Exercise 14.5

Assignment 21

Q7

$$\int_0^2\int_0^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x \, dz\,dy\,dx$$

$$\int_0^2\int_0^{\sqrt{4-x^2}} xz \Big|_{-5+x^2+y^2}^{3-x^2-y^2} dy\,dx$$

$$= \int_0^2\int_0^{\sqrt{4-x^2}} x(3-x^2-y^2) - x(-5+x^2+y^2)\, dy\,dx$$

$$= \int_0^2\int_0^{\sqrt{4-x^2}} 3x - x^3 - xy^2 + 5x - x^3 - xy^2 \, dy\,dx$$

$$= \int_0^2 3xy \Big|_0^{\sqrt{4-x^2}} - x^3 y \Big|_0^{\sqrt{4-x^2}} - \frac{xy^3}{3}\Big|_0^{\sqrt{4-x^2}} + 5xy\Big|_0^{\sqrt{4-x^2}} - x^3 y\Big|_0^{\sqrt{4-x^2}}$$

$$- \frac{xy^3}{3}\Big|_0^{\sqrt{4-x^2}}$$

$$= \int_0^2 3x\sqrt{4-x^2} - x^3\sqrt{4-x^3} - \frac{x(\sqrt{4-x^2})^3}{3} + 5x\sqrt{4-x^2}$$

$$- x^3\sqrt{4-x^2} - \frac{x(\sqrt{4-x^2})^3}{3} \, dx$$

$$= \int_0^2 8x\sqrt{4-x^2} - 2x^3\sqrt{4-x^2} - \frac{2x}{3}(\sqrt{4-x^2})^3 \, dx$$

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$$-4\int_0^2 -2x\sqrt{4-x^2}\,dx - \int_0^2 2x^3\sqrt{4-x^2}\,dx - \int_0^2 \frac{2x}{3}\left(\sqrt{4-x^2}\right)^3 dx$$

Appy substitution for middle part

let $4-x^2 = t^2$

$2x\,dx = 2t\,dt$

$dx = \frac{\cancel{t}}{\cancel{x}}\,dt$

$dx = \frac{t}{\sqrt{4-t^2}}$ $\quad \because x = \sqrt{4-t^2}$

$$= \int_0^2 2(4-t^2)^{3/2}\,\frac{t^2}{(4-t^2)^{1/2}}$$

$$= \int_0^2 2(4-t^2)\,t^2$$

$$= \int_0^2 (8t^2 - 2t^4)\,dt$$

$$= \frac{8t^3}{3} - \frac{2t^5}{5}\Big|_0^2$$

$$= \frac{8\left(\sqrt{4-x^2}\right)^3}{3} - \frac{2\left(\sqrt{4-x^2}\right)^5}{5}\Big|_0^2$$

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$$= \cancel{8}\frac{(4-x^2)^{3/2}}{3/2}\Big|_0^2 - \left(\frac{8(\sqrt{4-x^2})^3}{3} - \frac{2(\sqrt{4-x^2})^5}{5}\right)\Big|_0^2$$

$$- \frac{1}{3}\frac{(4-x^2)^{5/2}}{5/2}\cancel{dx}\Big|_0^2$$

$$= -4\left(\cancel{0}0 - \cancel{8}\frac{(4)^{3/2}}{3/2}\right) - \left[0 - \left(\frac{8(4)^{3/2}}{3} - \frac{2(4\cancel{-x^2})^{5/2}}{5}\right)\right]$$

$$- \left[0 - \frac{1}{3}\frac{(4)^{5/2}}{5/2}\right]$$

$$= \cancel{\frac{-128}{3}}^{64/3} + \frac{64}{3} - \frac{64}{5} + \frac{64}{15}$$

$$= \frac{128}{15}$$

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Q11

$= \iiint\limits_{G} x.yz \; dv$

z limit

$\boxed{z = 0}$ $\boxed{z = 2\cancel{x} - x^2}$

y - limit.

$\boxed{y = 0}$ $\boxed{y = x}$

x - limit

$\boxed{x = 0}$ $\because y = x$ & $y = 0$.

$z = 2 - x^2$

$0 = 2 - x^2$

$\boxed{x = 2}$

$= \int_0^2 \int_0^x \int_0^{2-x^2} xyz \; dz\,dy\,dx$

$= \int_0^2 \int_0^x \frac{xyz^2}{2} \Big|_0^{2-x^2} dy\,dx$

$$= \int_0^2 \int_0^x \frac{xy(2-x^2)^2}{2} \, dy \, dz$$

$$= \int_0^2 \left. \frac{xy^2(2-x^2)^2}{4} \right|_0^x$$

$$= \int_0^2 \frac{x^3(2-x^2)^2 \, dx}{4}$$

let $u^2 = 2-x^2$

$du \, 2u = -2x \, dx$

$dx = -\frac{u}{x} du$

$dx = \frac{-u}{\sqrt{2-u^2}} du$

$$= \frac{1}{4} \int_0^2 (2-u^2)^{3/2} \, u^4 \left(\frac{-u}{(2-u^2)^{1/2}} \right) du$$

