Table 2 – Helical Gear Set Manufacturing Data

	Pinion	Gear
Part name / Index	Helical Pinion	Helical Gear
Material	Grade 1 through hardened steel, 360 HB	Grade 1 through hardened steel, 360 HB
Normal pressure angle, ϕ_n	20	20
Normal module, m_n	4	4
Helix angle, ψ	30	30
Pitch diameter	$55.4256\ mm$	$249.4153\ mm$
Face width, F	55.4256 mm	55.4256 mm
Number of teeth	12	54

Table 3 – Bevel Gear Set Manufacturing Data

	Pinion	Gear
Part name / Index	Straight Bevel Gear	Straight Bevel Gear
Material	Grade 1 through hardened steel, 380 HB	Grade 1 through hardened steel, 380 HB
Pressure angle, ϕ_n	20	20
Module, m	9	9
Pitch diameter	117 mm	387 mm
Face width, F	60.65	60.65
Number of teeth	13	43

ME308

2024-2025 Spring Semester

Term Project

Data Set Number	: 02
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Contents

1 Introduction

Archimedes Screws, also known as screw conveyors, are widely used to transport granular material. They can be used in vertical, inclined or even horizontal alignments and are efficient in this regard. Typically for the vertical and inclined alignment screws a self-locking worm gear reducer is utilized to prevent unwanted material discharge, however for the horizontal alignment helical-bevel gear reducers are generally preferred.

The scope and focus project are on designing a power transmission system that transfers power from the electric motor to the input shaft. The gearbox will consist of helical-bevel gears that take the input torque and angular velocity from the motor and transfer it to the screw at an altered torque and angular velocity.

The design process of this gearbox will be carried out systematically, first selecting the gears according to the minimum required module to ensure secure power transmission and durability. The number of gear teeth will be selected such that there is no undercutting or interference to improve longevity. Subsequently, bearings will be selected for the shafts while keeping in mind constraints of minimum dimensions and adequate safety. Finally, mounting and lubrication considerations will be addressed to ensure safe operation of the assembly.

2 Gear Design

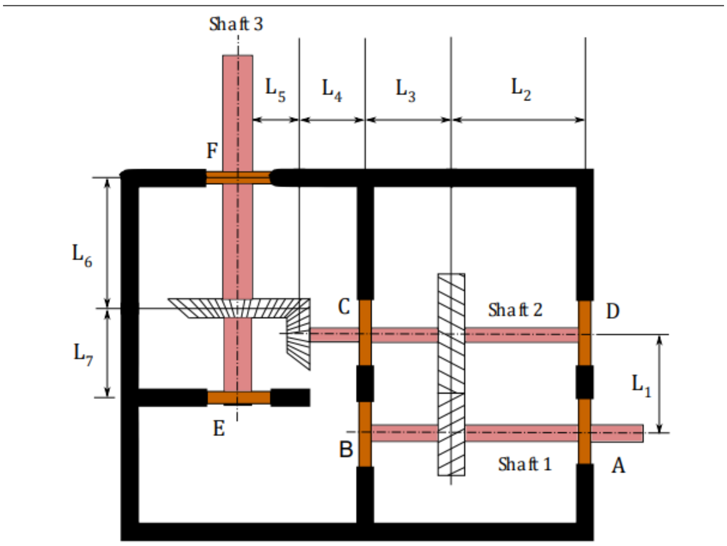

Figure 1. Schematic of Gearbox

Since we are given the speeds for shafts 1 and 3, we can find the reduction ratio for the entire gearbox as.

$$VR = \frac{N1}{N3} = \frac{1477}{100} = 14.77$$

To find the reduction ratio in the helical gear set, we have assumed midpoint of the range of the helical velocity ratio given:

$$VR_{helical} = 4.5$$

2.1 Helical Gear Design and stress analysis

We have assumed a normal pressure angle of **20 degrees** as they are generated using a rack cutter with an angle of 20 degrees. Apart from that, the helix angle is already specified as **30 degrees.**

$$\boldsymbol{\emptyset_n = 20^0}$$
$$\boldsymbol{\varphi = 30^0}$$

The helical angle and normal pressure angle are used to calculate the transverse pressure angle of this gear.

$$\emptyset_t = \tan^{-1}\frac{\tan\emptyset_n}{\cos\varphi} = \tan^{-1}\frac{\tan 20}{\cos 30} = 22.7959^0$$

Minimum virtual number of teeth for a 20-degree pinion are selected from notes to be used in examinations.

$$N_P' = 18$$

Since in helical gears, virtual number of teeth are used in **Slaymaker's inequality**, we need to calculate virtual number of teeth for gear as well.

$$N_P = N_P' * (\cos\varphi)^3 = 18 * (\cos 30)^3 = 12$$

$$N_g = N_P * VR_{helical} = 12 * 3.8 = 54$$

$$N_g' = \frac{N_g}{(\cos\varphi)^3} = \frac{46}{(\cos 30)^3} = 83.184$$

Next, we must employ the Slaymaker's equation to ensure there is no interference. In order to use it, we must know the value of k. Using the Notes to be Used in Examinations document, we find that when we have 20-degree full depth involute teeth, the value of k is 1 as defined below.

Since K=1,

$$\frac{(4*k)}{(\sin\emptyset_n)^2} * \left(N_g' + k\right) = 2877.0743$$

$$2 * N_P' * N_g' + {N_p}'^2 = 3316.9838$$

$$2877.0743 < 3316.9838$$

Since the left-hand side is less than the right-hand side of the equation, the inequality is satisfied and there is **no interference.** Next step involved in this gear design was finding a suitable module which fits our requirements for the safety factors of bending and contact stress given to us. This required an iterative approach, and the initial equations used in this design are stated below.

