

CSE 573: Artificial Intelligence

Heuristics & Pattern Databases

Dan Weld

With slides from

Dan Klein, Stuart Russell, Andrew Moore, Luke Zettlemoyer, Dana Nau...

Outline

- Constructing Heuristics
- Pattern Databases
- Bonus Search Algorithms (Incomplete Ones)

- Reading:
R&N through Section 3.6 and Sections 4.1 & 4.2

Search thru a Problem Space (aka State Space)

- Input:

- Set of states
- Operators [and costs]
- Start state
- Goal state [or test]

Functions: States → States

Aka “Successor Function”

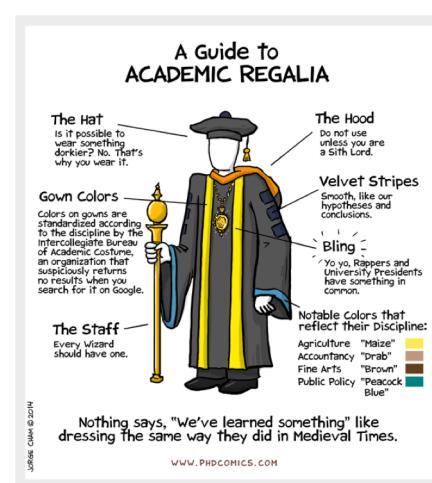
- Output:

- Path: start ⇒ a state satisfying goal test
[May require shortest path]
[Sometimes just need a state that passes test]

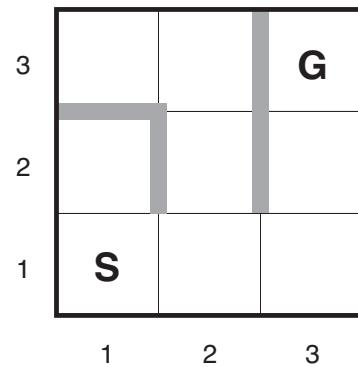
Getting a BS In CSE

Input:

- Set of states
- Operators [costs]
- Start state
- Goal state (test)



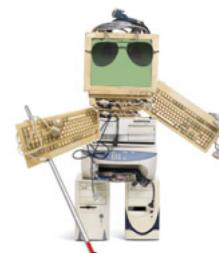
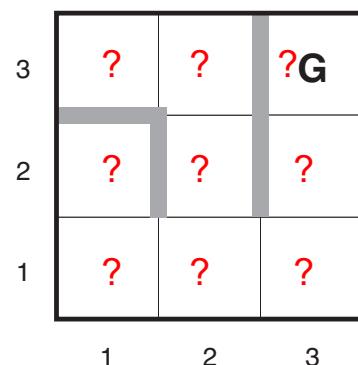
Search thru State Space



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What if Robot is Blind?

Moving into wall → noop

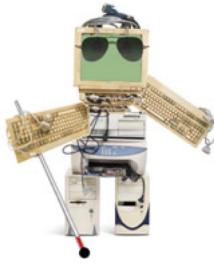


“Conformant Planning”

[Has a talking compass – knows which way is N]

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Conformant Planning



Sterilizing surgical gear

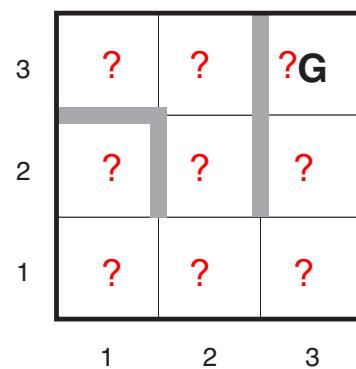
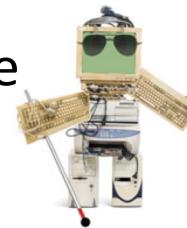


