CSE 573: Artificial Intelligence

Problem Spaces & Search

Dan Weld

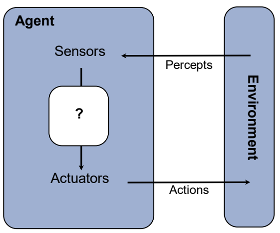
With slides from
Dan Klein, Stuart Russell, Andrew Moore, Luke Zettlemoyer, Dana Nau…

Outline

- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Iterative Deepening Search
  - Uniform-Cost Search
- Heuristic Search Methods
- Heuristic Generation

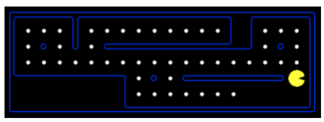
Agent *vs.* Environment

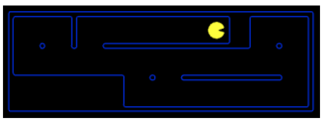
- An **agent** is an entity that *perceives* and *acts*.
- A **rational agent** selects actions that maximize its **utility function**.
- Characteristics of the **percepts, environment,** and **action space** dictate techniques for selecting rational actions.



Goal Based Agents

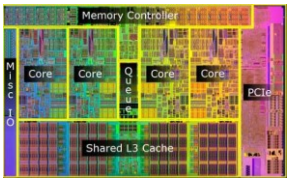
- Plan ahead
- Ask "what if"
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Act on how the world WOULD BE





Search: It's not just for Agents

Hardware verification



Planning optimal repair sequences

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Search thru a Problem Space (aka State Space)

- Input:
  - Set of states
  - Operators *[and costs]*
  - Start state
  - Goal state *[or test]*

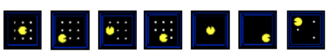
Functions: States → States

Aka "Successor Function"

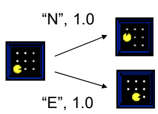
- Output:
  - Path: start $\Rightarrow$ a state satisfying goal test
    [May require shortest path]
    [Sometimes just need a state that passes test]

Example: Simplified Pac-Man

- Input:
  - A state space



  - Successor function



  - A start state
  - A goal test
- Output:

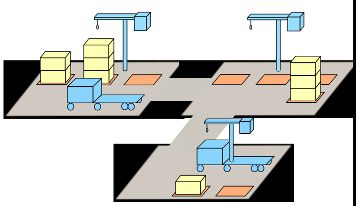
Ex: Route Planning: Arad → Bucharest

- Input:
  - Set of states
  - Operators [and costs]
  - Start state
  - Goal state (test)
- Output:

Different operators may be applicable in different states

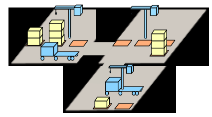
Ex: Dock Worker Robots

- A harbor with several locations
  - e.g., docks, docked ships, storage areas, parking areas
- Containers
  - going to/from ships
- Robot vehicles
  - can move containers
- Cranes
  - can load and unload containers



- Multiple robots can operate at the same time
- Move, load & other actions have **different durations**

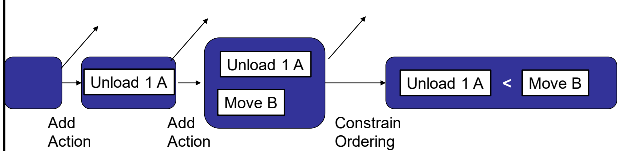
Dock Worker 2



Input:

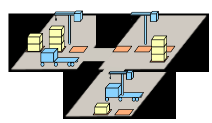
- Set of states
  - *Partially specified plans*
- Operators [and costs]
  - *Plan modification operators*
- Start state
  - *The null plan (no actions)*
- Goal test
  - *A plan which provably achieves the desired world configuration*

Plan Space



Blue boxes are plans = states in search space
Operators modify plans
Successors(p) = all possible ways of modifying p

Multiple Problem Spaces



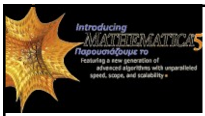
Real World

States of the world (e.g. loading dock configurations)
Actions (take one world-state to another)

Robot's Head

- **Problem Space 1**
  - PS states =
    - models of world states
  - Operators =
    - models of actions

