https://courses.cs.washington.edu/446

Machine Learning CSE446

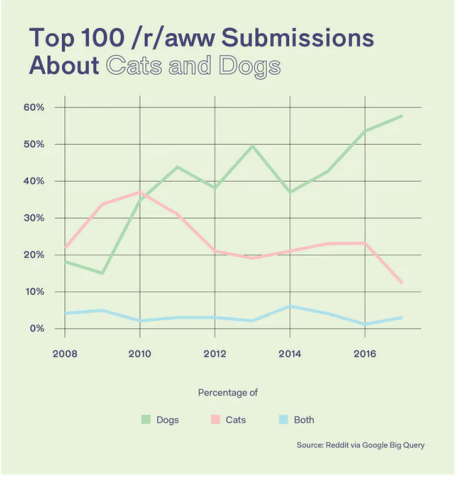
Kevin Jamieson and Anna Karlin
University of Washington

April 1, 2019

Traditional algorithms

Social media mentions of Cats vs. Dogs

Reddit



Google

Twitter?

Graphics courtesy of https://theoutline.com/post/3128/dogs-cats-internet-popularity?zd=1

Traditional algorithms

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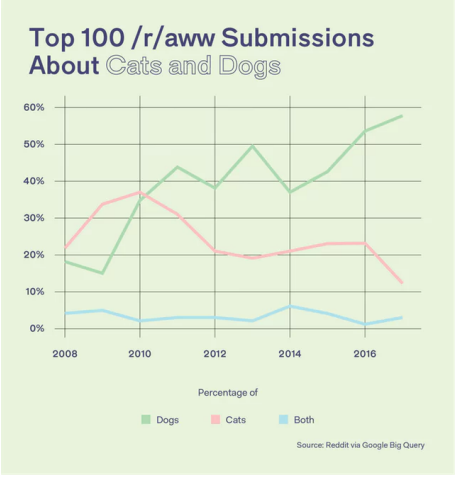

Google

Twitter?

Write a program that sorts tweets into those containing "**cat**", "**dog**", or ***other***

Graphics courtesy of https://theoutline.com/post/3128/dogs-cats-internet-popularity?zd=1

Traditional algorithms

Social media mentions of Cats vs. Dogs

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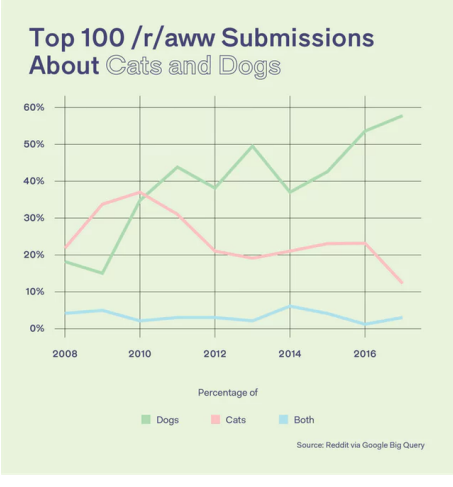
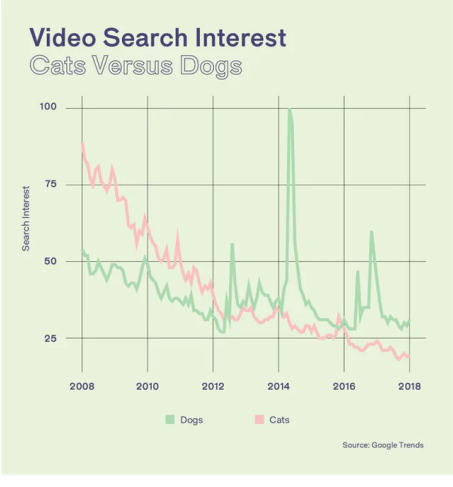


Google



Twitter?

```
cats = []
dogs = []
other = []
for tweet in tweets:
    if "cat" in tweet:
        cats.append(tweet)
    elseif "dog" in tweet:
        dogs.append(tweet)
    else:
        other.append(tweet)
return cats, dogs, other
```