Bowl feeder

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Search thru State Space

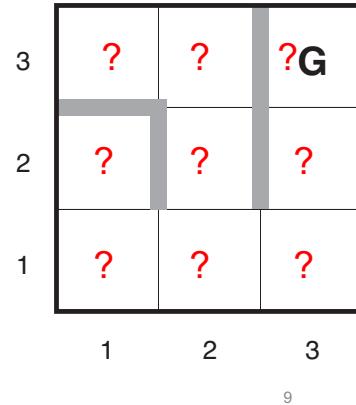
- States
 - SETS of states
 - “Belief state”
- Operators
 - Move actions
- Initial State
 - Set of all states
- Goal State
 - Set of just goal state(s)



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Soln: R, D, D, R, R, U, U

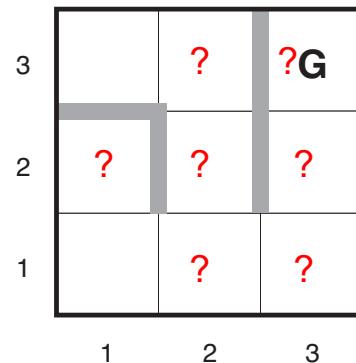
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Move Right

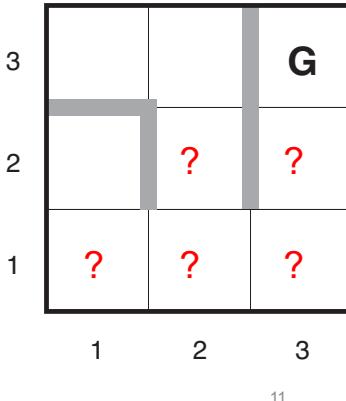
- States
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Move Down

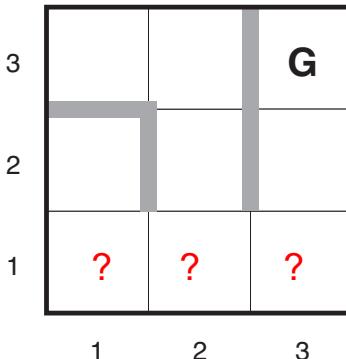
- **States**
 - SETS of states
 - “Belief state”
- **Operators**
 - Move actions
- **Initial State**
 - Set of all states
- **Goal State**
 - Set of just goal states



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Move Down

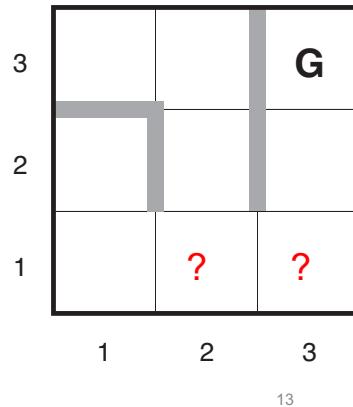
- **States**
 - SETS of states
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- **Operators**
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- **Initial State**
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 - Set of just goal states



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Move Right

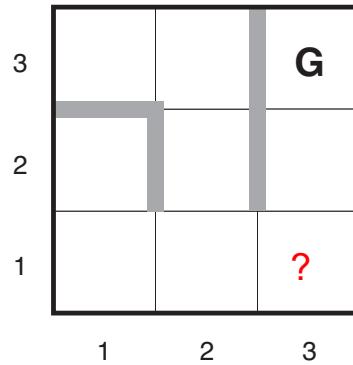
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Move Right

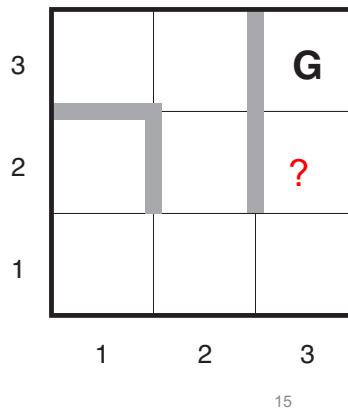
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Move Up

