UNIT 3

Illumination

INTRODUCTION

Study of illumination engineering is necessary not only to understand the principles of light control as applied to interior lighting design such as domestic and factory lighting but also to understand outdoor applications such as highway lighting and flood lighting. Nowaday, the electrically produced light is preferred to the other source of illumination because of an account of its cleanliness, ease of control, steady light output, low cost, and reliability. The best illumination is that it produces no strain on the eyes. Apart from its esthetic and decorative aspects, good lighting has a strictly utilitarian value in reducing the fatigue of the workers, protecting their health, increasing production, etc. The science of illumination engineering is therefore becoming of major importance.

Nature of light

Light is a form of electromagnetic energy radiated from a body and human eye is capable of receiving it. Light is a prime factor in the human life as all activities of human being ultimately depend upon the light.

Various forms of incandescent bodies are the sources of light and the light emitted by such bodies depends upon their temperature. A hot body about 500–800°C becomes a red hot and about 2,500–3,000°C the body becomes white hot. While the body is red-hot, the wavelength of the radiated energy will be sufficiently large and the energy available in the form of heat. Further, the temperature increases, the body changes from red-hot to white-hot state, the wavelength of the radiated energy becomes smaller and enters into the range of the wavelength of light. The wavelength of the light waves varying from 0.0004 to 0.00075 mm, i.e. 4,000-7,500 Å (1 Angstrom unit = 10^{-10} mm).

The eye discriminates between different wavelengths in this range by the sensation of color. The whole of the energy radiated out is not useful for illumination purpose. Radiations of very short wavelength varying from 0.0000156×10^{-6}m to 0.001×10^{-6} m are not in the visible range are called as rontgen or x-rays, which are having the property of penetrating through opaque bodies.

1. LAWS OF ILLUMINATION

Mainly there are two laws of illumination.

1. Inverse square law
2. Lambert's cosine law.

Inverse square law

This law states that 'the illumination of a surface is inversely proportional to the square of distance between the surface and a point source'.

Proof:

Let, '*S*' be a point source of luminous intensity '*I*' candela, the luminous flux emitting from source crossing the three parallel plates having areas $A_1 A_2$, and A_3 square meters, which are separated by a distances of *d*, 2*d*, and 3*d* from the point source respectively as shown in Fig. 6.10.

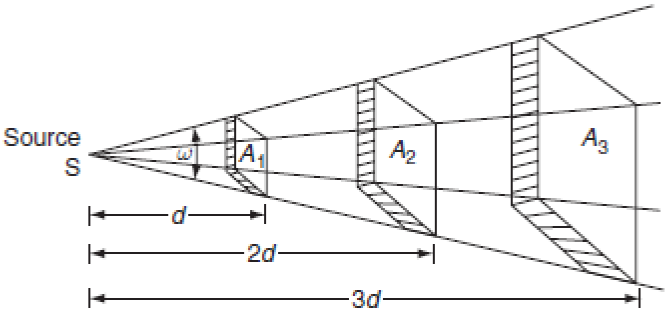


Fig. 6.10 Inverse square law

For area A_1, solid angle $\omega = \frac{A_1}{d^2}$.

Luminous flux reaching the area A_1 = luminous intensity × solid angle

$$= I \times \omega = I \times \frac{A_1}{d^2}.$$

∴ Illumination 'E_1' on the surface area 'A_1' is:

$$E_1 = \frac{\text{flux}}{\text{area}} = \frac{IA_1}{d^2} \times \frac{1}{A_1}$$

$$\therefore E_1 = \frac{I}{d^2} \quad \text{lux.} \qquad \textbf{(6.5)}$$

Similarly, illumination 'E_2' on the surface *area* A_2 is:

$$E_2 = \frac{I}{(2d)^2} \quad \text{lux} \qquad \textbf{(6.6)}$$

and illumination ‘E_3’ on the surface area A_3 is:

$$E_3 = \frac{\mathrm{I}}{(3d)^2} \quad \text{lux.} \qquad \textbf{(6.7)}$$

From Equations (6.5), (6.6), and (6.7)

$$E_1 : E_2 : E_3 = \frac{1}{d^2} : \frac{1}{(2d)^2} : \frac{1}{(3d)^2}. \qquad \textbf{(6.8)}$$

Hence, from Equation (6.8), illumination on any surface is inversely proportional to the square of distance between the surface and the source.

Lambert's cosine law

This law states that ‘illumination, *E* at any point on a surface is directly proportional to the cosine of the angle between the normal at that point and the line of flux’.

Proof:

While discussing, the Lambert's cosine law, let us assume that the surface is inclined at an angle ‘θ’ to the lines of flux as shown in Fig. 6.11.

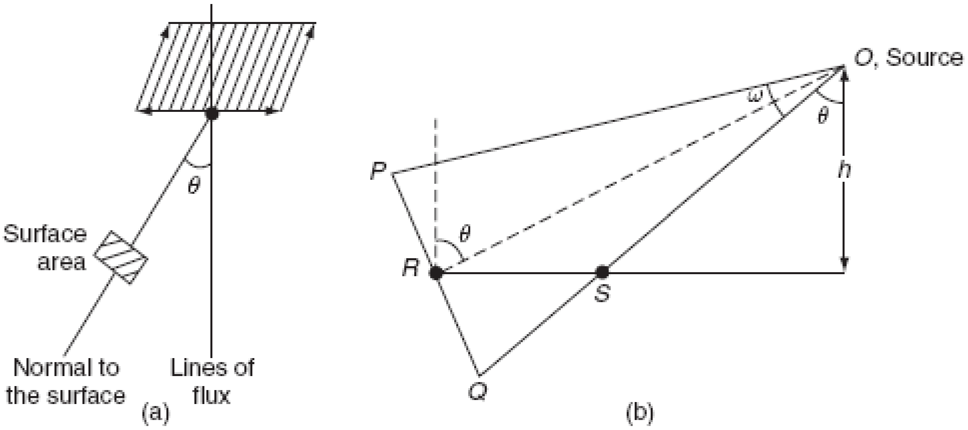


Fig. 6.11 Lambert's cosine law

Let

PQ = The surface area normal to the source and inclined at ‘θ’ to the vertical axis.

RS = The surface area normal to the vertical axis and inclined at an angle θ to the source ‘*O*’.

Therefore, from Fig. 6.11:

$$PQ = RS\cos\theta.$$

$$\therefore \text{The illumination of the surface } PQ, E_{PQ} = \frac{\text{flux}}{\text{area of } PQ}$$

$$= \frac{I \times \omega}{\text{area of } PQ} = \frac{I}{\text{area of } PQ} \times \frac{\text{area of } PQ}{d^2} \quad \left[\therefore \omega = \text{area}/(\text{radius})^2\right]$$

$$= \frac{I}{d^2}. \qquad \textbf{(6.9)}$$

$$\therefore \text{The illumination of the surface } RS, E_{RS} = \frac{\text{flux}}{\text{area of } RS} = \frac{\text{flux}}{\text{area of } PQ/\cos\theta}$$

$$[\therefore PQ = RS \cos\theta]$$

$$= \frac{I}{d^2}\cos\theta. \qquad \textbf{(6.10)}$$

From Fig. 6.11(b):

$$\cos\theta = \frac{h}{d}$$

$$\text{or } d = \frac{h}{\cos\theta}.$$

Substituting ‘*d*′ from the above equation in Equation (6.10):

$$\therefore E_{RS} = \frac{I}{(h/\cos\theta)^2} \times \cos\theta = \frac{I}{h^2}\cos^3\theta \qquad \textbf{(6.11)}$$

$$\therefore E_{RS} = \frac{I}{d^2}\cos\theta = \frac{I}{h^2}\cos^3\theta \qquad \textbf{(6.12)}$$

where *d* is the distance between the source and the surface in m, *h* is the height of source from the surface in m, and *I* is the luminous intensity in candela.

Hence, Equation (6.11) is also known as ‘cosine cube’ law. This law states that the ‘illumination at any point on a surface is dependent on the cube of cosine of the angle between line of flux and normal at that point’.

Note:

*From the above laws of illumination, it is to be noted that inverse square law is only applicable for the surfaces if the surface is normal to the line of flux. And Lambert's cosine law is applicable for the surfaces if the surface is inclined an angle ‘θ’ to the line of flux.

Example 6.6: The illumination at a point on a working plane directly below the lamp is to be 60 lumens/m^2. The lamp gives 130 CP uniformly below the horizontal plane. Determine:

1. The height at which lamp is suspended.
2. The illumination at a point on the working plane 2.8 m away from the vertical axis of the lamp.

Solution:

Given data:

Candle power of the lamp = 130 CP.

The illumination just below the lamp, E = 60 lumen/m^2.

