Priority Queues

Data Structures and Parallelism

Logistics

Project 1:

- Due Thursday July 11th at 11:59 PM
- One "Checkpoint" due next Friday
- Generics, Debugging, "out of memory" info on "handouts" webpage

Checkpoint:

- The project spec lists where we recommend you be (it represents being about halfway).
- You must answer the progress survey (each of you)
- You can also (optionally) sign up for a meeting with a staff member
- **As long as you answer the survey and have made a good faith effort to be half-way done, you'll "pass" the checkpoint.**

Tokens:

- 4 tokens per person. You can use them to either
- Get a late day on a project (max 2 per project)
- Redo an exercise in the last week of the quarter.

Exercise 1 is also out tonight (due same day as the project checkpoint)

Small Note from Wednesday

Worst Case != Big O

Best Case != Big Ω

Asymptotic analysis is what is done to mathematical expressions after best/worst/average case assumptions are already made.

If we get 3 + 4n algorithm operations *already assuming a worst-case*, we can find O and Ω for that expression just as viably.

A New ADT

Our previous worklists (stacks, queues, etc.) all choose the next element based on the order they were inserted.

That's not always a good idea.

Emergency rooms aren't first-come-first-served.

Sometimes our objects come with a **priority**, that tells us what we need to do next.

An ADT that can handle a line with priorities is a **priority queue.**

Priority Queue ADT

Min Priority Queue ADT

state

Set of comparable values
- Ordered based on "priority"

behavior

insert(value) – add a new element to the collection

removeMin() – returns the element with the smallest priority, removes it from the collection

peekMin() – find, but do not remove the element with the smallest priority

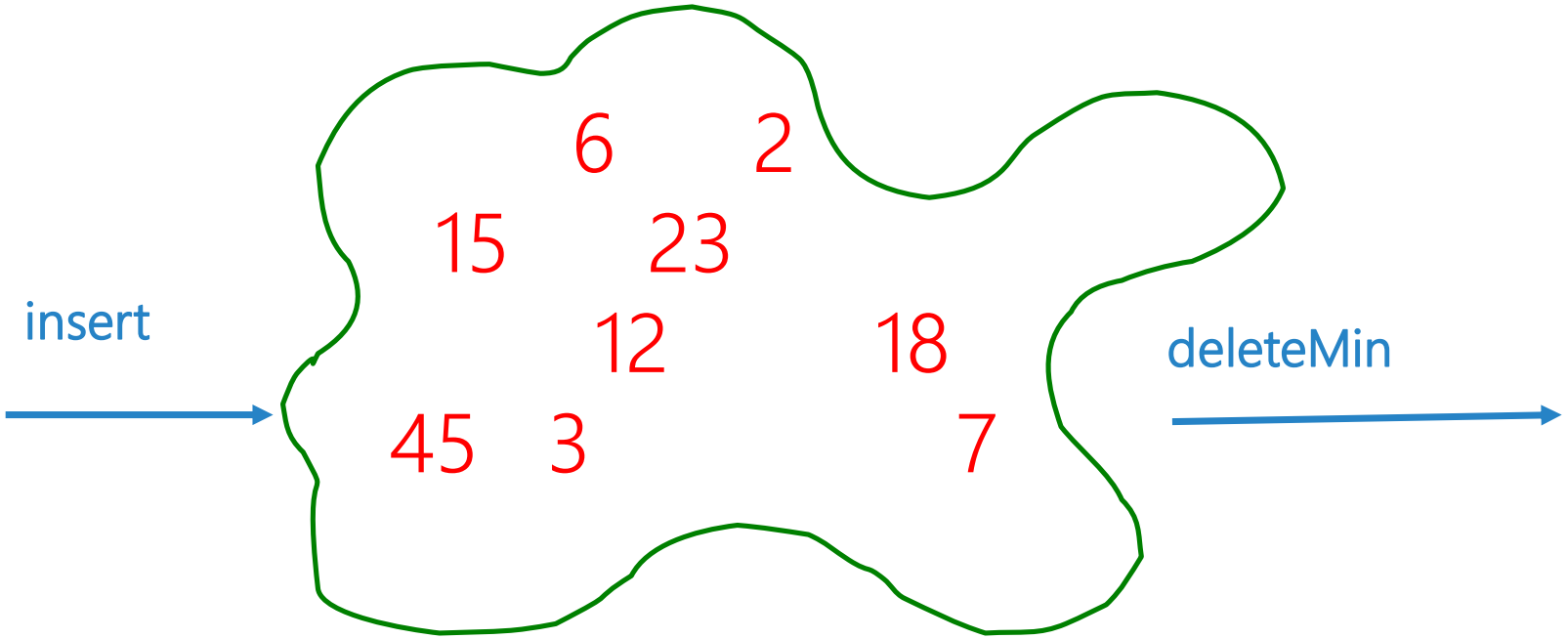
Uses:

