}_{\theta_{cand}} \models \Phi$  then
8:         Simulate  $D_{sim}$  from  $\mathcal{M}_{\theta_{cand}}$ 
9:         if  $Distance(D_{sim}, D_{obs}) < \epsilon$  then
10:          Update  $\theta_i \leftarrow \theta_{cand}$ 
11:        end if
12:      end if
13:    end for
14:  end for
15:  Return  $(\theta_1, \dots, \theta_N)$ 

```

References I

- [1] Joost-Pieter Katoen. “The probabilistic model checking landscape”. In: *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science*. 2016, pp. 31–45.