Write a program that sorts tweets into those containing "**cat**", "**dog**", or ***other***

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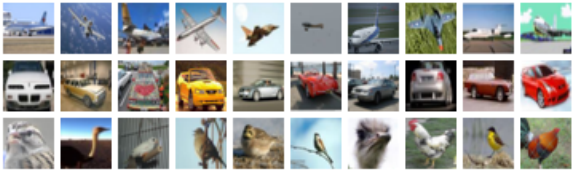
Machine learning algorithms

Write a program that sorts images into those containing "**birds**", "**airplanes**", or ***other.***



airplane

other

bird

Machine learning algorithms

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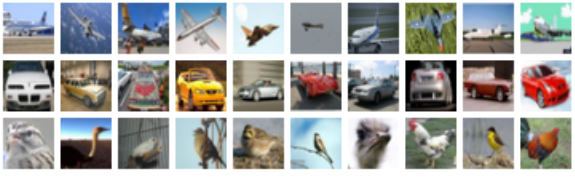

airplane

other

bird

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for image in images:
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   elseif plane in image:
      planes.append(image)
   else:
      other.append(tweet)
return birds, planes, other
```

Machine learning algorithms

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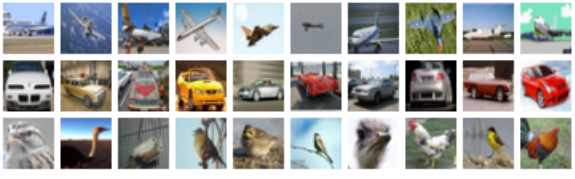
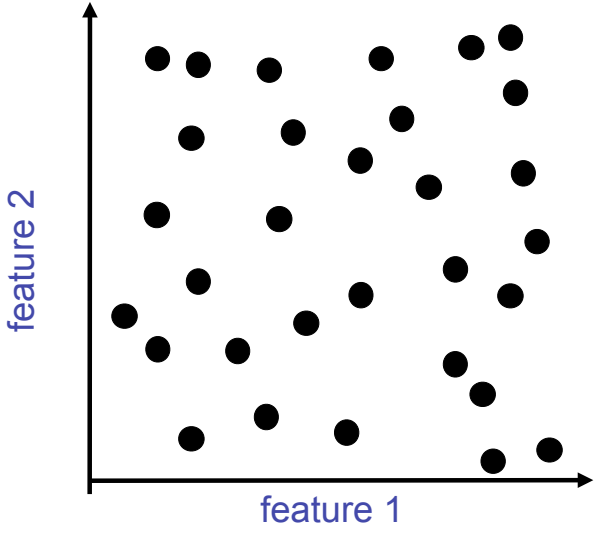


airplane
other
bird

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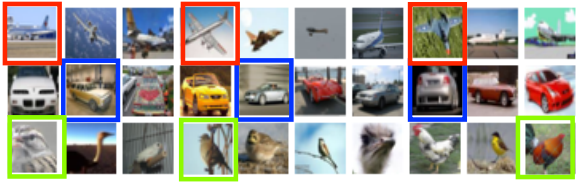
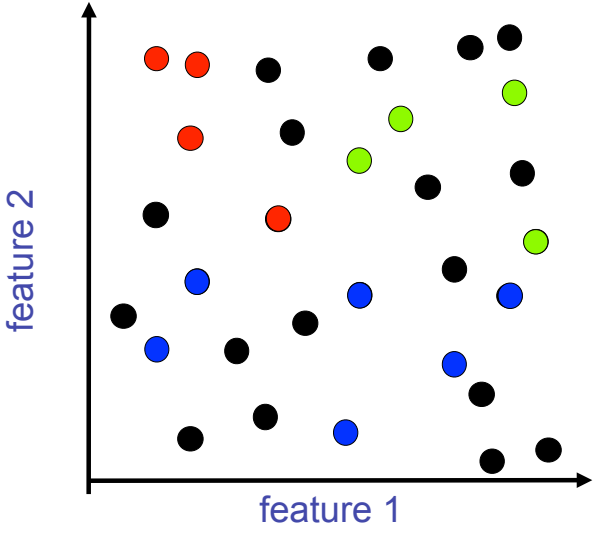


airplane
other
bird

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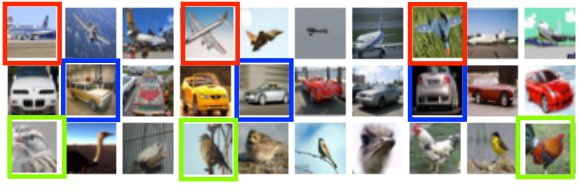
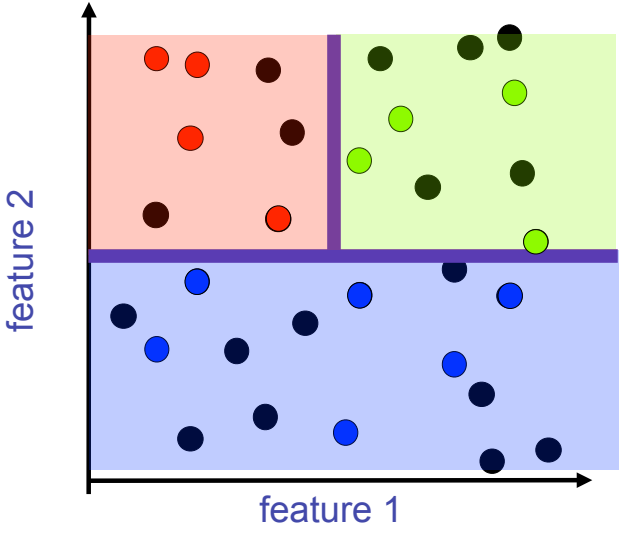


airplane
other
bird

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Machine learning algorithms

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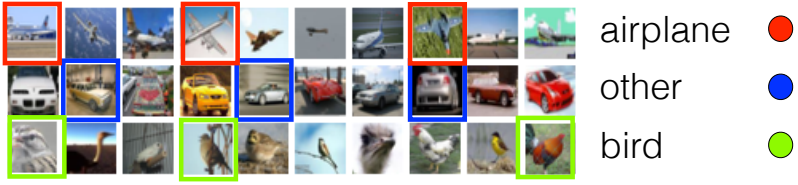

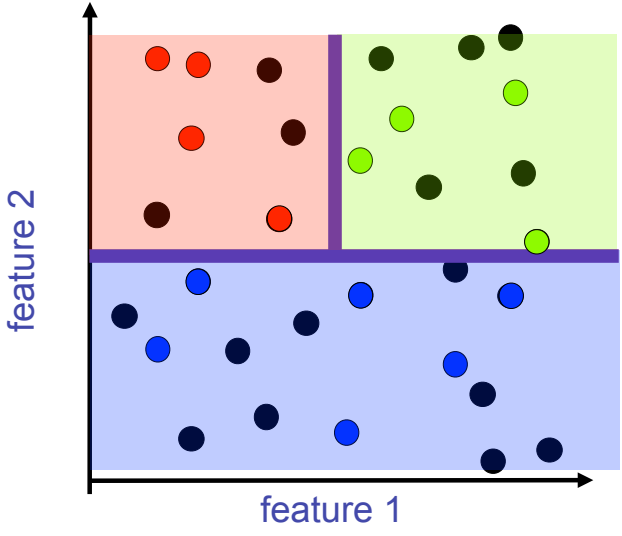


