Outline for Today

- Recap Q-Learning
- Approximate Q-Learning
- Hidden Markov Models
- Filtering
- Particle Filtering

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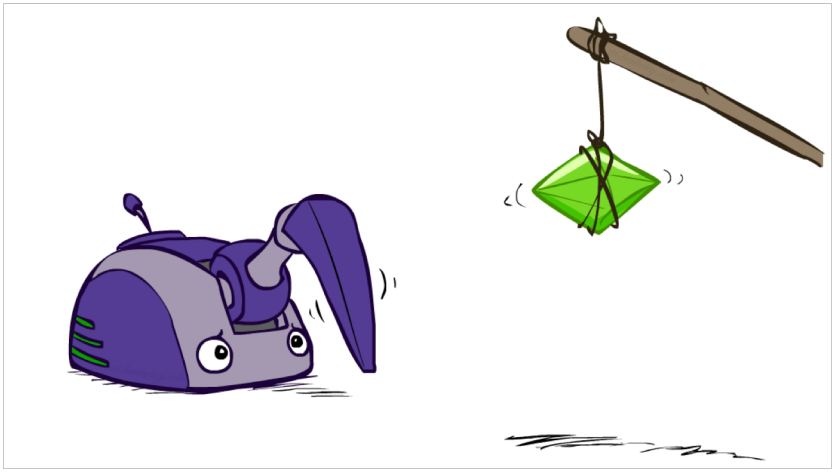
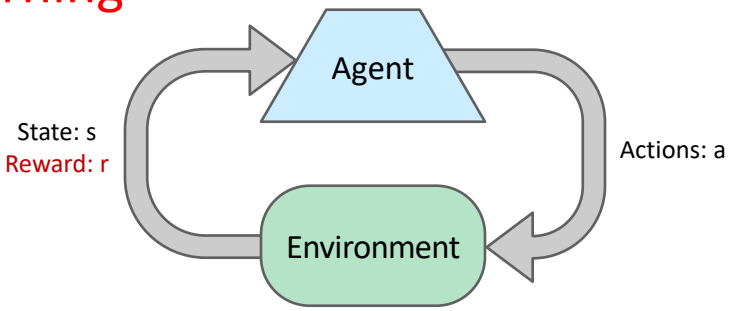
Reinforcement Learning

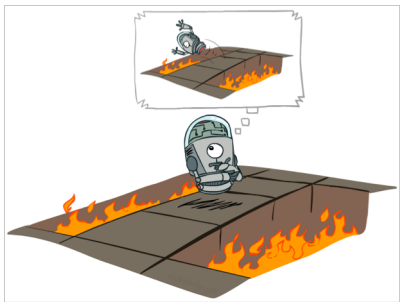


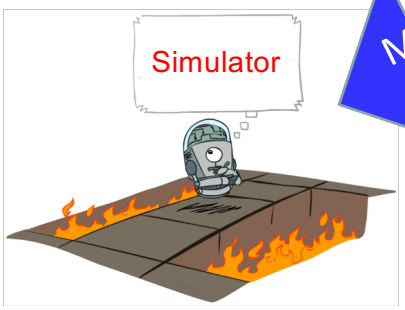
Reinforcement Learning



- MDP Framework, except: don't know transition funct. (T) or reward funct. (R)
- Must choose what action to execute so as to maximize expected rewards
- Receive feedback:
  - What state we end up in after executing a &
  - What reward we get
- Learn based on observed samples of outcomes!

Offline (MDPs) vs. Online (RL)





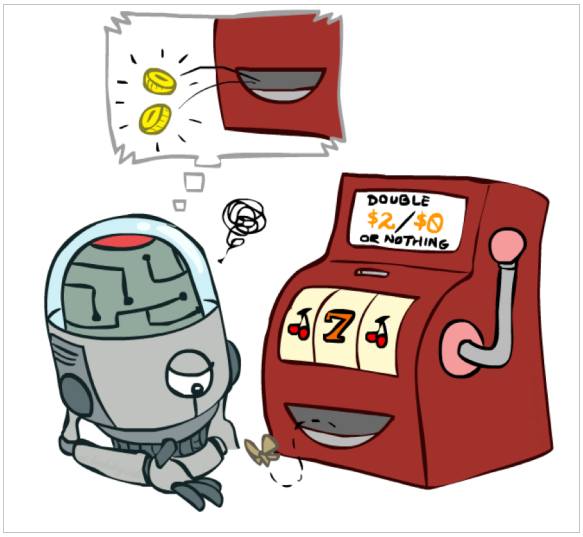
Offline Solution (Planning)

Monte Carlo Planning

Online Learning (RL)

Diff: 1) dying ok; 2) (re)set button

Model-Free Learning



Q Learning

- **Forall s, a**
  - **Initialize Q(s, a) = 0**
- **Repeat Forever**

  Where are you? s.

  Choose some action a

  Execute it in real world: *(s, a, r, s')*

  Do update:

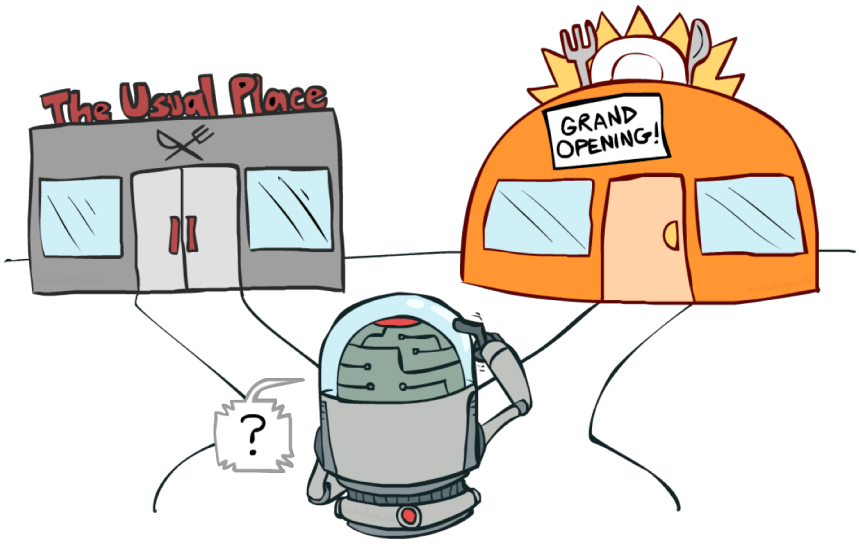
  difference ← $[r + \gamma \text{ Max}_{a'} Q(s', a')] - Q(s,a)$