1. From the Fig. P.6.2, the illumination just below the lamp, i.e., at point A:

$$E_A = \frac{I}{h^2}$$

$$\therefore h = \sqrt{\frac{I}{EA}} = \sqrt{\frac{130}{60}} = 1.471\,\text{m.}$$

2. The illumination at point ‘B':

$$E_B = \frac{I}{h^2}\cos^3\theta$$

$$= \frac{130}{(2.8)^2}\left\{\frac{2.8}{\sqrt{2.8^2 + 1.471^2}}\right\}^3 = 11.504\,\text{lux.}$$

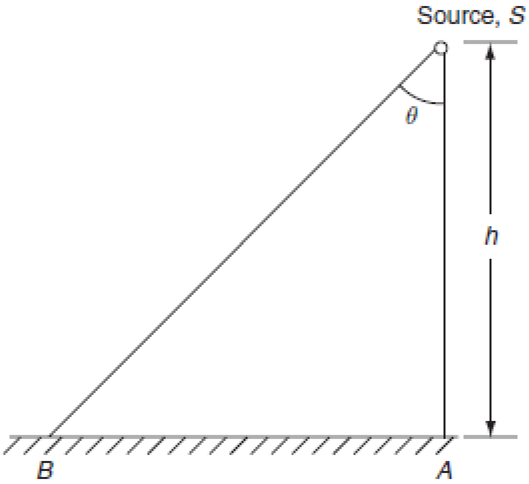


Fig. P.6.2

Example 6.7: A lamp having a candle power of 300 in all directions is provided with a reflector that directs 70% of total light uniformly on a circular area 40-m diameter. The lamp is hung at 15 m above the area.

1. Calculate the illumination.
2. Also calculate the illumination at the center.
3. The illumination at the edge of the surface without reflector.

Solution:

Given data:

Candle power of the lamp = 300 CP.

Circular area diameter (D) = 40 m.

Height of mounting = 15 m.

1. The illumination on the circular area (Fig. P.6.3):

$$E = \frac{\text{flux}}{\text{area}} = \frac{CP \times \omega}{A}.$$

Here, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 40^2 = 400\ \pi\text{m}^2.$

Solid angle 'ω' $= 2\pi\ (1 - \cos\theta)$

$$= 2\pi \left(1 - \frac{15}{\sqrt{15^2 + 20^2}}\right)$$

$$= 0.8\ \pi \text{ steradians.}$$

$$\therefore \text{Illumination } E = \frac{\text{flux}}{\text{area}} = \frac{CP \times \omega}{A}$$

$$= \frac{300 \times 0.8\pi}{400\pi}$$

$$= 0.6 \text{ lux.}$$

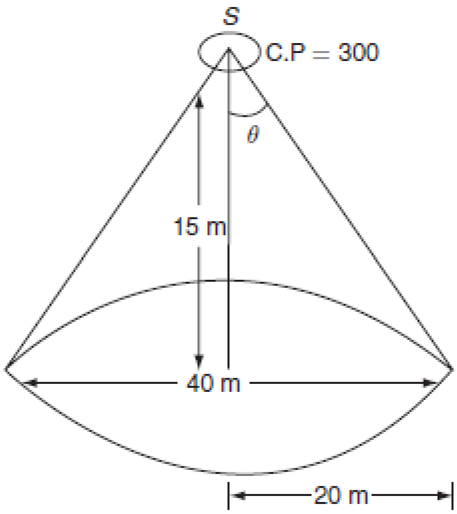


Fig. P.6.3

2. The illumination at the center with reflector 70%:

$$= \frac{\phi}{A} \times 0.7 = \frac{CP \times \omega}{A} \times 0.7$$

$$= \frac{300 \times 4\pi}{400\pi} \times 0.7$$

$$= 2.1 \text{ lux.}$$

3. The illumination at the edge without reflector:

$$= \frac{CP}{d^2} \times \cos\theta$$

$$= \frac{300}{(\sqrt{15^2+10^2})^2} \times \frac{15}{\sqrt{15^2+10^2}}$$

$$= 0.768 \text{ lux.}$$

Example 6.8: The luminous intensity of a source is 600 candela is placed in the middle of a 10 × 6 × 2 m room. Calculate the illumination:

1. At each corner of the room.
2. At the middle of the 6-m wall.

Solution:

Given data:

Luminous intensity, (*I*) = 600 cd.

Room area = 10 × 6 × 2 m.

1. From the Fig. P.6.4:

$$OB = BD = \frac{\sqrt{10^2+6^2}}{2} = 5.83 \text{ m}$$

$$\text{BS} = \text{d} = \sqrt{2^2+(5.38)^2} = 6.163 \text{ m.}$$

∴ The illumination at the corner ‘*B*':

$$E_B = E_A = E_C = E_D$$

$$\frac{I}{d^2}\cos\theta = \frac{600}{(6.163)^2} \times \frac{2}{(6.163)}$$

$$= 5.126 \text{ lux.}$$

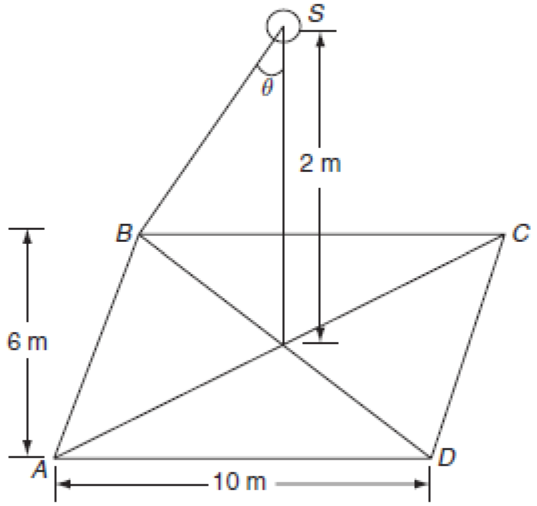


Fig. P.6.4

2. From Fig. P.6.5:

$$PS = \sqrt{2^2 + 5^2}$$

$$= 5.385 \text{ m.}$$

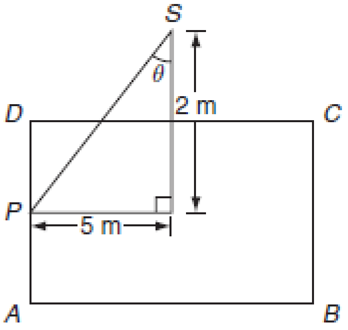


Fig. P.6.5

The illumintaion at the point ‘*P*’, $E_p = \dfrac{I}{d^2}\cos\theta$

$$= \frac{600}{(5.385)^2} \times \frac{2}{(5.385)}$$

$$= 7.684 \text{ lux.}$$

Example 6.9: The candle power of a source is 200 candela in all directions below the lamp. The mounting height of the lamp is 6 m. Find the illumination:

1. Just below the lamp.
2. 3 m horizontally away from the lamp on the ground.
3. The total luminous flux in an area of 1.5-m diameter around the lamp on the ground.

Solution:

The candle power of the source, I = 200 candela.

Mounting height (h) = 6 m.

1. The illumination just below the lamp, i.e., at point *'A'*:

$$E_p = \frac{I}{d^2} \cos\theta$$

$$= \frac{600}{(5.385)^2} \times \frac{2}{(5.385)}$$

$$= 7.684 \text{ lux.}$$

2. From Fig. P.6.6:

$$d = \sqrt{3^2 + 6^2} = 6.708.$$

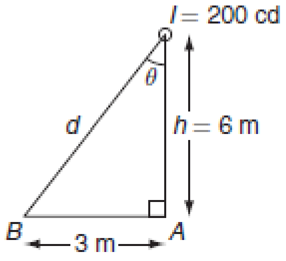


Fig. P.6.6

The illumination 3 m away from the lamp on the ground, i.e., at point '*B*' (Fig. P.6.7):

$$E_B = \frac{I}{d^2}\cos\theta$$

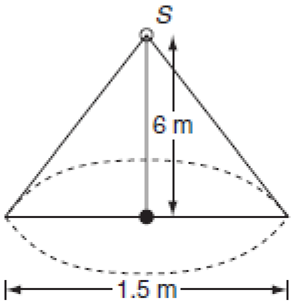


Fig. P.6.7

$$= \frac{200}{(6.708)^2} \times \frac{6}{(6.708)}$$

$$= 3.975 \text{ lux.}$$

3.

$$\text{Surface area } = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} \times (1.5)^2$$

$$= 1.767 \text{ m}^2.$$

The total flux reaching the area around the lamp:

= E_A × surface area

= 5.55 × 1.767

= 9.80 lumens.

Example 6.10: Two sources of candle power or luminous intensity 200 candela and 250 candela are mounted at 8 and 10 m, respectively. The horizontal distance between the lamp posts is 40 m, calculate the illumination in the middle of the posts.