```
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other = []
for image in images:
   if bird in image:
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   elseif plane in image:
      planes.append(image)
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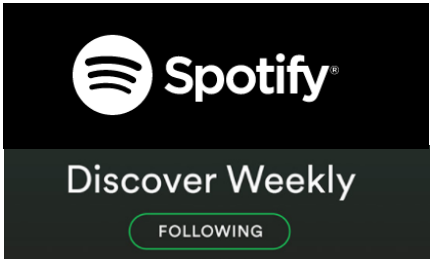
      other.append(tweet)
return birds, planes, other
```

The decision rule of
`if "cat" in tweet:`
is **hard coded by expert.**

The decision rule of
`if bird in image:`
is **LEARNED using DATA**

Machine Learning Ingredients

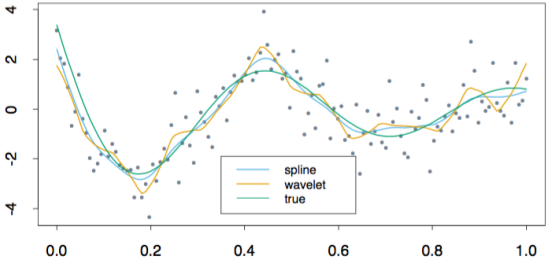
- **Data**: past observations
- **Hypotheses/Models**: devised to capture the patterns in data
- **Prediction**: apply model to forecast future observations



You may also like…

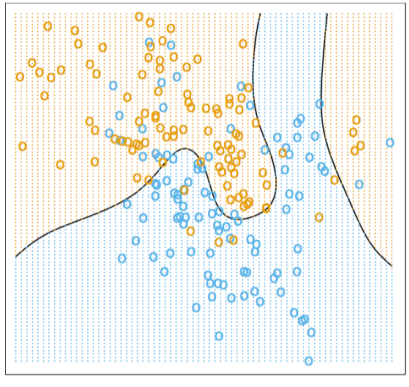
ML uses past data to make personalized predictions

Flavors of ML



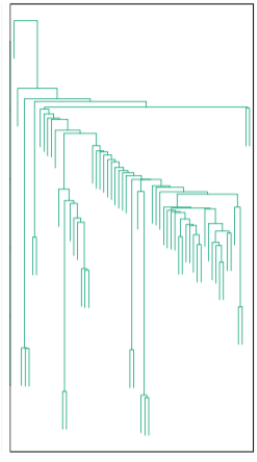
Regression

Predict continuous value:
ex: stock market, credit score, temperature, Netflix rating



Classification

Predict categorical value:
loan or not? spam or not? what disease is this?



Unsupervised Learning

Predict structure:
tree of life from DNA, find similar images, community detection

Mix of statistics (theory) and algorithms (programming)

CSE446: Machine Learning

Lecture: Monday, Wednesday 3:30-4:50 Room: GUG 220

Instructor: Kevin Jamieson and Anna Karlin

Contact: cse446-staff@cs.washington.edu

Website: https://courses.cs.washington.edu/courses/cse446/19sp/

What this class is:

- **Fundamentals of ML:** bias/variance tradeoff, overfitting, optimization and computational tradeoffs, supervised learning (e.g., linear, boosting, deep learning), unsupervised models (e.g. k-means, EM, PCA)
- **Preparation for further learning:** the field is fast-moving, you will be able to apply the basics and teach yourself the latest

What this class is not:

- **Survey course:** laundry list of algorithms, how to win Kaggle
- **An easy course:** familiarity with intro linear algebra and probability are assumed, homework will be time-consuming

Prerequisites

- Formally:
  - CSE 312, MATH 308, STAT 390 or equivalent
- Familiarity with:
  - Linear algebra
    - linear dependence, rank, linear equations
  - Multivariate calculus
  - Probability and statistics
    - Distributions, densities, marginalization, moments
  - Algorithms
    - Basic data structures, complexity
- “Can I learn these topics concurrently?”
- Use HW0 to judge skills
- **See website for review materials!**

Grading

- 5 homeworks (50%)
  - Each contains both theoretical questions and will have programming
  - Collaboration okay. You must write, submit, and understand your answers and code (which we may run)
  - Do not Google for answers.
- Midterm **May 8** (20%) and Final **June 13** (30%)

Homeworks