During the start of iteration, a normal module of 3mm was used which did not meet the factor of safety requirements. After solving, 2 modules which satisfied our design and were on the standard module table were found to be 4mm and 6mm. Since we were to design gears for smallest normal module, we chose 4 and the calculations below are performed for that.

$$m_n = 4mm$$

$$m_t = \frac{m_n}{\cos\varphi} = \frac{4}{\cos 30} = 4.6188mm$$

$$d_g = m_t * N_g = 249.4153mm$$

$$d_p = m_t * N_p = 55.4256mm$$

The relationship between the module and face width is given as below.

$$\boldsymbol{8 * m_n < F < 16 * m_n}$$

Hence to minimize the module, the upper limit of face width has been selected.

$$b = 16 * m_n \cos\varphi = 16 * 8 \cos 30 = 55.4256mm$$

2.1.1 Allowable Bending Stress:

Both gear and pinion are made of the same material, hardened grade 1 steel.

From equation 14-17 of notes in the equation,

$$\sigma_{all} = \frac{S_t * Y_N}{S_F * Y_\theta * Y_Z}$$

$$HB = 360\ HB$$

$$N = 1 * 10^7\ cycles$$

$$R_f = 0.92$$

$$S_f = 2.65$$

From Figure 14-2,

$$S_t = (0.533 * HB) + 88.3 = 280.18\ MPA\ from\ grade\ 1\ steel$$

From Figure 14-14,

$$Y_N = 1.6831 * (N)^{-0.0323} = 0.9916 \text{ stress cycle factors}$$

Since $T < 120$ as our given operating temperature is 75,

$$Y_\theta = 1$$

from Equation 14.38:

The reliability factor,

$$Y_Z = 0.658 - 0.0759 * ln(1 - R) = 0.8497$$

Hence using the above data, allowable bending stress in pinion is calculated as below.

$$\sigma_{all} = \frac{S_t * Y_N}{S_F * Y_\theta * Y_Z} = 123.3826\ MPA$$

Bending stresses in the pinion and gear can be calculated using Equation 14.16.

2.1.2 Bending Stress Calculation for Pinion and Gear

For Pinion:

Equation 14-16

$$\sigma_p = W_t * K_0 * K_v * K_s * K_H * \frac{K_b}{b * m_t * Y_J}$$

$$H = 7000\ W$$

$$V = \frac{\pi * d_p * n_1}{60{,}000} = \frac{4.286m}{s}$$

$$W_t = \frac{H}{V} = 1633.0832N$$

$$K_0 = 1.25\ since\ conveyor\ belts\ may\ have\ a\ moderate\ shocks.$$

The dynamic factor K_v can be calculated as follows:
From equation 14-28 where **Qv=9.**

$$B = 0.25 * (12 - 9)^{\frac{2}{3}} = 0.52$$

$$A = 50 + 56(1 - B) = 76.8788$$

$$K_v = \left(\frac{A + \sqrt{200 * V}}{A}\right)^B = 1.1827$$

$$K_s = 1$$

To calculate the load distribution factor K_H we have used Equations 14-30 to Equation 14-35,

$$C_e = 1$$

$$C_{pm} = 1$$

$$C_{mc} = 1 \; since\ teeth\ are\ uncrowned$$

$$C_{pf} = \left(\frac{b}{10 * d_p} - 0.0375 + 0.0125 * b\right) = 0.0898$$

However, to use the last equation, we need to convert our face width and diameter of pinion into inches.

$$b = \frac{b\ in\ mm}{25.4} = 2.1821\ inches$$

$$d = \frac{d\ in\ mm}{25.4} = 2.1821\ \ inches$$

Since commercial enclosed units

$$C_{ma} = A + F * B + C * F^2$$

Values of A, B, and C are chosen from Table 14.9,

$$C_{ma} = 0.127 + 0.0158 * B - 0.930 * 10^{-4} * F^2 = 0.161$$

$$K_H = 1.2508$$

Normally, the value of rim thickness factor K_b is assumed to be 1 however since we don't have enough information, we went to solve the equation 14.39 and equation 14.40.

$$t_r = \frac{d_p}{2} + dedendum = 32.7128$$

$$h_t = addendum + dedendum = 9$$

$$m_B = \frac{t_r}{h_t} = 3.63$$

$$since\ m_b \geq 1.2, We\ chose\ \ K_B = 1$$

We need to use fig. 14.7 and 14.8 to calculate the Bending-Strength Geometry Factor:

$$J' = 0.44$$
$$Mod_J = 0.99$$
$$Y_J = 0.4356$$

Hence stress in the pinion can be calculated as all the required values have been found calculated,

$$\sigma_p = W_t * K_0 * K_v * K_s * K_H * \frac{K_b}{b * m_t * Y_J} = 27.085\ MPa$$

$$since\ \sigma_p < \sigma_{all}$$

Since our pinion bending stress is smaller than the allowable stress, it is safe.

For Gear:

For gear, only the geometry modifying factors are changed.

$$J' = 0.54$$

$$Mod_J = 0.93$$

$$Y_J = 0.5022$$

$$\sigma_G = W_t * K_0 * K_v * K_s * K_H * \frac{K_b}{b * m_t * Y_J} = 23.493\ MPa$$

$$since\ \sigma_G < \sigma_{all}$$

Since our gear bending stress is smaller than the allowable stress, it is safe.

2.1.3 Allowable Contact (Pitting) Stress:

For Pinion:

From Equation 14-18,

$$\sigma_{call} = \frac{S_c * Z_N * Z_W}{S_H * Y_\theta * Y_Z}$$

From Figure 14.5,

$$S_c = 2.22 * HB + 200 = 999.2\ MPa$$

$$S_H = \sqrt{2.75}$$

Reliability and temperature factors remained unchanged.
from Figure 14.15:

$$Z_N = 2.466 * N^{-0.056} = 0.9854$$

$Z_w = 1$ for both gears and pinion since surface is finished.

$$\sigma_{\mathrm{call}} = \frac{\mathrm{S_c * Z_N * Z_w}}{\mathrm{S_H * Y_\theta * Y_z}} = 698.7652\ \mathrm{MPa}$$

For Gear:

Since the gears are made of the same material, the Brinell hardness-ratio factor of the pinion and gear is 1.

$$A' = 0$$
$$Z_{WG} = 1 + A'(m_G - 1) = 1$$

$$\sigma_{call} = \frac{S_c * Z_N * Z_w}{S_H * Y_\theta * Y_z} = 698.7652 \text{ MPa}$$

