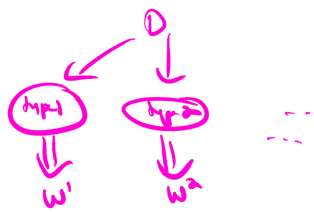
Using EM to compute MLE for mixture models

dataset: $y_1, \dots, y_n$

prob π_j of being type j [k possible types)

given of type j $\quad y_i \sim f_{\Theta_j}$

Given $y_1, \dots, y_n$ determine MLE's of π_j's Θ_j's



$$\hat{\pi}_j = \frac{\#\text{ of type } 1}{n}$$

EM: initialize Θ_j's, π_j params$_0$

for $t = 0, 1, \dots$ until convergence

E: given $\boxed{\text{params}_t}$ $E(z_{ij} \mid y_i)$ $\qquad \Theta_j^t, \pi_j^t$

$$= \Pr(z_{ij}=1 \mid y_i) = \frac{\Pr(y_i \mid z_{ij}=1)\Pr(z_{ij}=1)}{\Pr(y_i)}$$

$$= \frac{f_{\Theta_j^t}(y_i)\,\pi_j^t}{\sum_{\ell=1}^{k} \underbrace{\Pr(y_i \mid z_{i\ell}=1)}_{f_{\Theta_\ell^t}(y_i)} \underbrace{\Pr(z_{i\ell}=1)}_{\pi_\ell^t}}$$

M: [$z_{ij} = 1\{\text{point } i \text{ is of type } j\}$ Suppose knew z_{ij}'s

$$\text{Likelihood} = \prod_{i=1}^{n} \underbrace{\prod_{j=1}^{k} \left[f_{\Theta_j}(y_i)\right]^{z_{ij}}}$$

only 1 of $z_{i1}, \dots, z_{ik}$ is 1

$$\text{LogLike} = \sum_{i=1}^{n} \sum_{j=1}^{k} z_{ij} \ln\left(f_{\Theta_j}(y_i)\right) \Big]$$

find params$_{t+1}$ to max

$$\underset{z_{ij}\text{'s}}{E}\left(\text{LogLike}\right) = \sum_{i=1}^{n} \sum_{j=1}^{k} \underbrace{E(z_{ij} \mid y_i)}\, \ln\left(f_{\Theta_j}(y_i)\right)$$

$$\pi_j^k = \frac{\sum_i E(z_{ij} \mid y_i)}{n}$$

this is how you update π_j's.

you can use this without proof