- Operating System
- Well-designed printers
- Huffman Codes (in 143)
- Sorting (in Project 2)
- Graph algorithms

Priority Queue ADT

Main Operations:

- **`insert`**
- **`deleteMin`**



Key property: **`deleteMin`** returns and deletes from the queue the item with greatest priority (lowest priority value)

- Can resolve ties arbitrarily

Implementing Priority Queues: Take I

Maybe we already know how to implement a priority queue.
How long would insert and removeMin take with these data structures?

	Insert	removeMin
Unsorted Array		
Unsorted Linked List		
Sorted Linked List		
Sorted Circular Array		
Binary Search Tree		

For Array implementations, assume that the array is not yet full.
Other than this assumption, do **worst case** analysis.

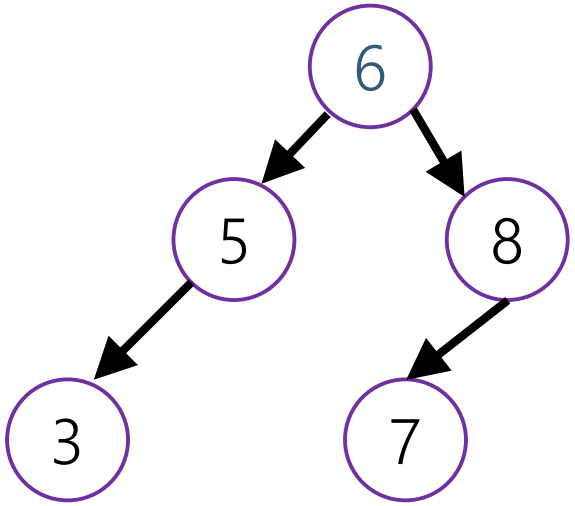
Review: Binary Search Trees

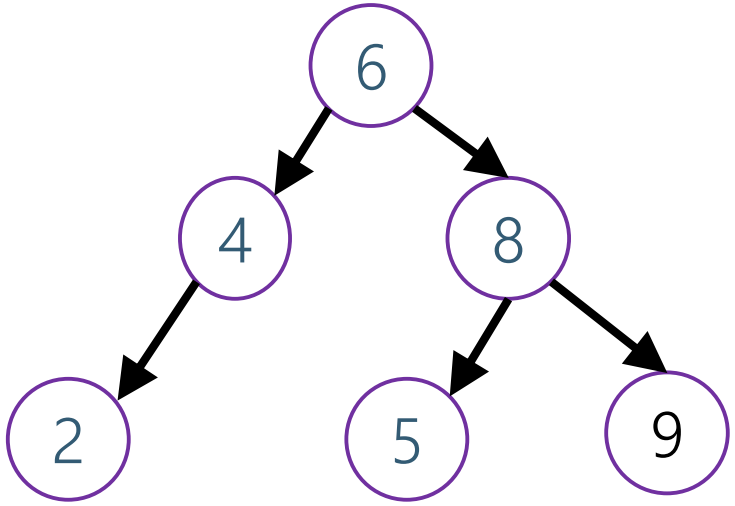
A BST is:

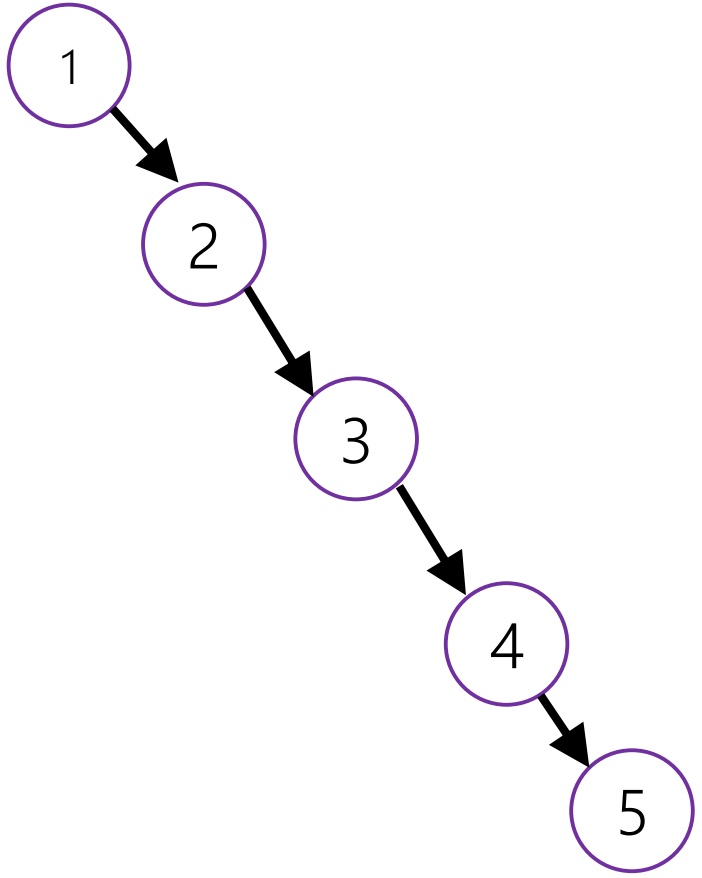
1. A binary tree

2. For each node, everything in its left subtree is smaller than it and everything in its right subtree is larger than it.

Are These BSTs?







Implementing Priority Queues: Take I

Maybe we already know how to implement a priority queue.
How long would insert and removeMin take with these data structures?

	Insert	removeMin
Unsorted Array	$\Theta(1)$	$\Theta(n)$
Unsorted Linked List	$\Theta(1)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(1)$
Sorted Circular Array	$\Theta(n)$	$\Theta(1)$
Binary Search Tree		

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Sorted Circular Array	$\Theta(n)$	$\Theta(1)$
Binary Search Tree	$\Theta(n)$	$\Theta(n)$

For Array implementations, assume that the array is not yet full.
Other than this assumption, do **worst case** analysis.

Implementing Priority Queues: Take II

BSTs have really bad behavior in the **worst** case, but is it actually a common problem? They seem like they would be a good fit…

Fact: On average, the height of a BST is $O(\log n)$

(for some suitable definition of "average")

Can we somehow get that behavior in the **worst case** for priority queues?

BST Properties

A BST is:

1. A binary tree

2. For each node, everything in its left subtree is smaller than it and everything in its right subtree is larger than it.

Point 2 is what causes the really bad behavior in the worst case.

We probably don't want exactly that requirement for implementing a priority queue.

Maybe we can explicitly enforce that we don't get a degenerate tree.

Binary Heaps

A Binary Heap is

1. A Binary Tree

2. Every node is less than or equal to all of its children
   - In particular, the smallest element must be the root!

3. The tree is **complete**
   - Every level of the tree is completely filled, except possibly the last row, which is filled from left to right.
   - No degenerate trees!