- **Problem Space 2**
  - PS states =
    - partially spec. plan
  - Operators =
    - plan modificat'n ops



Algebraic Simplification

$$
\begin{aligned}
\partial_r^2 u &= -\left[E' - \frac{l(l+1)}{r^2} - r^2\right] u(r) \\
e^{-2s}\left(\partial_s^2 - \partial_s\right) u(s) &= -\left[E' - l(l+1)e^{-2s} - e^{2s}\right] u(s) \\
e^{-2s}\left[e^{\frac{1}{2}s}\left(e^{-\frac{1}{2}s}u(s)\right)'' - \frac{1}{4}u\right] &= -\left[E' - l(l+1)e^{-2s} - e^{2s}\right] u(s) \\
e^{-2s}\left[e^{\frac{1}{2}s}\left(e^{-\frac{1}{2}s}u(s)\right)''\right] &= -\left[E' - \left(l+\frac{1}{2}\right)^2 e^{-2s} - e^{2s}\right] u(s) \\
v'' &= -e^{2s}\left[E' - \left(l+\frac{1}{2}\right)^2 e^{-2s} - e^{2s}\right] v
\end{aligned}
$$

- Input:
  - Set of states
  - Operators [and costs]
  - Start state
  - Goal state (test)
- Output:

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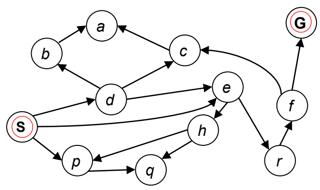
State Space Graphs

- State space graph:
  - Each node is a state
  - The operators are represented by arcs
  - Edges may be labeled with costs

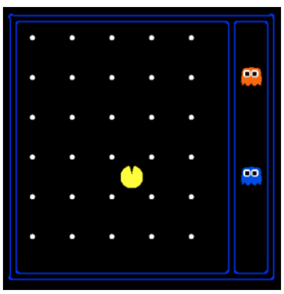


Ridiculously tiny search graph for a tiny search problem

- In contrast to other areas of CS…
- We can rarely build this graph in memory (so we don't try)
- … just a conceptual tool

State Space Sizes?

- Search Problem:
  Eat all of the food
- Pacman positions:
  10 x 12 = 120
- Pacman facing:
  up, down, left, right
- Food configurations: 2^{30}
- Ghost1 positions: 12
- Ghost 2 positions: 11

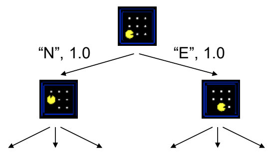


$120 \times 4 \times 2^{30} \times 12 \times 11 = 6.8 \times 10^{13}$

Search Methods

- Blind Search
  - Depth first search
  - Breadth first search
  - Iterative deepening search
  - Uniform cost search
- Local Search
- Informed Search
- Constraint Satisfaction
- Adversary Search

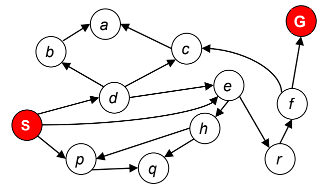
Search Trees



- A search tree:
  - Start state at the root node
  - Children correspond to successors
  - Nodes ***contain*** states, correspond to PLANS to those states
  - Edges are labeled with actions and costs
  - For most problems, we can never actually build the whole tree

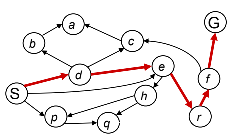
Example: Tree Search

State graph:



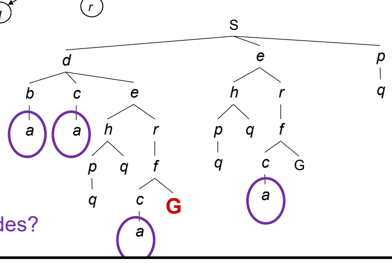
What is the search tree?

State Graphs vs. Search Trees



Each NODE in in the search tree denotes an entire PATH in the problem graph.