- HW 0 is out (**Due next Monday Midnight**, 37 points)
  - Short *review*
  - Work individually, treat as barometer for readiness
- HW 1,2,3,4 (100 points each)
  - They are not easy or short. Start early.
- Submit to Gradescope
- Regrade requests on Gradescope
- **There is no credit for late work, receives 0 points.**

1. All code must be written in Python
2. All written work must be typeset (e.g., LaTeX)

See course website for tutorials and references.

Communication Channels

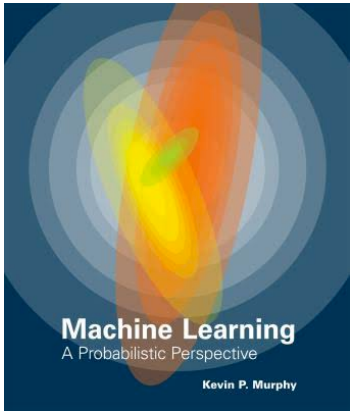
- **Announcements, questions about class, homework help**
  - Piazza (get code on Canvas)
  - Section
  - Office hours (start tomorrow)
- **Regrade requests**
  - Directly to Gradescope
- **Personal concerns**
  - Email: cse446-staff@cs.washington.edu
- **Anonymous feedback**
  - See website for link

Staff

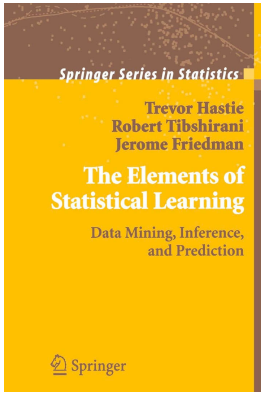
- 13 Experienced TAs, lots of office hours (see website)
- Satvik Agarwal, Kousuke Ariga, Eric Chan, Benjamin Evans, Shobhit Hathi, Zeyu Liu, Andrew Luo, Vardhman Mehta, Cheng Ni, Deric Pang, Robbie Weber, Kyle Zhang and Michael Zhang

Text Books

- Required Textbook:
  - ***Machine Learning: a Probabilistic Perspective*; Kevin Murphy**



- Optional Books (free PDF):
  - *The Elements of Statistical Learning: Data Mining, Inference, and Prediction;* Trevor Hastie, Robert Tibshirani, Jerome Friedman



Add code requests

Add codes are given out according to a centralized process organized by CSE. Do **not** email instructors, we cannot help.

Resources:
- https://www.cs.washington.edu/academics/ugrad/advising/
- https://www.cs.washington.edu/academics/ugrad/courses/petition

Enjoy!

- ML is becoming ubiquitous in science, engineering and beyond
- It's one of the hottest topics in industry today
- This class should give you the basic foundation for applying ML and developing new methods
- The fun begins…

Maximum Likelihood Estimation

Machine Learning – CSE446

Kevin Jamieson

University of Washington

April 1, 2019

Your first consulting job

- *Billionaire*: I have a special coin, if I flip it, what's the probability it will be heads?
- *You*: Please flip it a few times:

H H T H T

- *You*: The probability is: 3/5
- *Billionaire:* Why?

Coin – Binomial Distribution

- **Data**: sequence *D= (HHTHT…)*, **k heads** out of **n flips**
- **Hypothesis:** P(Heads) = θ, P(Tails) = 1-θ
  - Flips are i.i.d.:
    - Independent events
    - Identically distributed according to Binomial distribution

$P(X_2 = H \mid X_1 = T) = P(X_2 = H)$

H H T H T

- $P(\mathcal{D}|\theta) = \theta \cdot \theta \cdot (1-\theta)\,\theta\,(1-\theta)\cdots$

$= \theta^k (1-\theta)^{n-k}$

Maximum Likelihood Estimation

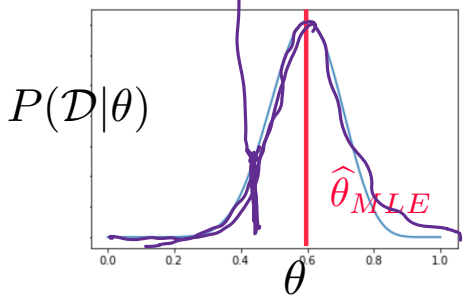
- **Data**: sequence *D= (HHTHT…),* **k heads** out of **n flips**
- **Hypothesis:** P(Heads) = θ, P(Tails) = 1-θ

