How to Calculate Neck Flexion Angle between two IMUs

Part 1. Calibration

Aim: Calibrate pitch angle measured by IMU against that measured by EM tracker

Data Collection: We use laptop stand to do calibration as shown in figure1. Adjust the laptop to 9 different position, every position should keep unmoved and last for a while in order to eliminate the noise. Collect 9 different angles' quaternion from IMU-1A, IMU-A6 and EM tracker at the same time. The quaternion data of 0 degree is used as reference, so 8 different angles could be calculated based on this reference.

Because in our measurement, the total angle derived from quaternion is all come from the pitch component, so in order to calibrate the pitch angles we only need to calibrate the total angle.

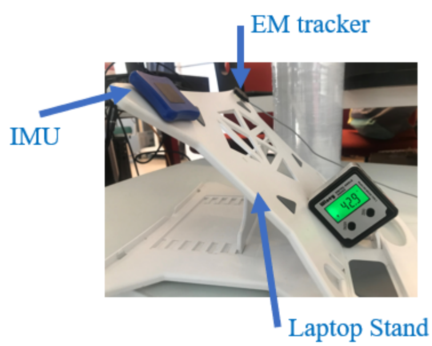

Figure1: The calibration environment setup.

Data Analysis (Angle Calculation):

Step 1: Load all measurement quaternion into MATLAB.

Step 2: Calculate the average quaternion for each IMU-1A, IMU-A6, EM tracker for each angle.

Step 3: Select the quaternion of 0 degree as reference. Then three averaged quaternions for each IMU-1A, IMU-A6, EM tracker at 0 degree are treated as reference quaternions.

Step 4: For each IMU-1A, IMU-A6, EM tracker, based on the following formula, calculate angles for 8 different averaged quaternions corresponding to their reference quaternions respectively. Then 8*3 = 24 angles are obtained.

Formula:

Assume $q1$ is the reference quaternion for one of the IMU-1A, IMU-A6, and EM tracker, and $q2$ is one of the 8 different averaged quaternions for the corresponding equipment, then the angle could be calculated as: (Note: every quaternion is [w, x, y, z])

$$Angle = 2 * atan\,2(||q_1^* q_2(x,y,z)||, q_1^* q_2(w)) \ * \frac{\pi}{180}(degree)$$

Take two quaternions measured from IMU-1A as an example.

Step 1. Load all measurement quaternion into MATLAB.

```
bag = rosbag('1AThird.bag');
bagselect = select(bag, 'Topic', '/imu_1A/imu');
msgStructs = readMessages(bagselect,'DataFormat','struct');
ts = timeseries(bagselect, 'Orientation.W', 'Orientation.X','Orientation.Y','Orientation.Z');
```

Step 2. Calculate the average quaternion for IMU-1A for each angle.

```
%% reference
reference = ts.Data(36620:41870,:);
eulZYX = quat2eul(reference);
quat = quaternion(eulZYX,'euler','ZYX','frame');
quatAverage = meanrot(quat)
%% next degree
first = ts.Data(34690:36270,:);
eulZYX_first = quat2eul(first);
quat_first = quaternion(eulZYX_first,'euler','ZYX','frame');
quatAverage_first = meanrot(quat_first)
```

Step 3. Select the first angle (0 degree) as reference

Step 4. Calculate angle between these two quaternions

```
%% angle
q1 =  [0.38673,  -0.0086113,-0.00060235,-0.92215];
q2 =  [0.40267,  0.0040498,-0.029676,-0.91485];
z = quatmultiply(quatconj(q1),q2)
a = 2* atan2(norm(z(2:4)),z(1))/pi*180
```

Linear regression:

8*1 angles from IMU_1A are treated as input x for the linear regression system, and 8*1 angles from EM tracker are treated as standard values y. The aim of the linear regression is to fit x to y.

By applying linear regression function in MATLAB to both IMU_1A and IMU_A6, two linear functions could be obtained separately.

