

Bayesian parameter synthesis of Markov population models.

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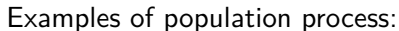
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Motivation

In this thesis, we study *population processes*. Kingman [1] defines *population processes* as stochastic models of discrete state spaces, in which:

- ▶ Each state in the state space represent the number of individuals in a category or a colony.
- ▶ Changing from a state to another state represents an increase or decrease of the number of individuals.



Question

As we study a Markov population process, the following questions are raised:

- ▶ *Parametric model*: How can we encompass unknown features into the population model?
- ▶ *Parameter synthesis*: Given a parametric population model and observed data of the population, how can we infer the model parameters?
- ▶ *Model composition*: How can we generalize aggregate a multi-agents population model from single-agent behaviour model?

In the scope of this thesis, we limit our study to discrete-time model.

Approach

- ▶ Model individual behaviour after a parametric Markov Decision Process.
- ▶ Model collective behaviour by compositing individual models to a parametric Discrete Time Markov Chain.
- ▶ Use data-informed Bayesian inference to synthesize the parametric Discrete Time Markov Chain parameters.

Data

As we use Bayesian inference, we need data.

- ▶ In the thesis, we use synthetic data, obtained by simulating the parametric model using a concrete assignment of parameters.
- ▶ Using *synthetic data* has an advantage over using real data. As the concrete parameters are known, it is possible to measure the distance between the synthesized parameters and true parameters.

Single agent model.

We model the behaviour of a single bee as a Markov Decision Process. Let \mathcal{S} be the individual model, we have

$$\mathcal{S} = (S, A, P_a, R_a)$$

in which

- ▶ S is the set of states.
- ▶ A is the set of actions.
- ▶ $P_a(s, s')$ is the probability of transitioning from state s to state s' given action a .
- ▶ $R_a(s, s')$ is the *reward* received after transitioning from state s to state s' given action a .

Single agent model.

Example of a single bee model

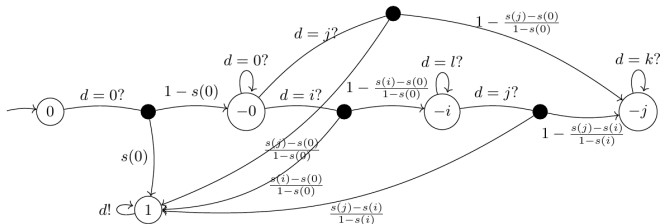


Figure: Generic single bee model (developed by Matej Hajnal and Tatjana Petrov)

Multi-agent model

To model collective behavior of multiple agents, we construct product of individual models. Let \mathcal{M} be the multiple agents pDTMC model, we constr

$$\mathcal{M} = (\mathcal{S}_1 || \mathcal{S}_2 || \dots || \mathcal{S}_k)$$

Composition of MDP are mentioned in [4]. Asynchronous and synchronous semantics for constructing pDTMC from multiple pMDP are currently developed by Tatjana Petrov.

Multi-agent model

Example of a multi-agent model.

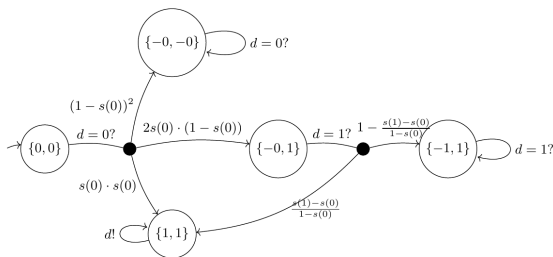


Figure: Example of 2 bees model (developed by Matej Hajnal and Tatjana Petrov)

Properties

Question: How can different parameters of single agent affect the population?

Answer: Given a model \mathcal{M}_Θ , to find the probability of having a certain number of individuals at the steady state, we

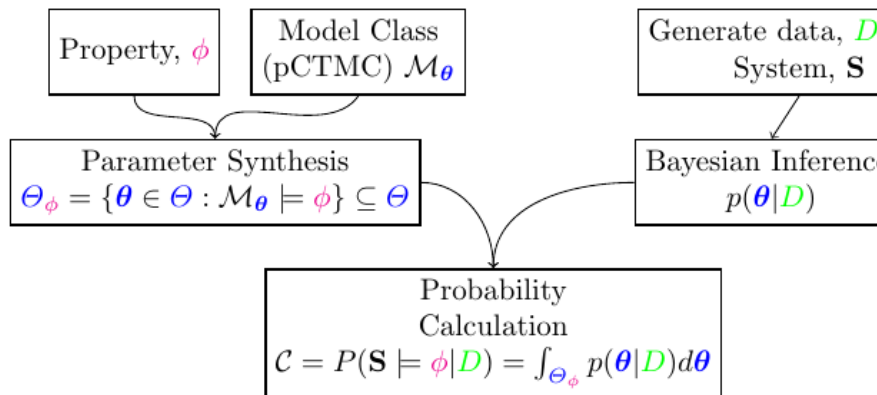
- ▶ represents population at steady state by a BSCC s_i , with i is the population size.
- ▶ checking the model of against $PCTL P_?(FGs_i)$