  $Q(s,a) \leftarrow Q(s,a) + \alpha(\text{difference})$

Trial

Note parallel to RTDP
- Both have trials
- But no T, R

Exploration vs. Exploitation



Want to Minimize ***Regret***

- **Cumulative Regret:**
  - Goal: achieve near optimal cumulative lifetime reward (in expectation)
  - Minimize this if acting in real world
- **Simple Regret:**
  - Goal: quickly identify policy with high reward (in expectation)
  - Minimize this if learning in simulator

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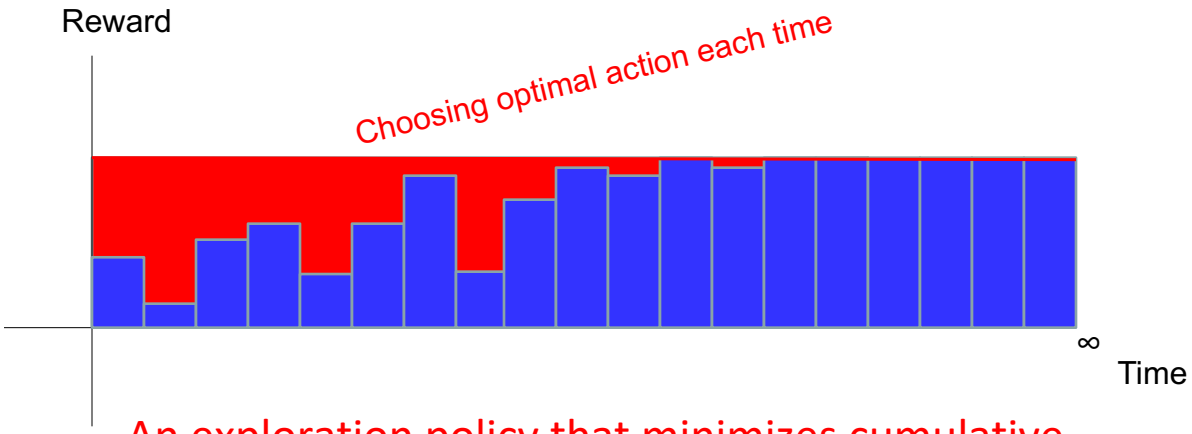
Cumulative Regret



An exploration policy that minimizes cumulative regret, minimizes the red area

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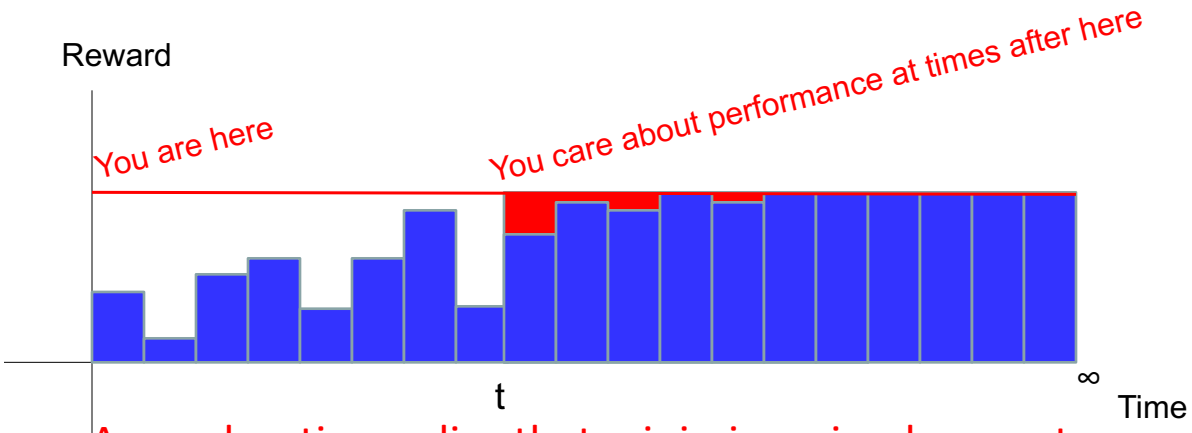
Simple Regret



An exploration policy that minimizes simple regret…
Is given a time, t, in the ***future***, and explores in order to ***minimize red area after t***

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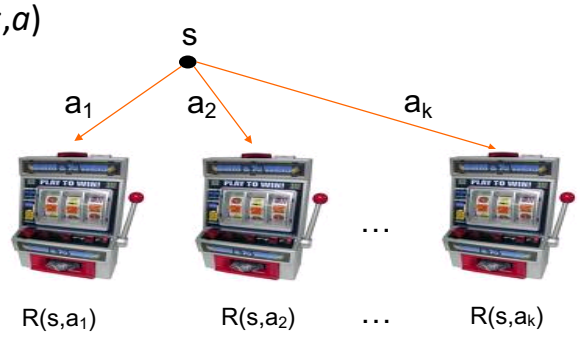
RL on Single State MDP

- Suppose MDP has a single state and k actions
  - Can sample rewards of actions using call to simulator
  - Sampling action *a* is like pulling slot machine arm with random payoff function *R*(*s*,*a*)



Multi-Armed Bandit Problem

Slide adapted from **Alan Fern** (OSU)

Upper Confidence Bound (UCB)

1. Execute each action (pull each arm) once.
2. For N = 1 to infinity, repeat:

   Execute the action, a_i, that maximizes:

$$ExpectedReward(a_i) + \sqrt{\frac{2\log(N)}{NumberOfTimesExecuted(a_i)}}$$

Theorem: No algorithm can achieve lower expected **cumulative regret**, up to constant factors!
(but Thompson Sampling also optimal and often better in practice)

Putting UCB into Q-Learning !!!!

