Assignment 03

Q5

$r = \sin 3\theta \qquad \theta = \pi/4$

$x = r\cos\theta \qquad y = r\sin\theta$

$\frac{dx}{d\theta} = -r\sin\theta + \frac{dr}{d\theta}\cos\theta \qquad \frac{dy}{d\theta} = r\cos\theta + \frac{dr}{d\theta}\sin\theta$

$$\frac{dy}{dx} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}$$

~~here $\theta = \frac{\pi}{4}$~~ here $r = \sin 3\theta$

$$\cancel{\frac{dy}{dx} = \sin 3\theta \cos\left(\frac{\pi}{4}\right) + \frac{dr}{d\theta}}$$

$$\frac{dy}{dx} = \frac{\sin 3\theta \cos\theta + 3\cos 3\theta \sin\theta}{-\sin 3\theta \sin\theta + 3\cos 3\theta \cos\theta}$$

here $\theta = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{\frac{1}{2} + \left(-\frac{3}{2}\right)}{-\frac{1}{2} + \left(-\frac{3}{2}\right)}$$

$$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{4}} = \frac{1}{2}$$

Q13

$r = 2\cos 3\theta$

Symmetric Test

For x-axis

$r = 2\cos 3(-\theta)$

$r = 2\cos 3\theta$

TRUE

For y-axis

$r = 2\cos(3\pi - 3\theta)$

$r = 2[\cos 3\pi \cos 3\theta + \sin 3\pi \sin 3\theta]$

$r = -2\cos 3\theta$

False

θ	r
0	2
$\pi/6$	0
$\pi/3$	-2
$\pi/2$	0
$2\pi/3$	2
$5\pi/6$	0
π	-2

$\theta = \frac{\pi}{6}$:

$r = 0$

$\frac{dr}{d\theta} = 6$

Scanned with CamScanner

$\underline{\theta = \pi/2 :}$

$r = 0$

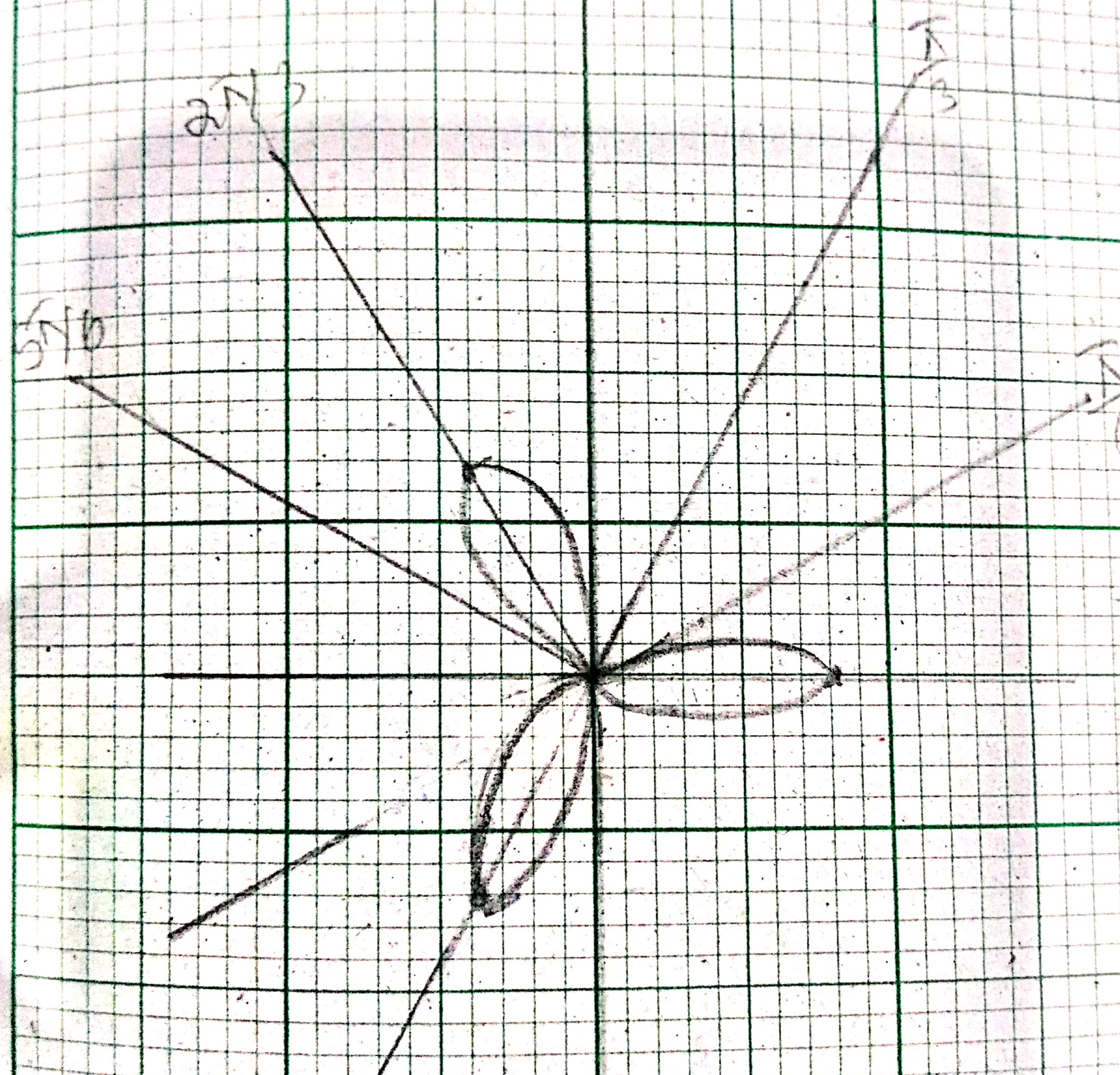
$$\frac{dr}{d\theta} = 6$$

$\underline{\theta = 5\pi/6}$

$r = 0$

$$\frac{dr}{d\theta} = -6$$

Q19



$r = 2\cos 3\theta$

$$\frac{dx}{d\theta} = a\cos\theta \sin\theta$$

$$= a\cos\theta \sin\theta - \sin\theta + \sin\theta$$

Q19

$r = a$

$$\frac{dr}{d\theta} = 0$$

$$L = \int_{0}^{2\pi} \sqrt{a^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$L = a\int_{0}^{2\pi} d\theta$$

$$L = a[\theta]_0^{2\pi}$$

$$\boxed{L = 2\pi a}$$

Q21

$$r = a(1-\cos\theta)$$

$$\frac{dr}{d\theta} = a\sin\theta$$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$L = \int_{0}^{2\pi} \sqrt{a^2(1-\cos\theta)^2 + a^2\sin^2\theta}\, d\theta$$

$$L = a\int_{0}^{2\pi} \sqrt{1-2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta$$

$$L = a\int_{0}^{2\pi} \sqrt{2(1-\cos\theta)}\, d\theta$$

$$L = \sqrt{2}a\int_{0}^{2\pi} \sqrt{1-\cos\theta}\, d\theta$$

$$L = \sqrt{2}a\int_{0}^{2\pi} \sqrt{2}\sin\frac{\theta}{2}\, d\theta$$

Let

$$x = \frac{\theta}{2}$$

$$\frac{dx}{d\theta} = \frac{1}{2}$$

$$d\theta = 2dx$$

$$L = \sqrt{2}a \int_0^{2\pi} 2\sqrt{2} \sin x \, dx$$

$$L = 4a \int_0^{2\pi} \sin x \, dx$$

$$L = \cancel{4a \left[-\cos x\right]_0^{2\pi}}$$

$$=$$

$$\cancel{= 4a \left[-(1-1)\right]}$$

$$= 4a \left[-\cos x\right]_0^{2\pi}$$

$$L = 4a \left[-\cos \frac{\theta}{2}\right]_0^{2\pi}$$

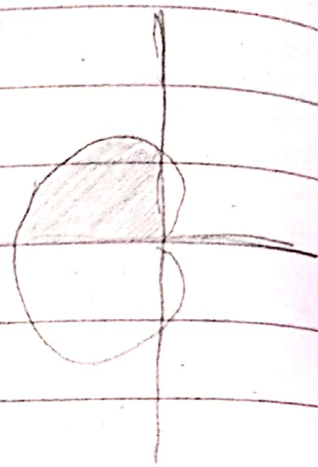
$$L = 4a(+1-(-1))$$

$$\boxed{L = 8a}$$

Q25

a) $r = 1 - \cos\theta$

$$A = \int_{\pi/2}^{\pi} \frac{1}{2}(1-\cos\theta)^2 \, d\theta$$



b) $r = 2\cos\theta$

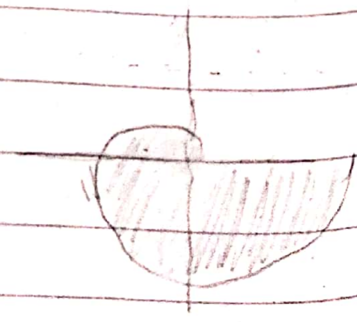
$$A = \int_{0}^{\frac{\pi}{2}} \frac{1}{2}(2\cos\theta)^2 \, d\theta$$