2.1.4 Contact Stress Calculation for Pinion and Gear

Equation 14-16.

$$\sigma_{Cp} = Z_E * \left(\left(W_t * K_0 * K_v * K_s * \frac{K_H * Z_R}{d_P * b * Z_I}\right) * 1\right)^{0.5}$$

From table 14-18, we can find elastic coefficient for this question.

$$Z_E = 191\sqrt{MPa}$$

$$Z_R = 1 \ which\ is\ the\ surface\ factor$$

$$r_p = \frac{d_p}{2} = 27.7128mm$$

$$r_g = \frac{d_g}{2} = 124.7077mm$$

$$r_{pbase} = r_p * \cos\emptyset = 25.5482mm$$

$$r_{gbase} = r_g * \cos\emptyset = 114.9669mm$$

From Equation 14-24,

$$P_N = \pi * m_n * \cos\emptyset_n = 11.8085mm$$

By using the shown equation, we will find the length of action,

$$Z := \left(\left(r_p + a\right)^2 - r_{basep}{}^2\right)^{0.5} + \left(\left(r_g + a\right)^2 - r_{baseg}{}^2\right)^{0.5} - \left(r_p + r_g\right) \cdot \sin\left(\phi_t\right)$$

$$Z_1 = 18.7881mm$$

$$Z_2 = 57.8643\text{mm}$$

$$Z_3 = 59.0552mm$$

Since both Z_1 and Z_2 are smaller than Z_3, they do not need to be replaced. Hence the length of contact is found out as.

$$Z = 17.5972mm$$

$$m_N = \frac{P_N}{0.95 * Z} = 0.7064$$

$$Z_I := \left(\frac{\cos\left(\phi_t\right) \cdot \sin\left(\phi_t\right) \cdot m_G}{2 \cdot m_N \cdot \left(m_G + 1\right)} \right)$$

The surface strength geometry factor is calculated as 0.2069 using the above equation.

Finally, the pitting stress for both pinion and gear are calculated as.

$$\sigma_{Cp} = Z_E * \left(\left(W_t * K_0 * K_v * K_s * \frac{K_H * Z_R}{d_P * b * Z_I} \right) * 1 \right)^{0.5} = 416.29\ MPa$$

$$\sigma_{Cg} = Z_E * \left(\left(W_t * K_0 * K_v * K_s * \frac{K_H * Z_R}{d_P * b * Z_I} \right) * 1 \right)^{0.5} = 416.29\ MPa$$

$$\boldsymbol{Since\ \sigma_c < \sigma_{call}}$$

The module selected which is 4mm satisfies both the bending and pitting stress of both gears. Hence it is selected as our final module in consideration.

2.2 Bevel Gear design and stress analysis

Note that all the equations that are numbered are taken from the 'Notes to be Used in the Examination'.
Initially we will use the rotation speed of shaft 1 and shaft 3 to obtain the Velocity ratio in the acceptable range

$$VR_{total} = \frac{n_1}{n_3}$$

We first defined the velocity ratio for helical gear,

$$VR_H = 4.5$$

Using this velocity ratio, we found the velocity ratio for the straight bevel gears as,

$$VR_b = 3.3$$

The bevel gears used are straight bevel gears with a 20-degree pressure angle and full depth involute tooth profiles.

$$\emptyset = 20°$$

Since we are dealing with straight bevel gears with a 20-degree pressure angle we will take the minimum number of pinion teeth as 13 from Table 13-3:

$$N_p = 13$$

Then we can compute the number of gear teeth using the velocity ratio we found:

$$N_G = VR_b \cdot N_P = 43$$

Gear Ratio is found as:

$$m_G = \frac{N_G}{N_P} = 3.3$$

Then we can find the respective pitch angles:

$$\gamma = \operatorname{atan}\left(\frac{N_P}{N_G}\right) = 16.82°$$

$$\Gamma = \operatorname{atan}\left(\frac{N_P}{N_G}\right) = 73.18°$$

Then we can use the Slaymaker's Equation to ensure that there is no interference or undercutting. For full depth involute teeth profile, we will take k as 1 from the 'Notes to be used in Examination'.

$$N'_p = \frac{N_P}{\cos(\gamma)} \cdot \frac{1}{cos^3(\varphi)} = 3700$$

$$N'_G = \frac{N_G}{\cos(\Gamma)} \cdot \frac{1}{cos^3(\varphi)} = 40485$$

$$\frac{4k}{sin^2(\emptyset_n)}(N_G' + k) \le 2N_G' \cdot N_P' + N_P'^2$$

The right-hand side of the equation equates to:

$$\frac{4k}{sin^2(\emptyset_n)}(N_G{}' + k) = 1.38 \cdot 10^6$$

While the left-hand size comes out to:

$$2N_G{}' \cdot N_P{}' + N_P{}'^2 = 3.13 \cdot 10^8$$

Since the right-hand side of the equation is lesser than the left-hand side of the equation the Slaymaker's is satisfied, hence there will be no interference or undercutting. We initially assumed a module of 8 mm which does not satisfy the condition of the given safety factor. We then iterated for different values until we got a satisfactory value which is $m = 9$.

$$m = 9\, mm$$

$$d_P = m \cdot N_P = 117\, mm$$

$$d_G = m \cdot N_G = 387\, mm$$

Our overall constraint is to design for the minimum dimensions possible and so we need to minimize the face width, for this we need to the cone distance from Equation 15-25 from the 'Notes to be Used in Examinations' document.

$$A_0 = \sqrt{(\frac{d_P}{2})^2 + (\frac{d_G}{2})^2} = 202.15\, mm$$

From which the face width will be:

$$b = \min(0.3 \cdot A_0, 10 \cdot m)$$

$$0.3 \cdot A_0 = 60.65\, mm$$

$$10 \cdot m = 90\, mm$$

So, we can select b as the minimum value:

$$b = 60.65\, mm$$

Then the average pinion and gear radii are found as:

$$r_{avP} = \frac{d_P - bcos(\Gamma)}{2} = 49.725\, mm$$

$$r_{avG} = \frac{d_G - bcos(\gamma)}{2} = 164.475\, mm$$

2.2.1 Allowable Bending Stress calculation

The number of load cycles for the helical pinion have been given to us in the problem:

$$N_L = 1.3 * 10^7$$

Then the number of life cycles for bevel pinion and gear will be found as follows:

$$n_{LP} = \frac{N}{3.3} = 3.94 * 10^6$$
$$n_{LG} = \frac{N}{(3.3)^2} = 1.19 * 10^6$$