- Dealing with multiple states
- (Multi armed bandit assumes ONE state)

UCB Balances Exploration & Exploitation

Let N_{sa} be number of times one has executed a in s; let $N = \sum_{sa} N_{sa}$

Let $Q^e(s,a) = Q(s,a) + \sqrt{2\log(N)/(1+n_{sa})}$

- **Forall s, a**
  - Initialize Q(s, a) = 0, $n_{sa} = 0$
- **Repeat Forever**
  Where are you? s.
  Choose action with highest Q^e
  Execute it in real world: *(s, a, r, s')*
  Do update:

N_{sa} += 1;
difference ← [r + γ $Max_{a'}$ $Q^e(s', a')$] - $Q^e(s,a)$
$Q(s,a) \leftarrow Q^e(s,a) + \alpha$(difference)

Term rewards exploration, but converges to zero

$$ExpReward(a_i) + \sqrt{\frac{2\log(N)}{NumberOfTimesExecuted(a_i)}}$$

Summary of Bandits in Theory

PAC Objective:

- **UniformBandit** is a simple PAC algorithm
- **MedianElimination** improves by a factor of log(k) and is optimal up to constant factors

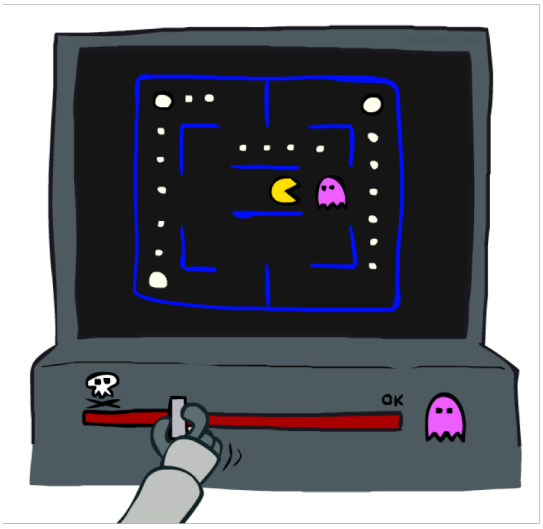
Cumulative Regret:

- **Uniform** is very bad!
- **UCB** is optimal (up to constant factors)
- **Thomson Sampling** also optimal; often performs better in practice

Simple Regret:

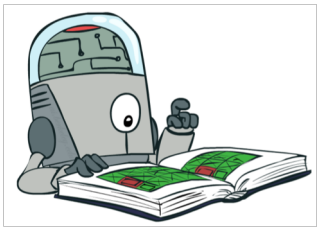
- **UCB** shown to reduce regret at polynomial rate
- **Uniform** reduces at an exponential rate
- **0.5-Greedy** may have even better exponential rate

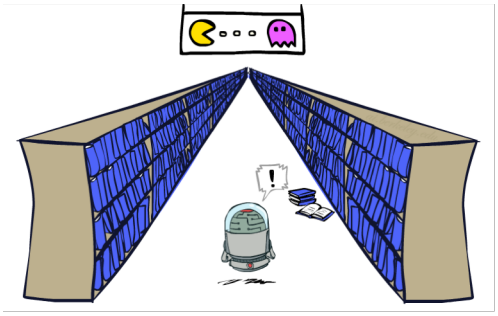
Approximate Q-Learning



Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again

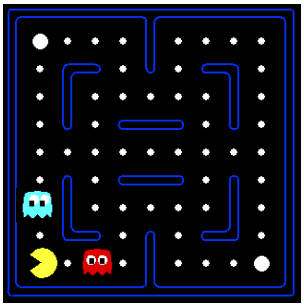




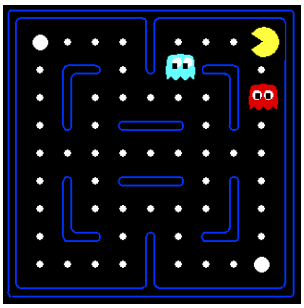
[demo – RL pacman]

Ex: Pacman – Failure to Generalize

Let's say we discover through experience that this state is bad:

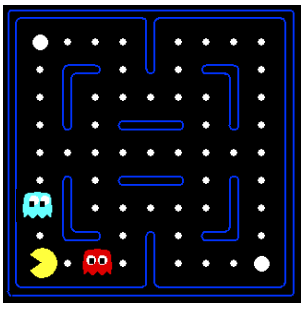


In naïve q-learning, we know nothing about this state:

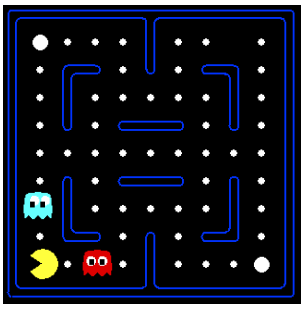


Ex: Pacman – Failure to Generalize

Let's say we discover through experience that this state is bad:



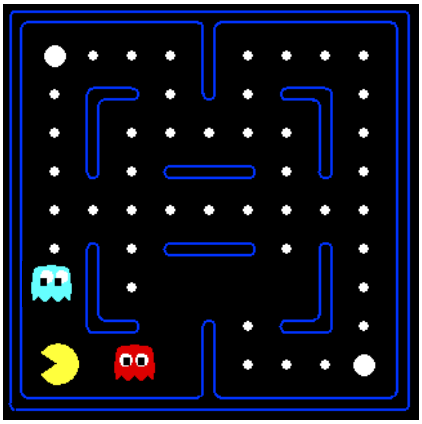
Or even this one!



Feature-Based Representations

Solution: describe a state using a **vector of features** (aka "properties")

- Features = functions from states to R (often 0/1) capturing important properties of the state
- Example features:
  - Distance to closest ghost or dot
  - Number of ghosts
  - 1 / (dist to dot)^2
  - Is Pacman in a tunnel? (0/1)
  - …… etc.
  - Is it the exact state on this slide?
    - Features -> inductive bias; ability to generalize
- Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Combination of Features

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states sharing features may actually have very different values!

Q Learning

- **Forall s, a**
  - **Initialize Q(s, a) = 0**
- **Repeat Forever**

  Where are you? s.

  Choose some action a ; *e.g. using $\in$-greedy or by maximizing $Q_e(s, a)$*

  Execute it in real world: *(s, a, r, s')*

  Do update:

  difference ← [r + γ $\text{Max}_{a'}$ Q(s', a')] - Q(s,a)

  Q(s,a) ← Q(s,a) + α(difference)

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- **Forall *i***
  - Initialize $w_i = 0$
- **Repeat Forever**

  Where are you? s.

