DOI 10.1007/s10778-017-0808-4
International Applied Mechanics, Vol. 53, No. 2, March, 2017

CALIBRATION OF AN INERTIAL MEASUREMENT UNIT

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A new method for the calibration of inertial measurement units of strapdown inertial technology is proposed. Such a unit consists of accelerometers, gyroscopes, and a signal processing system. The method of test turns and rotations on a rotary table is used to calibrate the inertial measurement unit. The new method involves measurement of the full angle of turn or final rotation. In fact, it is proposed to turn the inertial measurement unit around the axis of final rotation. To solve the calibration equation, it is necessary to make the rank of the matrix of the calibration equation equal to its order. The results of modeling data demonstrate the efficiency of the new calibration method

Keywords: inertial measurement unit, calibration, accelerometer, gyroscope

Introduction. An inertial measurement unit (IMU) is the core of inertial positioning and navigation systems. Each IMU consists of at least three accelerometers and three gyroscopes (angular-rate sensors) [5].

Before IMUs are assembled, gyroscopes and accelerometers are subject to independent tests [15, 20, 22]. However, it is important to determine their parameters when they are components of IMUs because the output parameters of gyroscopes and accelerometers are fixed to the reference axes of the IMU. Therefore, calibration is an obligatory stage in preparing IMUs for operation or aligning inertial systems [16]. By calibration is meant determination of the parameters or errors of an IMU for further use in the inertial system.

IMUs are usually calibrated by the method of test turns [6, 8, 15, 16, 20]. When calibrating accelerometers as components of IMUs, an optical dividing head (ODH) is used, which allows precise turns about the horizontal spin axis. To calibrate an accelerometer unit by the method of test turns, it is necessary to measure output signals by first turning the unit about the *ox*-axis, then, after readjusting it on the ODH, about the *oy*-axis, and, finally, about the *oz*-axis [6]. In practice, this orientation of the unit is a special case. Actually, a turn occurs through some finite angle that results from turns about two or three orthogonal axes.

To calibrate gyroscopes as components of IMUs, a rotary table is usually used. The standard method for the calibration of a gyroscope unit [6] involves successive rotations about one axis, say *ox* with angular rate ω_ξ, then about the *oy*-axis with angular rate ω_{yi}, and, finally, about the *oz*-axis with angular rate ω_{zi} (i is the test number). It should be noted that this method takes a good deal of time and, in actual practice, rotation can occur around several axes or around the axis of finite turn.

To this point, we have addressed the calibration of the determinate parameters of IMUs. However, there are also stochastic components such as noise of various nature at the output of IMUs.

There are several methods to evaluate the influence and balancing of noise at the output of sensors. The most popular method is the use of Kalman filter [13, 14, 17, 18]. Also, widely used is the Allan variance method [7] or, sometimes, the wavelet transform method [3, 19]. Fuzzy logic algorithms [12] and artificial neural networks [11, 21] are used even rarely.

In addition to the method of test turns, there is the scalar calibration method [1, 4, 9, 10] which assumes rotation around the vector of finite turn. In the scalar calibration method, a scalar quantity rather than a vector is used as a reference. In the Earth's

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gravity field, such scalar quantity is the acceleration of gravity g for accelerometers and the rotation rate Ω of the Earth or ω of the rotary table for gyroscopes. However, this method has its advantages and disadvantages.

Let us describe a new calibration method in which rotation (turn) occurs around an arbitrary axis that coincides with none of the coordinate axes.

1. Calibration of an Axial-Accelerometer Unit as a Component of an IMU. The output signals of an axial-accelerometer unit are expressed as follows [7]:

$$\begin{bmatrix} U_{ax} \\ U_{ay} \\ U_{az} \end{bmatrix} = \begin{bmatrix} B_{ax} \\ B_{ay} \\ B_{az} \end{bmatrix} + \mathbf{M}_{1a} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} n_{ax} \\ n_{ay} \\ n_{az} \end{bmatrix}, \tag{1.1}$$

where B_{ax}, B_{ay}, B_{az} are the zero signals of the accelerometers; a_x, a_y, a_z are the projections of apparent acceleration; n_{ax}, n_{ay}, n_{az} are the noise in the output signals of the accelerometers; $Oxyz$ is the frame of reference fixed to the accelerometer unit; $\mathbf{M}_{1a} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$, k_{11}, k_{22}, k_{33} are the scale factors of the accelerometers; all the other elements of the matrix are the cross-coupling coefficients.

