ECE368 Homework #9

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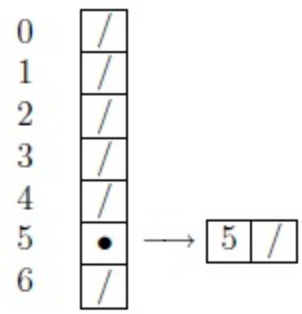
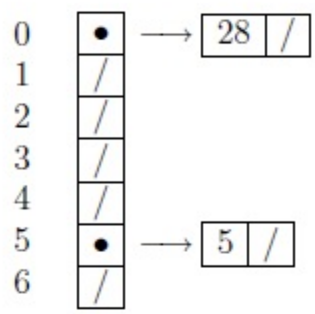
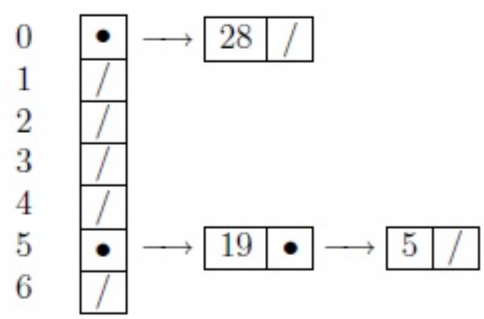
Please acknowledge those people who have helped you with this homework.

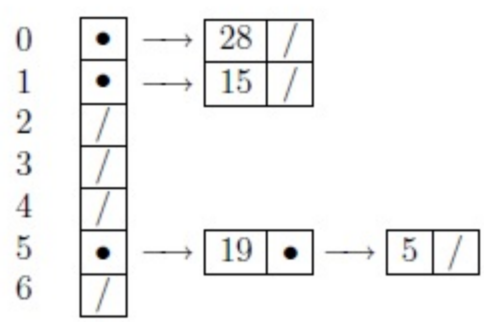
Question	Credits
1	
2	
3	

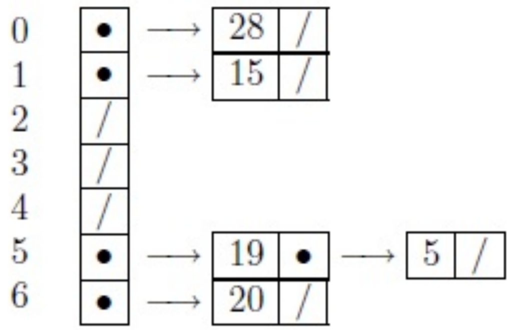
1. (30 points) Demonstrate the insertion of keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the hash table have 7 slots, and let the hash function be h(k) = k mod 7. Draw the hash table after each insertion.