  Choose some action a

  Execute it in real world: *(s, a, r, s')*

  Do update:

  difference ← [r + γ $\text{Max}_{a'}$ Q(s', a')] - Q(s,a)

  Q(s,a) ← Q(s,a) + α(difference)

Approximate Q Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- **Forall *i***
  - Initialize $w_i = 0$
- **Repeat Forever**

  Where are you? s.

  Choose some action a

  Execute it in real world: *(s, a, r, s')*

  Do update:

  difference ← [r + γ $\text{Max}_{a'}$ Q(s', a')] - Q(s,a)

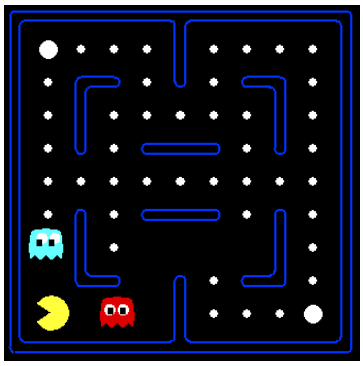
  Forall *i* do:

  $$w_i \leftarrow w_i + \alpha \, [\text{difference}] \, f_i(s,a)$$

Example: Pacman Features

$$Q(s,a) = w_1 f_{DOT}(s,a) + w_2 f_{GST}(s,a)$$

s



$$f_{DOT}(s,a) = \frac{1}{distance\ to\ closest\ food\ after\ taking\ a}$$

$$f_{DOT}(s, NORTH) = 0.5$$

$$f_{GST}(s,a) = distance\ to\ closest\ ghost\ after\ taking\ a$$

$$f_{GST}(s, NORTH) = 1.0$$

$$Q(s,a) = w_1 f_{DOT}(s,a) + w_2 f_{GST}(s,a)$$

Suppose

W1 = 4.0

W2 = -1.0

α = 0.004

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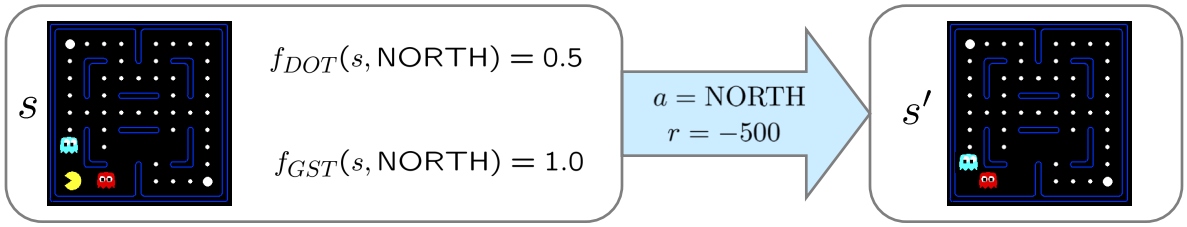
α = 0.004

Example: Q-Pacman

Forall *i* do:
$w_i \leftarrow w_i + \alpha$ [difference] $f_i(s,a)$

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$



$Q(s, \text{NORTH}) = +1$

$Q(s', \cdot) = 0$

$r + \gamma \max_{a'} Q(s', a') = -500 + 0$

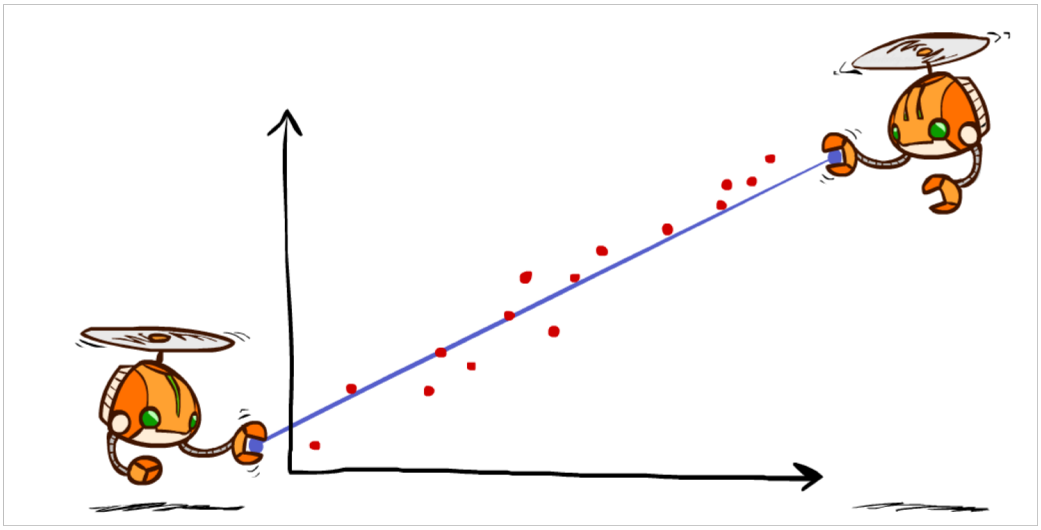
difference $= -501$ $\Rightarrow$ $w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\,0.5$, $w_{GST} \leftarrow -1.0 + \alpha\,[-501]\,1.0$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

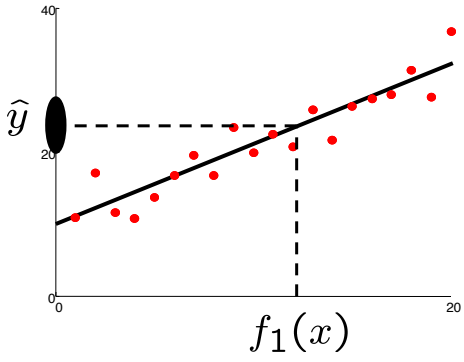
[Demo: approximate Q-learning pacman (L11D10)]

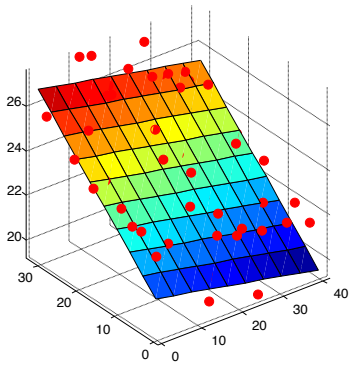
Video of Demo Approximate Q-Learning -- Pacman

