Introduction

This project entails the practical implementation of an RR planar robot. The Adeept 5-DoF Robotic Arm is used in a planar configuration whose rotatory joints are AD002 servo motors. By planar configuration, we mean that only 2 servo motors are utilized and connected to the Arduino board.

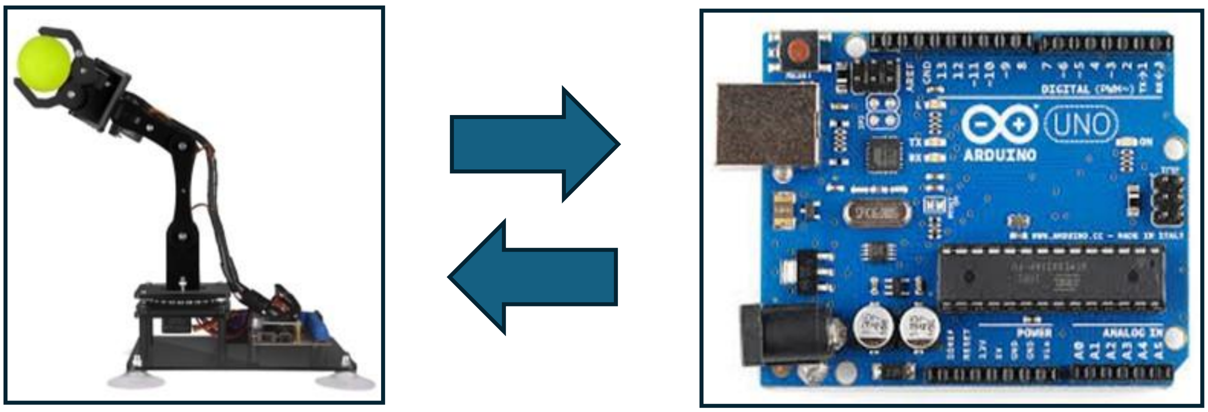

Figure 1

The focus of this project is to implement the inverse kinematics of an RR planar robot through Arduino programming. An RR planar robot is one of the simplest serial manipulators that is suitable for basic pick-and-place operations in a 2D space, for teaching robotics concepts (like Jacobian, Forward & Inverse Kinematics, etc.), and basic camera or sensor positioning etc. Though applications can be more vast, it is generally useful for controlling the position of any object within a 2D plane.

Its 2D positioning ability is subject to constraints imposed on its workspace. Workspace is the set of all points that the robot can reach. The workspace affects the reachability of the robot. The robot's reachability can be affected by obstacles in its environment, its parameters, such as link lengths, its own structure or assembly, and the limitations of actuators being used. In our case, we found that the servo motor can travel between 0-180^{o} that greatly reduced the workspace, limiting the robot's ability to perform complicated tasks. So, this robot may not be suitable for advanced position control or industrial use, but it still can be useful for education and validating Robotics theory on it.

This report will cover Inverse Kinematics, Workspace Analysis, and Path Planning of the said robot.

Controlling the Position of the End-Effector

The position of the end-effector is where the gripper of the robot is located. To control the position of any object in space, the robot must have the ability to grab the object, move it in its Workspace, and place it in the desired position. Microcontroller takes the heart of this. The desired coordinate position is given to the microcontroller, which then generates an appropriate signal for the actuators to let the end-effector move to the desired position.

The desired coordinate position of the end-effector is achieved by appropriate rotation or translation by joints (which can be rotatory, prismatic, etc.). Thus, for rotatory joints, the estimation of the numerical answers of each joint angle is the problem that is solved using Inverse Kinematics.

Figure 2

Inverse Kinematics can be done using the Forward Kinematics equations of the robot, or the Geometric Approach can also be utilized. In Forward Kinematics, we assign an independent XYZ coordinate frame to each joint of the robot and work our way by multiplying Homogeneous Transformation matrices. Forward Kinematics is more prioritized in Robotics because it can also describe the orientation of the gripper in space, which allows the Robot to grab objects in different ways.