Finally, for every pitch angle, a calibrated pitch angle could be obtained.

Part 2. Derive neck flexion angle

Firstly, we need to measure Quaternion and Euler Angles for both IMUs when surgeon is in reference position and in operating position. Here reference position means let the surgeon stands normally for around one minutes.

Step 1. Load measurement data

Load four CSV files which records two IMUs' Quaternions and Euler Angles when surgeon is in reference position and operating position respectively.

Step 2. Get reference position's Euler Angles for both IMUs with sequence XYZ

a. For both IMUs data, average reference position's Quaternions by using the *meanrot* function in MATLAB.

b. Convert the averaged Quaternions into Euler angles with sequence XYZ by using the -*quat2eul* function in MATLAB.

Step 3. Read other positions' Euler Angles from the loaded data for both IMUs with sequence XYZ

Step 4. Calibrate every pitch angle in all Euler Angles

For Euler angles obtained in Step2b and Step3, calibrate their pitch angle by using linear regression functions obtained before.

Step 5. Transfer all calibrated Euler Angles to Quaternions with sequence XYZ

Transfer all calibrated Euler angles obtained in Step4 to Quaternions by using *eul2quat* function in MATLAB with sequence XYZ.

Step 6. Transfer all Quaternions to Rotation Matrix

Convert all Quaternions obtained in Step5 to Rotation Matrix by using the following formula.

$$M = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_0q_2 + q_1q_3) \\ 2(q_0q_3 + q_1q_2) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_0q_1 + q_2q_3) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

The matrix can be simplified as followed since the norm of a rotation quaternion is 1:

$$|q| = q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1,$$

$$M = 2 \cdot \begin{bmatrix} q_0^2 + q_1^2 - 0.5 & q_1q_2 - q_0q_3 & q_0q_2 + q_1q_3 \\ q_0q_3 + q_1q_2 & q_0^2 + q_2^2 - 0.5 & q_2q_3 - q_0q_1 \\ q_1q_3 - q_0q_2 & q_0q_1 + q_2q_3 & q_0^2 + q_3^2 - 0.5 \end{bmatrix}$$

So far, we have four Rotation Matrices:

R_{A6o}, R_{1Ao}: stands for the Rotation Matrix for IMU_A6 and IMU_1A during operating;

R_{A6r}, R_{1Ar}: stands for the Rotation Matrix for IMU_A6 and IMU_1A during standing normally.

Step 7. Calculate Rotation Matrix for both IMU_1A and IMU_A6 w.r.t their reference: $\boldsymbol{R_{A6}} = \boldsymbol{R_{A6o}^{T} R_{A6r}}$, $\boldsymbol{R_{1\mathrm{A}}} = \boldsymbol{R_{1\mathrm{A}o}^{T} R_{1Ar}}$

In order to find how much every IMU rotates during operating against standing normally, we need to calculate Rotation Matrix R_{A6} and R_{1A} for IMU_A6 and IMU_1A with respect to their reference separately.

Step 8. Calculate Difference Rotation Matrix between two Rotation Matrix calculated from Step 7: $\boldsymbol{D} = \boldsymbol{R_{A6}^{T} R_{A1}}$.

In order to find the neck flexion angle, we need to calculate Rotation Matrix D.

Step 9. Transfer Difference Rotation Matrix to Euler Angle

Finally, the neck flexion angle (Pitch ϕ) could be calculated by using the following formula.

$$If\ Rotation\ Matrix\ D = \begin{bmatrix} r_{00} & r_{01} & r_{02} \\ r_{10} & r_{11} & r_{12} \\ r_{20} & r_{21} & r_{22} \end{bmatrix}, then\ Euler\ Angles \begin{bmatrix} Pitch\ \phi \\ Yaw\ \theta \\ Roll\ \psi \end{bmatrix} = \begin{bmatrix} atan2(r_{21}, r_{22}) \\ \operatorname{asin}(-r_{20}) \\ atan2(r_{10}, r_{00}) \end{bmatrix}$$