Sidebar: Q-Learning and Least Squares



Linear Approximation: Regression





Prediction:

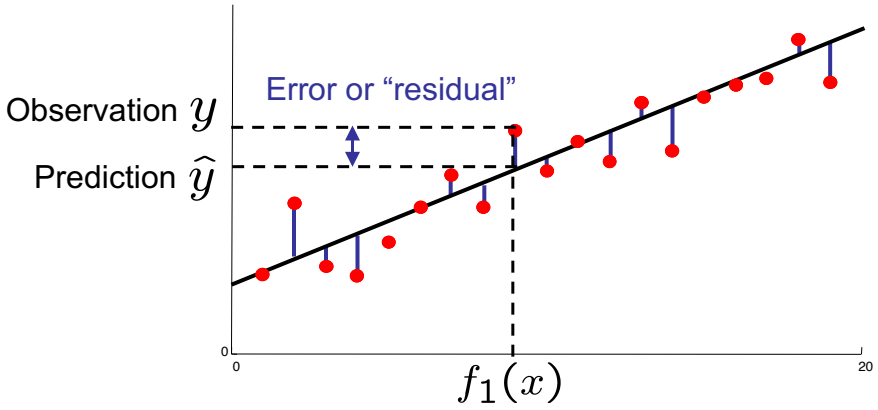
$\widehat{y} = w_0 + w_1 f_1(x)$

Prediction:

$\widehat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$

Optimization: Least Squares

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$

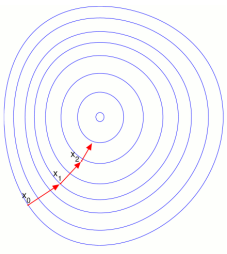


Minimizing Error

Suppose we have one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2} \left(y - \sum_k w_k f_k(x) \right)^2$$

$$\frac{\partial \text{ error}(w)}{\partial w_m} = - \left(y - \sum_k w_k f_k(x) \right) f_m(x)$$



So to shrink error, repeat this update:

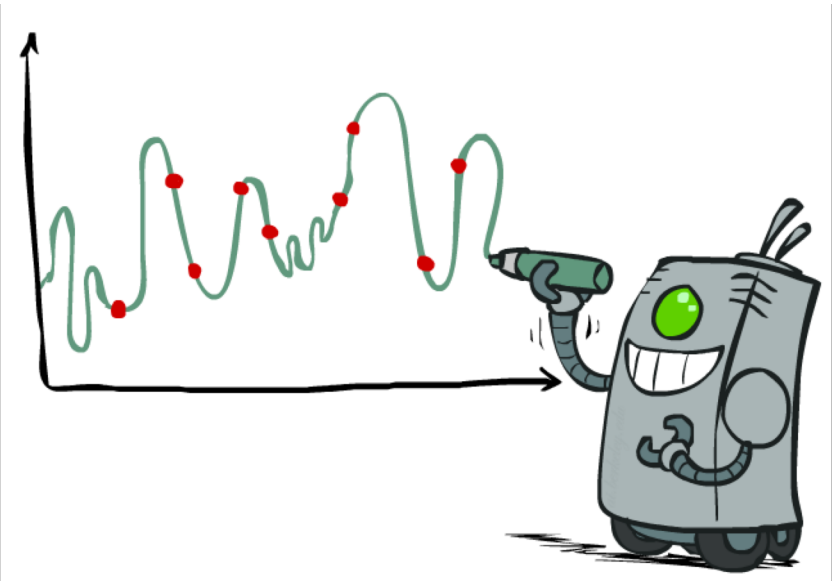
$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x) \right) f_m(x)$$

Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$

"target" "prediction"

Overfitting: Why Limiting Capacity Can Help



Simple Problem

Given: Features of current state
Predict: Will Pacman die on the next step?

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Just one feature. See a pattern?

- Ghost one step away, pacman dies
- Ghost one step away, pacman dies
- Ghost one step away, pacman dies
- Ghost one step away, pacman dies
- Ghost one step away, pacman lives
- Ghost more than one step away, pacman lives
- Ghost more than one step away, pacman lives
- Ghost more than one step away, pacman lives
- Ghost more than one step away, pacman lives
- Ghost more than one step away, pacman lives
- Ghost more than one step away, pacman lives

Learn: Ghost one step away → pacman dies!

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What if we add more features?

- Ghost one step away, score 211, pacman dies
- Ghost one step away, score 341, pacman dies
- Ghost one step away, score 231, pacman dies
- Ghost one step away, score 121, pacman dies
- Ghost one step away, score 301, pacman lives
- Ghost more than one step away, score 205, pacman lives
- Ghost more than one step away, score 441, pacman lives
- Ghost more than one step away, score 219, pacman lives
- Ghost more than one step away, score 199, pacman lives
- Ghost more than one step away, score 331, pacman lives
- Ghost more than one step away, score 251, pacman lives

Learn: Ghost one step away AND score is NOT 301 → pacman dies!

