Assignment # 08

Date

Q3

a) $v = 7i + 3j + 5k$, $v = -8i + 4j + 2k$

$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$

$$\|u\| = \sqrt{7^2 + 3^2 + 5^2} = \sqrt{83}$$

$$\|v\| = \sqrt{8^2 + 4^2 + 2^2} = 2\sqrt{21}$$

$$u \cdot v = (7 \times -8) + (3 \times 4) + (5 \times 2) = -34$$

$$\cos\theta = \frac{-34}{(\sqrt{83})(2\sqrt{21})}$$

$$\theta = 114.02^\circ$$

Angle is obtuse.

b) $u = 6i + j + 3k$, $v = 4i - 6k$

$$\|u\| = \sqrt{6^2 + 1^2 + 3^2} = \sqrt{46}$$

$$\|v\| = \sqrt{4^2 + 0^2 + 6^2} = 2\sqrt{13}$$

$$u \cdot v = (6 \times 4) + (1 \times 0) + (3 \times -6) = 6$$

$$\cos\theta = \frac{6}{(\sqrt{46})(2\sqrt{13})}$$

$$\theta = 82.95^\circ$$

angle is acute.

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c) $u = (1,1,1)$ $v = (-1,0,0)$

$\|u\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\|v\| = \sqrt{1^2} = 1$

$u \cdot v = (1 \times -1) + (1 \times 0) + (1 \times 0) = -1$

$\cos\theta = \dfrac{-1}{\sqrt{3}}$

$\theta = 125.26°$

angle is obtuse.

d) $u = (4,1,6)$, $v = (-3,0,2)$

$\|u\| = \sqrt{4^2+1^2+6^2} = \sqrt{53}$

$\|v\| = \sqrt{(-3)^2+0^2+2^2} = \sqrt{13}$

$u \cdot v = (4 \times -3) + (1 \times 0) + (6 \times 2) = 0$

$\cos\theta = \dfrac{0}{(\sqrt{53})(\sqrt{13})}$

$\theta = 90$

angle is orthogonal.

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Q13

let U be the AB vector and V be the AP vector.

when angle is orthogonal.

$$\boxed{U \cdot V = 0}$$

$$U = B - A = (3-1)i + (0-(-1))j + (5-3)k$$

$$U = 2i + j + 2k$$

$$V = P - A = (r-1)i + (r-(-1))j + (r-3)k$$

$$V = (r-1)i + (r+1)j + (r-3)k$$

$$U \cdot V = 0$$

$$(2 \times (r-1)) + (1 \times (r+1)) + (2 \times (r-3)) = 0$$

$$2r - 2 + r + 1 + 2r - 6 = 0$$

$$\boxed{r = \frac{7}{5}}$$

Q15

a) $v = i + j - k$

$\|v\| = \sqrt{3}$, $v_1 = 1$, $v_2 = 1$, $v_3 = -1$

$$\boxed{\cos\alpha = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos\beta = \frac{1}{\sqrt{3}}}$$

RC

No.

$$\cos\gamma = \frac{-1}{\sqrt{3}}$$

$$\alpha = \beta \approx 55^\circ$$
$$\gamma \approx 125^\circ$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2 = 1$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$1 = 1$$

b) $v = 2i - 2j + k$

$$\cos\alpha = \frac{V_1}{\|V\|} = \frac{2}{3} \qquad \alpha \approx 48^\circ$$

$$\cos\beta = \frac{V_2}{\|V\|} = \frac{-2}{3} \qquad \beta \approx 133^\circ$$

$$\cos\gamma = \frac{V_3}{\|V\|} = \frac{1}{3} \qquad \gamma \approx 71^\circ$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$(2/3)^2 + (-2/3)^2 + (1/3)^2 = 1$$

$$1 = 1$$

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Q39

$$W = \vec{F} \cdot d(\hat{V})$$

$$\hat{V} = \frac{V}{\|V\|} = \frac{i+j+k}{\sqrt{3}} = \frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$$

$$d\hat{V} = \frac{15}{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$d\hat{V} = 5\sqrt{3}\,\hat{i} + 5\sqrt{3}\,\hat{j} + 5\sqrt{3}\,\hat{k}$$

$$W = F \cdot d\hat{V}$$

$$W = (4i - 6j + k) \cdot (5\sqrt{3}\,\hat{i} + 5\sqrt{3}\,\hat{j} - 5\sqrt{3}\,\hat{k})$$

$$W = (4 \times 5\sqrt{3}) + (-6 \times 5\sqrt{3}) + (1 \times 5\sqrt{3})$$

$$\boxed{W = -5\sqrt{3}\ J}$$