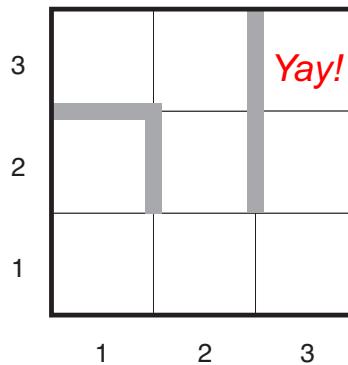
- **States**
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Move Up

- **States**
 - SETS of states
 - “Belief state”
- **Operators**
 - Move actions
- **Initial State**
 - Set of all states
- **Goal State**
 - Set of just goal states



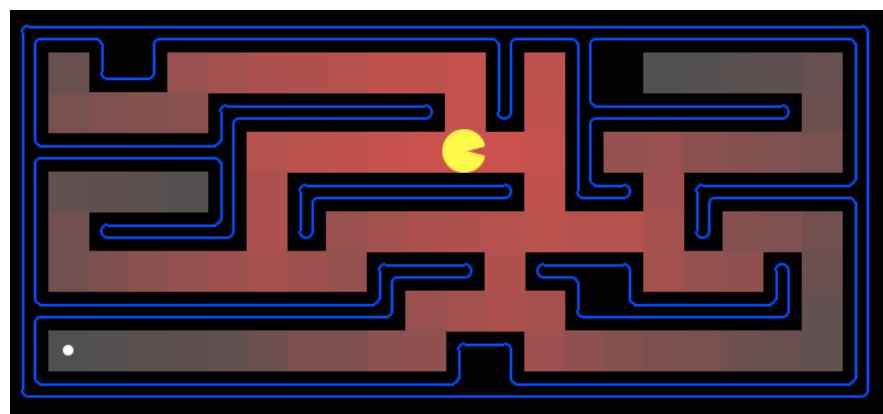
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Last Week's Methods

- Search Problems
- Uninformed Search Methods
 - Depth-First Search
 - Breadth-First Search
 - Iterative Deepening Search
 - Uniform-Cost Search
- Heuristic Search Methods
 - Uniform Cost = best first with $F(n) = G(n) = \text{cost-getting-to}(n)$
 - Greedy = best first with $F(n) = H(n) = \text{heuristic-est-distance}(n, \text{goal})$
 - A* = best first with $F(n) = G(n) + H(n)$
 - IDA*

Which Algorithm?

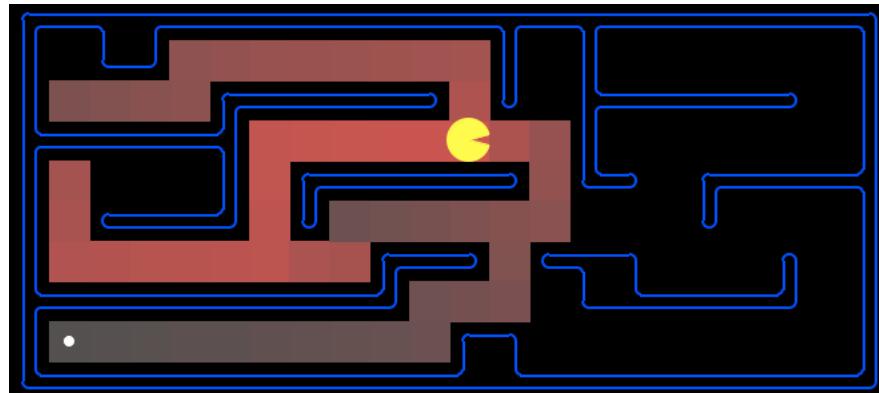
- Uniform cost search (UCS):