c) $r = \sin 2\theta$

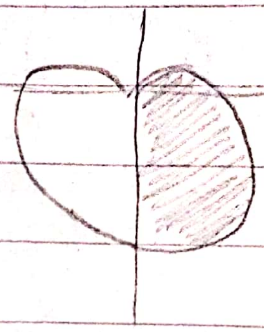
$$A = \int_{0}^{\frac{\pi}{2}} \frac{1}{2}(\sin 2\theta)^2 \, d\theta$$

d) $r = \theta$

$$A = \int_{0}^{2\pi} \frac{1}{2}\theta^2 \, d\theta$$

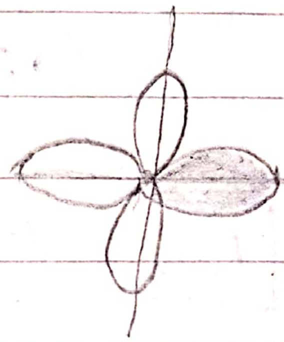


e) $r = 1 - \sin\theta$



$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1 - \sin\theta)^2 \, d\theta$$

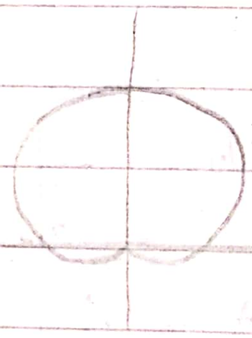
f) $r = \cos 2\theta$



$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(\cos 2\theta)^2 \, d\theta$$

Q29

$r = 2 + 2\sin\theta$



$$A = \int_{0}^{2\pi} \frac{1}{2}(2 + 2\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{0}^{2\pi} 4 + 8\sin\theta + 4\sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_{0}^{2\pi} 4 + 8\sin\theta + 2(1 - \cos 2\theta) \, d\theta$$

$$A = \frac{1}{2}\int_{0}^{2\pi} 4 + 8\sin\theta + 2 - 2\cos 2\theta \, d\theta$$

$$A = \frac{1}{2}\left[4\theta - 8\cos\theta + 2\theta - \sin 2\theta\right]_0^{2\pi}$$

$$= \frac{1}{2}\left[(12\pi - 8) + 8\right]$$

$$= \frac{12\pi - 0}{2}$$

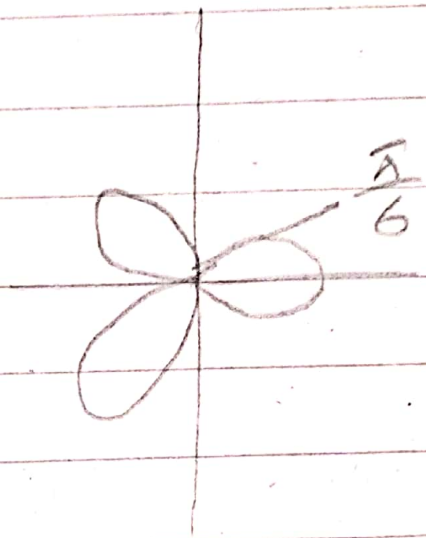
$$\boxed{A = 6\pi}$$

Q31

$r = 4\cos 3\theta$

Q31

$r = 4\cos 3\theta$



$$A = 3\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2}(4\cos 3\theta)^2$$