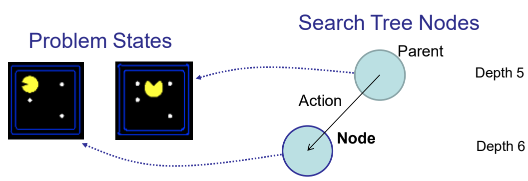
We construct both on demand – and we construct as little as possible.



Why duplicate nodes?

States vs. Nodes

- Vertices in state space graphs are problem states
  - Represent an abstracted state of the world
  - Have successors, can be goal / non-goal, have multiple predecessors
- Vertices in search trees ("Nodes") are plans
  - Contain a problem state and one parent, a path length, a depth & a cost
  - Represent a plan (sequence of actions) which results in the node's state
  - The same problem state may be achieved by multiple search tree nodes



Building Search Trees

- Search:
  - Expand out possible nodes (plans) in the tree
  - Maintain a fringe of unexpanded nodes
  - Try to expand as few nodes as possible

General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

Detailed pseudocode is in the book!

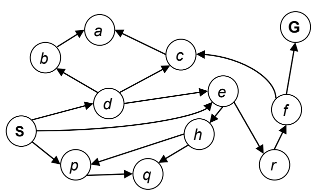
Important ideas:
- Fringe (leaves of tree)
- Expansion (adding successors of a leaf)
- Exploration strategy

which fringe node to expand next?

Review: Breadth First Search

***Strategy**: expand shallowest node first*

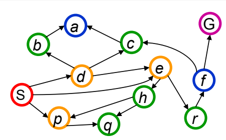
***Implementation**: Fringe is a queue - FIFO*

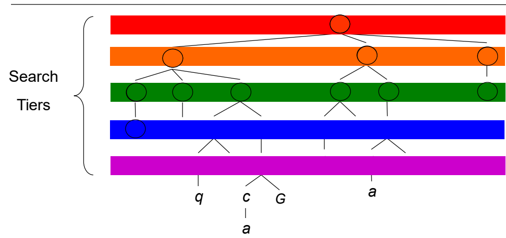


Review: Breadth First Search

Expansion order:

(S,d,e,p,b,c,e,h,r,q,a,a ,h,r,p,q,f,p,q,f,q,c,G)

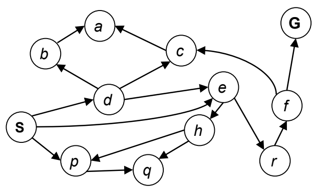




Review: Depth First Search

Strategy: expand ***deepest*** node first

Implementation: *Fringe is a stack - LIFO*

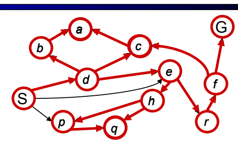


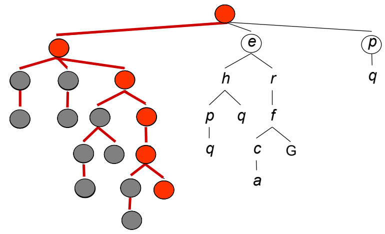
Wolog: assume we choose children in lexicographic order

Review: Depth First Search

Expansion ordering:

(d,b,a,c,a,e,h,p,q,q,r,f,c,a,G)





Search Algorithm Properties

- Complete? Guaranteed to find a solution if one exists?
- Optimal? Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

Variables:

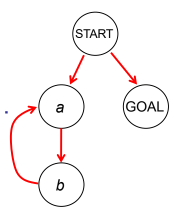
n	Number of states in the problem
b	The maximum branching factor *B* (the maximum number of successors for a state)
$C*$	Cost of least cost solution
d	Depth of the shallowest solution
m	Max depth of the search tree

Depth-First Search

Assuming finite tree

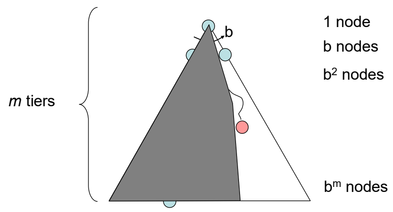
Algorithm		Complete	Optimal	Time	Space
DFS	Depth First Search	No	No	$O(b^m)$	O(b m)

- Infinite paths make DFS incomplete…
  - How can we fix this?
  - Check new nodes against ***path*** from S



d depth of solution
m max depth of tree

DFS Search (w/ cycle checking)