Solution:

From Fig. P.6.8:

$$d_1 = \sqrt{8^2 + 20^2}$$

$$= 21.54.$$

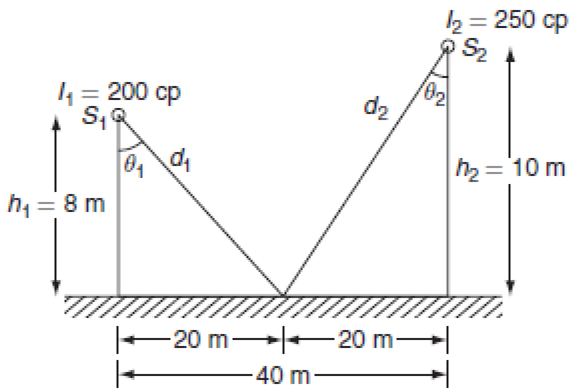


Fig. P.6.8

$$\cos\theta_1 = \frac{h_1}{d_1} = \frac{8}{21.54}$$

$$= 0.37.$$

∴ The illumination at the point 'P' due to the source 'S_1' $= \frac{I_1}{d_1^2}\cos\theta_1$

$$E_1 = \frac{200}{(21.54)^2} \times 0.37$$

$$= 0.159 \text{ lux}.$$

$$\text{and } d_2 = \sqrt{10^2 + 20^2} = 22.36$$

$$\cos\theta_2 = \frac{h_2}{d_2} = \frac{10}{22.36} = 0.447.$$

The illumination at the point '*P*' due to the source 'S_2':

$$E_2 = \frac{I_2}{d_2^2} \times \cos\theta_2$$

$$= \frac{250}{(22.36)^2} \times 0.447 = 0.2235 \text{ lux.}$$

∴ The total illumination at '*P*' due to both the sources S_1 and $S_2 = E_1 + E_2$

$= 0.159 + 0.2235$

$= 0.3825$ lux.

Example 6.11: Two sources of having luminous intensity 400 candela are hung at a height of 10 m. The distance between the two lamp posts is 20 m. Find the illumination (i) beneath the lamp and (ii) in the middle of the posts.

Solution:

Given data:

Luminous intensity = 400 CP.

Mounting height = 10 m.

Distance between the lamp posts = 20 m.

1. From Fig. P.6.9:

$$d_1 = \sqrt{10^2 + 20^2} = 22.36.$$

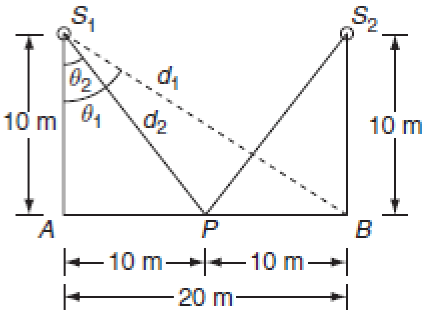


Fig. P.6.9

$$\cos\theta_1 = \frac{h}{d_1} = \frac{10}{22.36} = 0.4472.$$

The illumination at '*B*' due to '*S_1*':

$$E_1 = \frac{I}{d_1^2}\cos\theta_1$$

$$= \frac{400}{(22.36)^2} \times 0.4472$$

$$= 0.35778 \text{ lux.}$$

The illumination at '*B*' due to '*S_2*':

$$E_2 = \frac{400}{10^2} = 4 \text{ lux.}$$

$$\therefore \text{ The total illumination at 'B'} = E_1 + E_2$$

$$= 0.3577 + 4$$

$$= 4.3577 \text{ lux.}$$

$$d_2 = \sqrt{10^2 + 10^2} = 14.14.$$

$$\cos\theta_2 = \frac{10}{14.14} = 0.707.$$

The illumination at 'P' due to S_1 is:

$$E_1 = \frac{I}{d_2^2} \times \cos\theta_2$$

$$= \frac{400}{(14.14)^2} \times 0.707 = 1.414 \text{ lux.}$$

The illumination at 'P' due to S_2, 'E_2' will be same as E_1

∴ The illumination at 'P' due to both S_1 and S_2:

$= E_1 + E_2 = E_1 + E_1$

$= 2E_1 = 2 \times 1.414$

$= 2.828$ lux.

Example 6.12: In a street lighting, two lamps are having luminous intensity of 300 candela, which are mounted at a height of 6 and 10 m. The distance between lamp posts is 12 m. Find the illumination, just below the two lamps.

Solution:

1. The illumination at 'B' = the illumination due to L_1 + the illumination due to L_2. FormFig. P.6.10:

$$d_1 = \sqrt{6^2 + 12^2} = 13.416 \text{ m.}$$

$$\cos\theta_1 = \frac{h_1}{d_1} = \frac{6}{13.416} = 0.447.$$

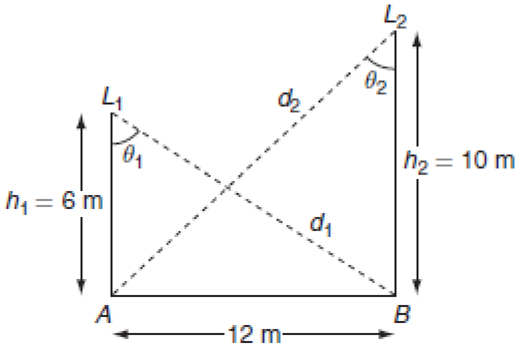


Fig. P.6.10

$$\therefore \text{The illumination at 'B' due to } L_1 = \frac{I}{d_1^2}\cos\theta_1$$

$$= \frac{300}{(13.416)^2} \times 0.447$$

$$= 0.745 \text{ lux.}$$

$$\text{Illumination at 'B' due to } L_2 = \frac{I}{h_2^2}$$

$$= \frac{300}{10^2}$$

$$= 3 \text{ lux.}$$

∴ The total illumination at '*B*' due to the two lamps = 0.745 + 3 = 3.745 lux.

2. The illumination at '*A*' = the illumination due to L_1+ the illumination due to L_2.

$$d_2 = \sqrt{10^2 + 12^2} = 15.62 \text{ m.}$$

$$\cos\theta_2 = \frac{h_2}{d_2} = \frac{10}{15.62} = 0.64.$$

$$\therefore \text{The illumination at 'A' due to lamp } L_1 = \frac{I}{d_2^2}\cos\theta_2$$

$$= \frac{300}{(15.62)^2} \times 0.64$$

$$= 0.786 \text{ lux.}$$

$$\text{Illumination at } A \text{ due to lamp 'L}_2\text{'} = \frac{I}{h_1^2}$$

$$= \frac{300}{6^2}$$

$$= 8.33 \text{ lux.}$$

∴ The total illumination at '*A*' due to both lamps = 0.786 + 8.33 = 9.116 lux.

Example 6.13: Four lamps 15 m apart are arranged to illuminate a corridor. Each lamp is suspended at a height of 8 m above the floor level. Each lamp gives 450 CP in all directions below the horizontal; find the illumination at the second and the third lamp.

Solution:

Given data:

Luminous intensity = 450 CP.

Mounting height = 8 m.

Distance between the adjacent lamps = 15 m (Fig. P.6.11).

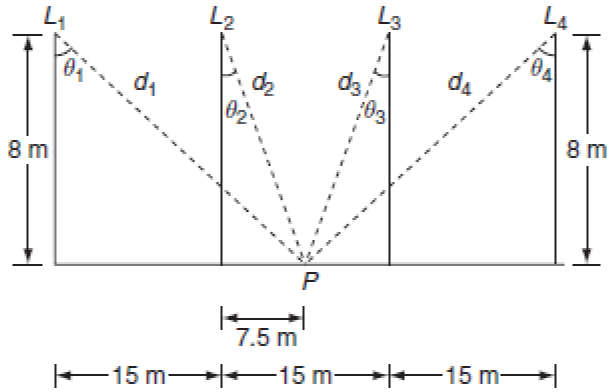


Fig. P.6.11

The illumination at '*P*' = the illumination due to L_1 + the illumination due to L_2

+ the illumination due to L_3 + the illumination due to L_4.

The illumination at '*P*' due to L_1, $E_1 = \frac{I}{d_1^2}\cos\theta_1$.

But, $d_1 = \sqrt{8^2 + 15^2} = 17$.

$\cos\theta_1 = \frac{h}{d_1} = \frac{8}{17} = 0.470.$

$$\therefore \mathrm{E}_1 = \frac{I}{d_1^2}\cos\theta_1$$

$$= \frac{450}{(17)^2} \times 0.47$$

$$= 0.73 \text{ lux.}$$

The illumination at '*P*' due to lamp 'L_2' is:

$$E_2 = \frac{I}{d_2^2}\cos\theta_2$$

$$= \frac{450}{\left[\sqrt{8^2+(7.5)^2}\right]^2} \times \frac{8}{\sqrt{8^2+7.5^2}}$$

$$= 2.73 \text{ lux.}$$

Similarly, the illumination at '*P*' due to the lamp L_3 'E_3' = the illumination at '*P*' due to the

the

lamp 'L_2', 'E_2',

and the illumination at '*P*' due to the lamp L_4, 'E_4' = illumination at '*P*' due to the lamp

'L_1', 'E_1.'