Tree Words

Root(T):

Leaves(T):

Children(B):

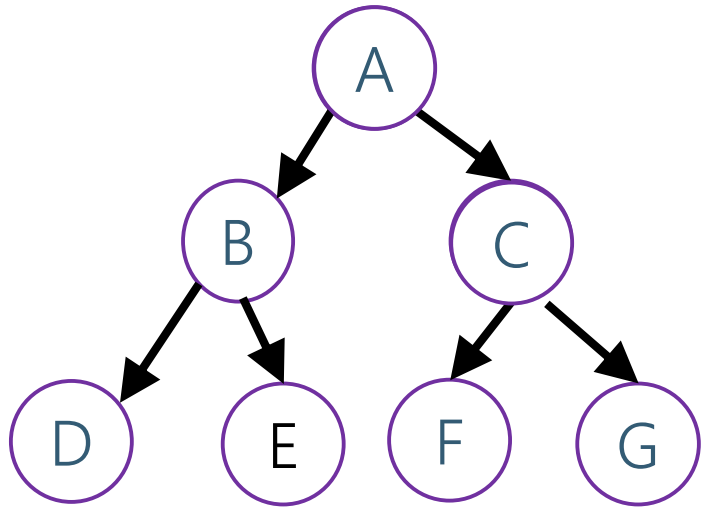
Parent(C):

Siblings(E):

Ancestors(F):

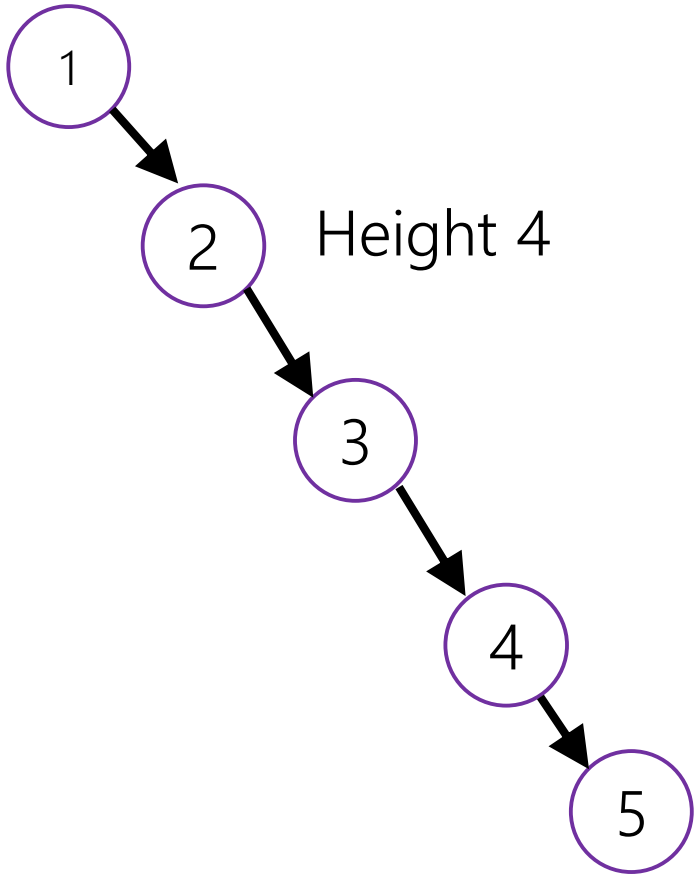
Descendents(A):

Subtree(C):

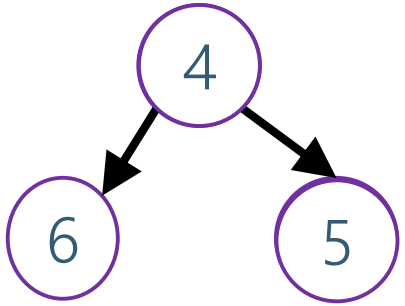


Tree Words

Height – the number of edges on the longest path from the root to a leaf.