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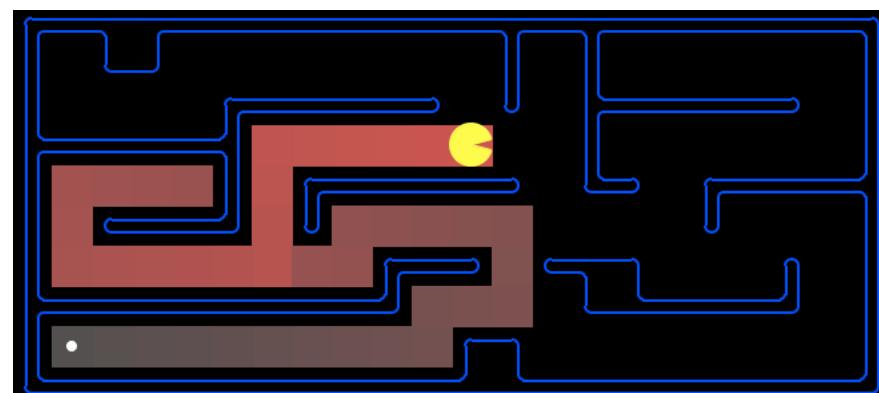
Which Algorithm?

- A*, Manhattan Heuristic:



Which Algorithm?

- Greedy Search, Manhattan Heuristic:



Demo

<http://qiao.github.io/PathFinding.js/visual/>

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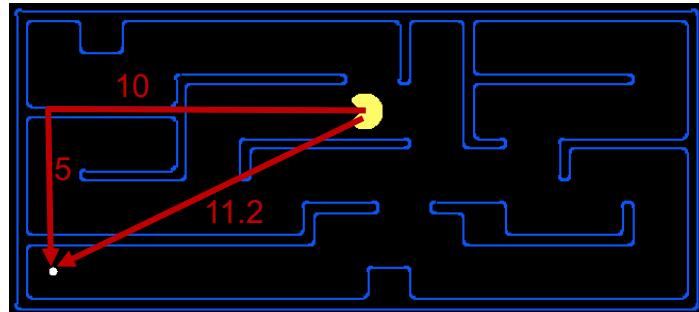
Heuristics

How we avoid exponential amounts of search



What is a *Heuristic*?

- An **estimate** of how close a state is to a goal
- Designed for a particular search problem



Admissible Heuristics

- $f(x) = g(x) + h(x)$
- g : cost so far
- h : underestimate of remaining costs

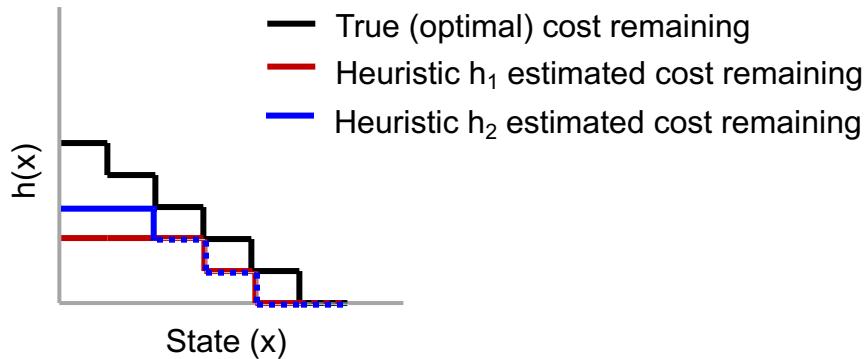
Questions

- 1) Which heuristic is better
- 2) Where do heuristics come from?

Dominance

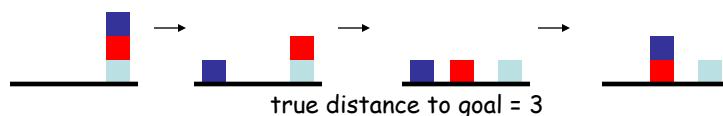
If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 dominates h_1

h_2 is better - guaranteed never to expand more nodes.