∴ The total illumination at '$P = E_1 + E_2 + E_3 + E_4$

$= 2E_1 + 2E_2$

$= 2(E_1 + E_2)$

$= 2\ (0.73 + 2.73)$

$= 6.92$ lux.

Example 6.15: Two lamps of each 500 CP are suspended 10 m from the ground and are separated by a distance of 20 m apart. Find the intensity of illumination at a point on the ground in line with the lamps and 12 m from the base on both sides of the lamps.

Solution:

Given data:

Luminous intensity, I = 500 CP.

Mounting height, h= 10 m.

Case (i):

From Fig. P.6.14:

$$d_1 = \sqrt{10^2 + 12^2}$$
$$= 15.62 \text{ m.}$$
$$\cos\theta_1 = \frac{h}{d_1} = \frac{10}{15.62} = 0.64.$$

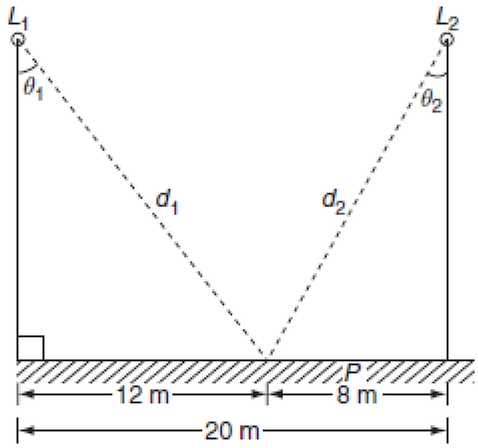


Fig. P.6.14

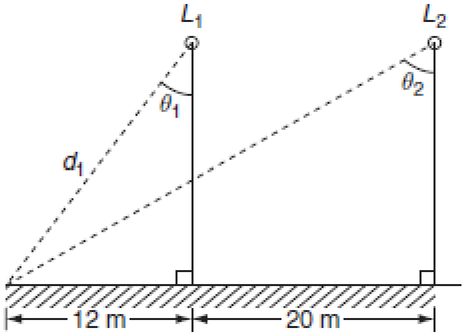


Fig. P.6.15

The illumination at '*P*' due to lamp L_1 is:

$$E_1 = \frac{I}{d_1^2}\cos\theta_1$$
$$= \frac{500}{(5.62)^2} \times 0.64$$
$$= 1.3115 \text{ lux.}$$

$$d_2 = \sqrt{8^2 + 10^2} = 12.806 \text{ m.}$$
$$\cos\theta_2 = \frac{h}{d_2} = \frac{10}{12.806} = 0.780.$$

The illumination at '*P*' due to lamp L_2 is:

$$E_2 = \frac{I}{d_2^2}\cos\theta_2$$
$$= \frac{500}{(12.806)^2} \times 0.78$$
$$= 2.378 \text{ lux.}$$

∴ The total illumination at the point '*P*' = $E_1 + E_2$

= 1.3115 + 2.378

= 3.689 lux.

Case (ii):

From Fig. P.6.15:

$$d_2 = \sqrt{8^2 + 10^2} = 12.806 \text{ m.}$$

$$\cos\theta_2 = \frac{h}{d_2} = \frac{10}{12.806} = 0.780.$$

The illumination at ‘*P*’ due to lamp L_1 is:

$$\begin{aligned} E_1 &= \frac{I}{d_1^2} \times \cos\theta_1 \\ &= \frac{500}{(15.62)^2} \times 0.64 \\ &= 1.3115 \text{ lux.} \end{aligned}$$

$$d_2 = \sqrt{10^2 + 32^2} = 33.52 \text{ m.}$$

$$\cos\theta_2 = \frac{I}{d_2} = \frac{10}{33.52} = 0.298.$$

The illumination at ‘*P*’ due to the lamp ‘L_2’ is:

$$\begin{aligned} E_2 &= \frac{I}{d_2^2} \cos\theta_2 \\ &= \frac{500}{(33.52)^2} \times 0.298 \\ &= 0.1326 \text{ lux.} \end{aligned}$$

∴ The total illumination at ‘*P*’ due to both lamps = $E_1 + E_2$

= 1.3115 + 0.1326

= 1.44 lux.

Example 6.16: Two similar lamps having luminous intensity 500 CP in all directions below horizontal are mounted at a height of 8 m. What must be the spacing between the lamps so that the illumination on the ground midway between the lamps shall be at least one-half of the illumination directly below the lamp.

Solution:

Given data:

The candle power of lamp, I = 600 CP.

The mounting height of lamps form the ground, H = 8 m.

Let, the maximum spacing between the lamps =x m.

From Fig. P.6.16:

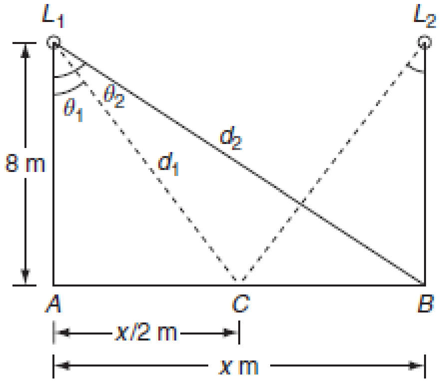


Fig. P.6.16

The illumination at 'C' due to the lamp 'L_1' is:

$$E_1 = \frac{I}{h^2}\cos^3\theta_1$$

$$= \frac{600}{8^2} \times \frac{(8)^3}{\left[8^2 + (x/2)^2\right]^{3/2}}.$$

The illumination ‘E_2’ at ‘C’ due to the lamp ‘L_2’ is same as to ‘E_1’.

∴ The total illumination at ‘C' due to the lamps, L_1 and L_2 is:

$$E_C = 2\,E_1$$

$$= 2 \times \left[\frac{600}{8^2} \times \frac{8^3}{\left[8^2 + (x/2)^2\right]^{3/2}}\right]$$

$$= \frac{9{,}600}{\left[8^2 + (x/2)^2\right]^{3/2}}.$$

The illumination just below the lamp, L_2 is:

E_B = the illumination due to lamp L_1 + the illumination due to lamp L_2:

$$= \frac{600}{8^2} \times \frac{8^3}{\left[8^2 + x^2\right]^{3/2}} + \frac{600}{8^2}.$$

But, given $E_C = \frac{1}{2} E_B$.

$$\therefore \frac{9,600}{\left[8^2 + \left(x/2\right)^2\right]^{3/2}} = \frac{1}{2}\left[\frac{4,800}{\left[8^2 + x^2\right]^{3/2}} + 9.375\right]$$

$$\frac{9,600}{\left[8^2 + \left(x/2\right)^2\right]^{3/2}} = \frac{2400}{\left[8^2 + x^2\right]^{3/2}} + 4.6875.$$

Example 6.17: Find the height at which a light source having uniform spherical distribution should be placed over a floor in order that the intensity of horizontal illumination at a given distance from its vertical line may be greatest.

Solution:

Let the luminous intensity of the lamp = '*I*' CP.

The illumination at the point '*A*' due to source is:

$$E_A = \frac{I}{\sqrt{h^2 + x^2}} . \cos\theta$$

$$= \frac{I}{h^2} \cos^3\theta.$$

But, from Fig. P.6.17:

$$\cos\theta = \frac{h}{\sqrt{h^2 + x^2}}.$$

$$\therefore \; E_A = \frac{I}{h^2} \times \left[\frac{h}{h^2 + x^2}\right]^3$$

$$= I \times \frac{h}{(h^2 + x^2)^{3/2}}.$$

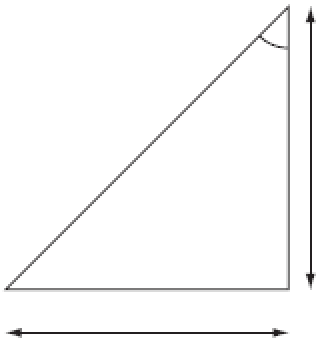


Fig. P.6.17

Given that, the illumination at a point away from the base of lamp may be the greatest:

$$\therefore \frac{dE_A}{dh} = 0$$

$$= I\left[\frac{h}{dh}\left[\frac{h}{(h^2+x^2)^{3/2}}\right]\right] = 0$$

$$= \frac{(h^2+x^2)^{3/2}\cdot 1 - h\cdot\frac{3}{2}(h^2+x^2)^{1/2}\cdot 2h}{\left\{(h^2+x^2)^{3/2}\right\}^2} = 0$$

$$= (h^2+x^2)^{1/2}\cdot\left[(h^2+x^2)-3h^2\right] = 0$$

$$= x^2 - 2h^2 = 0$$

$$\Rightarrow x^2 = 2h^2$$

$$\Rightarrow h = \frac{x}{\sqrt{2}} = 0.707x$$

$\therefore h = 0.707x.$

Example 6.18: A lamp of 250 candela is placed 2 m below a plane mirror that reflects 60% of light falling on it. The lamp is hung at 6 m above ground. Find the illumination at a point on the ground 8 m away from the point vertically below the lamp.