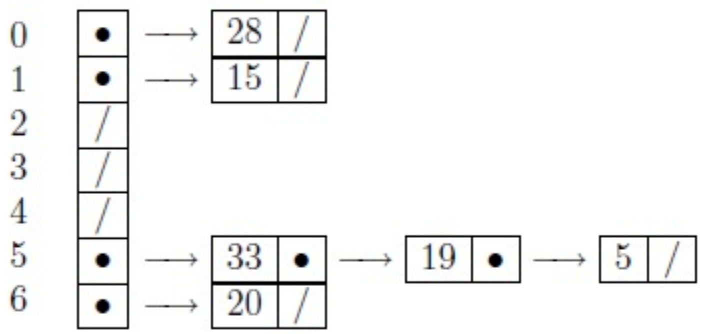









0 1 2 3 4 5 6
28
15
19
5
20


0 1 2 3 4 5 6
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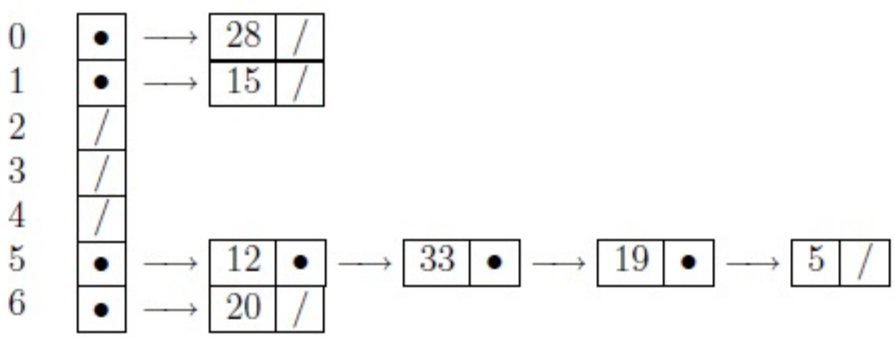
0 1 2 3 4 5 6
28
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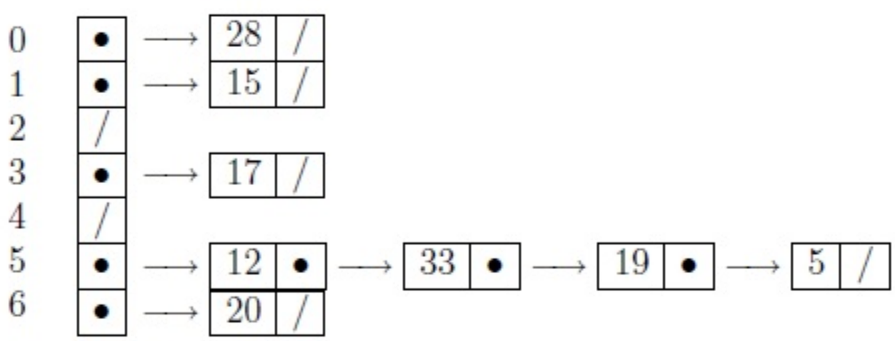
0 1 2 3 4 5 6
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33
19
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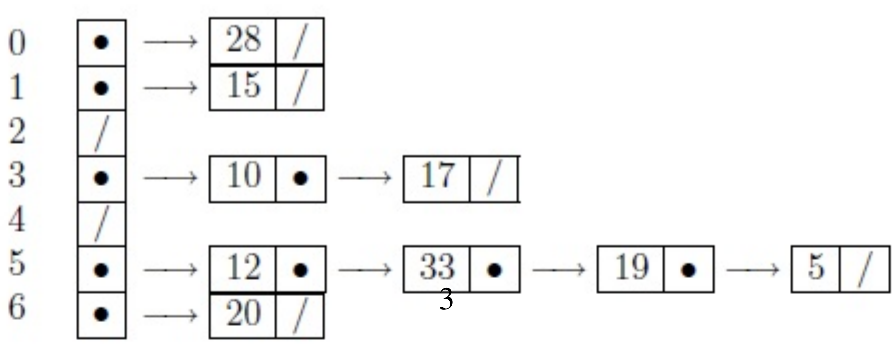
20


0 1 2 3 4 5 6
28
15
10
17
12
33
19
5
20

2. (50 points) Consider inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into a hash table of length m = 11 using open addressing with the primary hash function h'(k) = k mod m.
Illustrate the result of inserting these keys using the following probing methods; draw the hash table after each insertion.

1) Linear probing with $\alpha = 1$. (10 points)
2) Quadratic probing with $\alpha = 3$ and $\beta = 1$. (20 points)
3) Double hashing with $h_2(k) = 1 + (k \bmod (m - 1))$. (20 points)

Thus $h(x) = h(y)$.

Note that any permutation can be made by a sequence of interchanging of pairs of characters. Let x be a string derived from a string y by interchanging a pair of characters, if $h(x)$ is the same with $h(y)$ then any permutation of y hashes to $h(y)$ by mathematical induction.

Examples of applications

A dictionary which contains words expressed by ASCII code can be one of such example when each character of the dictionary is interpreted in radix $2^8 = 256$ and $m = 255$. The dictionary, for instance, might have words "STOP", "TOPS", "SPOT", "POTS" .. all of which are hashed into the same slot.

A DNA dictionary also can be one of such example. As we know, a DNA is a sequence of four bases, "Adenine", "Cytosin", "Guanine", "Thymine", which may be interpreted in radix 2^2. So following DNAs are all hased into the same slot.
"ACCTG", "GTCAC", "TCAGC", "GACCT"...