Relaxed Problems

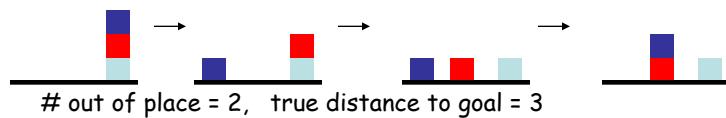
- Derive admissible heuristic from **exact** cost of a solution to a **relaxed** version of problem
 - For blocks world, distance = # move operations



- Cost of optimal soln to relaxed problem \leq cost of optimal soln for real problem

Relaxed Problems

- Derive admissible heuristic from **exact** cost of a solution to a **relaxed** version of problem
 - For blocks world, distance = # move operations
 - heuristic = number of misplaced blocks
 - What is relaxed problem?*



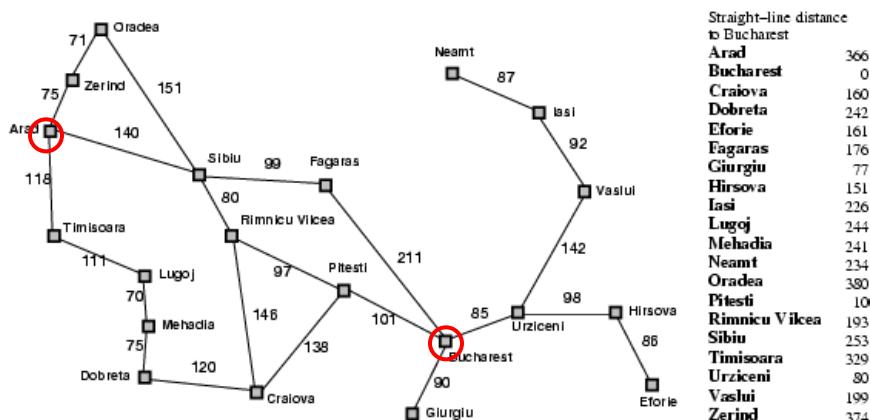
- Cost of optimal soln to relaxed problem \leq cost of optimal soln for real problem

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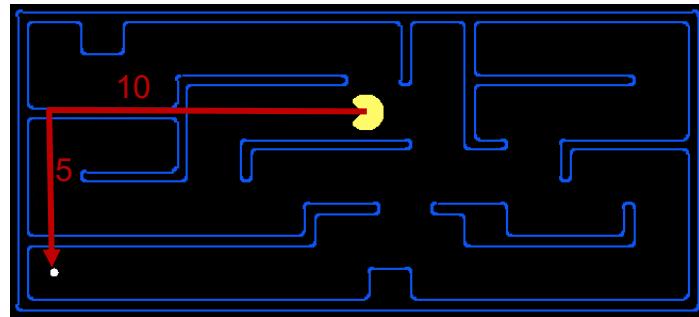
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What's being relaxed?

Heuristic = Euclidean distance



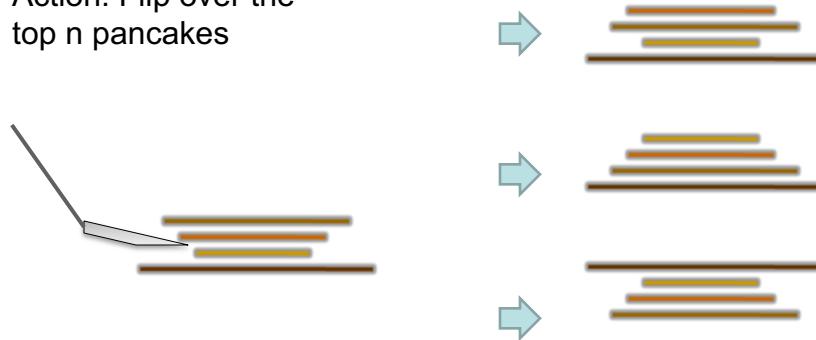
Manhattan Distance



What is Being Relaxed?

Example: Pancake Problem

Action: Flip over the top n pancakes



Cost: **Number** of pancakes flipped (not # flip actions)

Example: Pancake Problem

BOUNDS FOR SORTING BY PREFIX REVERSAL

William H. GATES

Microsoft, Albuquerque, New Mexico

Christos H. PAPADIMITRIOU*†

Department of Electrical Engineering, University of California, Berkeley, CA 94720, U.S.A.