To reduce the effect of noise, we will average the output signals over 30–60 sec. Therefore, we will neglect n_{ax}, n_{ay}, n_{az}. Let us write Eq. (1.1) for each accelerometer and for the ith position of IMU (test):

$$U_{axi} = B_{ax} + k_{11} a_{\xi} + k_{12} a_{yi} + k_{13} a_{zi},$$

$$U_{ayi} = B_{ay} + k_{21} a_{\xi} + k_{22} a_{yi} + k_{23} a_{zi},$$

$$U_{azi} = B_{az} + k_{31} a_{\xi} + k_{32} a_{yi} + k_{33} a_{zi}.$$

Let us perform a series of n tests (measurements) at different angles α, β, γ of turn around three axes.

Consider the output signal of the first accelerometer:

1st measurement: $U_{ax1} = B_{ax} + k_{11} a_{x1} + k_{12} a_{y1} + k_{13} a_{z1}$;

2nd measurement: $U_{ax2} = B_{ax} + k_{11} a_{x2} + k_{12} a_{y2} + k_{13} a_{z2}$;

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nth measurement: $U_{axn} = B_{ax} + k_{11} a_{xn} + k_{12} a_{yn} + k_{13} a_{zn}$.

The system of equations has the following matrix form:

$$\begin{bmatrix} U_{ax1} \\ U_{ax2} \\ \vdots \\ U_{axn} \end{bmatrix} = \begin{bmatrix} 1 & a_{x1} & a_{y1} & a_{z1} \\ 1 & a_{x2} & a_{y2} & a_{z2} \\ . & . & . & . \\ 1 & a_{xn} & a_{yn} & a_{zn} \end{bmatrix} \begin{bmatrix} B_{ax} \\ k_{11} \\ k_{12} \\ k_{13} \end{bmatrix}.$$

Similar equations can be derived for the other two other accelerometers.

Let us combine the matrix equations into one calibration equation:

$$\mathbf{U}_{a1} = \mathbf{G}_{n\times 4} \cdot \mathbf{X}_1, \tag{1.2}$$

where

$$\mathbf{U}_{a1} = \begin{bmatrix} U_{ax1} & U_{ay1} & U_{az1} \\ U_{ax2} & U_{ay2} & U_{az2} \\ . & . & . \\ U_{axn} & U_{ayn} & U_{azn} \end{bmatrix}, \quad \mathbf{G}_{n\times 4} = \begin{bmatrix} 1 & a_{x1} & a_{y1} & a_{z1} \\ 1 & a_{x2} & a_{y2} & a_{z2} \\ . & . & . & . \\ 1 & a_{xn} & a_{yn} & a_{zn} \end{bmatrix}, \quad \mathbf{X}_1 = \begin{bmatrix} B_{ax} & B_{ay} & B_{ax} \\ k_{11} & k_{21} & k_{31} \\ k_{12} & k_{22} & k_{32} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}.$$

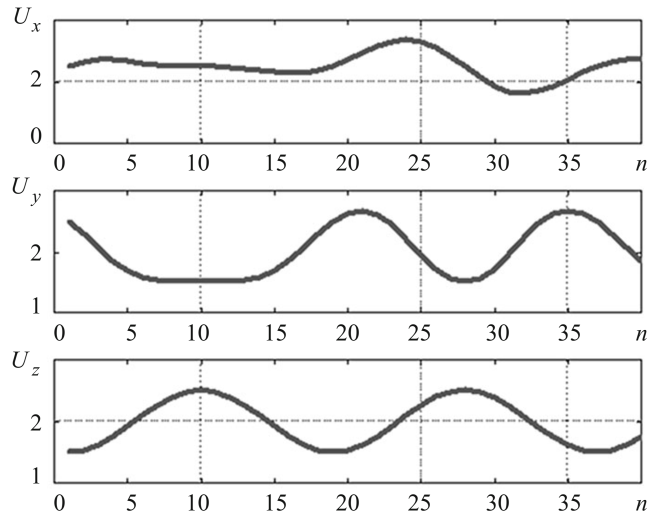

Fig. 1

This matrix equation is solved by the least-squares method:

$$\hat{\mathbf{X}}_1 = (\mathbf{G}_{n\times 4}^T \mathbf{G}_{n\times 4})^{-1} \mathbf{G}_{n\times 4}^T \mathbf{U}_{a1}, \tag{1.3}$$

where the superscript "T" denotes transposition.

Example 1. Consider an accelerometer unit with the following ratings:

$$B_{ax} = B_{ay} = B_{az} = 2.5 \text{ V}, \quad k_{11} = k_{22} = k_{33} = 1.0 \text{ V/g},$$

$$k_{12} = 0.01 \text{ V/g}, \quad k_{13} = -0.01 \text{ V/g}, \quad k_{21} = -0.01 \text{ V/g},$$

$$k_{23} = 0.01 \text{ V/g}, \quad k_{31} = 0.01 \text{ V/g}, \quad k_{32} = -0.01 \text{ V/g}.$$

The angles α, β, γ are varied from 0 to 400° at an increment of 10° (n = 40 positions total).

Figure 1 shows the calculated output signals of accelerometers (1.2) in volts for the ($n \times 4$)-matrix $\mathbf{G}_{n\times 4}$. Using formula (1.3), we obtain

$$\hat{\mathbf{X}}_1 = \begin{bmatrix} 2.5 & 2.5 & 2.5 \\ 1.0 & -0.01 & 0.009999 \\ 0.01 & 1.0 & -0.009999 \\ -0.009999 & 0.01 & 1.0 \end{bmatrix}.$$

Thus, using (1.2) and (1.3), we can estimate the drifts of the zero $\hat{\mathbf{B}}_a$, scale factors, and crosscoupling coefficients (elements of the matrix $\mathbf{M}_{1a}$).

Let us show that the method can also be applied to a pendulum-accelerometer unit.

2. Calibration of a Pendulum-Accelerometer Unit as a Component of an IMU. The output signals of a pendulum-accelerometer unit on a fixed platform in the gravity field are expressed as follows [20]:

$$\begin{bmatrix} U_{ax} \\ U_{ay} \\ U_{az} \end{bmatrix} = \begin{bmatrix} B_{ax} \\ B_{ay} \\ B_{az} \end{bmatrix} + \mathbf{M}_{1a} \cdot \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} + \mathbf{M}_{2a} \cdot \begin{bmatrix} g_x g_y \\ g_y g_z \\ g_x g_z \end{bmatrix} + \begin{bmatrix} n_{ax} \\ n_{ay} \\ n_{az} \end{bmatrix}, \tag{2.1}$$

where

$$\mathbf{M}_{1a} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}, \qquad \mathbf{M}_{2a} = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix}.$$

After performing n tests (measurements) and averaging the output signals over 30–60 sec, we transform the matrix calibration equation (2.1) to the form

$$\mathbf{U}_{a2} = \mathbf{G}_{n\times 7} \cdot \mathbf{X}_2, \tag{2.2}$$

where

$$\mathbf{U}_{a2} = \begin{bmatrix} U_{ax1} & U_{ay1} & U_{az1} \\ U_{ax2} & U_{ay2} & U_{az2} \\ . & . & . \\ U_{axn} & U_{ayn} & U_{azn} \end{bmatrix}, \qquad \mathbf{X}_2 = \begin{bmatrix} B_{ax} & k_{11} & k_{12} & k_{13} & l_{11} & l_{12} & l_{13} \\ B_{ay} & k_{21} & k_{22} & k_{23} & l_{21} & l_{22} & l_{23} \\ B_{az} & k_{31} & k_{32} & k_{33} & l_{31} & l_{32} & l_{33} \end{bmatrix}^T,$$