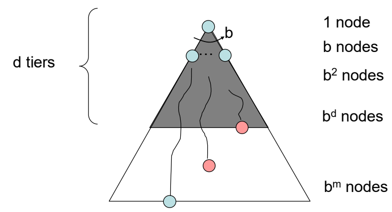


Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y if finite	N	$O(b^m)$	$O(b\ m)$

Only if finite tree

BFS Tree Search

Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	N unless finite	N	$O(b^m)$	$O(bm)$
BFS		Y*	Y*	$O(b^d)$	$O(b^d)$



* Assuming finite branching factor

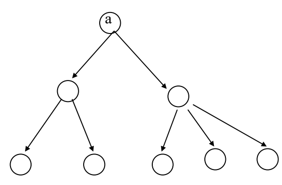
Memory a Limitation?

- Suppose:
  - 4 GHz CPU
  - 32 GB main memory
  - 100 instructions / expansion
  - 5 bytes / node

- 40 M expansions / sec
  - Memory filled in … 3 min

Iterative Deepening Search

- DFS with limit; incrementally grow limit
- Evaluation



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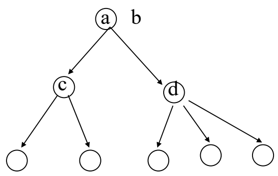
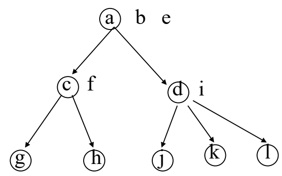
Iterative Deepening Search

- DFS Tree Search with limit; incrementally grow limit
- Evaluation



Iterative Deepening Search

- DFS Tree Search with limit; incrementally grow limit
- Evaluation
  - Complete?
  - Time Complexity?
  - Space Complexity?



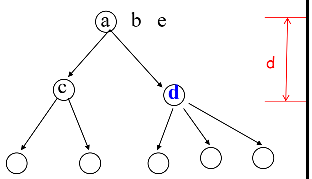
Iterative Deepening Search

- DFS with limit; incrementally grow limit
- Evaluation
  - Complete?

    *Yes **
  - Time Complexity?

    $O(b^d)$
  - Space Complexity?

    $O(bd)$



* Assuming branching factor is finite
Important Note: no cycle checking necessary!

Cost of Iterative Deepening

b	ratio ID to DFS
2	3
3	2
5	1.5
10	1.2
25	1.08
100	1.02

Puzzles

15 puzzle

3x3 Rubics cube

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Speed

Assuming 10M nodes/sec & sufficient memory

	BFS Nodes	BFS Time		Iter. Deep. Nodes	Iter. Deep. Time
8 Puzzle	10^5	.01 sec		10^5	.01 sec
2x2x2 Rubik's	10^6	.2 sec		10^6	.2 sec
15 Puzzle	10^{13}	6 days	1Mx	10^{17}	20k yrs
3x3x3 Rubik's	10^{19}	68k yrs	8x	10^{20}	574k yrs
24 Puzzle	10^{25}	12B yrs		10^{37}	10^{23} yrs

Why the difference?

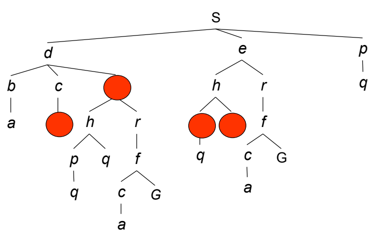
Rubik has higher branch factor
15 puzzle has greater depth

# of duplicates

Slide adapted from Richard Korf presentation

Tree vs Graph Search

In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



Graph Search

- Very simple fix: never expand a state type twice

```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST(problem, STATE[node]) then return node
      if STATE[node] is not in closed then
         add STATE[node] to closed
         fringe ← INSERTALL(EXPAND(node, problem), fringe)
   end
```

Some Hints

- On small problems
  - Graph search almost always better than tree search
  - Implement your closed list as a dict or set!
- On many real problems
  - Storage space is a huge concern
  - Graph search impractical

Search Methods

- Depth first search (DFS)
- Breadth first search (BFS)
- Iterative deepening depth-first search (IDS)

Blind search

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Search Methods

- Depth first search (DFS)
- Breadth first search (BFS)
- Iterative deepening depth-first search (IDS)
- Best first search
- Uniform cost search (UCS)
- Greedy search
- A*
- Iterative Deepening A* (IDA*)
- Beam search
- Hill climbing

Heuristic search

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Blind vs Heuristic Search

- Costs on Actions
- Heuristic Guidance

Separable Issues, but usually linked.