11.4-1

Consider inserting the keys $10, 22, 31, 4, 15, 28, 17, 88, 59$ into a hash table of length $m = 11$ using open addressing with the auxiliary hash function $h'(x) = k \bmod m$. Illustrate the result of inserting these keys using linear probing, using quadratic probing with $c_1 = 1$ and $c_2 = 3$, and using double hashing with $h_2(k) = 1 + (k \bmod (m-1))$.

Linear probing

$h(k, i) = (k + i) \bmod 11$.

1. Hasing 10
   $k = 10, i = 0,\ h(10, 0) = (10 + 0) \bmod 11 = 10$

										10

2. Hasing 22
   $k = 22, i = 0,\ h(22, 0) = (22 + 0) \bmod 11 = 0$

22										10

3. Hasing 31
   $k = 31, i = 0,\ h(31, 0) = (31 + 0) \bmod 11 = 9$

22									31	10

4. Hasing 4
   $k = 4, i = 0,\ h(4, 0) = (4 + 0) \bmod 11 = 4$

22				4					31	10

5. Hasing 15
   $k = 15, i = 0,\ h(15, 0) = (15 + 0) \bmod 11 = 4$, collision!
   $k = 15, i = 1,\ h(15, 1) = (15 + 1) \bmod 11 = 5$

22				4	15				31	10

6. Hasing 28
   $k = 28, i = 0,\ h(28, 0) = (28 + 0) \bmod 11 = 6$

22				4	15	28			31	10

7. Hasing 17
   $k = 17, i = 0,\ h(17, 0) = (17 + 0) \bmod 11 = 6$, collision!
   $k = 17, i = 1,\ h(17, 1) = (17 + 1) \bmod 11 = 7$

22				4	15	28	17		31	10

8. Hasing 88
   $k = 88, i = 0,\ h(88, 0) = (88 + 0) \bmod 11 = 0$, collision!
   $k = 88, i = 1,\ h(88, 1) = (88 + 1) \bmod 11 = 1$

22	88			4	15	28	17		31	10

9. Hasing 59
   $k = 59, i = 0,\ h(59, 0) = (59 + 0) \bmod 11 = 4$, collision!
   $k = 59, i = 1,\ h(59, 1) = (59 + 1) \bmod 11 = 5$, collision!
   $k = 59, i = 2,\ h(59, 2) = (59 + 2) \bmod 11 = 6$, collision!
   $k = 59, i = 3,\ h(59, 3) = (59 + 3) \bmod 11 = 7$, collision!
   $k = 59, i = 4,\ h(59, 4) = (59 + 4) \bmod 11 = 8$

22	88			4	15	28	17	59	31	10

Quadratic probing

$h(k, i) = (k + i + 3i^2) \bmod 11.$

1. Hasing 10
   $k = 10, i = 0,\ h(10, 0) = (10 + 0 + 0) \bmod 11 = 10$

										10

2. Hasing 22
   $k = 22, i = 0,\ h(22, 0) = (22 + 0 + 0) \bmod 11 = 0$

22										10

3. Hasing 31
   $k = 31, i = 0,\ h(31, 0) = (31 + 0 + 0) \bmod 11 = 9$

22									31	10

4. Hasing 4
   $k = 4, i = 0,\ h(4, 0) = (4 + 0 + 0) \bmod 11 = 4$

22				4					31	10

5. Hasing 15
   $k = 15, i = 0,\ h(15, 0) = (15 + 0 + 0) \bmod 11 = 4$, collision!
   $k = 15, i = 1,\ h(15, 1) = (15 + 1 + 3) \bmod 11 = 8$

22				4				15	31	10

6. Hasing 28
   $k = 28, i = 0,\ h(28, 0) = (28 + 0 + 0) \bmod 11 = 6$

22				4		28		15	31	10

7. Hasing 17
   $k = 17, i = 0,\ h(17, 0) = (17 + 0 + 0) \bmod 11 = 6$, collision!
   $k = 17, i = 1,\ h(17, 1) = (17 + 1 + 3) \bmod 11 = 10$, collision!
   $k = 17, i = 2,\ h(17, 2) = (17 + 2 + 12) \bmod 11 = 9$, collision!
   $k = 17, i = 3,\ h(17, 3) = (17 + 3 + 27) \bmod 11 = 3$

