Assignment 07

Q7

a) $P_1(3,5)$ $P_2(2,8)$

$P_1(x_1, y_1) = P_1(3,5)$

$P_2(x_2, y_2) = P_2(2,8)$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$= \langle 2-3, 8-5 \rangle$

$\overrightarrow{P_1P_2} = \langle -1, 3 \rangle$

b) $P_1(7,-2)$ $P_2(0,0)$

$P_1(x_1, y_1) = P_1(7,-2)$

$P_2(x_2, y_2) = P_2(0,0)$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$\overrightarrow{P_1P_2} = \langle -7, 2 \rangle$

c) $P_1(5,-2,1)$, $P_2(2,4,2)$

$P_1(x_1, y_1, z_1) = P_1(5,-2,1)$

$P_2(x_2, y_2, z_2) = P_2(2,4,2)$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

$\overrightarrow{P_1P_2} = \langle -3, 6, 1 \rangle$

~~A8~~ Q9

a) Find the terminal point of $V = 3i - 2j$ if the initial point is $(1, -2)$

$$P_1(x_1, y_1) = P_1(1, -2)$$
$$P_2(x_2, y_2) = ?$$

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$\langle 3, -2 \rangle = \langle x_2 - 1, y_2 + 2 \rangle$$

By compairing

$x_2 - 1 = 3$ $\quad\quad$ $y_2 + \cancel{2} = -\cancel{2}$

$x_2 = 4$ $\quad\quad$ $y_2 = -4$

$$P_2(x_2, y_2) = \langle 4, -4 \rangle$$

b) Find the initial point of $v = (-3, 1, 2)$ if the terminal point is $(5, 0, -1)$

$$P_1(x_1, y_1, z_1) = \cancel{(5, 0, -1)}) ?$$
$$P_2(x_2, y_2, z_2) = P_2(5, 0, -1)$$
$$\overrightarrow{P_1P_2} = V = \langle -3, 1, 2 \rangle$$

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$
$$\langle -3, 1, 2 \rangle = \langle 5 - x_1, -y_1, -1 - z_1 \rangle$$

By compairing

$-3 = 5 - x_1 \qquad 1 = -y_1 \qquad 2 = -1 - z_1$

$x_1 = 8 \qquad y_1 = -1 \qquad z_1 = -3$

Initial point $P_1(8, -1, -3)$

Q15

$U = i - 3j + 2k$

$V = i + j$

$w = 2i + 2j - 4k$

a) $\| U + V \|$

$U + V = 2i - 2j + 2k$

$\| P \| = \sqrt{x^2 + y^2 + z^2}$

$\| U + V \| = \sqrt{(2)^2 + (-2)^2 + (2)^2}$

$\| U + V \| = 2\sqrt{3}$

b) $\| U \| + \| V \|$

$\| U \| = \sqrt{1^2 + (-3)^2 + (2)^2} = \sqrt{14}$

$\| V \| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$

$\| U \| + \| V \| = \sqrt{14} + \sqrt{2}$

c) $\|-2u\| + 2\|v\|$

$-2u = -2i + 6j - 4k$

$\|-2u\| = \sqrt{(-2)^2 + (6)^2 + (-4)^2}$

$\|-2u\| = 8$

$\|v\| = \sqrt{1^2 + 1^2 + 0^2}$

$\|v\| = \sqrt{2}$

$\|-2u\| + 2\|v\| = 8 + 2\sqrt{2}$

$2\|u\| + 2\|v\|$

$\|u\| = \sqrt{1^2 + (-3)^2 + 2^2} = \sqrt{14}$

$\|v\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$

$2\|u\| + 2\|v\| = 2\sqrt{14} + 2\sqrt{2}$

d) $\|3u - 5v + w\|$

$3u = 3i - 9j + 6k$

$5v = 5i + 5j$

$3u - 5v + w = -12j + 2k$

$\|3u - 5v + w\| = \sqrt{(-12)^2 + (2)^2}$

$\boxed{\|3u - 5v + w\| = 2\sqrt{37}}$

f) $\left\| \frac{1}{\|w\|} w \right\|$

$\|w\| = \sqrt{2^2 + 2^2 + (-4)^2}$

$\|w\| = 2\sqrt{6}$

$\frac{1}{\|w\|} w = \frac{1}{\sqrt{6}} i + \frac{1}{\sqrt{6}} j - \frac{2}{\sqrt{6}} k$

$\left\| \frac{1}{\|w\|} w \right\| = \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{-2}{\sqrt{6}}\right)^2}$