Inverse Kinematics

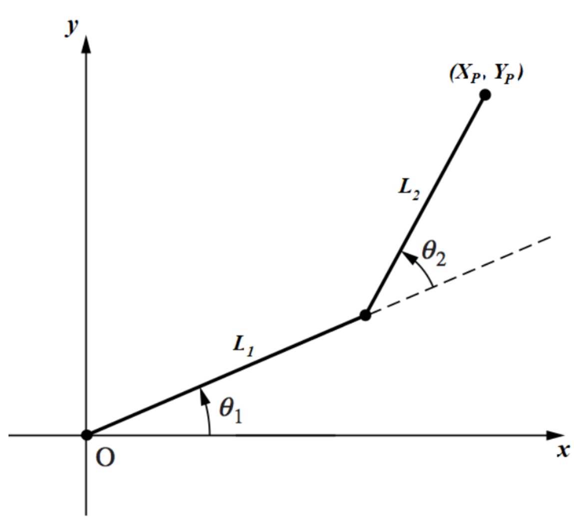

Figure 3

From the figure, one can deduce the position of X_P and Y_P with respect to the origin.

For a 2-link planar manipulator with link lengths L_1 and L_2, and joint angles θ_1 and θ_2, the equations for the x, y positions of the end effector are,

$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$ ----- (1)

$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$ ----- (2)

Two equations and two unknowns θ_1 and θ_2 are indeed solvable. We get θ_2 by squaring and adding equations (1) and (2).

$\cos\theta_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}$ ----- (3)

$\sin\theta_2 = \pm\sqrt{1-\cos^2\theta_2}$ ----- (4)

$\theta_2 = atan2(\sin\theta_2, \cos\theta_2)$ ----- (5)

Once θ_2 is known, the formula for θ_1 is obtained by applying trigonometric identities on (1) and (2).

$$x = L_1 \cos\theta_1 + L_2 \cos\theta_1 \cos\theta_2 - L_2 \sin\theta_1 \sin\theta_2$$

$$y = L_1 \sin\theta_1 + L_2 \sin\theta_1 \cos\theta_2 + L_2 \cos\theta_1 \sin\theta_2$$

We can separate constant values,

$$x = \cos\theta_1 (L_1 + L_2 \cos\theta_2) - \sin\theta_1 (L_2 \sin\theta_2)$$

$$y = \sin\theta_1 (L_1 + L_2 \cos\theta_2) + \cos\theta_1 (L_2 \sin\theta_2)$$

Taking $k_1 = L_1 + L_2 \cos\theta_2$ and $k_2 = L_2 \sin\theta_2$,

$$x = k_1 \cos\theta_1 - k_2 \sin\theta_1$$

$$y = k_1 \sin\theta_1 + k_2 \cos\theta_1$$

After some algebraic manipulation, one can verify that,

$\cos\theta_1 = \frac{k_1x+k_2y}{k_1^2+k_2^2}$ ----- (6)

$\sin\theta_1 = \frac{k_1y-k_2x}{k_1^2+k_2^2}$ ----- (7)

Thus,

$\theta_1 = \text{atan2}(\sin\theta_1, \cos\theta_1)$ ----- (8)

Multiple Solutions

It might be intuitive to think that a desired coordinate point can be reached in multiple ways by the robot. That is why the existence of multiple solutions for joint angles is indeed a possibility. It can be seen in our case that equation (4) has two answers, which can give us more than one solution for the desired coordinate position by the end-effector.

For multiple solutions, we choose the most suitable one. Suitable joint angles uses least amount of energy or avoid the most obstacles in its path. The joint angles also cannot be negative in most cases, and depend on how the microcontroller is coded to solve numerical issues.