Solution:

Figure P.6.18 shows the lamp and the mirror arrangements. Here, the lamp ‘*L*’ produces an image ‘*L*’, then the height of the image from the ground = 8 + 2 = 10 m.

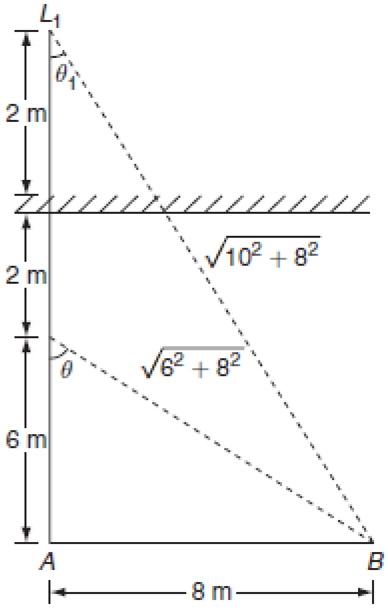


Fig. P.6.18

And L_1 acts as the secondary sources of light whose candle power is equals to 0.85 × CP of the lamp ‘*L*’.

i.e., 0.85 × 250 = 212.5 CP.

∴ The illumination at the point ‘*B*’, ‘8’ m away from the lamp = illumination at ‘*B*’ due to *L* + the illumination at ‘*B*’ due to L_1:

$$= \frac{250}{\left(\sqrt{6^2+8^2}\right)^2} \times \frac{6}{\sqrt{6^2+8^2}} + \frac{212.5}{\left(\sqrt{10^2+8^2}\right)^2} \times \frac{10}{\sqrt{10^2+8^2}}$$

$$= \frac{1500}{(6^2+8^2)^{3/2}} + \frac{2125}{(10^2+8^2)^{3/2}}$$

$$= 1.5 + 1.0117$$

$$= 2.5117 \text{ lux.}$$

Example 6.19: A light source with an intensity uniform in all direction is mounted at a height of 20 ms above a horizontal surface. Two points *'A'* and ‘*B*’ both lie on the surface with point *A* directly beneath the source. How far is *B* from *A* if the illumination at ‘*B*’ is only 1/15th as great as *A*?

Solution:

Let the luminous intensity of the lamp ‘*L*’ be ‘*I*’ candela and the distance of the point of illumination from the base of the lamp is ‘*x*’ m (Fig. P.6.19).

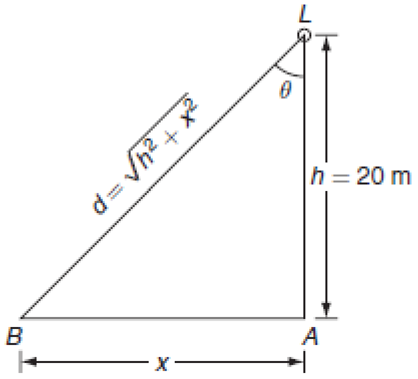


Fig. P.6.19

The illumination at the point *'A'* due to the lamp ‘*L*’ is:

$$E_A = \frac{I}{h^2} = \frac{I}{20^2} = \frac{I}{400}.$$

The illumination at the point *'B'* due to the lamp *'L'* is:

$$E_B = \frac{I}{h^2}\cos^3\theta$$

$$E_B = \frac{I}{(20)^2}\left[\frac{20}{\sqrt{(20^2+x^2)}}\right]^3.$$

$$\text{Given, } E_B = \frac{1}{15}E_A$$

$$\frac{20I}{(20^2+x^2)^{3/2}} = \frac{1}{15}\times\frac{I}{400}$$

$$20\times15\times400 = (20^2+x^2)^{3/2}$$

$$2143.98 = 20^2+x^2.$$

$$x^2 = 1743.98$$

$$x = 41.76\text{ m}.$$

Example 6.20: Two similar lamps having uniform intensity 500 CP in all directions below the horizontal are mounted at a height of 4 m. What must be the maximum spacing between the lamps so that the illumination on the ground midway between the lamps shall be at least one-half the illuminations directly under the lamps?

Solution:

The candle power of the lamp = 500 CP (Fig. P.6.20).

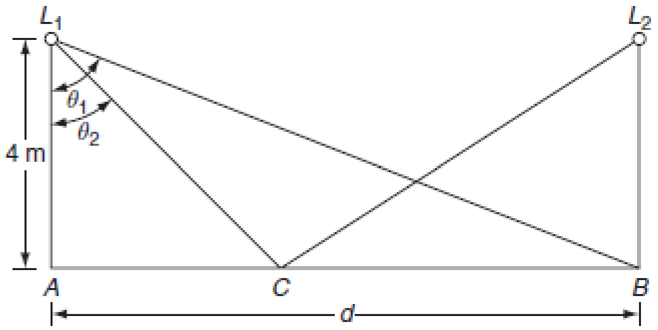


Fig. P.6.20

The height of the lamps from the ground, h = 4 m.

Let the maximum spacing between the lamps be of 'd' meters.

The illumination at the point 'C' in between the lamp post

= 2 × Illumination due to either L_1 or L_2

$$E_C = 2 \times \frac{500}{4^2} \times \frac{4^3}{\left[4^2 + (d/2)^2\right]^{3/2}} = \frac{4000}{\left[4^2 + d^2/4\right]^{3/2}}.$$

The illumination just below the lamp L_2 is:

E_B = the illumination due to the lamp L_1 + the illumination due to the lamp L_2

$$= \frac{500}{4^2} \times \frac{4}{\left[4^2 + d^2/2\right]^{3/2}} + \frac{500}{4^2}$$

$$= \frac{2{,}000}{(4^2 + d^2)^{3/2}} + 31.25.$$

Given:

$$E_C = \frac{1}{2} E_B$$

$$\frac{4000}{\left[4^2 + d^2/4\right]^{3/2}} = \frac{1}{2}\left[31.25 + \frac{200}{(4^2 + d^2)^{3/2}}\right]$$

$$\frac{4,000}{(4^2 + d^2/4)^{3/2}} = 15.625 + \frac{1,000}{(4^2 + d^2)^{3/2}}$$

$\therefore d$ = 9.56 m.

Example 6.21: A lamp with a reflector is mounted 10 m above the center of a circular area of 30-m diameter. If the combination of lamp and reflector gives a uniform CP of 1,200 over circular area, determine the maximum and minimum illumination produced.

Solution:

The mounting height of the lamp h = 10 m (Fig. P.6.21, P.6.22).

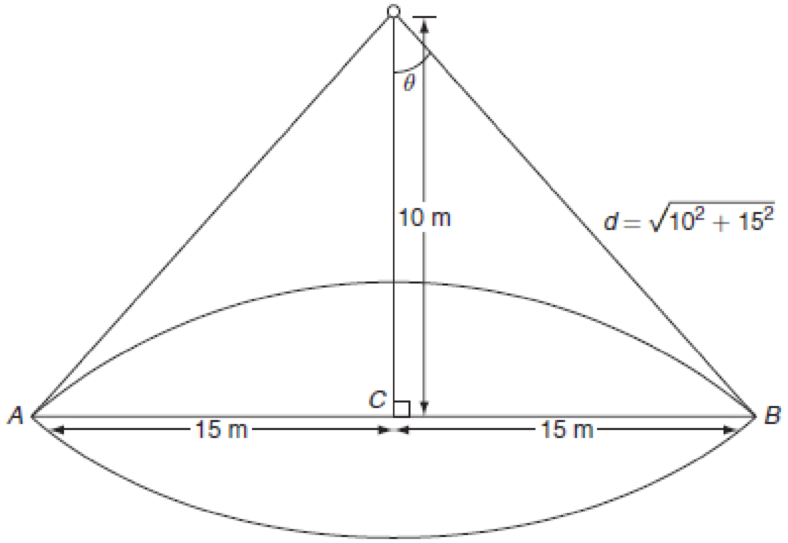


Fig. P.6.21

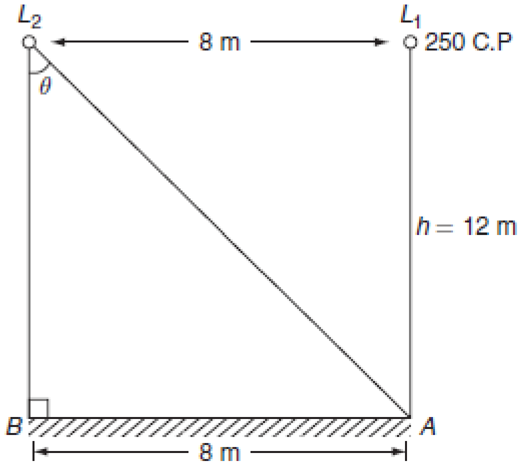


Fig. P.6.22

The diameter of the circular area = 30 m.

