MAT186H1 - Calculus I: Unit 1 Notes

Huzaim Malik

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1 Sets and Notation

A set is a collection of distinct objects. For example, the set of natural numbers is:

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

Set notation includes:

- Union: $A \cup B$
- Intersection: $A \cap B$
- Difference: $A \setminus B$

2 Set-building Notation

Set-builder notation is used to define sets concisely:

$$A = \{x \in \mathbb{R} \mid x > 0\}$$

This means A is the set of all x in $\mathbb{R}$ such that $x > 0$.

3 Quantifiers

Quantifiers are symbols that express the extent to which a predicate applies:

- Universal quantifier: $\forall$, meaning "for all"
- Existential quantifier: $\exists$, meaning "there exists"

4 Double Quantifiers

Statements can involve more than one quantifier:

$$\forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ such that } x + y = 0$$

This means for every real number x, there is a real number y such that $x+y = 0$.

5 Simple Proofs with Quantifiers

Proofs often involve manipulating quantifiers. For example, proving that $\forall x \in \mathbb{R}, x^2 \geq 0$.

6 Quantifiers and the Empty Set

If a statement involves the empty set, $\emptyset$, then:

$$\forall x \in \emptyset, P(x) \text{ is vacuously true.}$$

No element exists in the empty set to contradict $P(x)$.

7 Conditional Statements

A conditional statement has the form:

$$P \implies Q$$

This means "if P, then Q". It is false only when P is true and Q is false.

8 Negating Conditional Statements

The negation of a conditional statement $P \implies Q$ is:

$$\neg(P \implies Q) \equiv P \wedge \neg Q$$

This states that P is true and Q is false.

9 A Bad Proof

A proof is considered bad if it uses invalid logic. For example, assuming what is to be proven (circular reasoning).

10 How to Write a Rigorous Definition

A rigorous definition should clearly describe the conditions that define a concept. For example, a rigorous definition of a limit.

11 Proofs from Definitions

Some proofs follow directly from definitions. For example, proving the limit of a function by applying the epsilon-delta definition of a limit.

12 Proof by Induction

Proof by induction is used to prove statements about natural numbers. The steps are:

- Base case: Prove the statement for $n = 1$.
- Inductive step: Assume the statement holds for $n = k$, and prove it for $n = k + 1$.

13 One Theorem, Two Proofs

Some theorems can have multiple proofs using different techniques, such as direct proof and proof by contradiction.