$$\mathbf{G}_{n\times 7} = \begin{bmatrix} 1 & g_{x1} & g_{y1} & g_{z1} & g_{x1}g_{y1} & g_{y1}g_{z1} & g_{x1}g_{z1} \\ 1 & g_{x2} & g_{y2} & g_{z2} & g_{x2}g_{y2} & g_{y2}g_{z2} & g_{x2}g_{z2} \\ . & . & . & . & . & . & . \\ 1 & g_{xn} & g_{yn} & g_{zn} & g_{xn}g_{yn} & g_{yn}g_{zn} & g_{xn}g_{zn} \end{bmatrix}.$$

This matrix equation (2.2) is solved by the least-squares method:

$$\hat{\mathbf{X}}_2 = (\mathbf{G}_{n\times 7}^T \mathbf{G}_{n\times 7})^{-1} \mathbf{G}_{n\times 7}^T \mathbf{U}_{a2}. \tag{2.3}$$

Example 2. Consider a pendulum-accelerometer unit with the following ratings:

$$B_{ax} = B_{ay} = B_{az} = 2.5\ \text{V},$$

$$k_{11} = 1.0\ \text{B/g}, \quad k_{12} = 0.01\ \text{B/g}, \quad k_{13} = -0.01\ \text{B/g},$$

$$k_{21} = -0.01\ \text{B/g}, \quad k_{22} = 1.0\ \text{B/g}, \quad k_{23} = 0.01\ \text{B/g},$$

$$k_{31} = 0.01\ \text{B/g}, \quad k_{32} = -0.01\ \text{B/g}, \quad k_{33} = 1.0\ \text{B/g},$$

$$l_{11} = -0.001\ \text{B/g}^2, \quad l_{12} = 0.001\ \text{B/g}^2, \quad l_{13} = 0.001\ \text{B/g}^2,$$

$$l_{21} = 0.001\ \text{B/g}^2, \quad l_{22} = -0.001\ \text{B/g}^2, \quad l_{23} = 0.001\ \text{B/g}^2,$$

$$l_{31} = 0.001\ \text{B/g}^2, \quad l_{32} = 0.001\ \text{B/g}^2, \quad l_{33} = -0.001\ \text{B/g}^2.$$

The angles α, β, γ are varied from 0 to 400° at an increment of 10° ($n = 40$ positions total).

Figure 2 shows the calculated output signals of accelerometers (2.2) in volts for the ($n\times 7$)-matrix $\mathbf{G}_{n\times 7}$. Using formula (2.3), we get

$$\hat{\mathbf{X}}_2 = \begin{bmatrix} 2.5 & 2.5 & 2.5 \\ 1 & -0.01 & 0.01 \\ 0.01 & 1 & -0.01 \\ -0.01 & 0.01 & 1 \\ -0.001 & 0.001 & 0.001 \\ 0.001 & -0.001 & 0.001 \\ 0.001 & 0.001 & -0.001 \end{bmatrix}.$$

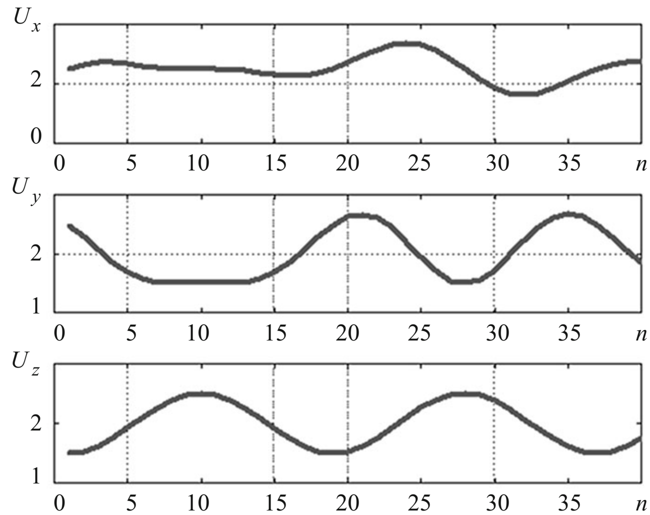

Fig. 2

Thus, using (2.2) and (2.3), we can estimate the drifts of the zero $\hat{\mathbf{B}}_a$ and coefficients of the matrices $\mathbf{M}_{1a}$ and $\mathbf{M}_{2a}$.