The candle power of the lamp I = 1,200 CP.

The maximum illumination occur just directly below the lamp, i.e., at point '*C*' is:

$$E_C = \frac{I}{d^2} = \frac{I}{h^2} = \frac{1200}{10^2} = 12 \text{ lux.}$$

Minimum Illumination will occur at the periphery of the circular area, i.e., at A (or) B.

$$\therefore E_A = E_B = \frac{I}{d^2}\cos\theta$$

$$= \frac{1200}{\left(\sqrt{10^2+15^2}\right)^2} \times \frac{10}{\sqrt{10^2+15^2}}$$

$$= \frac{12,000}{(10^2 + 15^2)^{3/2}}$$

$$= 2.048 \text{ lux.}$$

Example 6.22: Two lamps hung at a height of 12 m from the floor level. The distance between the lamps is 8 m. Lamp one is of 250 CP. If the illumination on the floor vertically below this lamp is 40 lux, find the CP of the second lamp.

Solution:

Given data:

The candle power of the lamp, I = 250 CP.

The intensity of L_1 illumination just below the lamp L_1 = 40 lux.

Let CP of L_2 = ICP.

∴ The illumination at the point A = the illumination due to the lamp L_1 +the illumination due to the lamp L_2:

$$40 = \frac{I_1}{h^2} + \frac{I}{h^2}\cos^3\theta$$

$$= \frac{250}{(12)^2} + \frac{I}{(12)^2}\left(\frac{12}{\sqrt{12^2 + 8^2}}\right)^3$$

$$= 1.736 + \frac{12I}{14.42}$$

$$\frac{12I}{14.42} = 38.263$$

$$I = 551.76 \text{ C.P.}$$

POLAR CURVES

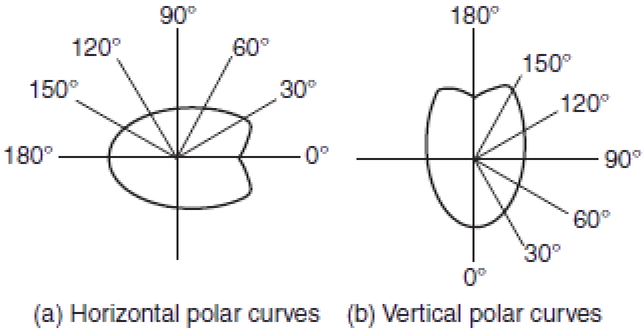
The luminous flux emitted by a source can be determined using the intensity distribution curve. Till now we assumed that the luminous intensity or the candle power from a source is distributed uniformly over the surrounding surface. But due to its s not uniform in all directions. The luminous intensity or the distribution of the light can be represented with the help of the polar curves.

The polar curves are drawn by taking luminous intensities in various directions at an equal angular displacement in the sphere. A radial ordinate pointing in any particular direction on a polar curve represents the luminous intensity of the source when it is viewed from that direction. Accordingly, there are two different types of polar curves and they are:

1. A curve is plotted between the candle power and the angular position, if the luminous intensity, i.e., candle power is measured in the horizontal plane about the vertical axis, called *'horizontal polar curve'*.

2. curve is plotted between the candle power, if it is measured in the vertical plane and the angular position is known as *'verticalpolar curve'*.

Figure 6.12 shows the typical polar curves for an ordinary lamp.



(a) Horizontal polar curves (b) Vertical polar curves

Fig Polar curves

Depression at 180° in the vertical polar curve is due to the lamp holder. Slight depression at 0° in horizontal polar curve is because of coiled coil filament.

Polar curves are used to determine the actual illumination of a surface by employing the candle power in that particular direction as read from the vertical polar curve. These are also used to determine mean horizontal candle power (MHCP) and mean spherical candle power (MSCP).

The mean horizontal candle power of a lamp can be determined from the horizontal polar curve by considering the mean value of all the candle powers in a horizontal direction.

The mean spherical candle power of a symmetrical source of a light can be found out from the polar curve by means of a Rousseau's construction.

Rousseau's construction

Let us consider a vertical polar curve is in the form of two lobes symmetrical about XOX^1 axis. A simple Rousseau's curve is shown in Fig. 6.13.

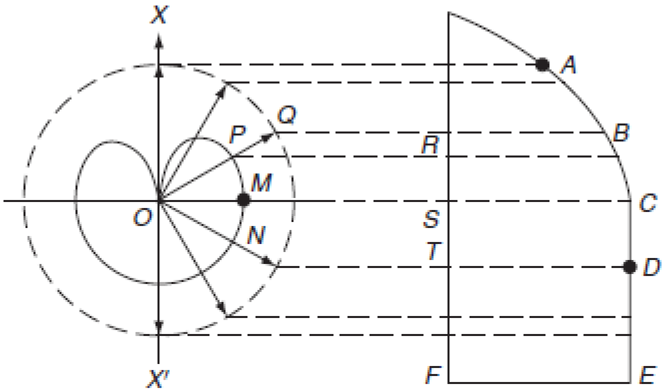


Fig. 6.13 Rousseau's curve

Rules for constructing the Rousseau's curve are as follows:

1. Draw a circle with any convenient radius and with '*O*' as center.
2. Draw a line *'AF'* parallel to the axis XOX^1 and is equal to the diameter of the circle.
3. Draw any line '*OPQ'* in such a way that the line meeting the circle at point '*Q*'. Now let the projection be '*R*' onto the parallel line *'AF'*.
4. Erect an ordinate at '*R*' as, *RB* = *OP*.
5. Now from this line *'AF'* ordinate equals to the corresponding radius on the polar curve are setup such as *SC* = *OM, TD* = *ON,* and so on.
6. The curve *ABC DEFA* so obtained by joining these ordinates is known as Rousseau's curve.

The mean ordinate of this curve gives the mean spherical candle power (MSCP) of the lamp having polar curve given in Fig. 6.13.

The mean ordinate of the curve:

$$= \frac{\text{area of } ABCDEFA}{\text{length of } AF}.$$

The area under the Rousseau's curve can be determined by Simpson's rule.

TYPES OF LIGHTING SCHEMES

Usually, with the reflector and some special diffusing screens, it is possible to control the distribution of light emitted from lamps up to some extent. A good lighting scheme results in an attractive and commanding presence of objects and enhances the architectural style of the interior of a building. Depending upon the requirements and the way of light reaching the surface, lighting schemes are classified as follows:

1. direct lighting,
2. semidirect lighting,
3. indirect lighting,
4. semi-indirect lighting, and
5. general lighting.

Direct lighting schemes

Direct lighting scheme is most widely used for interior lighting scheme. In this scheme, by using deep reflectors, it is possible to make 90% of light falls just below the lamp. This scheme is more efficient but it suffers from hard shadows and glare. Hence, while designing such schemes, all the possibilities that will cause glare on the eye have to be eliminated. It is mainly used for industrial and general outdoor lighting.

Semidirect lighting schemes

In semidirect lighting scheme, about 60–90% of lamps luminous flux is made to fall downward directly by using some reflectors and the rest of the light is used to illuminate the walls and ceiling. This type of light scheme is employed in rooms with high ceiling. Glare can be avoided by employing diffusing globes. This scheme will improve not only the brightness but also the efficiency.

Indirect lighting schemes

In this lighting scheme, 90% of total light is thrown upwards to the ceiling. In such scheme, the ceiling acts as the lighting source and glare is reduced to minimum.

This system provides shadowless illumination, which is very useful for drawing offices and in workshops where large machines and other difficulties would cause trouble some shadows if direct lighting schemes were used.

Semi-indirect lighting schemes

In semi-indirect lighting scheme, about 60–90% of light from the lamp is thrown upwards to the ceiling and the remaining luminous flux reaches the working surface. Glare will be completely eliminated with such type of lighting scheme. This scheme is widely preferred for indoor lighting decoration purpose.

General lighting scheme

This scheme of lighting use diffusing glasses to produce the equal illumination in all directions. Mounting height of the source should be much above eye level to avoid glare. Lamp fittings of various lighting schemes are shown in Fig. 7.20.