$$= 24\int_{-\pi/6}^{\pi/6} \cos^2 3\theta$$

$$= 24\int_{\pi/6}^{\pi/6} \frac{1+\cos 6\theta}{2}$$

$$= 12\left[\theta + \frac{\sin 6\theta}{6}\right]_{-\pi/6}^{\pi/6}$$

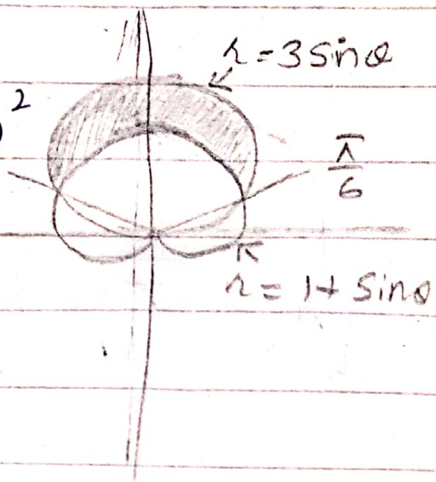
$$= 12\left[\frac{\pi}{6} + \frac{\pi}{6}\right]$$

$$= \cancel{12}^{4} \times \frac{\pi}{\cancel{3}}$$

$$\boxed{A = 4\pi}$$

Q39

$$A = \cancel{\int_0^{\pi} \frac{1}{2}(3\sin\theta)^2} - \cancel{\int_0^{\pi} \frac{1}{2}(1+\sin\theta)^2}$$



$$3\sin\theta = 1+\sin\theta$$

$$2\sin\theta = 1$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{6}$$

$$A = 2\left[\int_{\pi/6}^{\pi/2} \frac{1}{2}(3\sin\theta)^2\,d\theta - \int_{\pi/6}^{\pi/2} \frac{1}{2}(1+\sin\theta)^2\,d\theta\right]$$

$$= 2\int_{\pi/6}^{\pi/2} \frac{1}{2}\left[(3\sin\theta)^2 - (1+\sin\theta)^2\right]\,d\theta$$

$$= 2\int_{\pi/6}^{\pi/2} \frac{1}{2}\left(9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta\right)\,d\theta$$

$$= \int_{\pi/6}^{\pi/2} \overset{4}{\cancel{8}}\left(\frac{1-\cos 2\theta}{\cancel{2}}\right) - 1 - 2\sin\theta\,d\theta$$

$$A = \int_{\pi/6}^{\pi/2} 3 - 4\cos 2\theta - 2\sin\theta\,d\theta$$

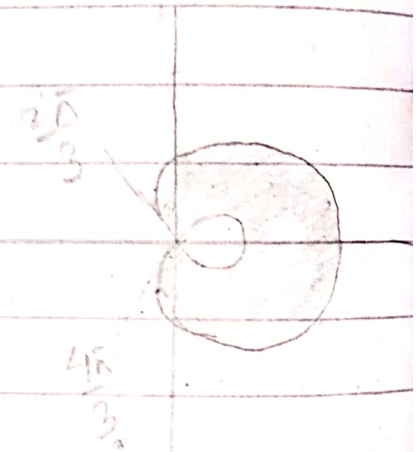
$$= \left[3\theta - 2\sin 2\theta + 2\cos\theta\right]_{\pi/6}^{\pi/2}$$

$$= \frac{3\pi}{2} - \left(\frac{\pi}{2} - \cancel{\sqrt{3}} + \cancel{\sqrt{3}}\right)$$

$$\boxed{A = \pi}$$

Q43

$$r = \frac{1}{2} + \cos\theta$$



$$A = 2\left[\int_0^{2\pi/3} \frac{1}{2}\left(\frac{1}{2}+\cos\theta\right)^2 - \int_\pi^{4\pi/3} \frac{1}{2}\left(\frac{1}{2}+\cos\theta\right)^2\right].$$

$$= \int_0^{\frac{2\pi}{3}} \frac{1}{4} + \cos\theta + \frac{1+\cos 2\theta}{2} - \int_\pi^{\frac{4\pi}{3}} \frac{1}{4} + \cos\theta + \frac{1+\cos 2\theta}{2}.$$

$$= \left[\frac{1}{4}\theta + \sin\theta + \frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{\frac{2\pi}{3}} - \left[\frac{1}{4}\theta + \sin\theta + \frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_\pi^{\frac{4\pi}{3}}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{8} - \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3} + \frac{\sqrt{3}}{8} - \frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$= \frac{\pi}{2} + \frac{3\sqrt{3}}{8} - \left(\frac{\pi}{4} - \frac{3\sqrt{3}}{8}\right)$$

$$A = \frac{\pi}{4} + \frac{3\sqrt{3}}{4} \Rightarrow \boxed{A = \frac{\pi + 3\sqrt{3}}{4}}$$