$\boxed{\left\| \frac{1}{\|w\|} w \right\| = 1}$

e) $\frac{1}{\|w\|} w$

$\|w\| = \sqrt{2^2 + 2^2 + 4^2}$

$\|w\| = 2\sqrt{6}$

$\boxed{\frac{1}{\|w\|} w = \frac{1}{\sqrt{6}} i + \frac{1}{\sqrt{6}} j - \frac{2}{\sqrt{6}} k}$

Q21

a) same direction as $-i+4j$

$$\vec{U} = -i+4j$$
$$\|U\| = \sqrt{(-1)^2+(4)^2} = \sqrt{17}$$
$$\hat{U} = \frac{-i+4j}{\|U\|}$$
$$\boxed{\hat{U} = \frac{-1}{\sqrt{17}}i + \frac{4}{\sqrt{17}}j}$$

b) Opposite directed to $6i-4j+2k$

$$\vec{U} = 6i-4j+2\hat{k}$$
$$\|U\| = \sqrt{(6)^2+(-4)^2+(2)^2} = \cancel{8}\,2\sqrt{14}$$
$$\hat{U} = \frac{\vec{U}}{\|U\|} = \frac{6i-4j+2\hat{k}}{2\sqrt{14}}$$
$$\hat{U} = \frac{3}{\sqrt{14}}i - \frac{2}{\sqrt{14}}j + \frac{1}{\sqrt{14}}\hat{k}$$
$$\boxed{-\hat{V} = \frac{-3}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}j - \frac{1}{\sqrt{14}}\hat{k}}$$

c) same direction as the vector from the point $A(-1,0,2)$ to the point $B(3,1,1)$

Let $\vec{U}$ be the vector

$\vec{U} = \vec{AB}$

$= \langle 3+1, 1-0, 1-2 \rangle$

$\vec{U} = 4i + j - \hat{k}$

$\|U\| = \sqrt{4^2 + 1^2 + (-1)^2} = 3\sqrt{2}$

$\hat{U} = \dfrac{\vec{U}}{\|U\|} = \dfrac{4i + j - \hat{k}}{3\sqrt{2}}$

$\hat{U} = \dfrac{4}{3\sqrt{2}}\hat{i} + \dfrac{1}{3\sqrt{2}}\hat{j} - \dfrac{1}{3\sqrt{2}}\hat{k}$

Q23

a) oppositely directed to $v = (3,-4)$ and half the length of v

let $\vec{x}$ be the vector

$\vec{x} = -\dfrac{v}{2}$

$\boxed{\vec{x} = \dfrac{-3i}{2} + 2j}$

b) Length $\sqrt{17}$ and same direction as $v = (7, 0, -6)$

Let $\vec{x}$ be the vector

$\vec{x} = \sqrt{17}\,\hat{v}$

$\vec{x} = \sqrt{17}\dfrac{v}{\|v\|} \rightarrow ①$

$\|v\| = \sqrt{7^2+0^2+(-6)^2} = \sqrt{85}$

$\vec{x} = \dfrac{\sqrt{17}}{\sqrt{85}}(7i - 6k)$

$= \dfrac{1}{\sqrt{5}}(7i - 6k)$

$$\boxed{\vec{x} = \frac{7}{\sqrt{5}}i - \frac{6}{\sqrt{5}}k}$$

Q31

$2u - v + x = 7x + w$

$2(1,3) - (2,1) + x = 7x + (4,-1)$

$(2,6) - (2,1) - (4,-1) = 6x$

$6x = (-4, 6)$

$$\boxed{x = \left(-\frac{2}{3}, 1\right)}$$

$$\boxed{x = -\frac{2}{3}i + j}$$

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Q57

a)

$$w = c_1 v_1 + c_2 v_2$$

$$4j = c_1(2i - j) + c_2(4i + 2j)$$

$$4j = (2c_1 + 4c_2)\,i + (-c_1 + 2c_2)\,j$$

$$2c_1 + 4c_2 = 0 \qquad -c_1 + 2c_2 = 4$$

$$c_1 = -2c_2 \qquad 2c_2 + 2c_2 = 4$$

$$c_1 = -2(1) \qquad \boxed{c_2 = 1}$$

$$\boxed{c_1 = -2}$$

b)

$$w = c_1 v_1 + c_2 v_2$$

$$\cancel{2i+j-k}\ (3,5) = c_1(1,-3) + c_2(-2,6)$$

$$(3,5) = (c_1 - 2c_2,\ -3c_1 + 6c_2)$$

$$c_1 - 2c_2 = 3 \qquad -3c_1 + 6c_2 = 5$$

$$c_1 - 2c_2 = -\frac{5}{3}$$

The scalar equation has same LHS but different RHS, so the vector "w" cannot be as linear combination of vectors v_1 & v_2