3. Calibration of a Gyroscope Unit. The output signals of a gyroscope unit are expressed as follows [7]:

$$\begin{bmatrix} U_{\omega x} \\ U_{\omega y} \\ U_{\omega z} \end{bmatrix} = \begin{bmatrix} B^*_{\omega x} \\ B^*_{\omega y} \\ B^*_{\omega z} \end{bmatrix} + M_{1\omega} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \begin{bmatrix} n_{\omega x} \\ n_{\omega y} \\ n_{\omega z} \end{bmatrix}, \tag{3.1}$$

where $B^*_{\omega x}, B^*_{\omega y}, B^*_{\omega z}$ are the zero signals of the gyroscopes that can contain drifts depending on g and g^2; ω_x, ω_y, ω_z are the projections of the angular rate of the IMU onto the axes of $oxyz$; $n_{\omega x}, n_{\omega y}, n_{\omega z}$ are the noise in the output signals of the gyroscopes; $\mathbf{M}_{1\omega}$ is a (3×3)-matrix,

$$\mathbf{M}_{1\omega} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}.$$

Let us consider a calibration method for a gyroscope unit in which rotation occurs around several axes simultaneously. Performing n tests (measurements) at angular rates ω_ξ, ω_{yi}, ω_{zi} and averaging the output signals over 30–60 sec, we obtain the calibration equation

$$\mathbf{U}_{\omega 3} = \boldsymbol{\omega}_{n\times 4} \cdot \mathbf{X}_3, \tag{3.2}$$

where

$$\mathbf{U}_{\omega 3} = \begin{bmatrix} U_{\omega x1} & U_{\omega y1} & U_{\omega z1} \\ U_{\omega x2} & U_{\omega y2} & U_{\omega z2} \\ . & . & . \\ U_{\omega xn} & U_{\omega yn} & U_{\omega zn} \end{bmatrix}, \quad \boldsymbol{\omega}_{n\times 4} = \begin{bmatrix} 1 & \omega_{x1} & \omega_{y1} & \omega_{z1} \\ 1 & \omega_{x2} & \omega_{y2} & \omega_{z2} \\ . & . & . & . \\ 1 & \omega_{xn} & \omega_{yn} & \omega_{zn} \end{bmatrix}, \quad \mathbf{X}_3 = \begin{bmatrix} B^*_{\omega x} & B^*_{\omega y} & B^*_{\omega z} \\ n_{11} & n_{21} & n_{31} \\ n_{12} & n_{22} & n_{32} \\ n_{13} & n_{23} & n_{33} \end{bmatrix}.$$

Equation (3.2) seems to be very similar to Eq. (1.2). The difference is in that the matrix $\boldsymbol{\omega}_{n\times 4}$ consists of the projections of angular rates, while the matrix $\mathbf{G}_{n\times 4}$ consists of the projections of apparent acceleration. While we used a sequence of turns of the platform without any constraints to calibrate the accelerometer unit, the projections of angular rates should be set so that rank $\boldsymbol{\omega}_{n\times 4} = 4$.