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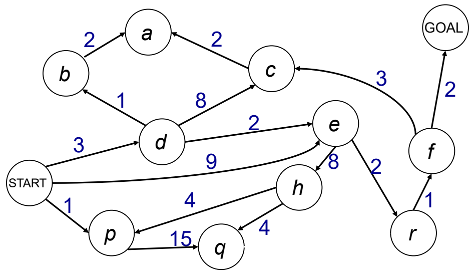
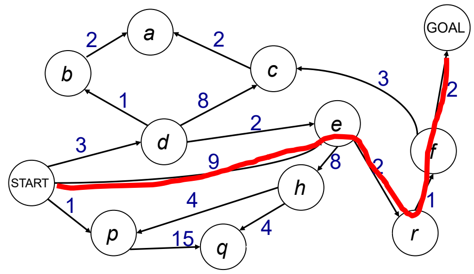
Costs on Actions



Objective: Path with smallest overall cost

Costs on Actions



What will BFS return?

… finds the shortest path in terms of number of transitions. It does ***not*** find the least-cost path.

Best-First Search

- Generalization of breadth-first search
- Fringe = ***Priority*** queue of nodes to be explored
- Cost function f(n) applied to each node

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Priority Queue Refresher

A priority queue is a data structure in which you can insert and retrieve (key, value) pairs with the following operations:

pq.push(key, value)	inserts *(key, value)* into the queue.
pq.pop()	returns the key with the lowest value, and removes it from the queue.

- You can decrease a key's priority by pushing it again
- Unlike a regular queue, insertions aren't constant time, usually O(log *n*)
- We'll need priority queues for cost-sensitive search methods

Best-First Search

- Generalization of breadth-first search
- Fringe = ***Priority*** queue of nodes to be explored
- Cost function f(n) applied to each node

```
Add initial state to priority queue
While queue not empty
      Node = head(queue)
      If goal?(node) then return node
      Add new children of node to queue
```

"expanding the node"

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Old Friend

Breadth First = Best First
with f(n) = depth(n)

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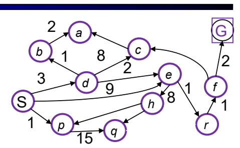
Uniform Cost Search

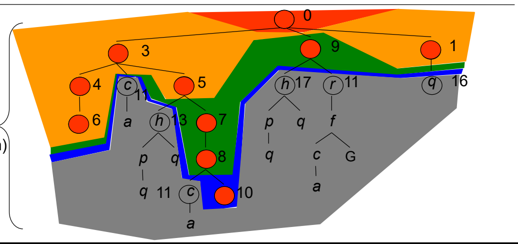
Best first, where
f(n) = "cost from **start** to **n**"

aka "Dijkstra's Algorithm"

Uniform Cost Search

Expansion order:
S, p, d, b, e, a, r, f, e, G

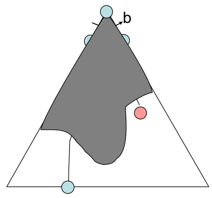




Uniform Cost Search

Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y if finite	N	$O(b^m)$	$O(bm)$
BFS		Y	Y*	$O(b^d)$	$O(b^d)$
UCS		Y*	Y	$O(b^{C*/\varepsilon})$	$O(b^{C*/\varepsilon})$