We now have to calculate the modifying factors according to the problem statement and the constraints provided:
Since no temperature information is provided, according to our working conditions we can safely assume $K_\theta = 1$
For the reliability factor Y_Z we can use Eq. 15-20 with R = 0.92:

$$Y_Z = 0.7 - 0.15\log(1 - R) = 0.8645$$

Then using Eq. 15-23 for Grade 1 through hardened steel with a 380 Brinell hardness:

$$\sigma_{film} = 0.30 \cdot H_B + 14.48 = 128.48\ MPa$$

For the Stress Cycle factor for Bending Moment by using Eq. 15-15:

$$Y_{NTP} = 1.6831 n_{LP}{}^{-0.0323} = 1.0306$$
$$Y_{NTG} = 6.1514 n_{LG}{}^{-0.1192} = 1.1604$$

The safety factor given to us is:

$$S_F = 2.65$$

From Eq 15-14:

$$\sigma_{allP} = \frac{\sigma_{film}}{S_F} \cdot \frac{Y_{NTP}}{K_\theta \cdot Y_Z} = 57.79\ MPa$$

$$\sigma_{allG} = \frac{\sigma_{film}}{S_F} \cdot \frac{Y_{NTG}}{K_\theta \cdot Y_Z} = 65.07\ MPa$$

2.2.2 Bending Stress Calculation for Bevel Gear and Pinion

For Pinion:

$$\sigma_P = 1000 \cdot \frac{W_T \cdot K_A \cdot K_v \cdot Y_X \cdot K_{HB}}{b \cdot m \cdot Y_B \cdot Y_{Jp}}$$

$$\sigma_G = 1000 \cdot \frac{W_T \cdot K_A \cdot K_v \cdot Y_X \cdot K_{HB}}{b \cdot m \cdot Y_B \cdot Y_{JG}}$$

$$H = 7000\, W$$

The angular velocity of the bevel pinion is found as follows:

$$n_P = \frac{n_1}{4.5} = 328.33\, rpm$$

$$T = \frac{H}{2 \cdot n_2 \cdot \frac{\pi}{60}} = 203.66\, Nmm$$

$$W_T = \frac{T}{d_P/2} = 3481.3\, N$$

Considering the application for which the gears are being designed we can take the overload factor as $K_A = 1.25$

For the dynamic factor K_v:
Then,

$$v_{et} = 5.236 * 10^{-5} \cdot d_P \cdot n_P = 2.0107\, \frac{m}{s}$$

From Eq 15-6,

$$Q_v = 9$$

$$B = 0.25(12 - Q_v)^{\frac{2}{3}} = 0.52$$

$$A = 50 + 56(1 - B) = 76.88$$

$$K_v = (\frac{A + \sqrt{200 \cdot v_{et}}}{A})^B = 1.1281$$

The size factor $Y_X = 1$ as is given in the question.

Both bevel gears in the problem statement are straddle mounted,
Then from Eq 15-11:

$$K_{mb} = 1.00$$

And,

$$K_{HB} = K_{mb} + 5.6 \cdot 10^{-6} \cdot b^2 = 1.0206$$

From Eq 15-13, for straight bevel gears:

$$Y_\beta = 1$$

From Figure 15-7:

$$Y_{JP} = 0.23$$

Then,

$$\sigma_P = 1000 \cdot \frac{W_T \cdot K_A \cdot K_v \cdot Y_X \cdot K_{HB}}{b \cdot m \cdot Y_B \cdot Y_{JP}} = 39.91\ MPa$$

<u>For Gear:</u>

The only difference for the gear in the bending stress equation is Y_{JG}.

$$Y_{JG} = 0.172$$

$$\sigma_G = 1000 \cdot \frac{W_T \cdot K_A \cdot K_v \cdot Y_X \cdot K_{HB}}{b \cdot m \cdot Y_B \cdot Y_{JG}} = 53.37\ MPa$$

The stresses calculated for pinion as well as gear are smaller than the allowable stress values

2.2.3 Allowable Contact Stress Calculation

From Eq 15-2:

$$\sigma_{callP} = \frac{\sigma_{Hlim}}{S_H} \cdot \frac{Z_{NTP} \cdot Z_W}{K_\theta \cdot Z_Z}$$

$$\sigma_{callG} = \frac{\sigma_{Hlim}}{S_H} \cdot \frac{Z_{NTG} \cdot Z_W}{K_\theta \cdot Z_Z}$$

$$S_H = \sqrt{2.75}$$

From Eq 15-22:

$$\sigma_{Hlim} = 2.35 H_B + 162.89\ MPa = 1055.89\ MPa$$

Stress Cycle factor for pitting resistance can be found using equation 15-14:

$$Z_{NTP} = 3.4822 \cdot n_{Lpinion}{}^{-0.0602} = 1.3958$$

$$Z_{NTG} = 3.4822 \cdot n_{Lgear}{}^{-0.0602} = 1.4998$$

Since the pinion and gear are made from the same material:

$$Z_W = 1$$

$$K_\theta = 1$$

From Eq 15-20:

$$Z_Z = {Y_Z}^{0.5} = 0.9298$$

$$\sigma_{callP} = \frac{\sigma_{Hlim}}{S_H} \cdot \frac{Z_{NTP} \cdot Z_W}{K_\theta \cdot Z_Z} = 955.81\ MPa$$

$$\sigma_{callG} = \frac{\sigma_{Hlim}}{S_H} \cdot \frac{Z_{NTG} \cdot Z_W}{K_\theta \cdot Z_Z} = 1027.03\ MPa$$

As can be seen, the allowable contact stress of the pinion is lower than the contact stress of the gear, so only pinion needs to be checked as it is the more critical member.
Contact Stress Calculation for Bevel Pinion:

From Eq 15-1,

$$\sigma_H = Z_E \cdot \sqrt{\frac{1000 \cdot W_T}{d_P \cdot b \cdot Z_T} \cdot K_A \cdot K_V \cdot K_{HB} \cdot Z_X \cdot Z_{XC}}$$

From Eq 15.9,

$$Z_X = 0.00492b + 0.4375 = 0.7359$$

From Eq 15-12,

$$Z_{XC} = 2$$

From Table 14-8,

$$Z_E = 191\sqrt{MPa}$$

From Fig 15-6,

$$Z_I = 0.072$$

Which is the contact geometry factor found using respective number of teeth,

$$\sigma_H = Z_E \cdot \sqrt{\frac{1000 \cdot W_T}{d_P \cdot b \cdot Z_T} \cdot K_A \cdot K_V \cdot K_{HB} \cdot Z_X \cdot Z_{XC}} = 725.64\ MPa$$

$$\sigma_H < \sigma_{callP}$$

3 Shaft Force Analysis

3.1 Shaft 1:

Shaft 1 is connected to the input motor and there is a helical gear mounted on this shaft. With these details, the free body diagram of this shaft is as shown below.

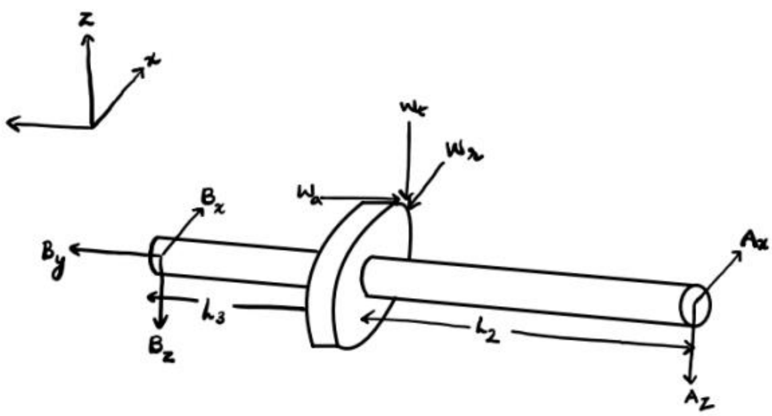

Figure 2. Shaft 1 free body diagram

Since forces of gears are all linked to transmitted force and helix angle, they are found out as below.

$$W_t = 1633.1\ N$$

$$W_a = W_t * \tan\varphi = 942.9N$$

$$W_r = W_t * \tan\emptyset_t = 686.3N$$

L2= 90mm

L3= 90mm

Hence 5 equations with 5 unknowns are required to solve the system. Equations shown in the figure below are solved using MATLAB, and answers are given below.

eq. 1: $A_x + B_x - W_r = 0$

eq. 2: $B_y - W_a = 0$

eq. 3: $W_t - B_z - A_z = 0$

eq. 4: $-W_r \cdot L3 + A_x \cdot (L3 + L2) - W_a \cdot \frac{d_p}{2} = 0$

eq. 5: $A_z \cdot (L_2 + L_3) - W_t \cdot L_3 = 0$

Table 1. Unknowns from shaft analysis of shaft 1

Variable	***Value***
A_x	488.319 N
A_z	816.550 N
B_x	197.981 N
B_y	942.900 N
B_z	816.550 N

3.2 Shaft 3:

Figure 3 shows free body diagram of shaft 3. Points are represented as E and F.

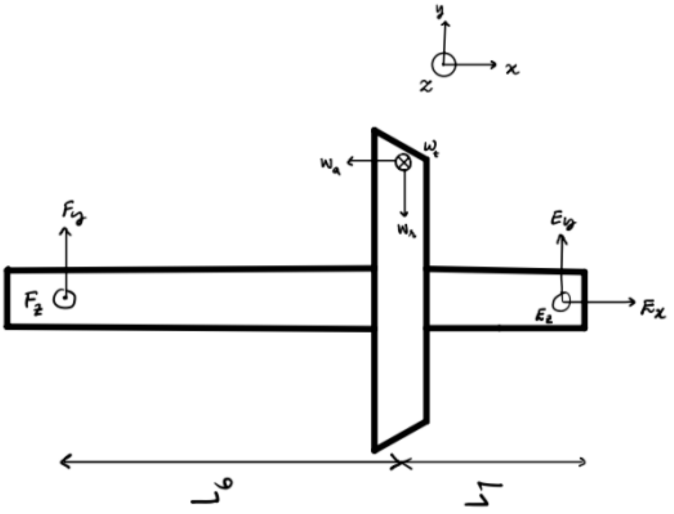

Figure 3. Free body Diagram of shaft 3

$$W_{tp} = \frac{H}{v_{et}} = 3481.32\ N$$

$$W_{rp} = W_{tp} \cdot \tan(\emptyset) \cdot \cos(\gamma) = 1212.88\ N$$

$$W_{ap} = W_{tp} \cdot \tan(\emptyset) \cdot \sin(\gamma) = 366.69\ N$$

We have found forces on bevel gear by using action reaction principle on forces of bevel pinion

$$W_{tg} = W_{tp} = 3481.32\ N$$

$$W_{ag} = W_{rp} = 1212.88\ N$$

$$W_{rg} = W_{ap} = 366.69\ N$$

To find the forces on the bearings e and f:

$$L_6 = 70\ mm \qquad L_7 = 120\ mm$$

$$E_x - W_{ag} = 0$$

$$F_y + E_y - W_{rg} = 0$$

$$E_z + F_z - W_{tg} = 0$$

$$F_z(L_6 + L_7) - W_{tg} * L_7 = 0$$

$$W_{rg} * L_7 - F_y(L_6 + L_7) + W_{ag} * r_{avgear} = 0$$

Solving these equations using MATLAB:

Table 2. Unknowns from shaft analysis of shaft 3

Variable	***Value***
F_y	1281.53 N
F_z	2198.73 N
E_x	1212.88 N
E_y	-914.85 N
E_z	1282.59 N

4 Bearing Selection

4.1 Shaft 1

As specified in the project document there are two places where bearings need to be used in shaft 1 that are at A and B. At both places, deep groove ball bearings are used and the process employed while choosing these bearings is described below.

$$f_z = 1.1$$
$$f_d = 1.3$$
$$R_{bearing} = 90\%$$
$$T_{op} = 70$$
$$p = 3 \; since \; ball \; bearings$$
$$L_h = 3500 \; hours$$
$$\rho = 870 \frac{kg}{m^3}$$
$$a_1 = 1 \; since \; reliablity \; is \; 90\%$$

For Bearing at A:

$$F_r = \sqrt{A_X{}^2 + A_Z{}^2} = 951.4249 \; N$$

$$F_a = 0$$
$$P_{eff} = f_z * f_d * F_r = 1360.5376N$$

Since the diameter of shaft 1 is given to us as 30mm, we have selected the bearings with d=30mm for our iterations only.

