# Bayesian parameter synthesis for markov population model.

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# Introduction.

# Preliminaries

# Parameter synthesis

Katoen [1] definition

### Definition (Parameter synthesis)

Given a finite-state parametric Markov model, find the parameter values for which a given reachability property exceeds (or is below) a given threshold $\beta$.

# Parameter synthesis

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# Parameter synthesis and Parameter inference

What are the differences between parameter inference and parameter synthesis?

| | Input | Ouput |
|---|---|---|
| Parameter inference | Model $\mathcal{M}_\theta$<br>Observed data $D_{obs}$ | Parameter estimation<br>$\hat{\theta}$ |
| Parameter synthesis | Model $\mathcal{M}_\theta$<br>Reachability property $\Phi$ | $(\theta_1, \ldots, \theta_N)$<br>$\forall \theta_i \in (\theta_1, \ldots, \theta_N) : \mathcal{M}_{\theta_i} \models \Phi$ |

Synthesis: input: model $\mathcal{M}_\theta$ and property $\Phi$, output is the set of satisfying parameters $\theta$ Inference: input: model $\mathcal{M}_\theta$ and observed data $D_{obs}$, output is the set of parameter estimation $\hat{\theta}$ This thesis: combines parameter inference and synthesis into data-informed, Bayesian parameter synthesis frameworks.

# Problem description

# Frameworks

## Model checking step

A property $\Phi$ is

- a bounded reachability property and
- specified by PCTL.

Checking a model $\mathcal{M}_\theta$ against $\Phi$

- Rational fucntion evaluation
- Statistical model checking

# Parameter synthesis step

**Algorithm 1** Markov Chain Monte-Carlo with rational functions

```
procedure MCMC-RF
    Init θ from prior distribution π(θ).
    i ← 1
    while i ≤ N do
        Draw θ_cand from Q(θ|θ_1, ..., θ_i)
        if P(D_obs|θ_cand) > P(D_obs|θ_i) then
            Accept θ_cand if M_{θ_cand} ⊨ Φ
            i ← i + 1
        end if
    end while
    Return (θ_1, ..., θ_N)
end procedure
```

# Parameter synthesis step

**Algorithm 2** Sequential Monte-Carlo with rational functions

**procedure** SMC-RF
  Init $\theta_1, \dots, \theta_N$ from prior distribution $\pi(\theta)$.
  Set $w_i \leftarrow P(D_{obs}|\theta_i), 1 \leq i \leq N$
  **for** $t \in (1, \dots, M)$ **do**
    Normalize $w_1^t, \dots, w_N^t$
    Sample with replacement $\theta_1^t, \dots, \theta_N^t$ from $\theta_1^{t-1}, \dots, \theta_N^{t-1}$
    **for** $\theta_i \in (\theta_1^t, \dots, \theta_N^t)$ **do**
      $\theta_i \leftarrow MCMC - RF(\theta_i),\ Q = F_i(\theta|\theta_1^{t-1}, \dots, \theta_N^{t-1})$
    **end for**
  **end for**
  Return $(\theta_1, \dots, \theta_N)$
**end procedure**

# Parameter synthesis step

**Algorithm 3** Sequential Monte-Carlo with simulations

**procedure** RF-SMC<br>
    Init $\theta_1, \dots, \theta_N$ from prior distribution $\pi(\theta)$.<br>
    **for** $t \in (1, \dots, M)$ **do**<br>
        Sample with replacement $\theta_1^t, \dots, \theta_N^t$ from $\theta_1^{t-1}, \dots, \theta_N^{t-1}$<br>
        **for** $\theta_i \in (\theta_1^t, \dots, \theta_N^t)$ **do**<br>
            Draw $\theta_{cand}$ from $F_i(\theta | \theta_1^{t-1}, \dots, \theta_N^{t-1})$<br>
            **if** Statistical Model Checking $\mathcal{M}_{\theta_{cand}} \models \Phi$ **then**<br>
                Simulate $D_{sim}$ from $\mathcal{M}_{\theta_{cand}}$<br>
                **if** $Distance(D_{sim}, D_{obs} < \epsilon)$ **then**<br>
                    Update $\theta_i \leftarrow \theta_{cand}$<br>
                **end if**<br>
            **end if**<br>
        **end for**<br>
    **end for**<br>
    Return $(\theta_1, \dots, \theta_N)$<br>
**end procedure**

# References I

[1] Joost-Pieter Katoen. "The probabilistic model checking landscape". In: *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science*. 2016, pp. 31–45.