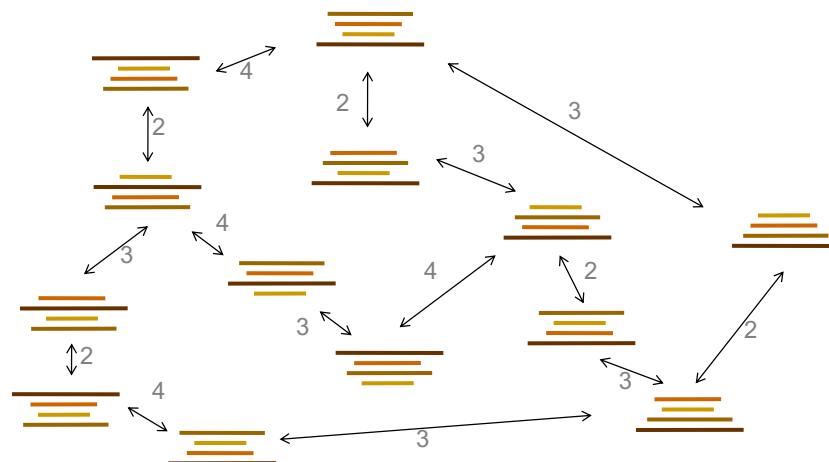
Received 18 January 1978

Revised 28 August 1978

For a permutation σ of the integers from 1 to n , let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n . We show that $f(n) \leq (5n+5)/3$, and that $f(n) \geq 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leq g(n) \leq 2n + 3$.

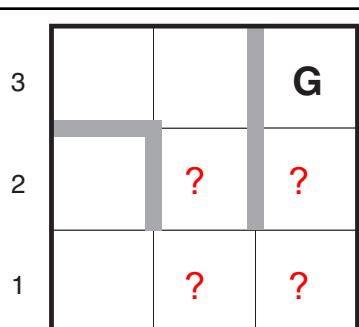
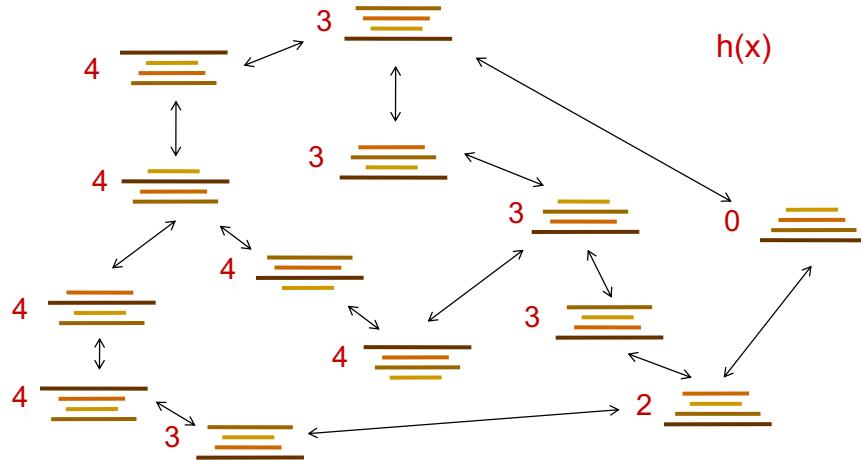
Example: Pancake Problem

State space graph with costs as weights

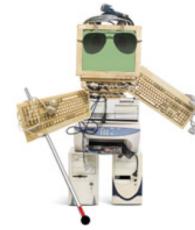


Pancake Heuristic?

Heuristic: the largest pancake that is still out of place



- States
 - SETS of states
 - “Belief state”
- Goal State
 - Set of just goal state(s)



Heuristics?

Relaxed Problem?