<u>Assuming 160 06</u>

$$d = 30mm$$
$$D = 55mm$$
$$C_{ISO} = 9.55\ KN$$
$$d_m = 42.5mm$$

From figure 2 in the STEYR bearing catalog,

$$v_1 = 19\ \frac{mm^2}{s}$$

Since oil used is SAE 40, the following equation is used for viscosity calculations.

$$\mu := 0.083369 \cdot \mathbf{e}^{\frac{1474.4}{1.8 \cdot T_{op} + 127}}$$

$$\mu = 23.1733\ mPa * s$$
$$v = \frac{\mu}{\rho} = 2.6575 * 10^{-5}\ \frac{m^2}{s}$$
$$x = \frac{v}{v_1 * 10^{-6}} = 1.3987$$

From figure 4 in the catalog,

$$1.2 < a_{23} < 1.9$$

To be more conservative in design and safety, lower value of this is selected. Hence C_{ISOr} is calculated with the formula given below.

$$C_{ISOr} := \left(\frac{L_h \cdot 60 \cdot n_1}{a_1 \cdot a_{23} \cdot 10^6} \right)^{\frac{1}{p}} \cdot P_{eff} :$$

$$C_{ISOr} = 8666.63\ N < 9550\ N$$

Although the required value is less than the specified value for bearing. However, they are very close to each other. If other factors are considered, this bearing may not be sufficient to give life of 3500 hours. Hence another bearing is tried and tested on this current system for better results.

Assuming 60 06

$$d = 30mm$$

$$D = 55mm$$

$$C_{ISO} = 11.3\ KN$$

$$d_m = 42.5mm$$

From figure 2 in the STEYR bearing catalog,

$$v_1 = 19\ \frac{mm^2}{s}$$

Since oil used is SAE 40, the following equation is used for viscosity calculations.

$$\mu := 0.083369 \cdot \mathbf{e}^{\frac{1474.4}{1.8 \cdot T_{op} + 127}}$$

$$\mu = 23.1733\ mPa * s$$

$$v = \frac{\mu}{\rho} = 2.6575 * 10^{-5}\ \frac{m^2}{s}$$

$$x = \frac{v}{v_1 * 10^{-6}} = 1.3987$$

From figure 4 in the catalog,

$$1.2 < a_{23} < 1.9$$

$$C_{ISOr} = 8666.63\ N < 11{,}300\ N$$

Since for this bearing, the required value is almost 2/3 of the specified value for the bearing, this one is more suitable in terms of safety and life. Hence, this bearing is chosen at A.

For Bearing at B:

Assuming 160 06:

$$C_{0,ISO} = 7320\ N$$

$$C_{ISO} = 9550\ N$$

$$F_r = \sqrt{{B_X}^2 + {B_Z}^2} = 840.209\ N$$

$$F_a = B_y = 942.9\ N$$

$$\frac{F_a}{F_r} = 1.122$$

$$\frac{F_a}{C_{0,ISO}} = 0.1288$$

From Table 2.2.2:

$$e_{ratio} = 0.3125$$

Since $e < \frac{F_a}{F_r}$:

$$X = 0.56$$

$$Y = 1.403$$

$$P = X_x * F_r + Y_x * F_r = 1793.41\ N$$

$$P_{eff} = f_z * f_d * P = 2564.57\ N$$

Diameter Values for 160 06,

$$d = 30mm$$

$$D = 55mm$$

$$d_m = 42.5mm$$

$$n_1 = 1477\ rpm$$

From Fig 2 in the catalog,

$$v_1 = 18.5\frac{mm^2}{s}$$

From Fig 12.13, since oil used is SAE 40, the following equation is used for viscosity calculations,

$$\mu := 0.083369 \cdot \mathbf{e}^{\frac{1474.4}{1.8 \cdot T_{op} + 127}}$$

$$\mu = 23.1733\ mPa * s$$

$$v = \frac{\mu}{\rho} = 2.7263 * 10^{-5}\frac{m^2}{s}$$

$$x = \frac{v}{v_1} * 10^{-6} = 1.474$$

Which gives us,

$$a_{23} = 1.9$$

Then,

$$C_{ISOr} := \left(\frac{L_h \cdot 60 \cdot n_1}{a_1 \cdot a_{23} \cdot 10^6} \right)^{\frac{1}{p}} \cdot P_{eff} :$$

$$C_{ISOr} = 14016.2 \ N$$

Since,

$$C_{ISOr} > C_{ISO}$$

This bearing selection is **unsuitable** at B.