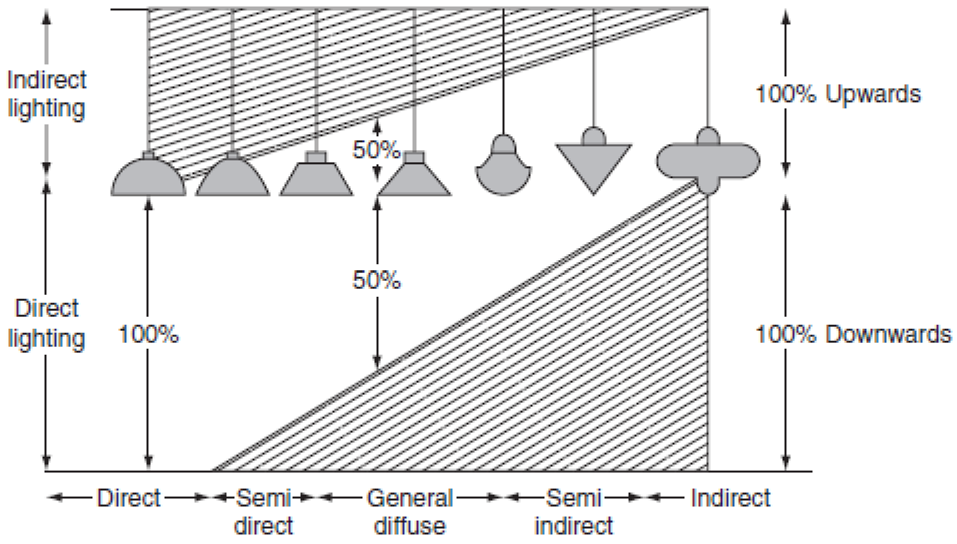


Fig. 7.20 Lighting schemes

DESIGN OF LIGHTING SCHEMES

The lighting scheme should be such that:

- It should be able to provide sufficient illumination.
- It should be able to provide the uniform distribution of light throughout the working plane.
- It should be able to produce the light of suitable color.
- It should be able to avoid glare and hard shadows as much as possible.

While designing a lighting scheme, the following factors should be taken into consideration.

1. Illumination level.
2. The size of the room.
3. The mounting height and the space of fitting.

STREET LIGHTING

Street lighting not only requires for shopping centers, promenades, etc. but also necessary for the following.

- In order to make the street more attractive, so that obstructions on the road clearly visible to the drivers of vehicles.
- To increase the community value of the street.
- To clear the traffic easily in order to promote safety and convenience.

The basic principles employed for the street lighting are given below.

1. Diffusion principle.
2. The specular reflection principle.

Diffusion principle

In this method, light is directed downwards from the lamp by the suitably designed reflectors. The design of these reflectors are in such a way that they may reflect total light over the road surface uniformly as much as possible. The reflectors are made to have a cutoff between 30° and 45°, so that the filament of the lamp is not visible expect just below the source, which results in eliminating glare. Illumination at any point on the road surface is calculated by applying inverse square low or point-by-point method.

Specular reflection principle

The specular reflection principle enables a motorist to see an object about 30 m ahead. In this case, the reflectors are curved upwards, so that the light is thrown on the road at a very large angle of incidence. This can be explained with the help of Fig. 7.21. An object resides over the road at ‘P’ in between the lamps S_1, S_2, and S_3 and the observer at ‘*Q*’.

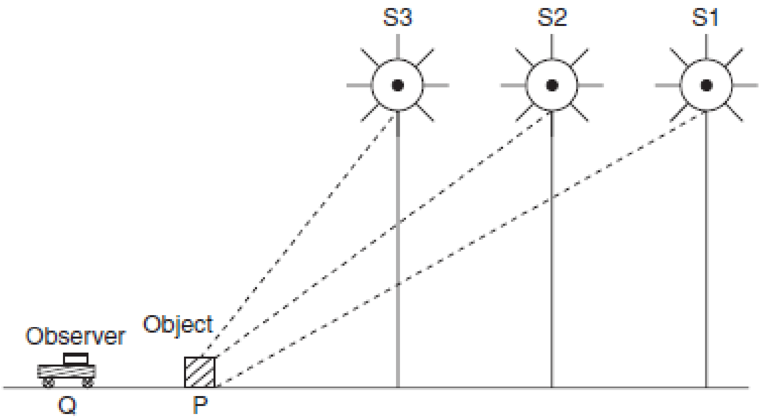


Fig. 7.21 Specular reflection for street lighting

Thus, the object will appear immediately against the bright road surface due to the lamps at a longer distance. This method of lighting is only suitable for straight sections along the road. In this method, it is observed that the objects on the roadway can be seen by a smaller expenditure of power than by the diffusion method of lighting.

Illumination level, mounting height, and the types of lamps for street lighting

Normally, illumination required depends upon the class of street lighting installation. The illumination required for different areas of street lighting are given in Table 7.3.

Table 7.3 Illumination required for different areas of street lighting

	Area	***Illumination (lumen/m^2)***
1.	Road junctions and important shopping centers.	30
2.	Poorly lighted sub-urban streets.	4
3.	Average well-lighted street.	8–15

Mercury vapor and sodium vapor discharge lamps are preferable for street lighting since the overall cost of the installation of discharge lamps are less than the filament lamps and also the less power consumption for a given amount of power output. Normal spacing for the standard lamps is 50 m with a mounting height of 8 m. Lamp posts should be fixed at the junctions of roads.

FLOODLIGHTING

Floodlighting means flooding of large surface areas with light from powerful projectors. A special reflector and housing is employed in floodlighting in order to concentrate the light emitted from the lamp into a relatively narrow beam, which is known as floodlight projector. This projector consists of a reflecting surface that may be a silvered glass or chromium plate or stainless steel. The efficiency of silvered glass and polished metal are 85–90% and 70%, respectively. Usually metal reflectors are robust; therefore, they can be preferred. An important application of illumination engineering is the floodlighting of large and open areas. It is necessary to employ floodlighting to serve one or more of the following purposes.

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METHODS OF LIGHTING CALCULATIONS

There are so many methods have been employed for lighting calculation, some of those methods are as follows.

1. Watts-per-square-meter method.
2. Lumen or light flux method
3. Point-to-point method

.

Example 7.1: A room 20 × 10 m is illuminated by 60 W incandescent lamps of lumen output of 1,600 lumens. The average illumination required at the workplace is 300 lux. Calculate the number of lamps required to be fitted in the room. Assume utilization and depreciation factors as 0.5 and 1, respectively.

Solution:

The area of the room (A) = 20 × 10 m

$= 200\ \text{m}^2$.

Total illumination required (E) = 300 lux.

The wattage of each lamp = 60 W

The luminous output of the lamp (φ) = 1,600 lumens

UF = 0.5, DF = 1.

∴ Maintenance factor, $\text{MF} = \frac{1}{DF} = \frac{1}{1} = 1.$

∴ The number of lamps required:

$$N = \frac{F \times A}{\phi \times \text{UF} \times \text{MF}}$$

$$= \frac{300 \times 200}{1{,}600 \times 1 \times 0.5} = 7.5 \text{ lamps.}$$

Example 7.2: The front of a building 35 × 18 m is illuminated by 15 lamps; the wattage of each lamp is 80 W. The lamps are arranged so that uniform illumination on the surface is obtained. Assuming a luminous efficiency of 20 lumens/W, the coefficient of utilization is 0.8, the waste light factor is 1.25, DF = 0.9. Determine the illumination on the surface.

Solution:

Area = (A) = 35 × 18 = 630 m^2.

The number of lamps, $N = 15$.

Luminous efficiency, $\eta = 20$ lumens/W.

UF = 0.8, DF = 0.9.

Waste light factor = 1.25, E = ?

$$\therefore\ N = \frac{A \times E \times \text{DF} \times \text{waste light factor}}{\text{UF} \times \eta \times \text{wattage of each lamp}}$$

$$15 = \frac{630 \times E \times 1.25 \times 0.9}{0.8 \times 20 \times 80}$$

$$= 0.554\ E.$$

$$\therefore \text{E} = 27.07 \text{ lux (or) lumens/m}^2.$$

Example 7.3: A room of size 10 × 4 m is to be illuminated by ten 150-W lamps. The MSCP of each lamp is 300. Assuming a depreciation factor of 0.8 and a utilization factor of 0.5. Find the average illumination produced on the floor.

Solution:

The area of the room (A) = 10 × 4 = 40 m^2.

The total luminous flux emitted by ten lamps (φ)

= 10 × 150 × 4π = 18, 849.5 lumens.

The total luminous flux reaching the working plane

$$= \frac{\phi \times \text{utilization factor}}{\text{depreciation factor}}$$

$$= \frac{18{,}849.5 \times 0.5}{0.8} = 11{,}780.97 \text{ lumens.}$$

The illumination on the working plane

$$E = \frac{\text{lumens on the working plane}}{\text{total area to be illuminated}}$$

$$= \frac{11{,}780.97}{40} = 294.52 \text{ lux.}$$

Example 7.4: The front of a building 25 × 12 m is illuminated by 20 1,200-W lamps arranged so that uniform illumination on the surface is obtained. Assuming a luminous efficiency of 30 lumens/W and a coefficient of utilization of 0.75. Determine the illumination on the surface. Assume DF = 1.3 and waste light factor 1.2.

Solution:

Area to be illuminated = 25 × 12 = 300 m^2.

The total lumens given out by 20 lamps is:

φ = number of lamps × wattage of each lamp × efficiency of each lamp

= 20 × 30 × 1,200 = 720,000 lumens.