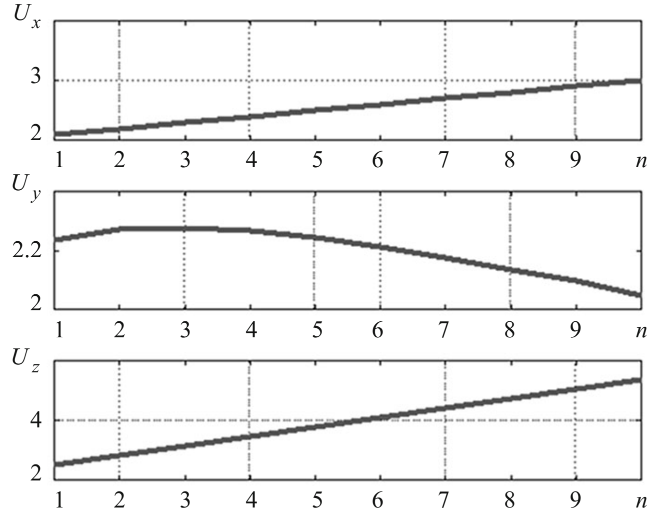

Fig. 3

This condition can be obtained for nonlinear dependence of the last three columns of the matrix $\boldsymbol{\omega}_{n\times 4}$. Indeed, according to the Kronecker–Capelli theorem [2], if the rank of the matrix $\boldsymbol{\omega}_{n\times 4}$ is equal to the rank of the augmented matrix consisting of the matrix $\boldsymbol{\omega}_{n\times 4}$ and the matrix of the right-hand sides $\mathbf{U}_{\omega 3}$,

$$\operatorname{rank} \boldsymbol{\omega}_{n\times 4} = \operatorname{rank} [\boldsymbol{\omega}_{n\times 4}, \mathbf{U}_{\omega 3}],$$

then the system of equations has a solution.

It is established by check that if the dependence of the last three columns of the matrix $\boldsymbol{\omega}_{n\times 4}$ is nonlinear, then rank $\boldsymbol{\omega}_{n\times 4} = 4$ and rank $[\boldsymbol{\omega}_{n\times 4}, \mathbf{U}_{\omega 3}] = 4$.

Thus, for the calibration equation to have a solution, it is necessary that the rank of the matrix of the calibration equation be equal to its order or number of columns.

This matrix equation is solved by the least-squares method:

$$\hat{\mathbf{X}}_3 = (\boldsymbol{\omega}_{n\times 4}^T \boldsymbol{\omega}_{n\times 4})^{-1} \boldsymbol{\omega}_{n\times 4}^T \mathbf{U}_{\omega 3}. \tag{3.3}$$

Example 3. Consider a gyroscope unit with the following ratings:

$$B_{\omega x}^* = B_{\omega y}^* = B_{\omega z}^* = 2.0 \text{ B}, \quad n_{11} = n_{22} = n_{33} = 1.0 \text{ B}/(d/s),$$

$$n_{12} = 0.01 \text{ B}/(d/s), \quad n_{13} = -0.01 \text{ B}/(d/s), \quad n_{21} = -0.01 \text{ B}/(d/s),$$

$$n_{23} = 0.01 \text{ B}/(d/s), \quad n_{31} = 0.01 \text{ B}/(d/s), \quad n_{32} = -0.01 \text{ B}/(d/s).$$

To make the rank of the matrix $\boldsymbol{\omega}_{n\times 4}$ equal to its order, we set the projections of angular rates of the table as

$$\omega_\xi = \omega_i, \quad \omega_{yi} = \omega_i^{1/2}, \quad \omega_{zi} = \omega_i^{1/3}.$$

The angular rate ω_i is varied from 0 to 100 °/sec at an increment of 10 °/sec (n = 10 values total).

Figure 3 shows the calculated output signals of the gyroscope unit. Solving the calibration equation according to (3.3), we obtain

$$\hat{\mathbf{X}}_3 = \begin{bmatrix} 2.0 & 2.0 & 2.0 \\ 1.0 & -0.01 & 0.03 \\ 0.01 & 1.0 & -0.01 \\ -0.02 & 0.01 & 1.0 \end{bmatrix}.$$

Thus, this calibration method allows us to determine the zero signals of the gyros and the elements of the matrix $\mathbf{M}_{1\omega}$.