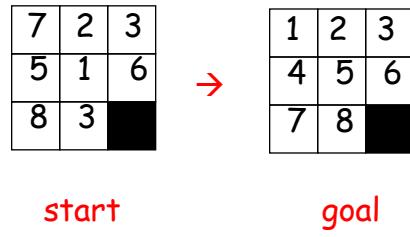
- What if it weren't blind?
- Max # moves from any state in belief state

Also... (admissible?)

- Number of states in belief state

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Heuristics for eight puzzle



- What can we relax?

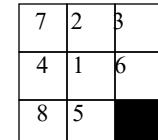
h_1 = number of tiles in wrong place

h_2 = Σ distances of tiles from correct loc

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Importance of Heuristics

h_1 = number of tiles in wrong place



D	A*(h1)
2	6
4	13
6	20
8	39
10	93
12	227
14	539
18	3056
24	39135

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Importance of Heuristics

h_1 = number of tiles in wrong place

7	2	3
4	1	6
8	5	

h_2 = Σ distances of tiles from correct loc

D	A*(h1)	A*(h2)
2	6	6
4	13	12
6	20	18
8	39	25
10	93	39
12	227	73
14	539	113
18	3056	363
24	39135	1641

Decrease effective branching factor

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Starship Raises \$25 Million to Roll Out More Delivery Robots

By [Chris Albrecht](#) - June 8, 2018 0



Starship Technologies, makers of squat, autonomous wheeled delivery robots, announced yesterday that the company has raised \$25 million in additional "seed"

Make your own Admissible Heuristic

- **Delivery robot**

- Can carry multiple packages
- States = location of robot & packages
- Cost = total distance traveled by robot



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Need More Power!

Performance of Manhattan Distance Heuristic

- 8 Puzzle < 1 second
- 15 Puzzle 1 minute
- 24 Puzzle 65000 years

Need even better heuristics!

Subgoal Interactions

- Manhattan distance assumes
 - Each tile can be moved independently of others
- Underestimates because
 - Doesn't consider interactions between tiles

1	2	3
4	6	5
7	8	

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Pattern Databases

[Culberson & Schaeffer 1996]

- Pick any subset of tiles
 - E.g., tiles 1-8
- Precompute a table
 - Optimal cost of solving just these tiles
 - For all possible configurations
 - ~ 50 Million in this case
 - Use A* or IDA*
 - State = position of just these tiles (& blank)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

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Using a Pattern Database

- As each state is generated
 - Use position of chosen tiles as index into DB
 - Use lookup value as heuristic, $h(n)$
- Admissible?

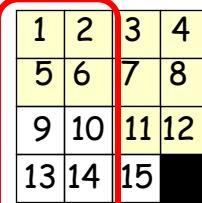
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Combining Multiple Databases

- Can choose another set of tiles
 - Precompute multiple tables
- How combine table values?
- E.g. Optimal solutions to Rubik's cube
 - First found w/ IDA* using pattern DB heuristics
 - Multiple DBs were used (dif cubie subsets)
 - Most problems solved optimally in 1 day
 - Compare with 574,000 years for IDDFS



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

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Drawbacks of Standard Pattern DBs

- Since we can only take *max*
 - Diminishing returns on additional DBs
 - Would like to be able to *add* values

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Disjoint Pattern DBs

- Partition tiles into disjoint sets
 - For each set, precompute table
 - E.g. 8 tile DB has 519 million entries
 - And 7 tile DB has 58 million
- During search
 - Look up heuristic values for each set
 - *Can add values without overestimating!*
- Manhattan distance is a special case of this idea where each set is a single tile

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

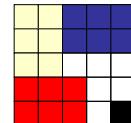
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Performance

- **15 Puzzle:** 2000x speedup vs Manhattan dist
 - IDA* with the two DBs shown previously solves 15 Puzzles optimally in 30 milliseconds
- **24 Puzzle:** 12 million x speedup vs Manhattan
 - IDA* can solve random instances in 2 days.
 - Requires 4 DBs as shown
 - Each DB has 128 million entries
 - Without PDBs: 65,000 years



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