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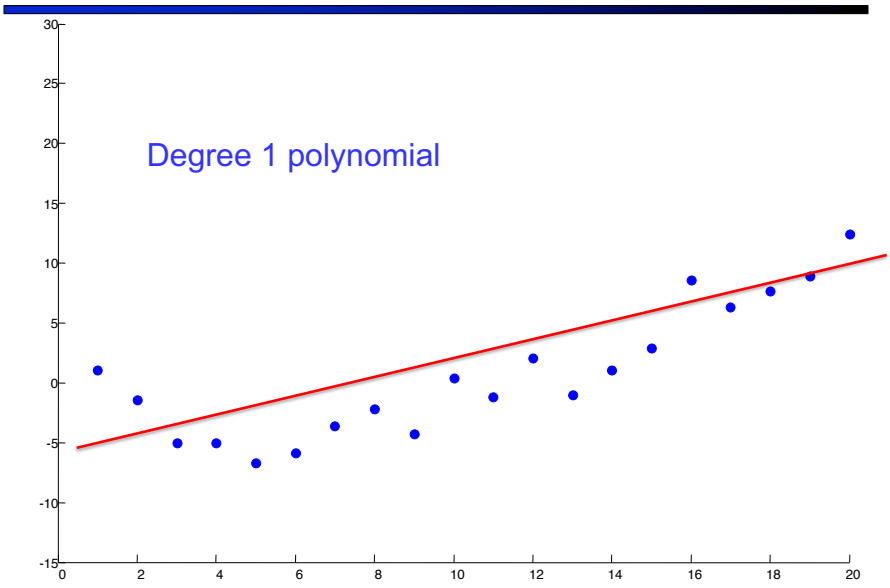
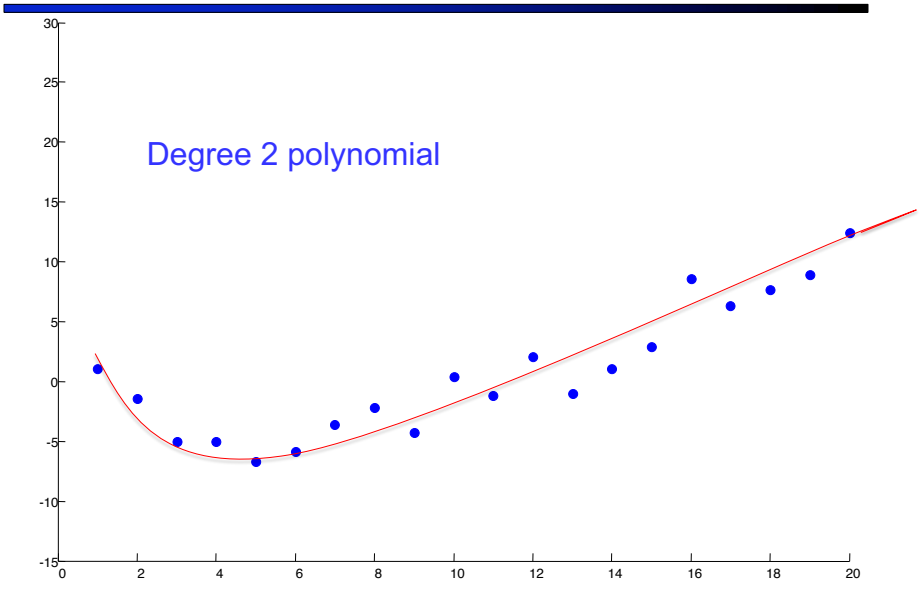
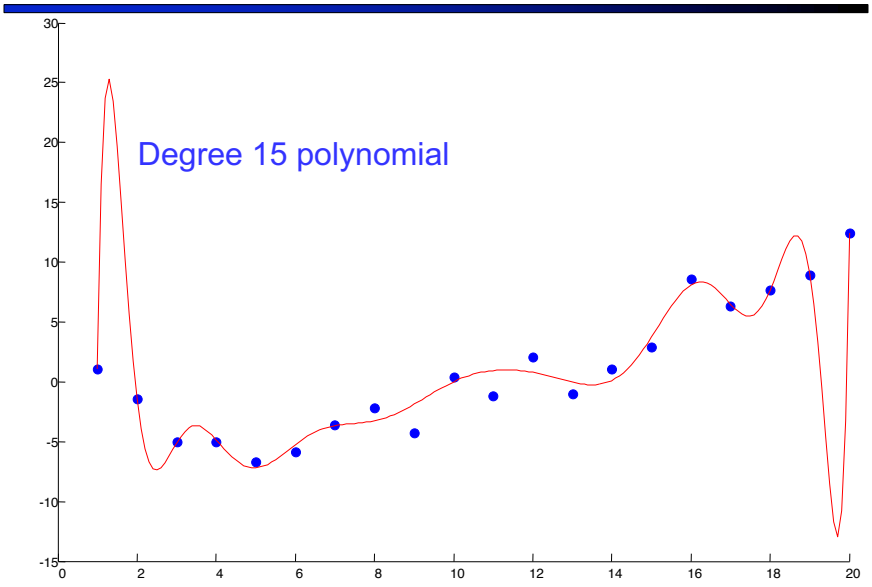
There's fitting, and there's



There's fitting, and there's

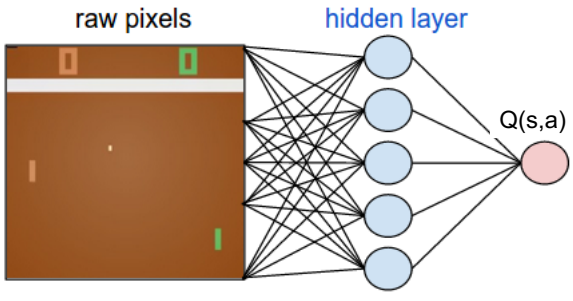


Overfitting



Approximating Q Function

- Linear Approximation
- Could also use Deep Neural Network
  - https://www.nervanasys.com/demystifying-deep-reinforcement-learning/



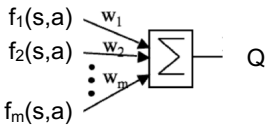
Deepmind Atari

https://www.youtube.com/watch?v=V1eYniJ0Rnk

Approximating the Q Function

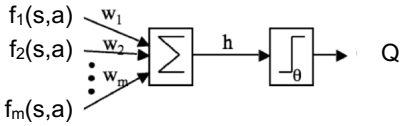
Linear Approximation

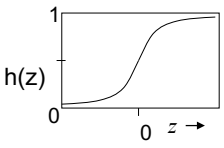
$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$



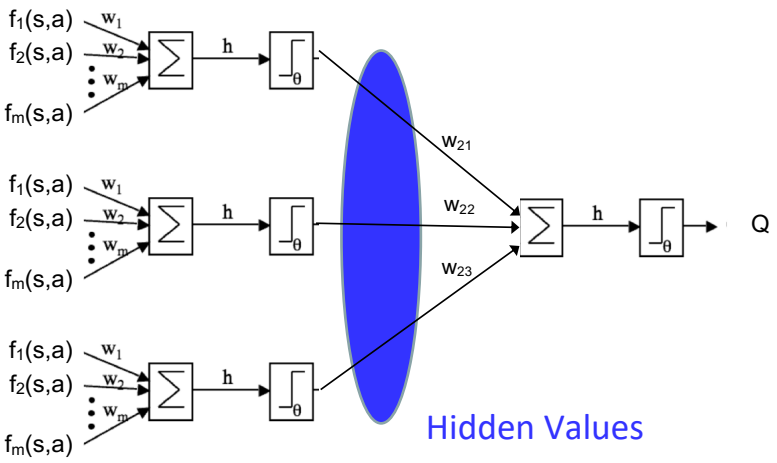
Neural Approximation (nonlinear)

$$h(z) = \frac{1}{1+e^{-z}}$$

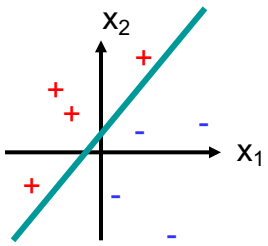




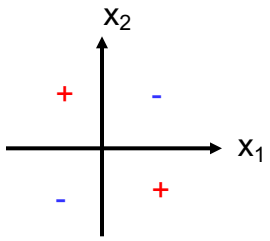
Deep Neural Networks



Decision Surface of a Perceptron



Linearly separable



Non-Linearly separable

- Perceptron is able to represent ***many*** useful functions
  - E.g., for AND(x_1, x_2) choose weights w_0=-1.5, w_1=1, w_2=1
- But functions that are not linearly separable are not representable
  - E.g. XOR(x_1, x_2)

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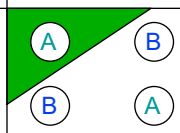
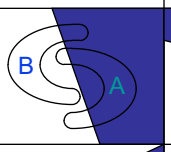
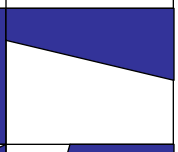
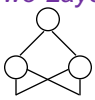
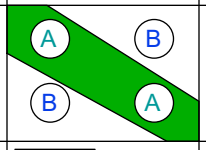
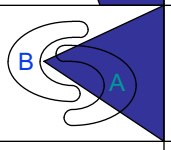
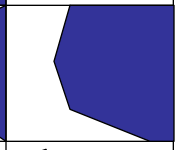
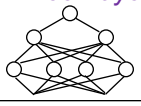
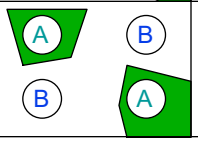
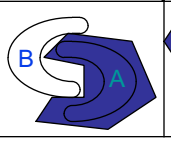
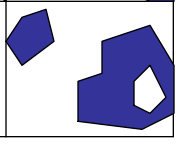
Slide by Moshe Sipper
www.cs.bgu.ac.il/~sipper/courses/ecal061/**ann-ics320Part4.ppt**

Hyperplane partitions

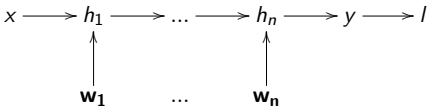
- An extra layer models a convex hull
  - "An area with no dents in it"
  - Perceptron models, but can't learn
  - Sigmoid function learning of convex hulls
  - Two layers add convex hulls together
  - Sufficient to classify anything "sane".
- In theory, further layers add nothing
- In practice, extra layers may be better

Different Non-Linearly Separable Problems

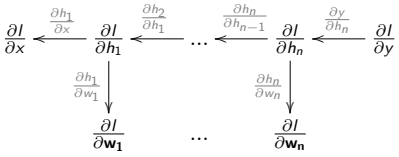
Structure	*Types of Decision Regions*	*Exclusive-OR Problem*	*Classes with Meshed regions*	*Most General Region Shapes*
Single-Layer 	*Half Plane Bounded By Hyperplane*			
Two-Layer 	*Convex Open Or Closed Regions*			
Three-Layer 	Arbitrary (Complexity Limited by No. of Nodes)			

Deep Representations

- A deep representation is a composition of many functions



- Its gradient can be backpropagated by the chain rule



Slide adapted from David Silver

Training via Gradient Descent

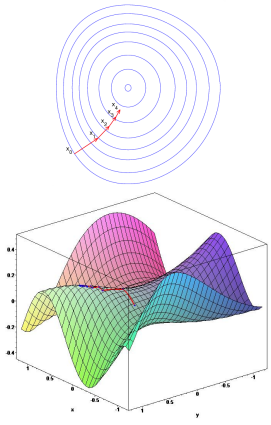
Loss = ML word for "error"

- Calculate gradient of expected loss $(\mathbf{w}) = \mathbb{E}[l]$

$$\frac{\partial l}{\partial \mathbf{w}} \sim \mathbb{E}\left[\frac{\partial l}{\partial \mathbf{w}}\right] = \frac{\partial L(\mathbf{w})}{\partial \mathbf{w}}$$

- Adjust **w** down the gradient

$$\Delta w \propto \frac{\partial l}{\partial \mathbf{w}}$$



Slide adapted from David Silver

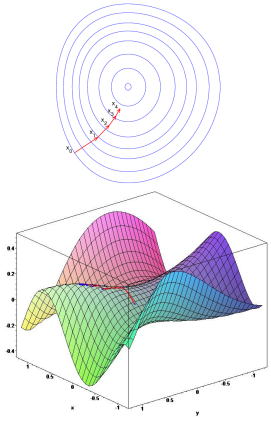
Training *via* ***Stochastic*** Gradient Descent

- Sample gradient of expected loss $L(\mathbf{w}) = \mathbb{E}[l]$

$$\frac{\partial l}{\partial \mathbf{w}} \sim \mathbb{E}\left[\frac{\partial l}{\partial \mathbf{w}}\right] = \frac{\partial L(\mathbf{w})}{\partial \mathbf{w}}$$

- Adjust $\mathbf{w}$ down the Sampled gradient

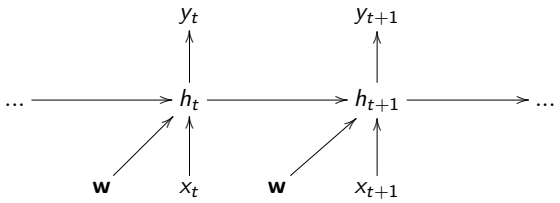
$$\Delta w \propto \frac{\partial l}{\partial \mathbf{w}}$$



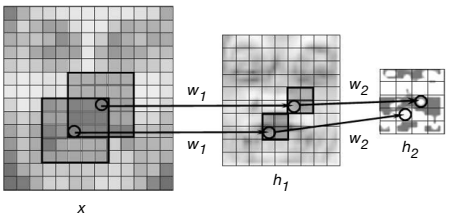
Slide adapted from David Silver

Weight Sharing

Recurrent neural network shares weights between time-steps



Convolutional neural network shares weights between local regions

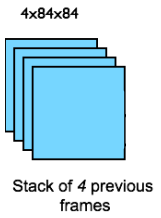


Slide adapted from David Silver

How Do Authors Represent an Atari State?