$$P(\mathcal{D}|\theta) = \theta^k(1-\theta)^{n-k}$$

- Maximum likelihood estimation (MLE): Choose θ that maximizes the probability of observed data:

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} P(\mathcal{D}|\theta)$$

$$= \arg\max_{\theta} \log P(\mathcal{D}|\theta)$$



Your first learning algorithm

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} \log P(\mathcal{D}|\theta)$$

$$= \arg\max_{\theta} \log \theta^k (1-\theta)^{n-k}$$

$\log(ab) = \log(a) + \log(b)$

- Set derivative to zero:

$$\frac{d}{d\theta} \log P(\mathcal{D}|\theta) = 0$$

$\log(a^b) = b \log(a)$

$$= \log(\theta^k) + \log((1-\theta)^{n-k})$$

$$= k \log(\theta) + (n-k) \log(1-\theta)$$

$$\frac{\partial}{\partial \theta}[\cdot] = \frac{k}{\theta} + \frac{n-k}{1-\theta}(-1) = 0$$

$$(1-\theta)k + \theta(n-k)(-1) = 0$$

$$k - \theta(n) = 0 \qquad \theta = \frac{k}{n}$$

How many flips do I need?

$$\widehat{\theta}_{MLE} = \frac{k}{n}$$

- *You*: flip the coin 5 times. *Billionaire*: I got 3 heads.

$$\widehat{\theta}_{MLE} = 3/5$$

- *You*: flip the coin 50 times. *Billionaire*: I got 20 heads.

$$\widehat{\theta}_{MLE} = 20/50 = 2/5$$

- *Billionaire:* Which one is right? Why?

Simple bound (based on Hoeffding's inequality)

An estimator $\hat{\theta}: \mathcal{D} \rightarrow \mathbb{R}$ $\quad \mathbb{E}[\hat{\theta}] = \theta^*$

- For **n flips** and **k heads** the MLE is **unbiased** for true θ^*:

$$\widehat{\theta}_{MLE} = \frac{k}{n} \qquad \mathbb{E}[\widehat{\theta}_{MLE}] = \theta^*$$

$k = \sum_{i=1}^{n} \mathbb{1}\{\text{ith coin came up heads}\}$

$\mathbb{E}[k] = n\theta^*$

- Hoeffding's inequality says that for any $\varepsilon > 0$:

$$P(|\widehat{\theta}_{MLE} - \theta^*| \geq \epsilon) \leq 2e^{-2n\epsilon^2}$$

$\epsilon = 0.1$ $\qquad \leq 2e^{-.02 \cdot n}$

PAC Learning

- PAC: Probably Approximate Correct
- *Billionaire*: I want to know the parameter θ^*, within $\varepsilon = 0.1$, with probability at least $1-\delta = 0.95$. How many flips?

$$P(|\widehat{\theta}_{MLE} - \theta^*| \geq \epsilon) \leq 2e^{-2n\epsilon^2}$$

What about continuous variables?

- *Billionaire*: What if I am measuring a **continuous variable**?
- ***You*: Let me tell you about Gaussians…**

$$P(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Some properties of Gaussians

- affine transformation (multiplying by scalar and adding a constant)
  - $X \sim N(\mu, \sigma^2)$
  - $Y = aX + b \quad \rightarrow \quad Y \sim N(a\mu+b, a^2\sigma^2)$

  $\mathbb{E}[Y] = a\,\mathbb{E}[X] + b$
- Sum of Gaussians
  - $X \sim N(\mu_X, \sigma^2_X)$
  - $Y \sim N(\mu_Y, \sigma^2_Y)$
  - $Z = X+Y \quad \rightarrow \quad Z \sim N(\mu_X+\mu_Y, \sigma^2_X+\sigma^2_Y)$

MLE for Gaussian

- Prob. of i.i.d. samples $D=\{x_1,\ldots,x_N\}$ (e.g., exam scores):

