Exercise 14.3

Assignment 20

Date

Q5

$$= \int_0^{\pi} \int_0^{1-\sin\theta} r^2 \cos\theta \, dr \, d\theta$$

$$= \int_0^{\pi} \left. \frac{r^3}{3} \cos\theta \right|_0^{1-\sin\theta} d\theta$$

$$= \int_0^{\pi} \frac{(1-\sin\theta)^3}{3} \cos\theta \, d\theta$$

$$= -\frac{1}{3} \int_0^{\pi} -\cos\theta \, (1-\sin\theta)^3 \, d\theta$$

$$= -\frac{1}{3} \times \left. \frac{(1-\sin\theta)^4}{4} \right|_0^{\pi}$$

$$= -\frac{1}{3} \left(\frac{1-\sin\pi}{4} - \frac{1-\sin 0}{4} \right)$$

$$= +\frac{1}{3} (0)$$

$$= 0$$

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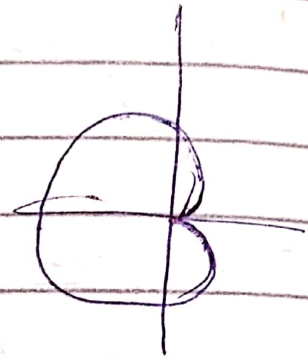
Date

Q7

$$= \iint_R dA$$



$$dA = r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta$$

$$\int_0^{2\pi} \frac{r^2}{2}\Big|_0^{1-\cos\theta} d\theta$$

$$\int_0^{2\pi} \frac{(1-\cos\theta)^2}{2}\, d\theta$$

$$\int_0^{2\pi} \frac{1-2\cos\theta+\cos^2\theta}{2}\, d\theta$$

$$\because \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\int_0^{2\pi} \frac{1}{2}\, d\theta - \int_0^{2\pi} \cos\theta + \frac{1}{2}\int_0^{2\pi} \frac{1+\cos 2\theta}{2}$$

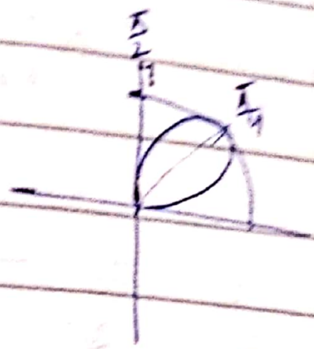
$$\frac{\theta}{2}\Big|_0^{2\pi} - \sin\theta\Big|_0^{2\pi} + \frac{1}{4}\theta\Big|_0^{2\pi} + \frac{\sin 2\theta}{4\times 2}\Big|_0^{2\pi}$$

No.

$= \frac{2\pi}{2} - 0 + \frac{2\pi}{4} - 0$

$= \frac{3\pi}{2}$

Q9



$$A = \iint_R r\,dr\,d\theta$$

$$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$$

$$\sin 2\theta \le r \le 1$$

$$\int_{\pi/4}^{\pi/2}\int_{\sin 2\theta}^{1} r\,dr\,d\theta$$

$$\int_{\pi/4}^{\pi/2} \frac{r^2}{2}\Big|_{\sin 2\theta}^{1} d\theta$$

$$\int_{\pi/4}^{\pi/2} \frac{1}{2} - \frac{\sin^2 2\theta}{2}\, d\theta$$

$$\frac{1}{2}\int_{\pi/4}^{\pi/2} d\theta - \frac{1}{2}\int_{\pi/4}^{\pi/2} \sin^2 2\theta\, d\theta$$

$$\because \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$= \frac{1}{2}\theta\Big|_{\pi/4}^{\pi/2} - \frac{1}{2}\int_{\pi/4}^{\pi/2} \frac{1-\cos 4\theta}{2}\, d\theta$$

$$= \frac{1}{2}\theta\Big|_{\pi/4}^{\pi/2} - \frac{1}{4}\int_{\pi/4}^{\pi/2} d\theta + \frac{1}{4}\int_{\pi/4}^{\pi/2} \cos 4\theta \, d\theta$$

$$= \frac{\pi}{8} - \frac{\pi}{16} + \frac{\sin 4\theta}{16}\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{16}$$

Q29

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

$$r = \sqrt{x^2+y^2}$$

$$y = \sqrt{2x-x^2}$$

$\because x = r\cos\theta \qquad \because y = r\sin\theta$

$$r\sin\theta = \sqrt{2r\cos\theta - r^2\cos^2\theta}$$

$$r^2\sin^2\theta = 2r\cos\theta - r^2\cos^2\theta$$

$$r^2\sin^2\theta + r^2\cos^2\theta = 2r\cos\theta$$

Date

$$r^2 = 2r\cos\theta$$

$$r = 2\cos\theta$$

$$\underline{y=0}$$

$$r\sin\theta = 0$$

$$r = \theta$$

$$x = 0$$

$$r\cos\theta = 0$$

$$\cos\theta = 0$$

$$\cos^{-1}(0) = \theta$$

$$\theta =$$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\cos\theta} r \cdot r\,dr\,d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{r^3}{3}\Big|_0^{2\cos\theta} d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{8\cos^3\theta}{3}\, d\theta$$

$$\frac{8}{3}\int_0^{\frac{\pi}{2}} \cos\theta \cdot \cos^2\theta \quad d\theta$$

$$= \frac{8}{3}\int_0^{\frac{\pi}{2}} \cos(1-\sin^2\theta)\, d\theta$$

$$= \frac{8}{3}\int_0^{\frac{\pi}{2}} \cos\theta - \cos\sin^2\theta\, d\theta$$

$$= \frac{8}{3}\left[\sin\theta \Big|_0^{\frac{\pi}{2}} - \frac{\sin^3\theta}{3}\Big|_0^{\frac{\pi}{2}}\right]$$

$$= \frac{8}{3}\left[(1-0) - \left(\frac{1}{3} - 0\right)\right]$$

$$= \frac{8}{3} \times \frac{2}{3}$$

$$= \frac{16}{9}$$

Q31

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{dy\,dx}{(1+x^2+y^2)^{3/2}} \quad (a>0)$$

$$1+x^2+y^2 = 1+r^2$$

$$y = \sqrt{a^2-x^2}$$

$$r\sin\theta = \sqrt{a^2 - r^2\cos^2\theta}$$

$$r^2\sin^2\theta + r^2\cos^2\theta = a^2$$

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$x = a$

$$\int_0^{\frac{\pi}{2}}\int_0^a \frac{1}{(1+r^2)^{3/2}}\, r\,dr\,d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}}\int_0^a 2r(1+r^2)^{-3/2}\,dr\,d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \left.\frac{(1+r^2)^{-1/2}}{-1/2}\right|_0^a d\theta$$

$$\cancel{\frac{1}{2}\int_0^{\frac{\pi}{2}}(1+a^2)^{-1/2}}\ \cancel{\theta} = 1$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(-2(1+a^2)^{-1/2} + 2(1)^{-1/2}\right)d\theta$$

$$\int_0^{\frac{\pi}{2}} -(1+a^2)^{-1/2}\,d\theta + \int_0^{\frac{\pi}{2}} d\theta$$

$$-\theta(1+a^2)^{-1/2}\Big|_0^{\frac{\pi}{2}} + \theta\Big|_0^{\pi/2}$$

$$-\frac{\pi}{2}(1+a^2)^{-1/2}\ \cancel{+0} + \frac{\pi}{2}$$

$$\boxed{\frac{\pi}{2}\left[1-(1+a^2)^{-1/2}\right]}$$

No.