Natural Language Processing (CSE 447/547M): Word Representations

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Preliminaries

Token: an instance of a word observed in text

Type: the word in the abstract

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   - More methods today

The Word-Document Matrix

Let $\mathbf{A} \in \mathbb{R}^{V \times C}$ contain statistics of association between words in $\mathcal{V}$ and C documents.

N is the total number of word tokens.

Tiny example, three documents:

- `yes , we have no bananas`
- `say yes for bananas`
- `no bananas , we say`

	1	2	3
`,`	1	0	1
`bananas`	1	1	1
`for`	0	1	0
`have`	1	0	0
`no`	1	0	1
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`we`	1	0	1
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Count matrix: $[\mathbf{A}]_{v,c} = c_{\boldsymbol{x}_c}(v)$

A Word-Context Matrix

Let $\mathbf{A} \in \mathbb{R}^{V \times C}$ contain statistics of association between words in $\mathcal{V}$ and symbols that occur just before them

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- `yes , we have no bananas`
- `say yes for bananas`
- `no bananas , we say`

	○ _	, _	bananas _	for _	have _	no _	say _	we _	yes _
,	0	0	1	0	0	0	0	0	1
bananas	0	0	0	1	0	2	0	0	0
for	0	0	0	0	0	0	0	0	1
have	0	0	0	0	0	0	0	1	0
no	1	0	0	0	1	0	0	0	0
say	1	0	0	0	0	0	0	1	0
we	0	2	0	0	0	0	0	0	0
yes	1	0	0	0	0	0	1	0	0

Count matrix: $[\mathbf{A}]_{v,v'} = c_{\boldsymbol{x}}(v'v)$

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Intuition: consider the ratio of *observed* frequency ($c_{\boldsymbol{x}_c}(v)$) to "chance" under independence ($\frac{c_{\boldsymbol{x}_{1:C}}(v)}{N} \cdot \ell_c$).

Pointwise Mutual Information

A common starting point is positive **pointwise mutual information**:

$$[\mathbf{A}]_{v,c} = \left[\log \frac{c_{\boldsymbol{x}_c}(v)}{\frac{c_{\boldsymbol{x}_{1:C}}(v)}{N} \cdot \ell_c}\right]_+ = \left[\log \frac{N \cdot c_{\boldsymbol{x}_c}(v)}{c_{\boldsymbol{x}_{1:C}}(v) \cdot \ell_c}\right]_+$$

From our example:

$$[\mathbf{A}]_{\text{bananas},1} = \log \frac{15 \cdot 1}{3 \cdot 6} \approx -0.18 \to 0$$

$$[\mathbf{A}]_{\text{for},2} = \log \frac{15 \cdot 1}{1 \cdot 4} \approx 1.32$$

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- One way to think about PMI: it's telling us where a unigram model is most wrong.
- We could use $\mathbf{A}$ as $\mathbf{M}$ (though d is usually much smaller than C) ...

Topic Models: Latent Semantic Indexing/Analysis

(Deerwester et al., 1990)

LSI/A seeks to solve:

$$\underset{V \times C}{\mathbf{A}} \approx \hat{\mathbf{A}} = \underset{V \times d}{\mathbf{M}} \times \underset{d \times d}{\operatorname{diag}(\mathbf{s})} \times \underset{d \times C}{\mathbf{C}}^{\top}$$

where $\mathbf{M}$ contains embeddings of words, $\mathbf{C}$ contains embeddings of documents.

$$[\mathbf{A}]_{v,c} \approx \sum_{i=1}^{d} [\mathbf{m}_v]_i \cdot [\mathbf{s}]_i \cdot [\mathbf{c}_c]_i$$