We can survey further constraints on model and properties:

- ▶ Surviving percentage: lumping BSCCs.
- ▶ Reduce parameter space: apply a linear/sigmoidal constraints over parameter space Θ .

Related work

Gareth Molyneux et al. [3] presented *ABCSeq* framework for Bayesian Verification of CTMC.



Related work

In [2], the authors present $ABC(SMC)2$ to improve $ABCSeq$ framework for better performance.

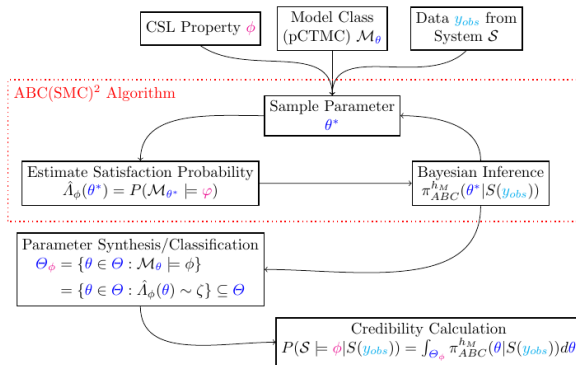


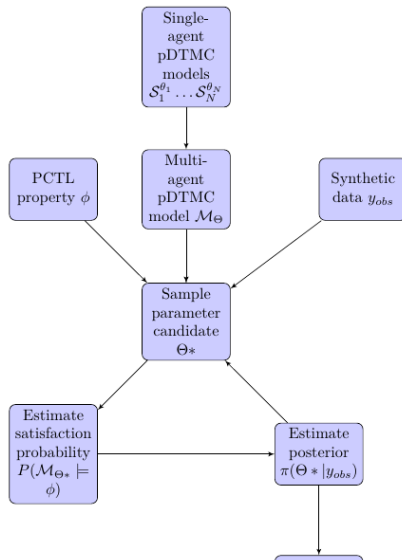
Fig. 1. Bayesian Verification via $ABC(SMC)^2$.

Proposed Framework

We develop a similar framework for parameter synthesis of pDTMC based on [2] and [3]. Compare to the work by Gareth Molyneux et al., our framework should

- ▶ Works with discrete time model.
- ▶ Verify PCTL property.
- ▶ Since the model is of discrete-time, the closed form solution (symbolic) for a PCTL property is in some cases obtainable. In such cases, we can compute the exact likelihood without simulation.

Proposed Framework



Tools

For probabilistic model checking and parameter synthesis, we use STORM probabilistic model checker

- ▶ **Extensible:** High quality C++ APIs <- this is important.
- ▶ **Scalable:** Faster compare to PRISM.
- ▶ **Capable:** STORM factorizes symbolic results on-the-fly.

Case study

We study the defensive behaviour of a bee colony.

- ▶ Bees response to stimulations from the environment by *stinging*.
- ▶ After *stinging*, an individual bee releases *pheromone* and dies.

Our questions concerns the relation between the concentration of pheromone in the environment and the aggressiveness of each individual in the colony.

Case study

As we study the biological system, we have the following research questions:

1. Given a population of bee, how many individuals left in the steady state.
2. How does an individual's behaviour change the collective behaviour? Does each individual become more aggressive given the

Timeline

Thesis milestones

1. **07.12.2020**: Model and properties lists.
2. **21.12.2020**: Framework implementation and results.
3. **10.01.2021**: Model improvement.
4. **30.01.2021**: Thesis submission.

Progress is reported weekly.

References

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: Simultaneous Inference and Model Checking of Chemical Reaction Networks”. In: *International Conference on Computational Methods in Systems Biology*. Springer. 2020, pp. 255–279.
- [3] Gareth W Molyneux, Viraj B Wijesuriya, and Alessandro Abate. “Bayesian verification of chemical reaction networks”. In: *International Symposium on Formal Methods*. Springer. 2019, pp. 461–479.
- [4] Ana Sokolova and Erik P De Vink. “Probabilistic automata: