

CSE 544

Principles of Database

Management Systems

Lecture 3 –Schema Normalization

Announcements

- Project groups due on Friday
- First review due on Tuesday (makeup lecture)
 - Run ‘git pull’ to get the ‘review’ subdirectory
 - Place your review there
 - Commit, push (see instructions in hw1.md); no need to tag
- Homework 1 due next Friday
 - Run turnInHw.sh hw1
 - Or manually: ‘git commit...’, ‘git tag...’, ‘git push...’

Database Design

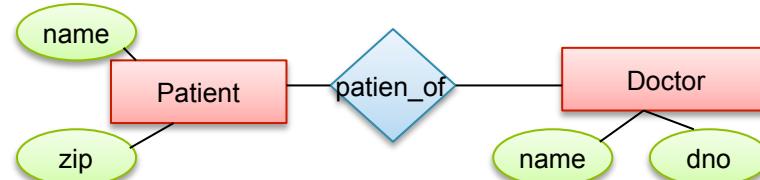
- The relational model is great, but how do I design my database schema?

Outline

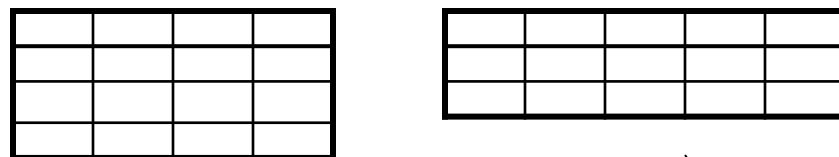
- Conceptual db design: entity-relationship model
- Problematic database designs
- Functional dependencies
- Normal forms and schema normalization

Conceptual Schema Design

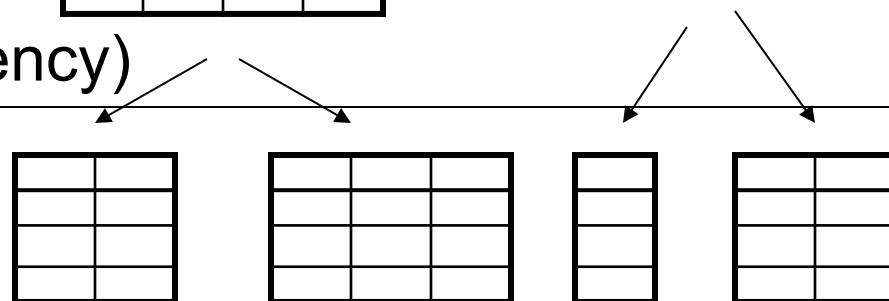
Conceptual Model:



Relational Model:
plus FD's
(FD = functional dependency)



Normalization:
Eliminates anomalies



Entity-Relationship Diagram

Attributes
 name

Entity sets
 Patient

Relationship sets
 patient_of

Entity-Relationship Diagram

Patient

Doctor

Attributes

name

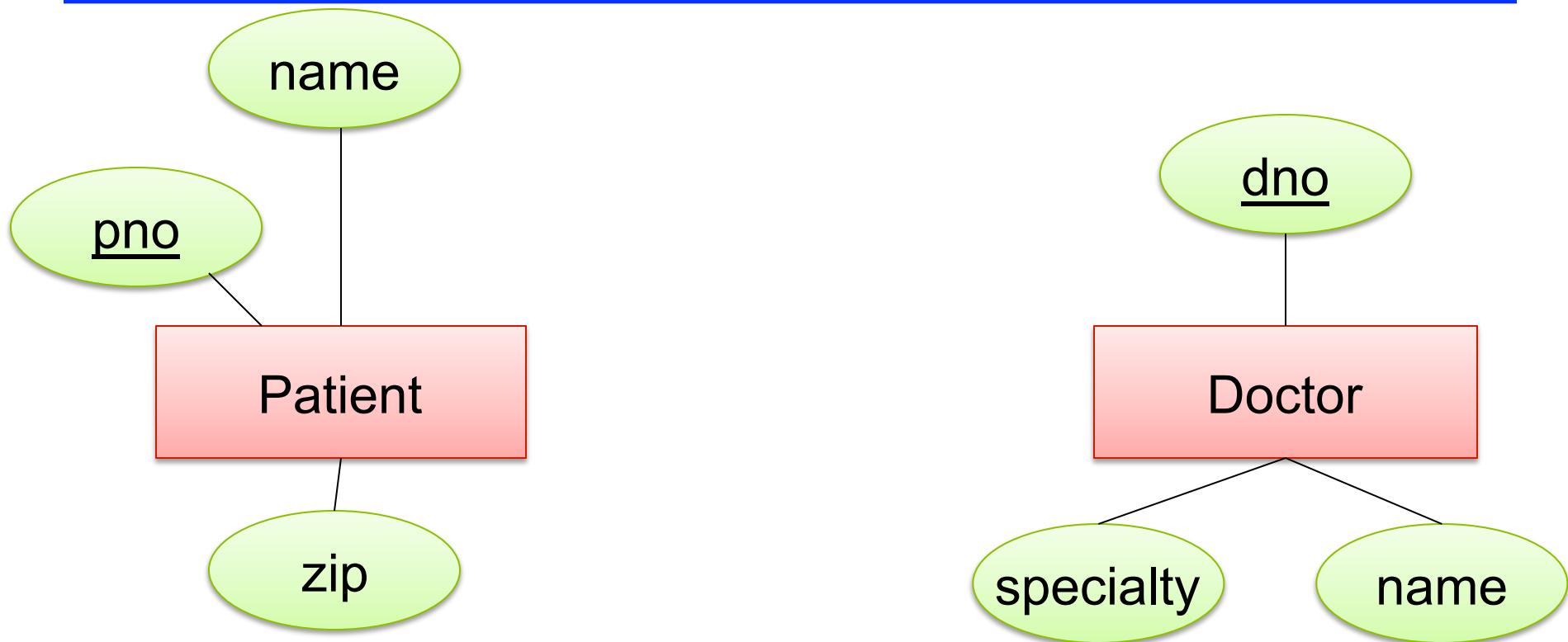
Entity sets

Patient

Relationship sets

patient_of

Entity-Relationship Diagram



Attributes



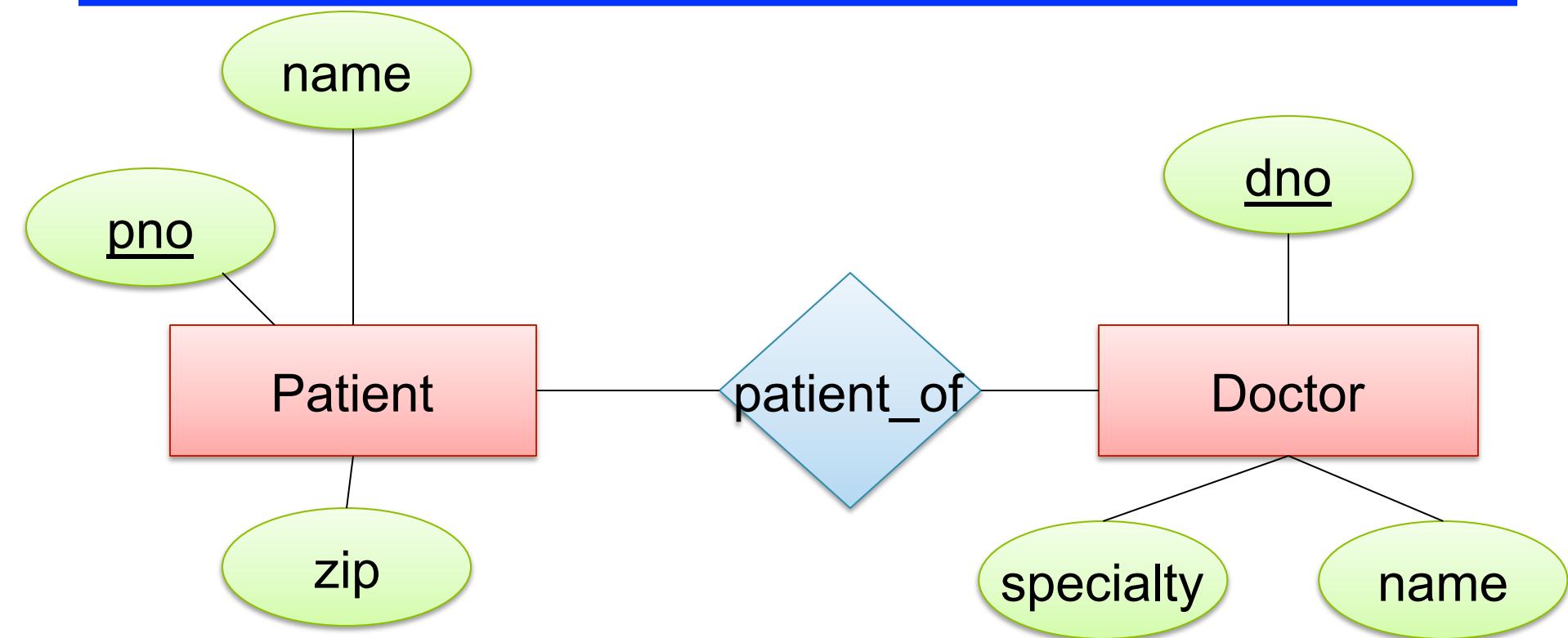
Entity sets



Relationship sets



Entity-Relationship Diagram



Attributes



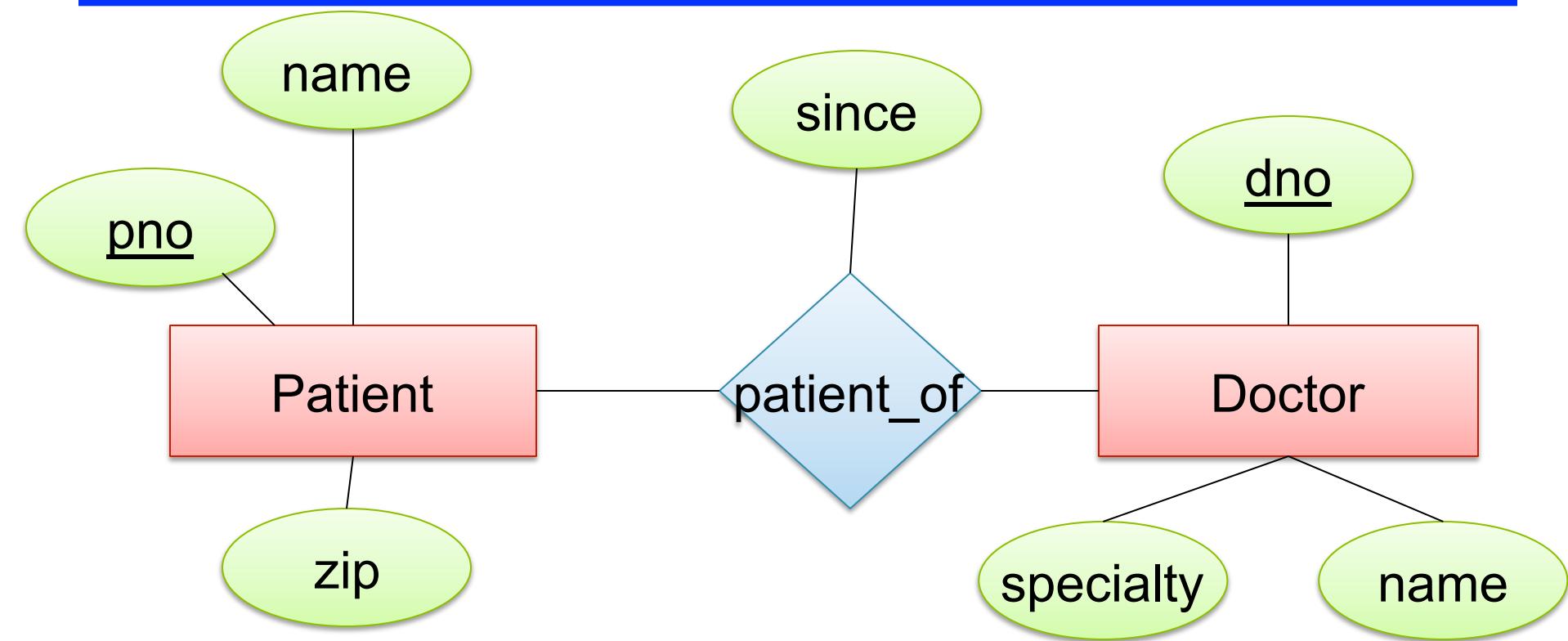
Entity sets



Relationship sets



Entity-Relationship Diagram



Attributes



Entity sets

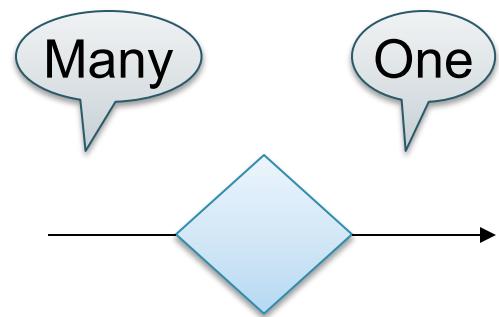


Relationship sets

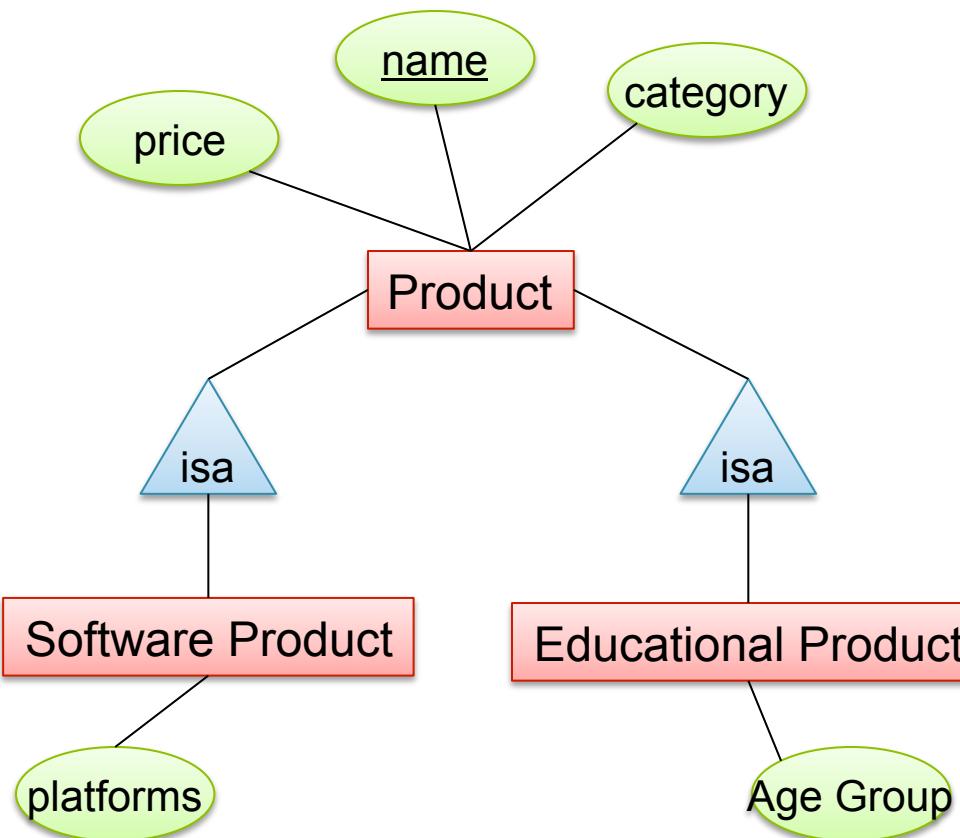


Entity-Relationship Model

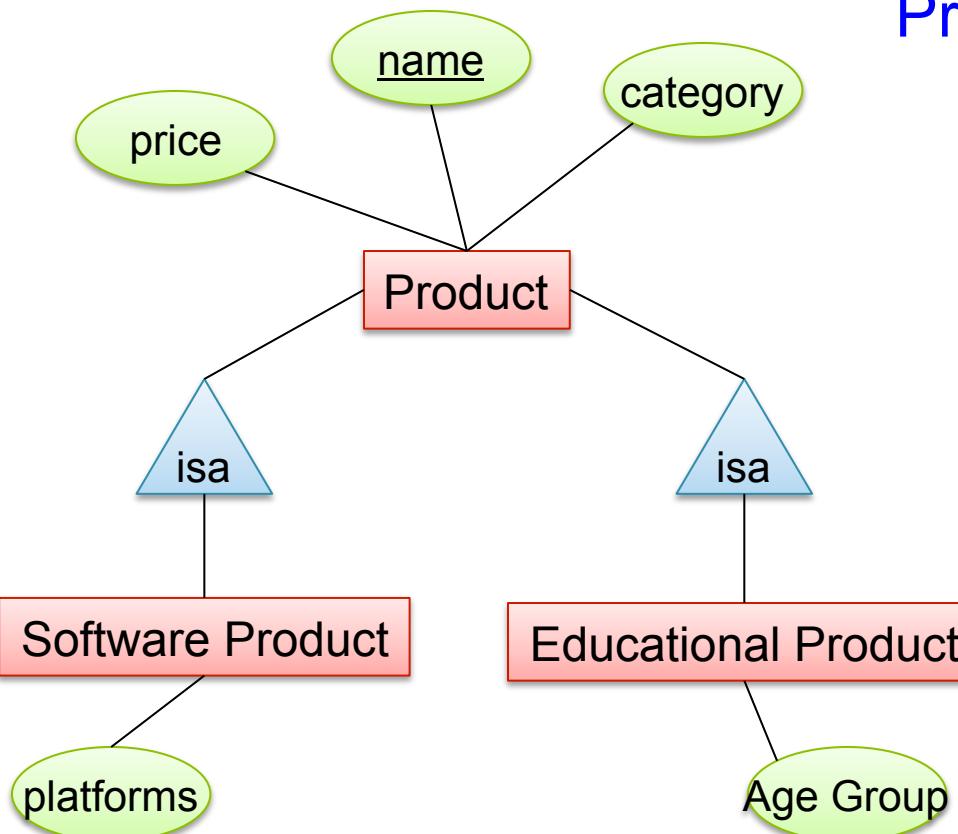
- Typically, each entity has a key
- ER relationships can include multiplicity
 - One-to-one, one-to-many, etc.
 - Indicated with arrows
- Can model multi-way relationships
- Can model subclasses
- And more...



Subclasses to Relations



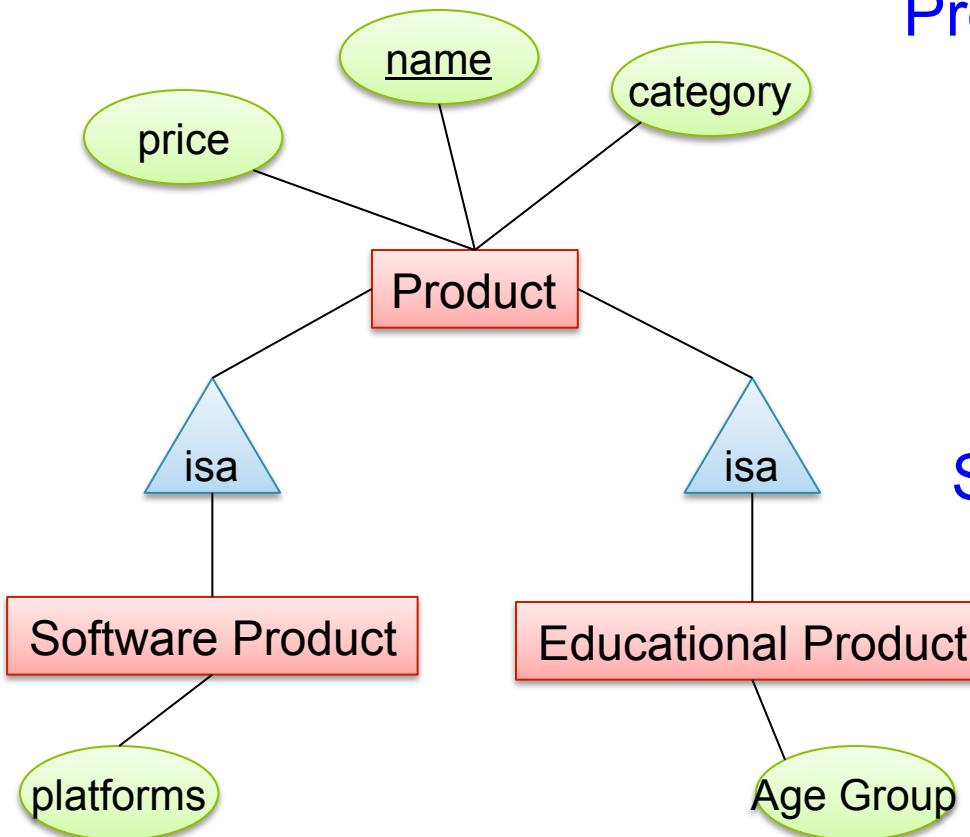
Subclasses to Relations



Product

Name	Price	Category
Gizmo	99	gadget
Camera	49	photo
Toy	39	gadget

Subclasses to Relations



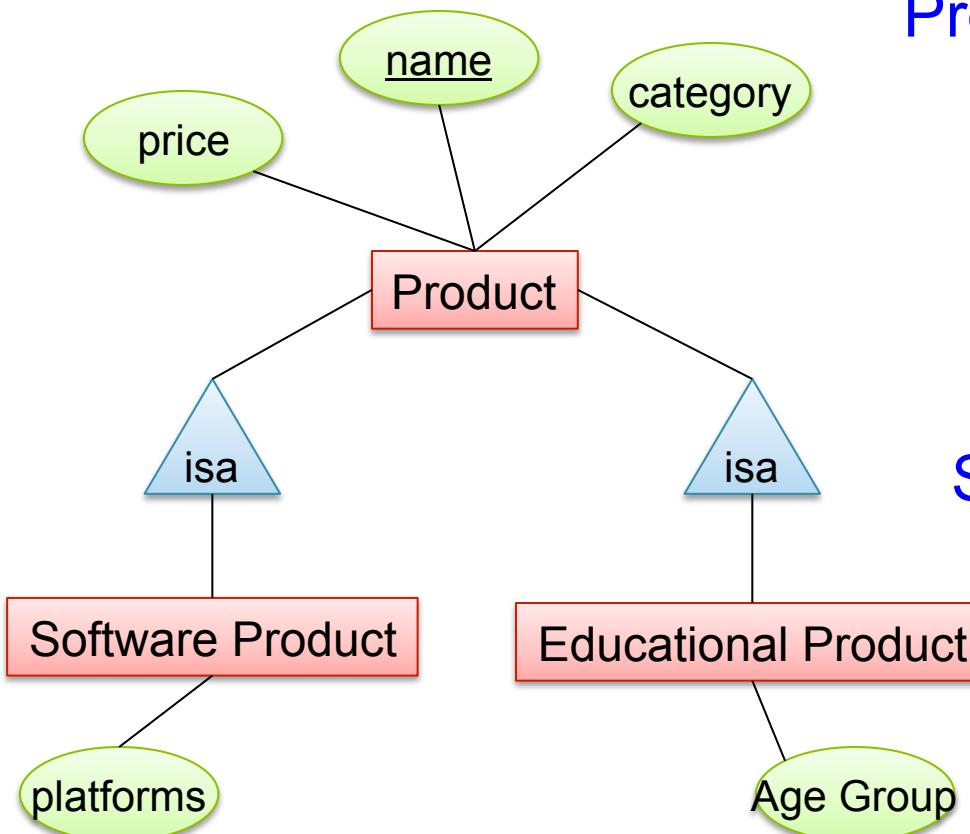
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Sw.Product

Name	platforms
Gizmo	unix

Subclasses to Relations



Product

Name	Price	Category
Gizmo	99	gadget
Camera	49	photo
Toy	39	gadget

Sw.Product

Name	platforms
Gizmo	unix

Ed.Product

Name	Age Group
Gizmo	toddler
Toy	senior

General approach to Translating Diagram into Relations

- Each entity set becomes a relation with a key
- Each relationship set becomes a relation with foreign keys except many-one relationships: just add a fk
- Each isA relationship becomes another relation, with both a key and foreign key

Outline

- Conceptual db design: entity-relationship model
- Problematic database designs
- Functional dependencies
- Normal forms and schema normalization

Relational Schema Design

Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

One person may have multiple phones, but lives in only one city

Primary key is thus (SSN, PhoneNumber)

What is the problem with this schema?

Relational Schema Design

Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

Anomalies:

- **Redundancy** = repeat data for Fred
- **Update anomalies** = what if Fred moves to “Bellevue”?
- **Deletion anomalies** = what if Joe deletes his phone number?

Relation Decomposition

Break the relation into two:

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

Name	<u>SSN</u>	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

<u>SSN</u>	<u>PhoneNumber</u>
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121

Anomalies have gone:

- No more repeated data
- Easy to move Fred to “Bellevue” (how ?)
- Easy to delete all Joe’s phone numbers (how ?)

Relational Schema Design (or Logical Design)

How do we do this systematically?

- Start with some relational schema
- Find out its *functional dependencies* (FDs)
- Use FDs to *normalize* the relational schema

Functional Dependencies (FDs)

Definition

If two tuples agree on the attributes

$$A_1, A_2, \dots, A_n$$

then they must also agree on the attributes

$$B_1, B_2, \dots, B_m$$

Formally:

$A_1 \dots A_n$ determines $B_1 \dots B_m$

$$A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$$

Functional Dependencies (FDs)

Definition $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$ holds in R if:

$\forall t, t' \in R,$

$(t.A_1 = t'.A_1 \wedge \dots \wedge t.A_m = t'.A_m \rightarrow t.B_1 = t'.B_1 \wedge \dots \wedge t.B_n = t'.B_n)$

R	A_1	\dots	A_m	B_1	\dots	B_n	
t							
t'							

if t, t' agree here then t, t' agree here

Example

An FD holds, or does not hold on an instance:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

EmpID → Name, Phone, Position

Position → Phone

but not Phone → Position

Example

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
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Position → Phone

Example

EmpID	Name	Phone	Position
E0045	Smith	1234 →	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

But not Phone → Position

Example

name → color
category → department
color, category → price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Green	Toys	99

Which FD's hold?

Buzzwords

- FD **holds** or **does not hold** on an instance
- If we can be sure that *every instance of R* will be one in which a given FD is true, then we say that **R satisfies the FD**
- If we say that R satisfies an FD, we are **stating a constraint on R**

An Interesting Observation

If all these FDs are true:

$\text{name} \rightarrow \text{color}$
 $\text{category} \rightarrow \text{department}$
 $\text{color, category} \rightarrow \text{price}$

Then this FD also holds:

$\text{name, category} \rightarrow \text{price}$

Find out from application domain some FDs,
Compute all FD's implied by them

Closure of a set of Attributes

Given a set of attributes A_1, \dots, A_n

The **closure** is the set of attributes B, denoted $\{A_1, \dots, A_n\}^+$,
s.t. $A_1, \dots, A_n \rightarrow B$

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Example:

- 1. name \rightarrow color
- 2. category \rightarrow department
- 3. color, category \rightarrow price

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Example:

1. $\text{name} \rightarrow \text{color}$
2. $\text{category} \rightarrow \text{department}$
3. $\text{color, category} \rightarrow \text{price}$

Closures:

$$\text{name}^+ = \{\text{name, color}\}$$

Closure of a set of Attributes

Given a set of attributes A_1, \dots, A_n

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Example:

1. $\text{name} \rightarrow \text{color}$
2. $\text{category} \rightarrow \text{department}$
3. $\text{color}, \text{category} \rightarrow \text{price}$

Closures:

$$\text{name}^+ = \{\text{name}, \text{color}\}$$

$$\{\text{name}, \text{category}\}^+ = \{\text{name}, \text{category}, \text{color}, \text{department}, \text{price}\}$$

Closure of a set of Attributes

Given a set of attributes A_1, \dots, A_n

The **closure** is the set of attributes B, denoted $\{A_1, \dots, A_n\}^+$,
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Example:

1. $\text{name} \rightarrow \text{color}$
2. $\text{category} \rightarrow \text{department}$
3. $\text{color}, \text{category} \rightarrow \text{price}$

Closures:

$$\text{name}^+ = \{\text{name}, \text{color}\}$$

$$\{\text{name}, \text{category}\}^+ = \{\text{name}, \text{category}, \text{color}, \text{department}, \text{price}\}$$

$$\text{color}^+ = \{\text{color}\}$$

Keys

- A **superkey** is a set of attributes A_1, \dots, A_n s.t. for any other attribute B , we have $A_1, \dots, A_n \rightarrow B$
- A **key** is a minimal superkey
(no subset is a superkey)

Computing (Super)Keys

- For all sets X , compute X^+
- If $X^+ = [\text{all attributes}]$, then X is a superkey
- Try reducing to the minimal X 's to get the key

Example

Product(name, price, category, color)

name, category → price
category → color

What is the key ?

Example

Product(name, price, category, color)

name, category → price
category → color

What is the key ?

(name, category) + = { name, category, price, color }

Example

Product(name, price, category, color)

name, category → price
category → color

What is the key ?

(name, category) + = { name, category, price, color }

Hence (name, category) is a key

Key or Keys ?

Can we have more than one key ?

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A → B
B → C
C → A

what are the keys here ?

Key or Keys ?

Can we have more than one key ?

A → B
B → C
C → A

AB → C
BC → A

A → BC
B → AC

what are the keys here ?

Eliminating Anomalies

Main idea:

- $X \rightarrow A$ is OK if X is a (super)key
- $X \rightarrow A$ is not OK otherwise
 - Need to decompose the table, but how?

Boyce-Codd Normal Form

There are no
“bad” FDs:

Definition. A relation R is in BCNF if:

Whenever $X \rightarrow B$ is a non-trivial dependency,
then X is a superkey.

Equivalently:

Definition. A relation R is in BCNF if:

$\forall X$, either $X^+ = X$ or $X^+ = [\text{all attributes}]$

BCNF Decomposition Algorithm

Normalize(R)

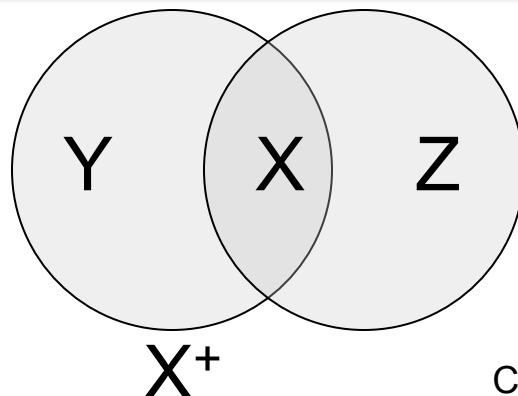
 find X s.t.: $X \neq X^+$ and $X^+ \neq [\text{all attributes}]$

if (not found) then “ R is in BCNF”

let $Y = X^+ - X$; $Z = [\text{all attributes}] - X^+$

 decompose R into $R1(X \cup Y)$ and $R2(X \cup Z)$

 Normalize($R1$); Normalize($R2$);



Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

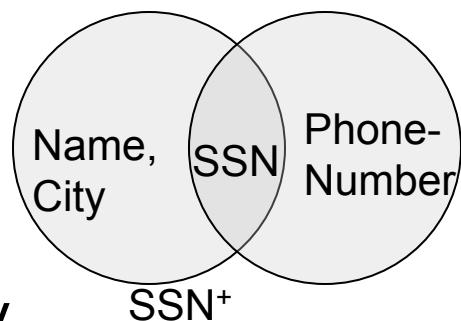
$\text{SSN} \rightarrow \text{Name, City}$

The only key is: $\{\text{SSN, PhoneNumber}\}$

Hence $\text{SSN} \rightarrow \text{Name, City}$ is a “bad” dependency

In other words:

$\text{SSN}^+ = \text{SSN, Name, City}$ and is neither SSN nor All Attributes



Find X s.t.: $X \neq X^+$ and $X^+ \neq [all\ attributes]$

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

$SSN \rightarrow name, age$

$age \rightarrow hairColor$

Find X s.t.: $X \neq X^+$ and $X^+ \neq [\text{all attributes}]$

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

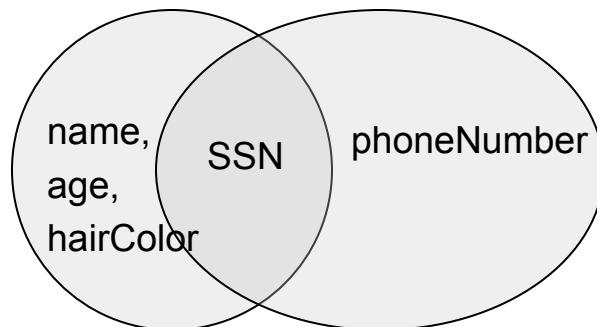
$\text{SSN} \rightarrow \text{name, age}$

$\text{age} \rightarrow \text{hairColor}$

Iteration 1: Person: $\text{SSN}^+ = \text{SSN, name, age, hairColor}$

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)



Find X s.t.: $X \neq X^+$ and $X^+ \neq [all\ attributes]$

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

$SSN \rightarrow name, age$

$age \rightarrow hairColor$

What are
the keys ?

Iteration 1: Person: $SSN^+ = SSN, name, age, hairColor$

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)

Iteration 2: P: $age^+ = age, hairColor$

Decompose: People(SSN, name, age)

Hair(age, hairColor)

Phone(SSN, phoneNumber)

Find X s.t.: $X \neq X^+$ and $X^+ \neq [all\ attributes]$

Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

$SSN \rightarrow name, age$

$age \rightarrow hairColor$

Note the keys!

Iteration 1: Person: $SSN^+ = SSN, name, age, hairColor$

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)

Iteration 2: P: $age^+ = age, hairColor$

Decompose: People(SSN, name, age)

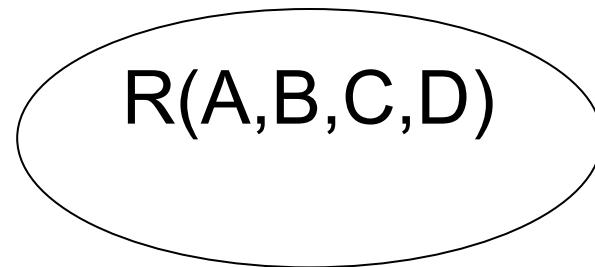
Hair(age, hairColor)

Phone(SSN, phoneNumber)

$R(A,B,C,D)$

Example: BCNF

$A \rightarrow B$
$B \rightarrow C$

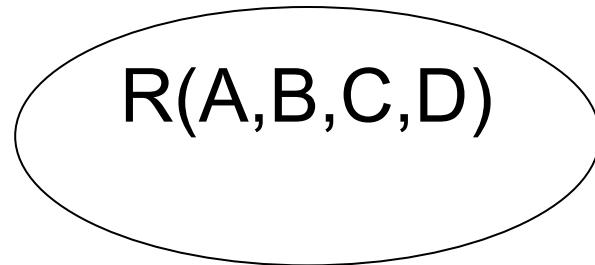


$R(A,B,C,D)$

Example: BCNF

$A \rightarrow B$
 $B \rightarrow C$

Recall: find X s.t.
 $X \subsetneq X^+ \subsetneq [\text{all-attrs}]$



$R(A,B,C,D)$

Example: BCNF

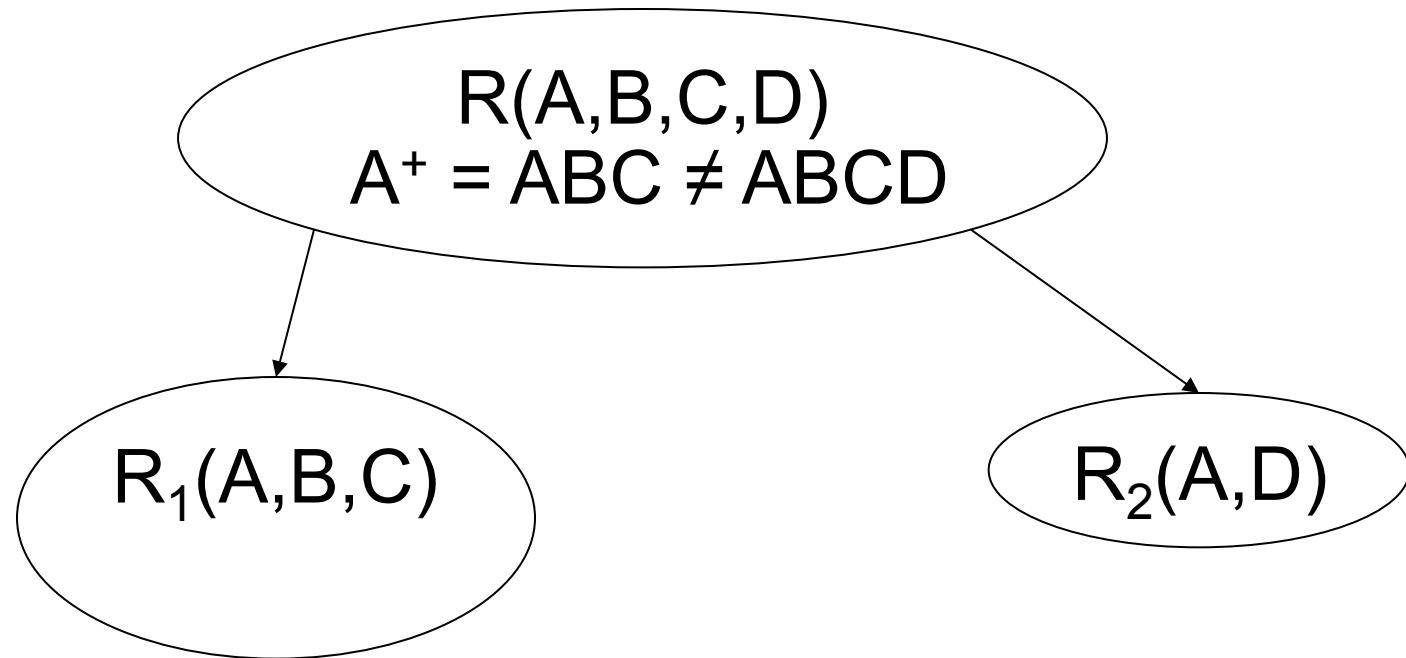
$A \rightarrow B$
$B \rightarrow C$

$R(A,B,C,D)$
 $A^+ = ABC \neq ABCD$

$R(A,B,C,D)$

Example: BCNF

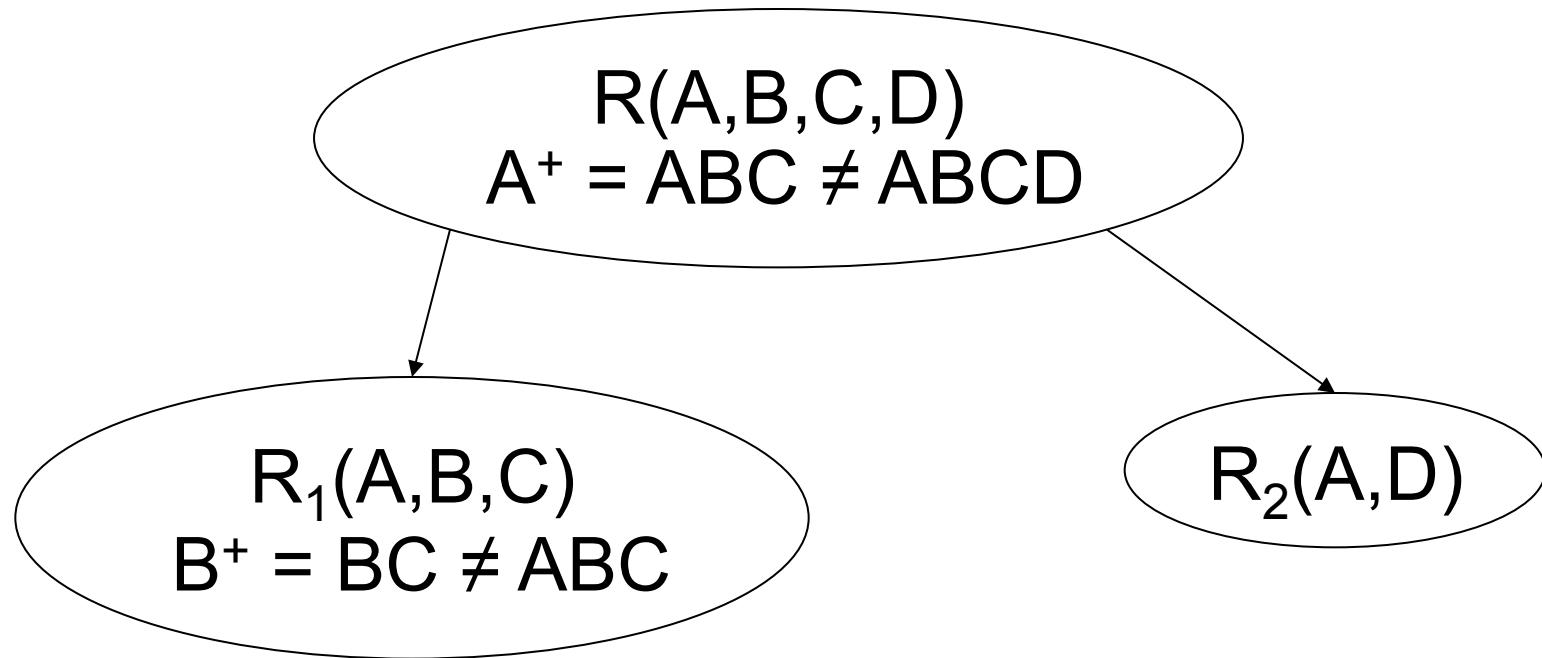
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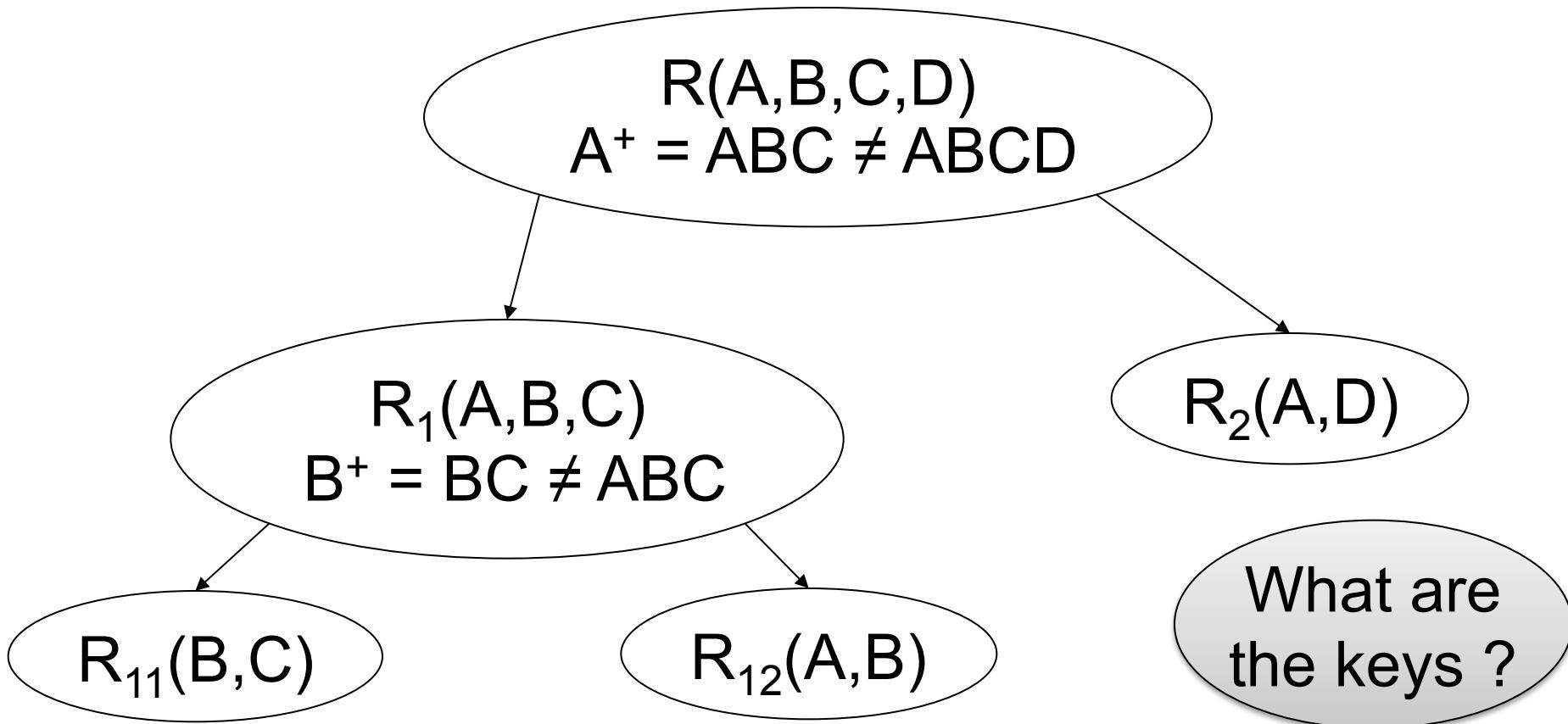
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$R(A,B,C,D)$

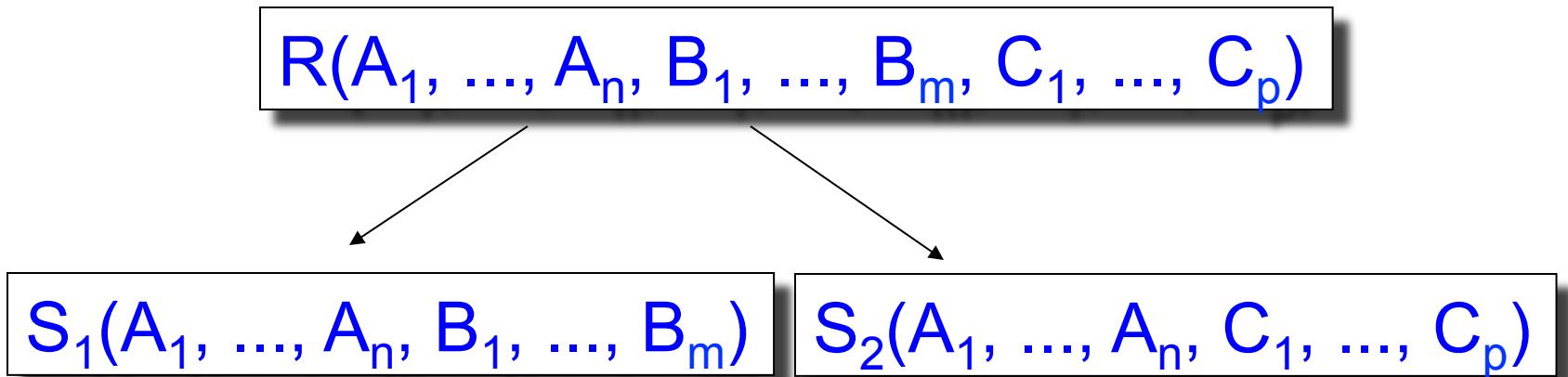
Example: BCNF

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array}$$



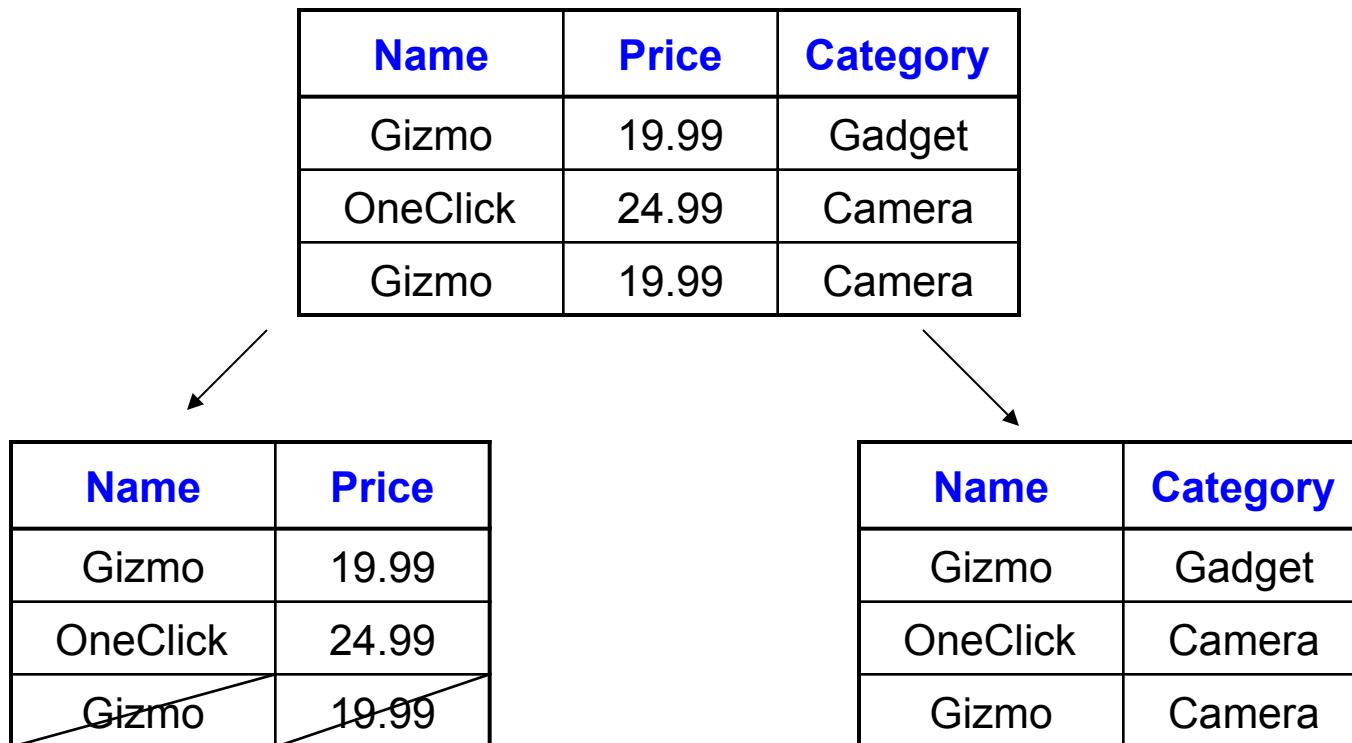
What happens if in R we first pick B^+ ? Or AB^+ ?

Decompositions in General



S_1 = projection of R on $A_1, \dots, A_n, B_1, \dots, B_m$
 S_2 = projection of R on $A_1, \dots, A_n, C_1, \dots, C_p$

Lossless Decomposition



Lossy Decomposition

What is
lossy here?

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Price	Category
19.99	Gadget
24.99	Camera
19.99	Camera

Lossy Decomposition

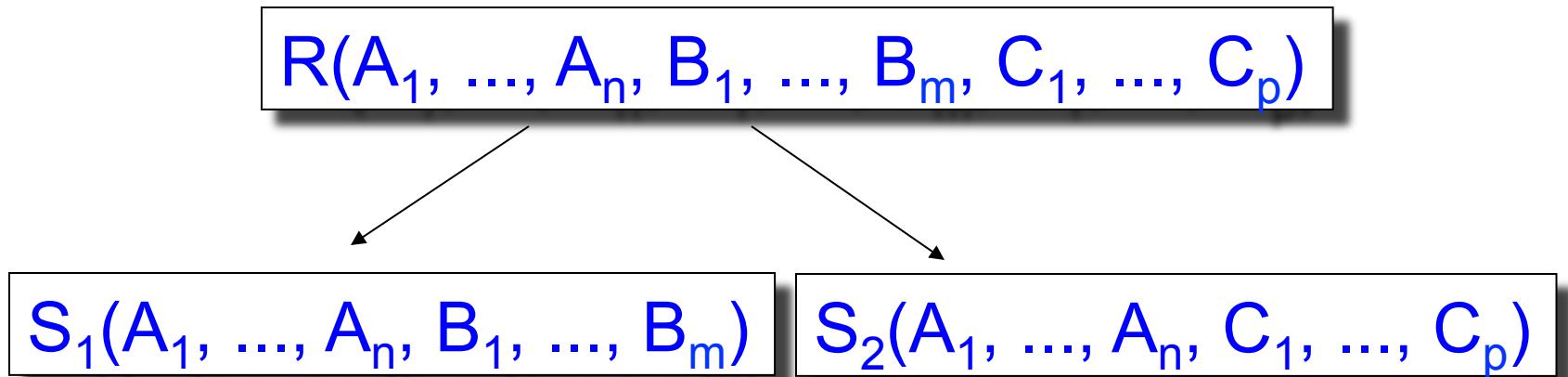
Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

A diagram illustrating lossy decomposition. Three arrows point from the original table above to two separate tables below. The left arrow points to a table with columns Name and Category. The right arrow points to a table with columns Price and Category.

Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Price	Category
19.99	Gadget
24.99	Camera
19.99	Camera

Decomposition in General



Let: S_1 = projection of R on $A_1, \dots, A_n, B_1, \dots, B_m$
 S_2 = projection of R on $A_1, \dots, A_n, C_1, \dots, C_p$

The decomposition is called lossless if $R = S_1 \bowtie S_2$

Fact: If $A_1, \dots, A_n \rightarrow B_1, \dots, B_m$ then the decomposition is lossless

It follows that every BCNF decomposition is lossless

Testing for Lossless Join

If we decompose R into $\Pi_{S_1}(R)$, $\Pi_{S_2}(R)$, $\Pi_{S_3}(R)$, ...
Is it true that $S_1 \bowtie S_2 \bowtie S_3 \bowtie \dots = R$?

That is true if we can show that:

$R \subseteq S_1 \bowtie S_2 \bowtie S_3 \bowtie \dots$ always holds (why?)

$R \supseteq S_1 \bowtie S_2 \bowtie S_3 \bowtie \dots$ neet to check

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$
R satisfies: $A \rightarrow B$, $B \rightarrow C$, $CD \rightarrow A$

$S1 = \Pi_{AD}(R)$, $S2 = \Pi_{AC}(R)$, $S3 = \Pi_{BCD}(R)$,
hence $R \subseteq S1 \bowtie S2 \bowtie S3$

Need to check: $R \supseteq S1 \bowtie S2 \bowtie S3$

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$$R(A,B,C,D) = S_1(A,D) \bowtie S_2(A,C) \bowtie S_3(B,C,D)$$

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$S_1 = \Pi_{AD}(R)$, $S_2 = \Pi_{AC}(R)$, $S_3 = \Pi_{BCD}(R)$,
hence $R \subseteq S_1 \bowtie S_2 \bowtie S_3$

Need to check: $R \supseteq S_1 \bowtie S_2 \bowtie S_3$

Suppose $(a,b,c,d) \in S_1 \bowtie S_2 \bowtie S_3$ Is it also in R?

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$$R(A,B,C,D) = S_1(A,D) \bowtie S_2(A,C) \bowtie S_3(B,C,D)$$

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hence $R \subseteq S_1 \bowtie S_2 \bowtie S_3$

Need to check: $R \supseteq S_1 \bowtie S_2 \bowtie S_3$

Suppose $(a,b,c,d) \in S_1 \bowtie S_2 \bowtie S_3$ Is it also in R?

R must contain the following tuples:

A	B	C	D
a	b1	c1	d

Why ?

$(a,d) \in S_1 = \Pi_{AD}(R)$

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$$R(A,B,C,D) = S_1(A,D) \bowtie S_2(A,C) \bowtie S_3(B,C,D)$$

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Need to check: $R \supseteq S_1 \bowtie S_2 \bowtie S_3$

Suppose $(a,b,c,d) \in S_1 \bowtie S_2 \bowtie S_3$ Is it also in R?

R must contain the following tuples:

A	B	C	D
a	b1	c1	d
a	b2	c	d2

Why ?

$(a,d) \in S_1 = \Pi_{AD}(R)$

$(a,c) \in S_2 = \Pi_{BD}(R)$

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$$R(A,B,C,D) = S_1(A,D) \bowtie S_2(A,C) \bowtie S_3(B,C,D)$$

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Need to check: $R \supseteq S_1 \bowtie S_2 \bowtie S_3$

Suppose $(a,b,c,d) \in S_1 \bowtie S_2 \bowtie S_3$ Is it also in R?

R must contain the following tuples:

A	B	C	D
a	b1	c1	d
a	b2	c	d2
a3	b	c	d

Why ?

$(a,d) \in S_1 = \Pi_{AD}(R)$

$(a,c) \in S_2 = \Pi_{BD}(R)$

$(b,c,d) \in S_3 = \Pi_{BCD}(R)$

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$$R(A,B,C,D) = S_1(A,D) \bowtie S_2(A,C) \bowtie S_3(B,C,D)$$

R satisfies: $A \rightarrow B$, $B \rightarrow C$, $CD \rightarrow A$

$S_1 = \Pi_{AD}(R)$, $S_2 = \Pi_{AC}(R)$, $S_3 = \Pi_{BCD}(R)$,
hence $R \subseteq S_1 \bowtie S_2 \bowtie S_3$

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“Chase” them (apply FDs):

$A \rightarrow B$

A	B	C	D
a	b1	c1	d
a	b1	c	d2
a3	b	c	d

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$$

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$(a,d) \in S1 = \Pi_{AD}(R)$

$(a,c) \in S2 = \Pi_{BD}(R)$

$(b,c,d) \in S3 = \Pi_{BCD}(R)$

“Chase” them (apply FDs):

$A \rightarrow B$

A	B	C	D
a	b1	c1	d
a	b1	c	d2
a3	b	c	d

$B \rightarrow C$



A	B	C	D
a	b1	c	d
a	b1	c	d2
a3	b	c	d

Example from textbook Ch. 3.4.2

The Chase Test for Lossless Join

$$R(A,B,C,D) = S1(A,D) \bowtie S2(A,C) \bowtie S3(B,C,D)$$

R satisfies: $A \rightarrow B$, $B \rightarrow C$, $CD \rightarrow A$

$S1 = \Pi_{AD}(R)$, $S2 = \Pi_{AC}(R)$, $S3 = \Pi_{BCD}(R)$,
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$(a,d) \in S1 = \Pi_{AD}(R)$

$(a,c) \in S2 = \Pi_{BD}(R)$

$(b,c,d) \in S3 = \Pi_{BCD}(R)$

“Chase” them (apply FDs):

$A \rightarrow B$

A	B	C	D
a	b1	c1	d
a	b1	c	d2
a3	b	c	d

$B \rightarrow C$

A	B	C	D
a	b1	c	d
a	b1	c	d2
a3	b	c	d

$CD \rightarrow A$

A	B	C	D
a	b1	c	d
a	b1	c	d2
a	b	c	d

Hence R
contains (a,b,c,d)

Schema Refinements = Normal Forms

- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce Codd Normal Form = no bad FDs
- 3rd Normal Form = see book
 - BCNF is lossless but can cause loss of ability to check some FDs
 - 3NF fixes that (is lossless and dependency-preserving), but some tables might not be in BCNF – i.e., they may have redundancy anomalies