Height 4

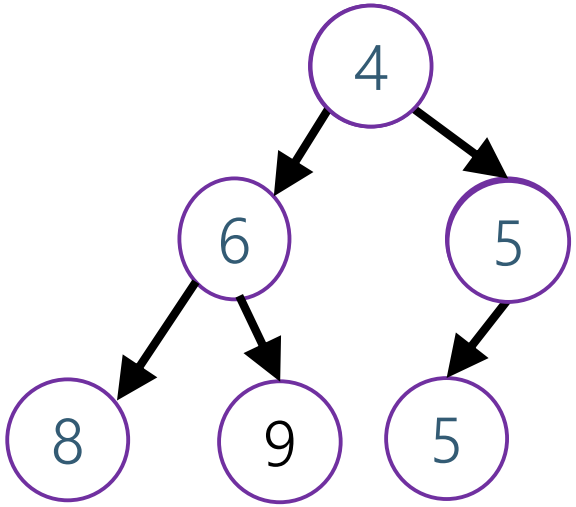


Height 1

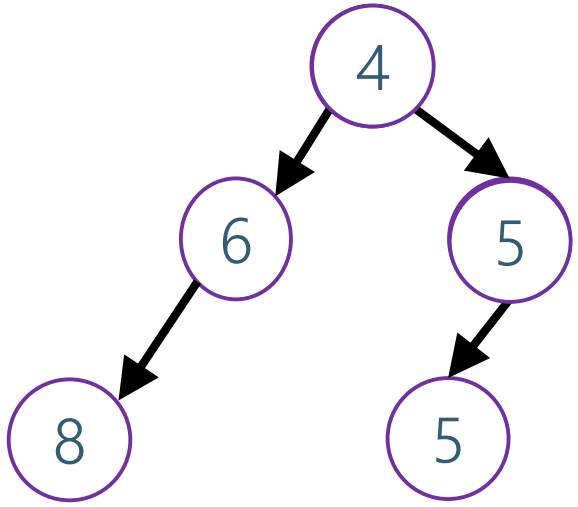
Tree Words

Complete – every row is completely filled, except possibly the last row, which is filled from left to right.

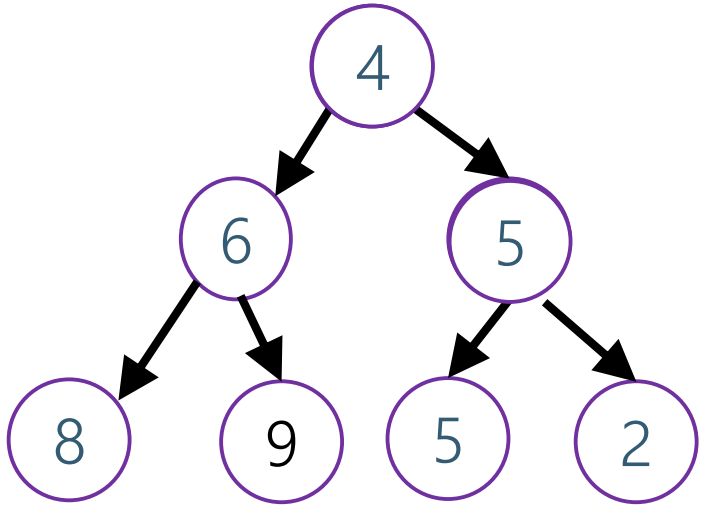
Perfect – every row is completely filled



Complete, but not perfect



Neither



Both Perfect and Complete

Heights of Perfect Trees

How many nodes are there in level i of a perfect binary tree?

2^i.

Thus the total number of nodes in a perfect binary tree of height h is

$\sum_{i=0}^{h} 2^i = 2^{h+1} - 1$.

So if we have n nodes in a perfect tree, we can use the formula

$n = 2^{h+1} - 1$ to conclude that $h = O(\log n)$,so

A perfect tree with n nodes has height $O(\log n)$.

A similar argument can show the same statement for complete trees.