For an RR planar robot, we get two possible positions, and we call them elbow-up and elbow-down configurations. In our case, our joint angles can't be negative and are up to 0-180° degrees, the configuration that can take the robot to a reachable point easily is used.

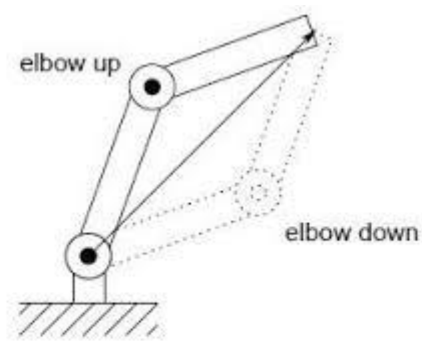

Figure 4

Workspace Analysis

The workspace is the set of all points that the robot can reach. The following workspace area is plotted using MATLAB using the following parameters,

- Link lengths: a_1 = 6.5 cm and a_2 = 13 cm
- Joint Angles: $0 \leq \theta_1 \leq 2\pi$ and $0 \leq \theta_2 \leq 2\pi$.

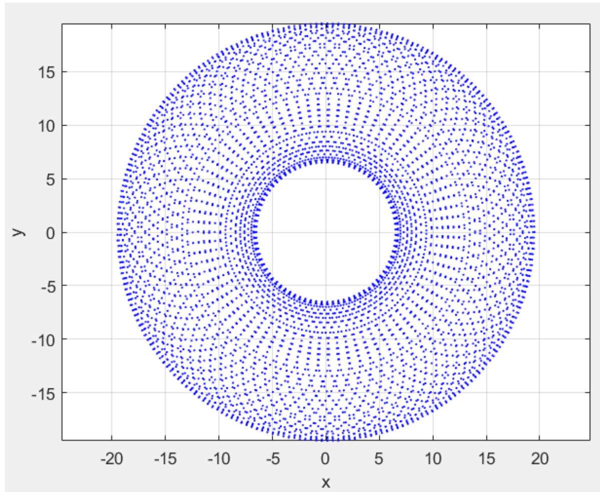

Figure 5

The observable workspace is a circular annulus with an inner radius $|a_1 - a_2|$ and outer radius $(a_1 + a_2)$. For the radius,

$$r = \sqrt{x^2 + y^2}$$

The robot can reach all points whose distance r satisfies:

$$|a_1 - a_2| \;\le\; r \;\le\; a_1 + a_2$$

The Adeept AD002 Micro Servo motors can reach only 0-180° positions. In the implemented structure, the second servo motor is mounted such that its 0° degree angle corresponds to –90° degrees using the mathematical model developed in Inverse Kinematics section. These deficiencies are common in practical systems which further limits the Workspace of the robot.

For our robot the workspace gets limited by the following additional constraints imposed by the own structure of the robot.

- Joint angles: $0 \le \theta_1 \le \pi$ and $-\pi/2 \le \theta_2 \le \pi/2$

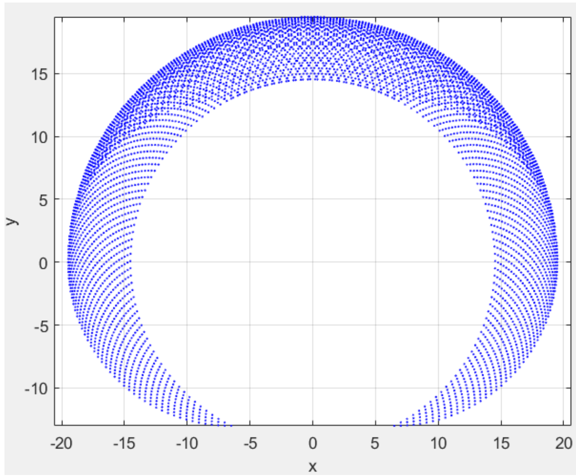

Figure 6

So, the said RR planar robot can only reach points indicated by Figure 6, the points outside the shaded region are unreachable.