Bayesian parameter synthesis for markov population model.

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Introduction.

Preliminaries

Parameter synthesis

Katoen [1] definition

Definition (Discrete-time Markov Chain)

Parameter synthesis

Katoen [1] definition

Definition (PCTL)

Model checking PCTL

Katoen [1] definition

Definition (PCTL)

Model checking PCTL

Katoen [1] definition

Definition (PCTL)

Parameter synthesis

Katoen [1] definition

Definition (Parametric Discrete-time Markov Chain)

Parameter synthesis

Katoen [1] definition

Definition (Parameter synthesis)

Given a finite-state parametric Markov model, find the parameter values for which a given reachability property exceeds (or is below) a given threshold β.

Parameter synthesis and Parameter inference

What are the differences between parameter inference and parameter synthesis?

	Input	Ouput
Parameter inference	Model $\mathcal{M}_\theta$ Observed data D_{obs}	Parameter estimation $\hat{\theta}$
Parameter synthesis	Model $\mathcal{M}_\theta$ Reachability property Φ	$(\theta_1, \ldots, \theta_N)$ $\forall \theta_i \in (\theta_1, \ldots, \theta_N) : \mathcal{M}_{\theta_i} \models \Phi$

Synthesis: input: model $\mathcal{M}_\theta$ and property Φ, output is the set of satisfying parameters θ Inference: input: model $\mathcal{M}_\theta$ and observed data D_{obs}, output is the set of parameter estimation $\hat{\theta}$ This thesis: combines parameter inference and synthesis into data-informed, Bayesian parameter synthesis frameworks.

Problem description

Frameworks

Model checking step

A property Φ is

- a bounded reachability property and
- specified by PCTL.

Checking a model $\mathcal{M}_\theta$ against Φ

- Rational fucntion evaluation
- Statistical model checking

Parameter synthesis step

Monte-Carlo Markov chain framework, basically modified Metropolis-Hasting algorithm.

Algorithm 1 Markov Chain Monte-Carlo with rational functions

procedure MCMC-RF
    Init θ from prior distribution $\pi(\theta)$.
    $i \leftarrow 1$
    while $i \leq N$ **do**
        Draw θ_{cand} from $Q(\theta|\theta_1, \dots, \theta_i)$
        if $P(D_{obs}|\theta_{cand}) > P(D_{obs}|\theta_i)$ **then**
            Accept θ_{cand} if $\mathcal{M}_{\theta_{cand}} \models \Phi$
            $i \leftarrow i + 1$
        end if
    end while
    Return $(\theta_1, \dots, \theta_N)$
end procedure

Parameter synthesis step

Algorithm 2 Sequential Monte-Carlo with rational functions

procedure SMC-RF
    Init $\theta_1, \dots, \theta_N$ from prior distribution $\pi(\theta)$.
    Set $w_i \leftarrow P(D_{obs}|\theta_i), 1 \leq i \leq N$
    for $t \in (1, \dots, M)$ **do**
        Normalize $w_1^t, \dots, w_N^t$
        Sample with replacement $\theta_1^t, \dots, \theta_N^t$ from $\theta_1^{t-1}, \dots, \theta_N^{t-1}$
        for $\theta_i \in (\theta_1^t, \dots, \theta_N^t)$ **do**
            $\theta_i \leftarrow MCMC - RF(\theta_i),\ Q = F_i(\theta|\theta_1^{t-1}, \dots, \theta_N^{t-1})$
        end for
    end for
    Return $(\theta_1, \dots, \theta_N)$
end procedure

Parameter synthesis step

Algorithm 3 Sequential Monte-Carlo with simulations

procedure RF-SMC
  Init $\theta_1, \ldots, \theta_N$ from prior distribution $\pi(\theta)$.
  for $t \in (1, \ldots, M)$ **do**
    Sample with replacement $\theta_1^t, \ldots, \theta_N^t$ from $\theta_1^{t-1}, \ldots, \theta_N^{t-1}$
    for $\theta_i \in (\theta_1^t, \ldots, \theta_N^t)$ **do**
      Draw θ_{cand} from $F_i(\theta | \theta_1^{t-1}, \ldots, \theta_N^{t-1})$
      if Statistical Model Checking $\mathcal{M}_{\theta_{cand}} \models \Phi$ **then**
        Simulate D_{sim} from $\mathcal{M}_{\theta_{cand}}$
        if $Distance(D_{sim}, D_{obs} < \epsilon)$ **then**
          Update $\theta_i \leftarrow \theta_{cand}$
        end if
      end if
    end for
  end for
  Return $(\theta_1, \ldots, \theta_N)$

References I

[1] Joost-Pieter Katoen. "The probabilistic model checking landscape". In: *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science*. 2016, pp. 31–45.