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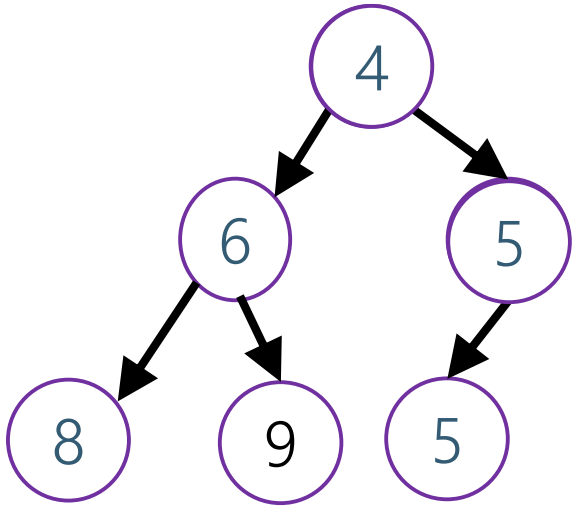
   - No degenerate trees!

Implementing Heaps

Let's start with removeMin.



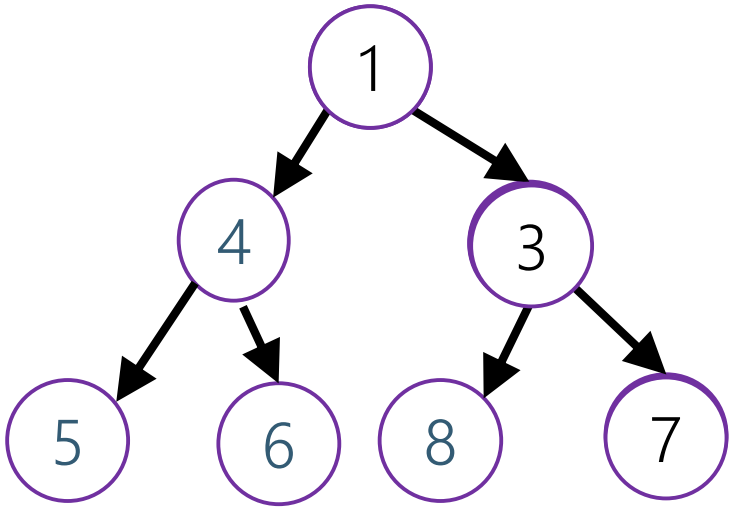
Idea: take the bottom right-most node and use it to plug the hole

Shape is correct now

But that value might be to big. We need to "percolate it down"

Implementing Heaps

Insertion



What is the shape going to be after the insertion?

Again, plug the hole first.

Might violate the heap property. Percolate it up

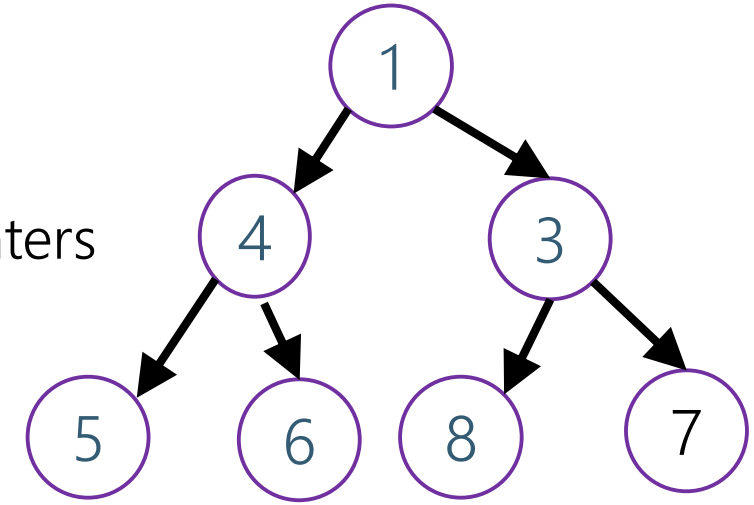
An Optimization

Pointers are annoying.

They're also slow.

Our tree is so nicely shaped, we don't need pointers

We can use an array instead.