$C*/\varepsilon$ tiers



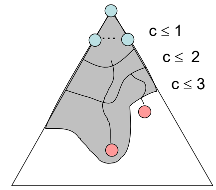
$C*$ = Optimal cost

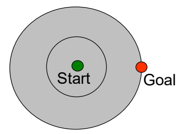
ε = Minimum cost of an action

* We'll keep assuming finite branching factor

Uniform Cost Issues

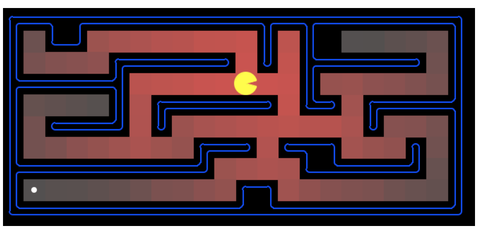
- Remember: explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about ***goal location***





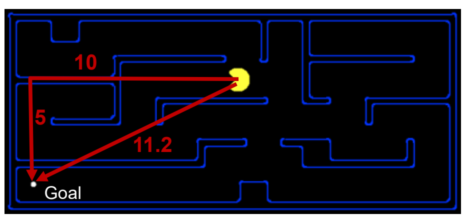
Uniform Cost: Pac-Man

- Cost of 1 for each action
- Explores all of the states, but one



What is a *Heuristic*?

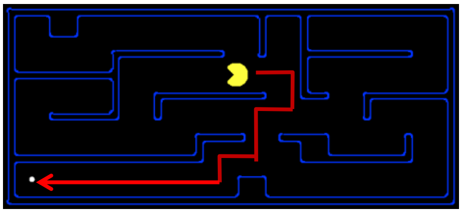
- An *estimate* of how close a state is to a goal
- Designed for a particular search problem



- Examples: Manhattan distance: 10+5 = 15
  Euclidean distance: 11.2

What is a *Heuristic*?

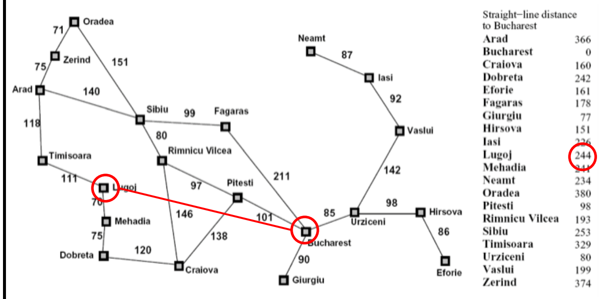
- An ***estimate*** of how close a state is to a goal
- Designed for a particular search problem



- Actual distance to goal: 2+4+2+1+8=

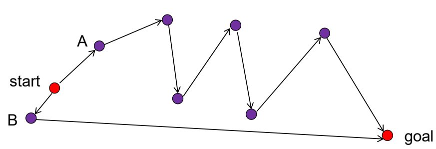
Greedy Search

Best first with f(n) = heuristic estimate of distance to goal



Greedy Search

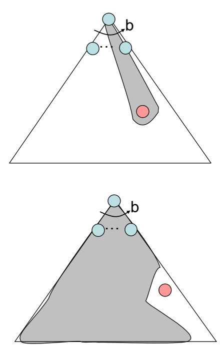
Expand the node that seems closest…



What can go wrong?

Greedy Search

- Common case:
  - Greedy search takes you straight to a (suboptimal) goal
- Worst-case: like a badly-guided DFS
  - Can explore everything
  - Can get stuck in loops if no cycle checking
- Like DFS wrt completeness
  - Complete w/ cycle checking
  - *If* finite # states



A* Search

Hart, Nilsson & Rafael 1968

Best first search with f(n) = g(n) + h(n)

- g(n) = sum of costs from start to n
- h(n) = (admissible) estimate of lowest cost path from n → goal   note: h(goal) = 0

Can view as cross-breed:

g(n) ~ uniform cost search

h(n) ~ greedy search

Nomenclature Confusion:
Is h the heuristic or is f?