$$P(\mathcal{D}|\mu,\sigma) = P(x_1,\ldots,x_n|\mu,\sigma) = \prod_{i=1}^{n} P(x_i|\mu,\sigma)$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \prod_{i=1}^{n} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

- Log-likelihood of data:

$$\log P(\mathcal{D}|\mu,\sigma) = -n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

Your second learning algorithm: MLE for mean of a Gaussian

- What's MLE for mean?

$$\frac{d}{d\mu} \log P(\mathcal{D}|\mu,\sigma) = \frac{d}{d\mu}\left[-n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}\right]$$

$$= \sum \frac{(x_i-\mu)}{\sigma^2}$$

$$= 0$$

$$\Rightarrow \sum_{i=1}^{n}(x_i-\mu) = 0 \Rightarrow \hat{\mu}_{MLE} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

MLE for variance

- Again, set derivative to zero:

$$\frac{d}{d\sigma} \log P(\mathcal{D}|\mu,\sigma) = \frac{d}{d\sigma}\left[-n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}\right]$$

$$= -\frac{n\sqrt{2\pi}}{\sigma\sqrt{2\pi}} + \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^3} = 0$$

$$\sigma^2 n = \sum_{i=1}^{n}(x_i-\mu)^2$$

$$\Rightarrow \quad \hat{\sigma}^2_{MLE} = \frac{1}{n}\sum(x_i-\hat{\mu}_{MLE})^2$$

Learning Gaussian parameters

- MLE:

$$\mathbb{E}[\hat{\mu}_{MLE}] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[x_i] = \mu$$

$x_i \sim N(\mu, \sigma^2)$

$$\widehat{\sigma^2}_{MLE} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu}_{MLE})^2$$

- MLE for the variance of a Gaussian is **biased**

$$\mathbb{E}[\widehat{\sigma^2}_{MLE}] \neq \sigma^2$$

- Unbiased variance estimator:

$$\widehat{\sigma^2}_{unbiased} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_{MLE})^2$$

Maximum Likelihood Estimation

Observe $X_1, X_2, \ldots, X_n$ drawn IID from $f(x;\theta)$ for some "true" $\theta = \theta_*$

Likelihood function $L_n(\theta) = \prod_{i=1}^{n} f(X_i;\theta)$

Log-Likelihood function $l_n(\theta) = \log(L_n(\theta)) = \sum_{i=1}^{n} \log(f(X_i;\theta))$

Maximum Likelihood Estimator (MLE) $\widehat{\theta}_{MLE} = \arg\max_{\theta} L_n(\theta)$

Maximum Likelihood Estimation

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Maximum Likelihood Estimator (MLE) $\widehat{\theta}_{MLE} = \arg\max_{\theta} L_n(\theta)$

Properties (under benign regularity conditions—smoothness, identifiability, etc.):

- Asymptotically consistent and normal: $\frac{\widehat{\theta}_{MLE} - \theta_*}{\widehat{se}} \sim \mathcal{N}(0,1)$
- Asymptotic Optimality, minimum variance (see Cramer-Rao lower bound)

Recap

- Learning is…
  - Collect some data
    - E.g., coin flips
  - Choose a hypothesis class or model
    - E.g., binomial
  - Choose a loss function
    - E.g., data likelihood
  - Choose an optimization procedure
    - E.g., set derivative to zero to obtain MLE
  - Justifying the accuracy of the estimate
    - E.g., Hoeffding's inequality