The total lumens reaching the surface to be illuminated

$$= \frac{\phi \times UF}{DF \times \text{waste light factor}}.$$

$$= \frac{7{,}20{,}000 \times 0.75}{1.3 \times 1.2}$$

$$= 3{,}46{,}153.84 \text{ lumens.}$$

The illumination on the surface:

$$E = \frac{3,46,153.84}{300} = 1,153.84 \text{ lux.}$$

Example 7.5: An illumination of 40 lux is to be produced on the floor of a room 16 × 12 m. 15 lamps are required to produce this illumination in the room; 40% of the emitted light falls on the floor. Determine the power of the lamp in candela. Assume maintenance factor as unity.

Solution:

Given data

E = 40 lux

A = 16 × 12 = 192 m^2

Number of lamps, N = 15

UF = 0.4, MF = 1

$$N = \frac{E \times A}{\phi \times \text{UF} \times \text{MF}}$$

$$15 = \frac{40 \times 192}{\phi \times 0.4 \times 1}$$

$$\therefore \phi = 1{,}280 \text{ lux.}$$

So, the lumen output of the lamp in candela $= \frac{1,280}{4\pi} = 101.85$ cd.

Example 7.6: A drawing, with an area of 18 × 12 m, is to be illuminated with an average illumination of about 150 lux. The lamps are to be fitted at 6 m height. Find out the number and size of incandescent lamps required for an efficiency of 20 lumens/W. UF = 0.6, MF = 0.75.

Solution:

Given data:

η = 120 lumens/W

E = 150 lux

A = 18 × 12 = 216 m^2

UF = 0.6

MF = 0.75

$$\text{The total gross lumens required } \phi = \frac{E \times A}{\text{UF}\times\text{MF}}.$$

$$= \frac{150\times 216}{0.6\times 0.75} = 72,000 \text{ lumens.}$$

$$\text{The total wattage required } = \frac{72,000}{\eta}$$

$$= \frac{72,000}{20} = 3,600 \text{ W.}$$

Let, if 24 lamps are arranged to illuminate the desired area. For space to height ratio unity, i.e., 6 lamps are taken along the length with a space of 18/6 = 3m, and 4 lamps are along the width giving a space of 12/4 = 3 m.

$$\therefore \text{The wattage of each lamp } = \frac{3,600}{24} = 150 \text{ W.}$$

The arrangement of 24 lamps in a hall of 18 × 12 m is shown in Fig. P.7.1

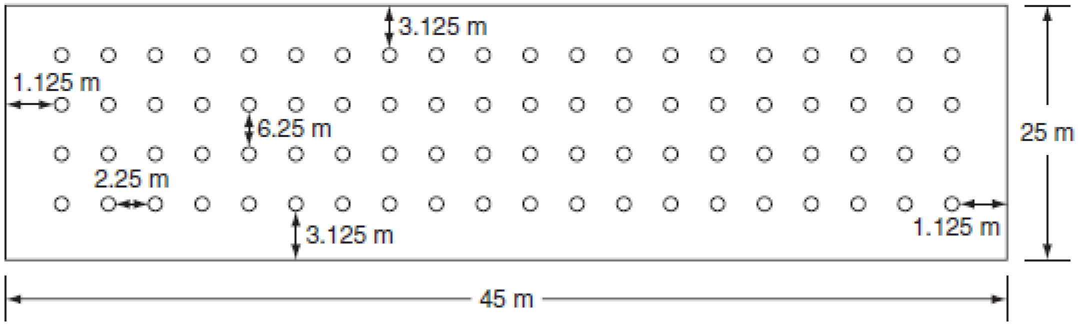


Fig. P.7.1 Lamp arrangement

Example 7.7: A hall of 30 × 20 m area with a ceiling height of 6 m is to be provided with a general illumination of 200 lumens/m^2, taking a coefficient of utilization of 0.6 and depreciation factor of 1.6. Determine the number of fluorescent tubes required, their spacing, mounting height, and total wattage. Take luminous efficiency of fluorescent tube as 25 lumens/W for 300-W tube.

Solution:

Given data:

Area of hall (A) = 30 × 20 m = 600 m^2

E = 200 lumens/m^2

CU = 0.6

DF = 1.6

The wattage of fluorescent tube = 300 W

Efficiency η = 25 lumens/W

$$\therefore \text{Gross lumens required, } \phi = \frac{A \times E \times \text{DF}}{\text{UF}}$$

$$= \frac{600 \times 200 \times 1.6}{0.6} = 320{,}000 \text{ lux.}$$

$$\text{The total wattage required } = \frac{\phi}{\eta} = \frac{320{,}000}{25}.$$

$$\text{The number of tubes required } = \frac{\text{total wattage required}}{\text{wattage of each tube}}$$

$$= \frac{12{,}800}{300}$$

$$= 42.666 \cong 44.$$

Let us arrange 44 lamps in a 30 × 30 m hall, by taking 11 lamps along the length with spacing 30/11 = 2.727 m and 4 lamps along the width with spacing 20/4 = 5m. Here the space to height ratio with this arrangement is, 2.727/5 = 0.545. Disposition of lamps is shown in Fig. P.7.2.

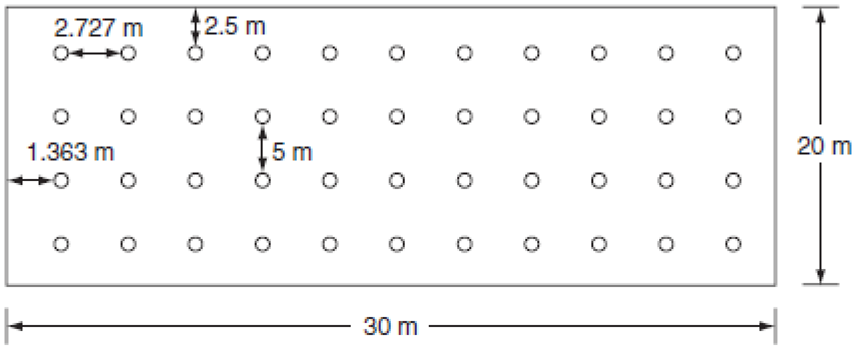
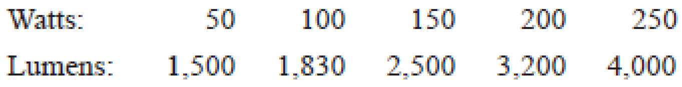


Fig. P.7.2 Lamp arrangement

Example 7.8: A hall 40-m long and 16-m wide is to be illuminated and illumination required is 70-m candles. Five types of lamps having lumen outputs, as given below are available.



Watts:	50	100	150	200	250
Lumens:	1,500	1,830	2,500	3,200	4,000

Taking a depreciation factor of 1.5 and a utilization coefficient of 0.7, calculate the number of lamps required in each case to produce required illumination. Out of above five types of lamps, select most suitable type and design, a suitable scheme, and make a sketch showing location of lamps. Assume a suitable mounting height and calculate space to height ratio of lamps.

Solution:

Given data:

Area (A) = 30 × 12 = 360 m^2

DF = 1.5

CU = 0.7

E = 50-m candle

Total gross lumens required:

$$\phi = \frac{A \times E \times \mathrm{DF}}{\mathrm{UF}}$$

$$= \frac{360 \times 50 \times 1.5}{0.7} = 38,572.42 \text{ lumens.}$$

1. If 50-W lamps are used, the number of lamps required $= \frac{38,571.42}{1,500} = 25.7 \cong 26.$
2. If 100-W lamps are used, the number of lamps required $= \frac{38,571.42}{1830} = 7.416 \cong 8.$
3. If 150-W lamps are used, the number of lamps required $= \frac{38,571.42}{2,500} = 15.42 \cong 16.$
4. If 200-W lamps are used, the number of lamps required $= \frac{38.571.42}{3,200} = 12.05 \cong 14.$
5. If 250-W lamps are used, the number of lamps required $= \frac{38,571.42}{4,000} = 9.642 \cong 10.$

Suitable type of lamp fitting will be 250-W lamps for a hall of 40 × 16 m.

Here, 10 lamps are arranged in two rows, each row having 5 lamps. By taking 5 lamps along the length with spacing 4058=m and 2 lamps along width side with spacing 16/2 = 8m, i.e., space to height ratio = 8/8 = 1.

The disposition of lamps is shown in Fig. P.7.3.

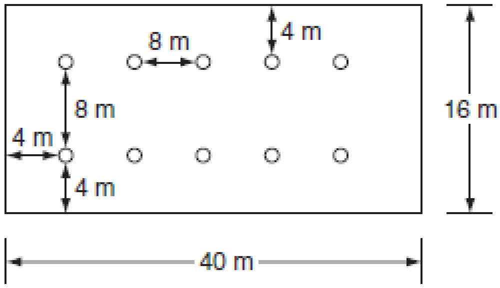


Fig. Lamp arrangement

Among the other lamps, some of wattage lamps require more number of lamp fittings and some other lamps will be few in requirement giving space–height ratio much more.