Assuming 64 06:

$$C_{0,ISO} = 27200 \ N$$

$$C_{ISO} = 37700 \ N$$

$$F_r = \sqrt{B_X^{\ 2} + B_Z^{\ 2}} = 840.209 \ N$$

$$F_a = B_y = 942.9 \ N$$

$$\frac{F_a}{F_r} = 1.122$$

$$\frac{F_a}{C_{0,ISO}} = 0.0347$$

From Table 2.2.2,

$$e_{ratio} = 0.2296$$

Since $e < \frac{F_a}{F_r}$,

$$X = 0.56$$

$$Y = 1.928$$

$$P = X_x * F_r + Y_x * F_r = 2288.43 \ N$$

$$P_{eff} = f_z * f_d * P = 3272.45 \ N$$

Diameter Values for 64 06,

$$d = 30mm$$

$$D = 90mm$$

$$d_m = 60mm$$

From Figure 2 in the catalog,

$$v_1 = 15.25\frac{mm^2}{s}$$

From Figure 12.13, Since oil used is SAE 40, the following equation is used for viscosity calculations.

$$\mu := 0.083369 \cdot \mathbf{e}^{\frac{1474.4}{1.8 \cdot T_{op} + 127}}$$

$$\mu = 23.1733 \; mPa * s$$

$$v = \frac{\mu}{\rho} = 2.7263 * 10^{-5}\frac{m^2}{s}$$

$$x = \frac{v}{v_1} * 10^{-6} = 1.7877$$

Which gives us,

$$a_{23} = 2.2$$

Then,

$$C_{ISOr} := \left(\frac{L_h \cdot 60 \cdot n_1}{a_1 \cdot a_{23} \cdot 10^6}\right)^{\frac{1}{p}} \cdot P_{eff}$$

$$C_{ISOr} = 17032 \; N$$

Since,

$$C_{ISO} > C_{ISOr}$$

This bearing selection is **suitable** at B.

4.2 Shaft 3

$f_z = 1.1$ $\quad f_d = 1.3$ $\quad R = 0.9$ $\quad L_h = 3500$ $\quad T_{op} = 70$

$p = \frac{10}{3}$ $\quad \rho = 850\frac{kg}{m^3}$ $\quad a_1 = 1$

For Bearing at E:

Assuming 320 08

$$d = 40\ mm$$
$$D = 68\ mm$$
$$d_m = 0.5(d + D) = 54\ mm$$
$$n_3 = 100\ rpm$$
$$C_{ISO} = 44100\ N$$
$$C_{0ISO} = 38300\ N$$

Using Figure 2:

$$v_1 = 130\frac{mm^2}{s}$$
$$\mu = 0.083369 * e^{\frac{1474.4}{1.8*T+127}} = 23.1733\ mPa.s$$
$$v = \frac{\mu * 10^{-3}}{\rho} = 2.726 * 10^{-5}\ \frac{m^2}{s}$$
$$\chi = \frac{v}{v_1 * 10^{-6}} = 0.2097$$

Using Figure 4

$$a_{23} = 0.85$$
$$F_{rE} = \sqrt{B_y^2 + B_z^2} = 1575.43\ N$$
$$F_a = B_x = 1212.88\ N$$
$$F_{rF} = \sqrt{A_y^2 + A_z^2} = 2544.94$$

To find appropriate bearing for F and E, we must figure out their radial and induced axial components. To do this, we follow the procedure given on page 42 in the Steyr Bearing Catalogue.

We assume O arrangement for the 2 bearings (i.e. in the first diagram on Pg 42 of the bearing catalogue Bearing A corresponds to Bearing F

Since we do not have any restrictions, we assume Y=1.75

$$\emptyset_F = \frac{F_{rF}}{2 * 1.75} = 727.13\ N$$

$$\emptyset_E = \frac{F_{rE}}{2 * 1.75} = 450.12\ N$$

$$\phi_F > \phi_E;\ hence\ the\ following\ procedure$$

$$F_{aF} = K_a + \phi_E = 1663\ N$$

$$F_{aE} = \phi_E = 450.12\ \text{N}$$

$$\frac{F_{aE}}{F_{rE}} = 0.2857$$

This value is smaller than the smallest value (0.39)

$$X_E = 1$$

$$Y_E = 0$$

$$P = F_{rE} * X_E + F_{aE} * Y_E = 1575.43\ N$$

$$P_{eff} = f_d * f_z * P = 2252.86\ N$$

$$C_{ISOr} = \left(\frac{L_h * 60 * n_3}{a_1 * a_{23} * 10^6}\right)^{\frac{1}{p}} * P_{eff} = 5896.23\ N$$

$$C_{ISO} > C_{ISOr}$$

Hence the selection is **suitable.**

Bearing at F:

Assuming 320 08

$$\frac{F_{aF}}{F_{rF}} = 0.6535$$

This value is larger than 0.39

$$X_F = 0.4$$

$$Y_F = 1.55$$

$$P = F_{rF} * X_F + F_{aF} * Y_F = 3595.63\ N$$

$$P_{eff} = f_d * f_z * P = 5141.75\ N$$

$$C_{ISOr} = \left(\frac{L_h * 60 * n_3}{a_1 * a_{23} * 10^6}\right)^{\frac{1}{p}} * P_{eff} = 15431.73\ N$$