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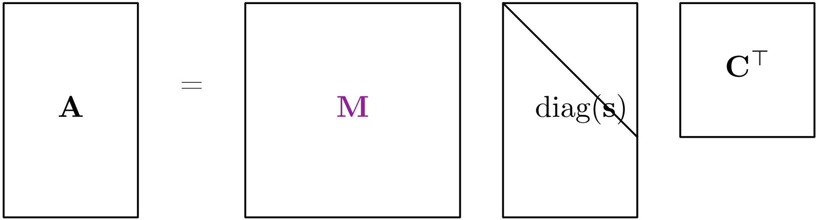
$$[\mathbf{A}]_{v,c} \approx \sum_{i=1}^{d} [\mathbf{m}_v]_i \cdot [\mathbf{s}]_i \cdot [\mathbf{c}_c]_i$$

This can be solved by applying singular value decomposition to $\mathbf{A}$, then truncating to d dimensions.

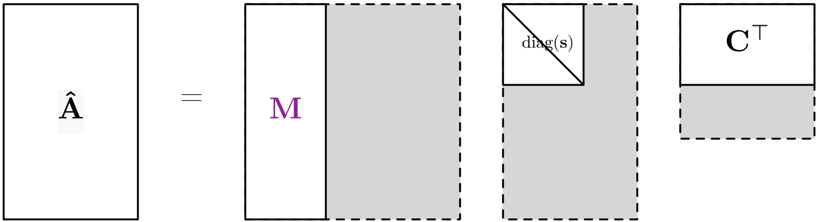
- $\mathbf{M}$ contains left singular vectors of $\mathbf{A}$
- $\mathbf{C}$ contains right singular vectors of $\mathbf{A}$
- $\mathbf{s}$ are singular values of $\mathbf{A}$; they are nonnegative and conventionally organized in decreasing order.

Truncated Singular Value Decomposition

SVD:



truncated at d:



A Nod to Linear Algebra

For (not truncated) singular value decomposition $\mathbf{A} = \mathbf{M} \times \operatorname{diag}(\mathbf{s}) \times \mathbf{C}^\top$:

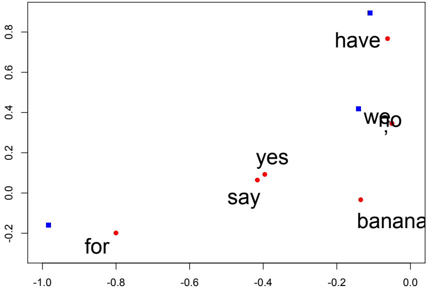
- The columns of $\mathbf{M}$ form an orthonormal basis, $\mathbf{M}$ are eigenvectors of $\mathbf{A}\mathbf{A}^\top$, with eigenvalues $\mathbf{s}^2$.
- The columns of $\mathbf{C}$ form an orthonormal basis, $\mathbf{C}$ are eigenvectors of $\mathbf{A}^\top\mathbf{A}$, with eigenvalues $\mathbf{s}^2$.

If some elements of $\mathbf{s}$ are zero, then $\mathbf{A}$ is "low rank."

Approximating $\mathbf{A}$ by truncating $\mathbf{s}$ equates to a "low rank approximation."

LSI/A Example

$k = 2$



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Words and documents in two dimensions.

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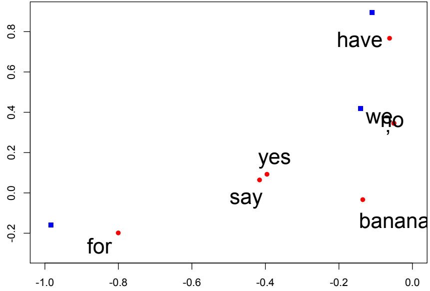

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Words and documents in two dimensions.

Note how `no`, `we`, and `,` are all in the exact same spot. Why?