Best of both worlds…

A* Search

Hart, Nilsson & Rafael 1968

Best first search with f(n) = g(n) + h(n)

- g(n) = sum of costs from start to n
- h(n) = admissible estimate of lowest cost path from n → goal   h(goal) = 0

If h(n) is admissible and monotonic

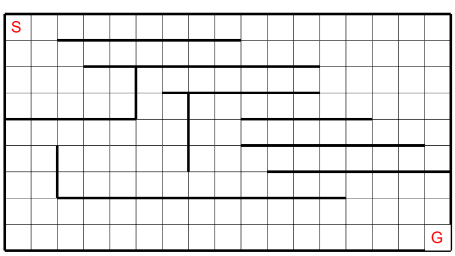
then A* is optimal

Underestimates (≤) cost of reaching goal from node

f values never decrease From node to descendants (triangle inequality)

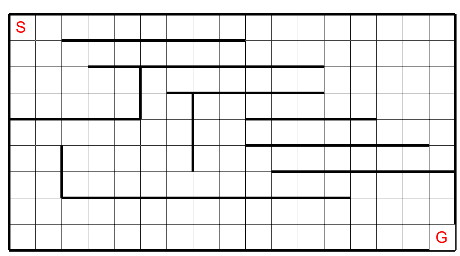
Is Manhattan distance **admissible**?

- Underestimate?

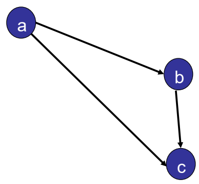


Is Manhattan distance **monotonic**?

- f values increase from node to children
- (triangle inequality)



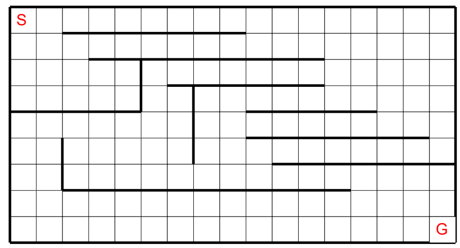
Monotonicity



F(a) ≥ F(b)
G(a)+H(a) ≥ G(b)+H(b)

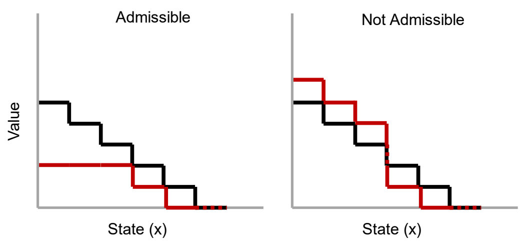
Euclidean Distance

- Admissible?
- Monotonic?



Admissible Heuristics

F(n) = G(n) + H(n)

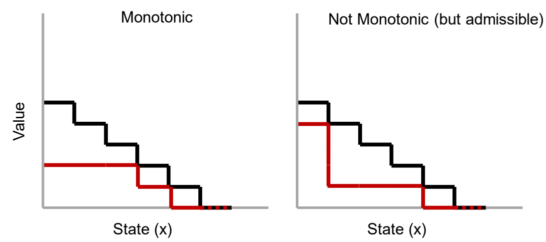


True (optimal) cost remaining
H(n) = H-estimate of cost remaining

Slide credit: Travis Mandel

Monotonic/Consistent Heuristics

F(n) = G(n) + H(n)

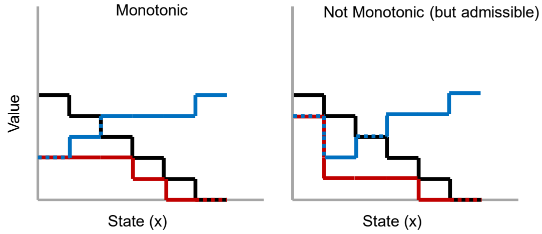


True (optimal) cost remaining
H(n) = H-estimate of cost remaining

Slide credit: Travis Mandel

Monotonic/Consistent Heuristics

F(n) = G(n) + h(n)



True (optimal) cost remaining
H(n) Heuristic-estimated cost remaining
F(n) Heuristic + cost so far

Slide credit: Travis Mandel

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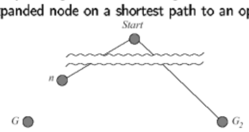
Optimality of A* (tree search)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Monotonicity required for proof in graph search version

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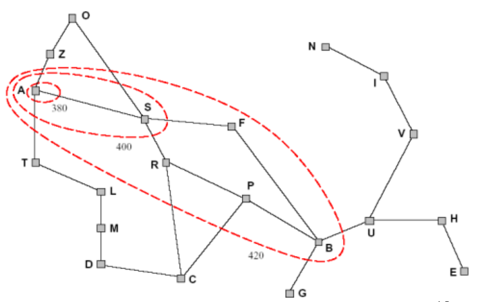
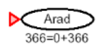
Optimality Continued