	1	4	3	5	6	8	7			
0	1	2	3	4	5	6	7	8	9	10

An Optimization

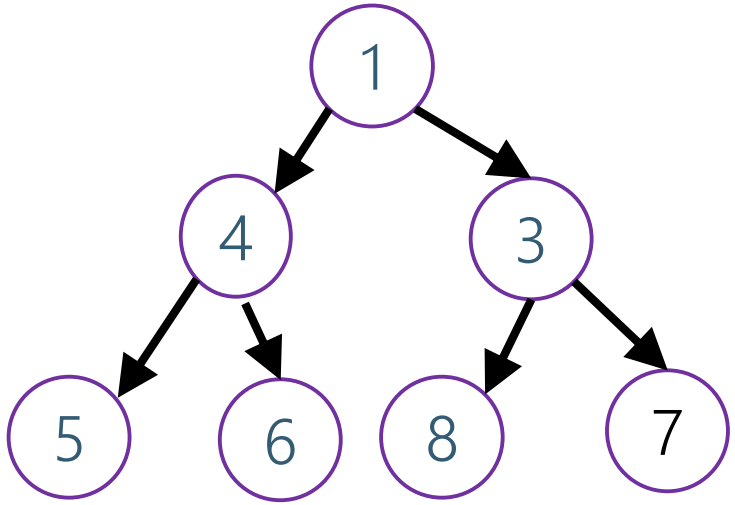
If I'm at index i, what is the index of:

My left child, right child and parent?

My left child: $2i$

My right child: $2i + 1$

My parent: $\left\lfloor \frac{i}{2} \right\rfloor$



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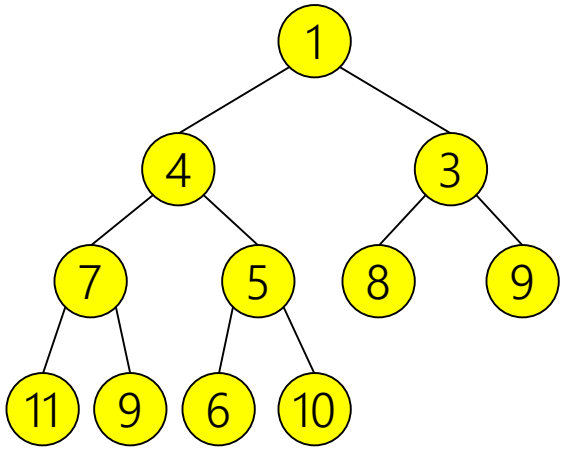
More DeleteMin Practice

Delete first 3 items



More DeleteMin Practice

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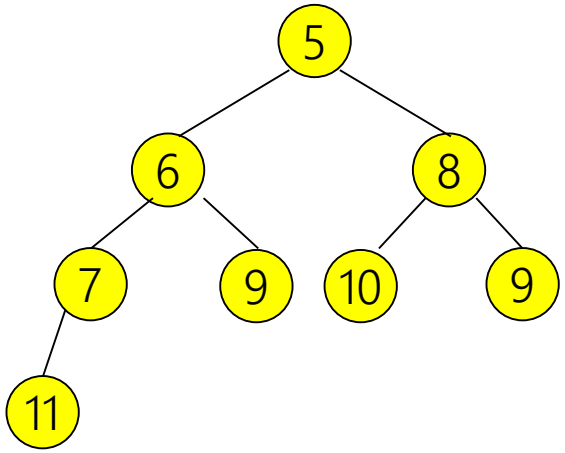
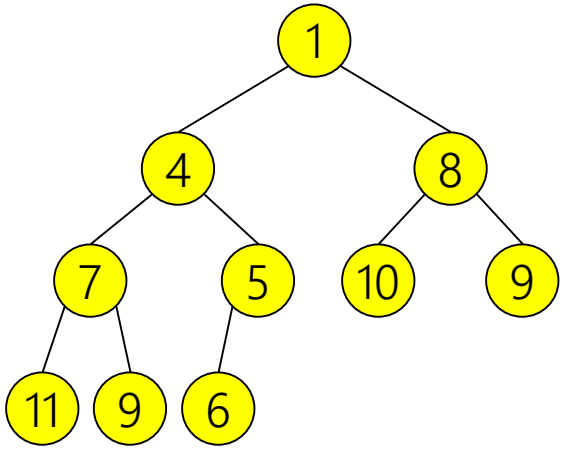


More Insert Practice

Insert 2, 4, 11



More Insert Practice

Insert 2, 4, 11