22			17	4		28		15	31	10

8. Hasing 88
   $k = 88, i = 0,\ h(88, 0) = (88 + 0 + 0) \bmod 11 = 0$, collision!
   $k = 88, i = 1,\ h(88, 1) = (88 + 1 + 3) \bmod 11 = 4$, collision!
   $k = 88, i = 2,\ h(88, 2) = (88 + 2 + 12) \bmod 11 = 3$, collision!
   $k = 88, i = 3,\ h(88, 3) = (88 + 3 + 27) \bmod 11 = 8$, collision!
   $k = 88, i = 4,\ h(88, 4) = (88 + 4 + 48) \bmod 11 = 8$, collision!
   $k = 88, i = 5,\ h(88, 5) = (88 + 5 + 75) \bmod 11 = 3$, collision!
   $k = 88, i = 6,\ h(88, 6) = (88 + 6 + 108) \bmod 11 = 4$, collision!
   $k = 88, i = 7,\ h(88, 7) = (88 + 7 + 147) \bmod 11 = 0$, collision!
   $k = 88, i = 8,\ h(88, 8) = (88 + 8 + 192) \bmod 11 = 2$

22		88	17	4		28		15	31	10

9. Hasing 59
   $k = 59, i = 0,\ h(59, 0) = (59 + 0 + 0) \bmod 11 = 4$, collision!
   $k = 59, i = 1,\ h(59, 1) = (59 + 1 + 3) \bmod 11 = 8$, collision!
   $k = 59, i = 2,\ h(59, 2) = (59 + 2 + 12) \bmod 11 = 7$

22		88	17	4		28	59	15	31	10

Double hashing

$h(k, i) = (k + i(1 + (k \bmod 10))) \bmod 11.$

1. Hasing 10
   $k = 10, i = 0,\ h(10, 0) = 10$

										10

2. Hasing 22
   $k = 22, i = 0,\ h(22, 0) = 0$

22										10

3. Hasing 31
   $k = 31, i = 0,\ h(31, 0) = 9$

22									31	10

4. Hasing 4
   $k = 4, i = 0,\ h(4, 0) = 4$

22				4					31	10

5. Hasing 15
   $k = 15, i = 0,\ h(15, 0) = 4$, collision!
   $k = 15, i = 1,\ h(15, 1) = 10$, collision!
   $k = 15, i = 2,\ h(15, 2) = 5$, collision!

22				4	15				31	10

6. Hasing 28
   $k = 28, i = 0,\ h(28, 0) = 6$

22				4	15	28			31	10

7. Hasing 17
   $k = 17, i = 0,\ h(17, 0) = 6$, collision!
   $k = 17, i = 1,\ h(17, 1) = 3$

22			17	4	15	28			31	10

8. Hasing 88
   $k = 88, i = 0,\ h(88, 0) = 0$, collision!

$k = 88, i = 1,\ h(88, 1) = 9$, collision!
$k = 88, i = 2,\ h(88, 2) = 7$

22			17	4	15	28	88		31	10

9. Hasing 59
   $k = 59, i = 0,\ h(59, 0) = 4$, collision!
   $k = 59, i = 1,\ h(59, 1) = 3$, collision!
   $k = 59, i = 2,\ h(59, 2) = 2$

22		59	17	4	15	28	88		31	10

3. (20 points) Suppose we wish to search a linked list of length n, where each element contains a key k along with a hash value h(k). Each key is a long character string. How might we take advantage of the hash values when searching the list for an element with a given key?

Answer: First compute the hash value of the given key. For each list element, we compare the hash value of the given key with the stored hash value. *i)* If there is a mismatch between hash values, we know there must be a mismatch between keys. So we don't have to examine the element's key. *ii)* Only when there is a match between hash values, we further compare the two keys.

Overall, we gain performance since comparing hash values is faster than comparing long strings (keys). Besides, the false positive from *ii)* should be low, as a good hash function can give us low collision rate.