$$C_{ISO} > C_{ISOr}$$

Selection is **suitable.**

Table 3 Bearings at their specified location

Location	Bearing
A	60 06
B	64 06
E	320 08
F	320 08

5 Assembly and Mounting Details

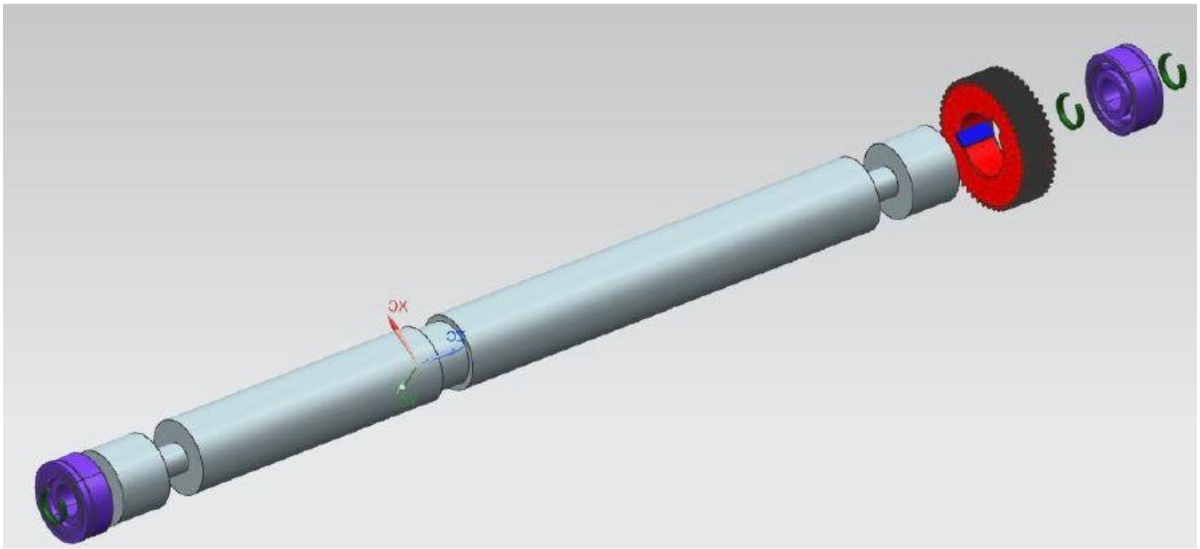

Figure 4 Shaft 1 assembly drawing and elements

The schematic above illustrates the assembly of Shaft 1. This setup includes the helical pinion, necessary bearings, and retaining rings mounted on both sides of the bearings. These components, along with machined shoulders on the shaft, are designed to restrict axial movement and ensure positional stability. Additionally, a key is fitted between the shaft and the helical pinion to facilitate effective torque transmission. The velocity at the pitch diameter of the helical gear is calculated as:

$$V_{pitchline} = n_1 * \frac{d_1}{2} * \frac{2\pi}{60 * 1000} = 1477 * 15 * \frac{2\pi}{60000} = \frac{2.32m}{s} < \frac{5m}{s}$$

As this surface speed remains below the 5 m/s limit, felt seals are employed to prevent the leakage of lubricants and to block the ingress of contaminants.

6 Conclusion

This project was carried out in several stages, beginning with the optimization of the helical and bevel gear selection. Our initial objective was to determine the minimum number of teeth required while satisfying the velocity ratio constraints, using Slaymaker's equation. Through iterative calculations, we identified the minimum viable module and corresponding number of teeth, as well as the minimum required face width. Notably, the pinion was identified as the more critical component, and its stress levels were incorporated into the final design to minimize failure risk.

Following this, static force analyses were conducted on Shafts 1 and 3 to determine the radial and axial loads acting on them. These results guided our selection of suitable bearings. Due to project constraints, deep groove ball bearings were used for Shaft 1 and tapered roller bearings for Shaft 3. Given the axial load-bearing nature of tapered bearings, additional axial stress calculations were required for Shaft 3. The resulting selections, as listed in Table 3, were found to be structurally adequate under the expected loads.

Finally, we addressed the mounting and assembly procedure for Shaft 1. Various design features were added to enhance the fit and function of the shaft, particularly in transmitting power through the helical pinion and supporting bearings. A detailed assembly was modeled in NX, accompanied by a video presentation of the process.

Despite a few unexpected findings during the analysis, all outcomes aligned with the constraints and objectives defined in the problem statement.

7 References

1] R. G. Budynas and J. K. Nisbett, Shigley's Mechanical Engineering Design, 10 ed.

[2] ME Department, ME 308 Machine Elements II Notes to be Used in The Examinations, Middle East Technical University.

[3] Styer Technical Manual 282 E, 1981

[4] Measuring Relative Density of Lubricants. (2013, March 13). Machinery Lubrication. https://www.machinerylubrication.com/Read/29319/measuring-relative-density#:~:text=The%20density%20of%20most%20oils,they%20are%20lighter%20by%20volume.

8 Appendix

Shaft 1 Analysis Code:

```
% Given constants
L2 = 0.09;     % L2 given
L3 = 0.09;     % L3 given
dp = 0.0554256;  % pinion diameter
Wt = 1633.1;  % Tangential gear force
Wr = 686.3;  % Radial gear force
Wa = 942.9;  % Axial gear force

% Define symbolic variables
syms Ax Az Bx By Bz

% Define equations
eq1 = Ax + Bx - Wr == 0;
eq2 = By - Wa == 0;
eq3 = Wt - Bz - Az == 0;
eq4 = -Wr*L3 + Ax*(L3 + L2) - Wa*(dp/2) == 0;
eq5 = Az*(L2 + L3) - Wt*L3 == 0;

% Solve system
sol = solve([eq1, eq2, eq3, eq4, eq5], [Ax, Az, Bx, By, Bz]);
% Display results in 3 decimal places
disp('Solution (rounded to 3 decimal places):')
fprintf('Ax = %.3f\n', double(vpa(sol.Ax, 6)));
fprintf('Az = %.3f\n', double(vpa(sol.Az, 6)));
fprintf('Bx = %.3f\n', double(vpa(sol.Bx, 6)));
fprintf('By = %.3f\n', double(vpa(sol.By, 6)));
fprintf('Bz = %.3f\n', double(vpa(sol.Bz, 6)));
```

Shaft 3 Analysis Code:

```
% Given values
W_ag = 1212.8813;  % N
W_rg = 366.685;    % N
W_tg = 3481.3249;  % N
L6 = 70;           % mm
L7 = 120;          % mm
ravgear = 164.475; % mm

% Convert all lengths to meters for consistency (optional if units match)
L6 = L6 / 1000;
L7 = L7 / 1000;
ravgear = ravgear / 1000;

% Define symbolic variables
syms Ex Ey Ez Fy Fz

% Define equations
eq1 = Ex - W_ag == 0;
eq2 = Fy + Ey - W_rg == 0;
eq3 = Ez + Fz - W_tg == 0;
eq4 = Fz*(L6 + L7) - W_tg*L7 == 0;
eq5 = W_rg*L7 - Fy*(L6 + L7) + W_ag*ravgear == 0;

% Solve system
sol = solve([eq1, eq2, eq3, eq4, eq5], [Ex, Ey, Ez, Fy, Fz]);

% Display results
Ex = double(sol.Ex);
Ey = double(sol.Ey);
Ez = double(sol.Ez);
Fy = double(sol.Fy);
Fz = double(sol.Fz);

fprintf('Ex = %.4f N\n', Ex);
fprintf('Ey = %.4f N\n', Ey);
fprintf('Ez = %.4f N\n', Ez);
fprintf('Fy = %.4f N\n', Fy);
fprintf('Fz = %.4f N\n', Fz);
```