$$= -\frac{1}{4} \int_0^2 (2-u^2) \, u^5$$

$$= -\frac{1}{4} \int_0^2 2u^5 - u^7 \, du$$

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$$= -\frac{1}{4}\left[\frac{u^6}{3} - \frac{u^8}{8}\right]$$

$$= -\frac{1}{4}\left[\frac{(2\cancel{x}-x^2)^3}{3} - \frac{(2-x^2)^4}{8}\right]_0^2$$

$$= -\frac{1}{4}\left[0 - \left(\frac{2^3}{3} - \frac{2^4}{8}\right)\right]$$

$$= -\frac{1}{\cancel{4}_2}\left(-\frac{2}{3}\right)$$

$$= \boxed{\frac{1}{6}}$$

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Q15

$$V = \iiint_G dz\,dy\,dx$$

$x = 0$, $y = 0$, $z = 0$

$3x + 6y + 4z = 12$

z - limits

$z = 0$

$$z = \frac{12 - 3x - 6y}{4}$$

y - limits

$y = 0$

$$y = \frac{12 - 3x}{6} \Rightarrow y = \frac{4 - x}{2}$$

x - limits

$x = 0$

$x = 4$

$$= \int_0^4 \int_0^{\frac{4-x}{2}} \int_0^{\frac{12-3x-6y}{4}} dz\,dy\,dx$$

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$$= \int_0^4 \int_0^{\frac{4-x}{2}} z \Big|_0^{\frac{12-3x-6y}{4}} dy\,dz$$

$$= \int_0^4 \int_0^{\frac{4-x}{2}} \frac{12-3x-6y}{4}\, dy\,dz$$

$$= \int_0^4 \frac{12}{4}y - \frac{3}{4}xy - \frac{6y^2}{8} \Big|_0^{\frac{4-x}{2}} dx$$

$$= \int_0^4 3y - \frac{3}{4}xy - \frac{3}{4}y^2 \Big|_0^{\frac{4-x}{2}} dx$$

$$= \int_0^4 \left[3\left(\frac{4-x}{2}\right) - \frac{3}{4}x\left(\frac{4-x}{2}\right) - \frac{3}{4}\left(\frac{4-x}{2}\right)^2\right] dx$$

$$\int_0^4 \frac{12-3x}{2} - \frac{3}{4}\left(\frac{4x-x^2}{2}\right) - \cancel{\left(\frac{12-3x}{8}\right)}$$

$$-\frac{3}{4}\left(\frac{4^2-8x+x^2}{4}\right)$$

$$\int_0^4 6 - \frac{3}{2}x - \cancel{\frac{3}{2}x} + \frac{3x^2}{8} - \frac{3}{2}$$

$$+ \cancel{\frac{3}{2}x} - \frac{3}{16}x^2$$

$$\int_0^4 \cancel{\frac{9}{2}}\,3 - \frac{3}{8}x + \frac{3x^2}{16}\, dx$$

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$$= 3x - \frac{3}{4}x^2 + \frac{3x^3}{16\times 3}\Big|_0^4$$

$$= \frac{3}{4}(4) - \frac{3}{4}(4)^2 + \frac{3(4)^3}{16\times 3}$$

$$= -4$$

$$= 4$$

Q17

$$y = x^2$$

$$y + z = 4$$

$$z = 0$$

<u>z limits</u>

$z = 0 \qquad z = 4 - y$

<u>y - limit</u>

$y = x^2 \qquad y = 4$

<u>x - limits</u>

$$x = \sqrt{y}$$

$$x = \sqrt{4}$$

$$x = \pm 2$$

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$$\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{4-y} dz\,dy\,dx$$

$$\int_{-2}^{2}\int_{x^2}^{4} z\Big|_{0}^{4-y} dy\,dx$$

$$\int_{-2}^{2}\int_{x^2}^{4} 4-y \quad dy\,dx$$

$$\int_{-2}^{2} 4y-\frac{y^2}{2}\Big|_{x^2}^{4} dx$$

$$\int_{-2}^{2}\left(16-\frac{16}{2}\right)-\left(4x^2-\frac{x^4}{2}\right) dx$$

$$\int_{-2}^{2} 8-4x^2+\frac{x^4}{2} \quad dx$$

$$8x-\frac{4x^3}{3}+\frac{x^5}{10}\Big|_{-2}^{2}$$

$$\left[8(2)-\frac{4(2)^3}{3}+\frac{(2)^5}{10}\right]-\left[8(-2)-\frac{4(-2)^3}{3}+\frac{(-2)^5}{10}\right]$$

$$=\frac{128}{15}-\left(\frac{-128}{15}\right)$$

$$=\frac{256}{15}$$

No.