Understanding LSI/A

- Mapping words and documents into the same k-dimensional space.
- Bag of words assumption (Salton et al., 1975): a document is nothing more than the distribution of words it contains.
- Distributional hypothesis (Harris, 1954; Firth, 1957): words are nothing more than the distribution of contexts (here, documents) they occur in. Words that occur in similar contexts have similar meanings.
- $\mathbf{A}$ is sparse and noisy; LSI/A "fills in" the zeroes and tries to eliminate the noise.
  - It finds the best rank-k approximation to $\mathbf{A}$.

Local Contexts: Distributional Semantics

Within NLP, emphasis has shifted from word/document associations to the relationship between $v \in \mathcal{V}$ and more local contexts.

For example: LSI/A, but replace documents with "nearby words." This is a way to recover word vectors that capture distributional similarity.

These models are designed to "guess" a word at position i given a word at a position in $\{i - w, \ldots, i - 1\} \cup \{i + 1, \ldots, i + w\}$. (My example on slide 12 used only position $i - 1$.)

Sometimes such methods are used to "pre-train" word vectors used in other, richer models (like neural language models).

word2vec

(Mikolov et al., 2013a,b)

Two models for word vectors designed to be computationally efficient.

- Continuous bag of words (CBOW): $p(v \mid c)$
  - Similar in spirit to the feedforward neural language model we saw before (Bengio et al., 2003)
- Skip-gram: $p(c \mid v)$

It turns out these are closely related to matrix factorization as in LSI/A (Levy and Goldberg, 2014).

Skip-Gram Model

$$p(C = c \mid X = v) = \frac{1}{Z_v} \exp \mathbf{c}_c^\top \mathbf{v}_v$$

- Two different vectors for each element of $\mathcal{V}$: one when it is "v" $(\mathbf{v})$ and one when it is "c" $(\mathbf{c})$.
- Normalization term Z_v is expensive, so approximations are required for efficiency.
- Can vary the context window, or expand this to be over the whole sentence or document, or otherwise choose which words "count" as contexts.

Word Vector Evaluations

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Word Vector Evaluations

Several popular methods for *intrinsic* evaluations:

- How well do (cosine) similarities of pairs of words' vectors correlate with judgments of word similarity by humans?
- TOEFL-like synonym tests, e.g., *rug* $\xrightarrow{?}$ $\{$*sofa*, *ottoman*, *carpet*, *hallway*$\}$
- Syntactic analogies, e.g., "*walking* is to *walked* as *eating* is to what?" Solved via:

$$\min_{v \in \mathcal{V}} \cos\left(\mathbf{v}_v, \mathbf{v}_{\mathit{walking}} - \mathbf{v}_{\mathit{walked}} + \mathbf{v}_{\mathit{eating}}\right)$$

Extrinsic Evaluations

1. Use large unannotated corpus to get your word vectors (called **pretraining**).
2. Use them in a text classifier (or some other NLP system, more examples to be introduced in future lectures). Two options:
   - Plug in word vectors as "frozen" features.
   - Treat them as parameters of the text classifier; pretraining gives initial values, but they get updated, or "finetuned" during supervised learning.
3. Does that system's performance improve?

Stepping Back

Big ideas:

- This is *unsupervised* learning: all you need is lots of raw text (no labels!)
- Large corpora $\rightarrow$ powerful word representations (the distributional hypothesis from linguistics, brought to life through engineering)
- It's all about the relationship between *words* and their *contexts*

Embeddings from Language Models (ELMo)

(Peters et al., 2018)

Why not give every word *token* (in context) its own vector?

- To do that, we need a function that maps contexts to vectors.
- An RNN language model can do that, for the entire left context.
  - $\mathbf{s}_i$ is x_i's history. $\boldsymbol{s}_{i+1} = f_{\text{recurrent}}(\mathbf{e}_{x_i}, \mathbf{s}_i)$ is a (left-)*contextualized* embedding of the token x_i.
  - To get the right-context, run an RNN language model from right to left, and extract the analogous state vector just after reading x_i. Concatenate the two.
- On extrinsic evaluations, this method gave big improvements to state of the art systems.

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