Raw pixels:
210 x 160 x 128 colors

Down-sampled to
84 x 84 x grey



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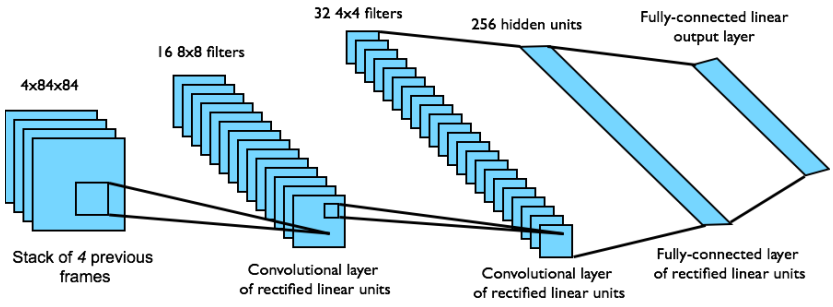
Represent Q Function Using DQN

- End-to-end learning of values $Q(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick/button positions
- Reward is change in score for that step

Raw pixels:
210 x 160 x 128 colors

Down-sampled to
84 x 84 x grey



Network architecture and hyperparameters fixed across all games

Slide adapted from David Silver

Recap: Approx Q-Learning

- Optimal Q-values should obey Bellman equation

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q(s',a')^* \mid s,a\right]$$

- Treat right-hand side $r + \gamma \max_{a'} Q(s',a',\mathbf{w})$ as a target
- Minimise MSE loss by stochastic gradient descent

$$I = \left(r + \gamma \max_{a} Q(s',a',\mathbf{w}) - Q(s,a,\mathbf{w})\right)^2$$

- Converges to Q^* using table lookup representation
- But diverges using neural networks due to:
  - Correlations between samples
  - Non-stationary targets

Slide adapted from David Silver

Why use Experience Replay?

- Maximize utility of data
- Avoid updates on correlated game states
  - I.I.D. assumption
- Avoid local maxima / divergence

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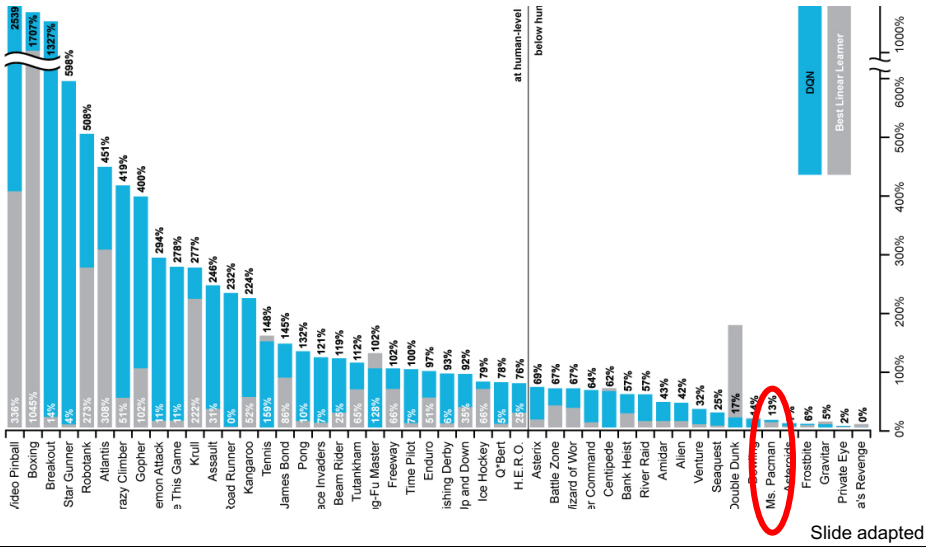
DQN Results on Atari

Which game characteristics → good / bad performance?



Slide adapted from David Silver

Evaluation?

Generally quite convincing…

Google bought Deepmind one month after paper appeared on ArXiv: $500M

- Fair comparison with human 'experts'?
- Effect of parameter values?
  - Action every k frames, clipping rewards, …
- Effect of experience replay?
- Effect of NN architecture (# layers etc)
- Does it work on go / chess?

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Takeaways

- Deep RL obviates hand features
- Approximate Q with nonlinear approx (NN)
  - Convergence?
- Use sequences (histories?) not individual actions
  - Markov?
- Challenge of sparse rewards, long time spans

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Deep Mind Resources

DQN paper
www.nature.com/articles/nature14236

DQN source code:
sites.google.com/a/deepmind.com/dqn/

See also: http://icml.cc/2016/tutorials/deep_rl_tutorial.pdf

Playing Atari with Deep RL

Breakout:
https://www.youtube.com/watch?v=V1eYniJ0Rnk

More games (less interesting)
https://www.youtube.com/watch?v=iqXKQf2BOSE

Paper
https://arxiv.org/pdf/1312.5602.pdf)

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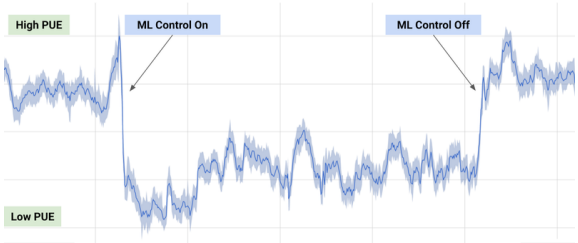
That's all for Reinforcement Learning!

- Very tough problem: How to perform any task well in an unknown, noisy environment!
- Traditionally used mostly for robotics, but…



Google DeepMind – RL applied to data center power usage

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