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



A* Example



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A* Example

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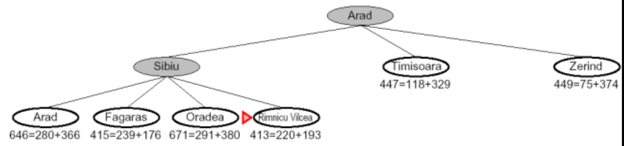
A* Example



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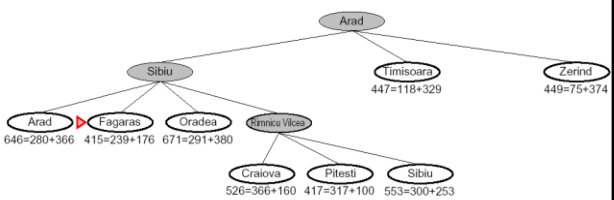
A* Example



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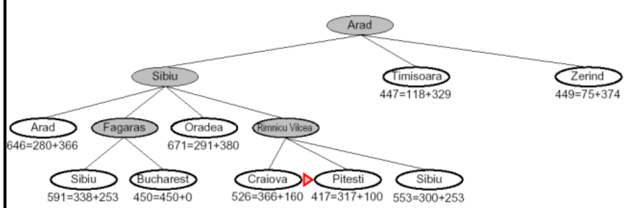
A* Example



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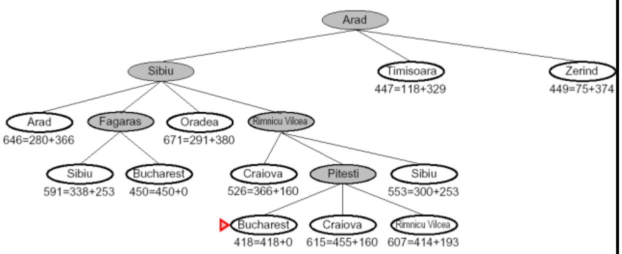
A* Example



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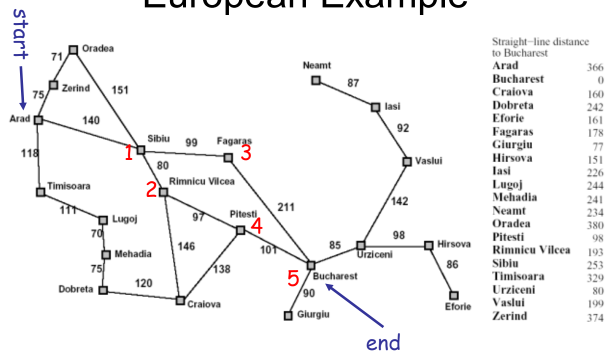
European Example



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A* Summary

- Pros

  Produces optimal cost solution!

  Does so quite quickly (focused)

- Cons

  Maintains priority queue…

  Which can get exponentially big ☹

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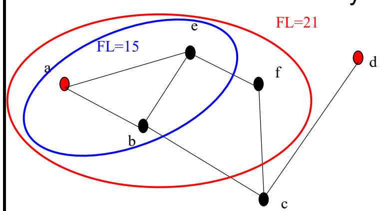
Iterative-Deepening A*

- Like iterative-deepening depth-first, but...
- Depth bound modified to be an f-limit
  - Start with f-limit = h(start)
  - Prune any node if f(node) > f-limit
  - Next f-limit = min-cost of any node pruned



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IDA* Analysis

- Complete & Optimal (ala A*)
- Space usage $\propto$ depth of solution
- Each iteration is DFS - no priority queue!
- # nodes expanded relative to A*
  - Depends on # unique values of heuristic function
  - In 8 puzzle: few values $\Rightarrow$ close to # A* expands
  - In traveling salesman: each f value is unique

    $\Rightarrow$ 1+2+…+n = $O(n^2)$ where n=nodes A* expands

    if n is too big for main memory, n^2 is too long to wait!

Forgetfulness

- A* used exponential memory
- How much does IDA* use?
  - During a run?
  - In between runs?
    - SMA*