4. Calibration of a Gyroscope Unit (Expanded Model). The output signals of a gyroscope unit can be represented in the following form [20]:

$$\begin{bmatrix} U_{\omega x} \\ U_{\omega y} \\ U_{\omega z} \end{bmatrix} = \begin{bmatrix} B^*_{\omega x} \\ B^*_{\omega y} \\ B^*_{\omega z} \end{bmatrix} + \mathbf{M}_{1\omega} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \mathbf{M}_{2\omega} \cdot \begin{bmatrix} \omega_x \omega_y \\ \omega_y \omega_z \\ \omega_x \omega_z \end{bmatrix} + \begin{bmatrix} n_{\omega x} \\ n_{\omega y} \\ n_{\omega z} \end{bmatrix}, \tag{4.1}$$

where $\mathbf{M}_{1\omega}$ and $\mathbf{M}_{2\omega}$ are matrices such that

$$\mathbf{M}_{1\omega} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}, \quad \mathbf{M}_{2\omega} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}.$$

Let us again consider a calibration method for a gyroscope unit in which rotation occurs around several axes simultaneously.

Performing n tests (measurements) at angular rates ω_{ξ}, ω_{yi}, ω_{zi} and averaging the output signals over 30–60 sec, we obtain the calibration equation

$$\mathbf{U}_{\omega 4} = \boldsymbol{\omega}_{n\times 7} \cdot \mathbf{X}_4, \tag{4.2}$$

where

$$\mathbf{U}_{\omega 4} = \begin{bmatrix} U_{\omega x1} & U_{\omega y1} & U_{\omega z1} \\ U_{\omega x2} & U_{\omega y2} & U_{\omega z2} \\ . & . & . \\ U_{\omega xn} & U_{\omega yn} & U_{\omega zn} \end{bmatrix}, \quad \mathbf{X}_4 = \begin{bmatrix} \mathbf{B}^T_{\omega} \\ \mathbf{M}^T_{1\omega} \\ \mathbf{M}^T_{2\omega} \end{bmatrix}, \quad \mathbf{B}^T_{\omega} = \left[B^*_{\omega x} \quad B^*_{\omega y} \quad B^*_{\omega z} \right],$$

$$\boldsymbol{\omega}_{n\times 7} = \begin{bmatrix} 1 & \omega_{x1} & \omega_{y1} & \omega_{z1} & \omega_{x1}\omega_{y1} & \omega_{y1}\omega_{z1} & \omega_{x1}\omega_{z1} \\ 1 & \omega_{x2} & \omega_{y2} & \omega_{z2} & \omega_{x2}\omega_{y2} & \omega_{y2}\omega_{z2} & \omega_{x2}\omega_{z2} \\ . & . & . & . & . & . & . \\ 1 & \omega_{xn} & \omega_{yn} & \omega_{zn} & \omega_{xn}\omega_{yn} & \omega_{yn}\omega_{zn} & \omega_{xn}\omega_{zn} \end{bmatrix}.$$

This matrix equation (4.2) is solved by the least-squares method:

$$\hat{\mathbf{X}}_4 = (\boldsymbol{\omega}^T_{n\times 7} \boldsymbol{\omega}_{n\times 7})^{-1} \boldsymbol{\omega}^T_{n\times 7} \mathbf{U}_{\omega 4}. \tag{4.3}$$

Example 4. Consider a gyroscope unit with the following ratings:

$$B^*_{\omega x} = B^*_{\omega y} = B^*_{\omega z} = 2.0 \text{ B}, \quad n_{11} = n_{22} = n_{33} = 1.0 \text{ B}/(d/s),$$

$$n_{12} = 0.01 \text{ B}/(d/s), \quad n_{13} = -0.01 \text{ B}/(d/s), \quad n_{21} = -0.01 \text{ B}/(d/s),$$

$$n_{23} = 0.01 \text{ B}/(d/s), \quad n_{31} = 0.01 \text{ B}/(d/s), \quad n_{32} = -0.01 \text{ B}/(d/s),$$

$$p_{11} = -0.001 \text{ B}/(d/s)^2, \quad p_{12} = 0.001 \text{ B}/(d/s)^2, \quad p_{13} = 0.001 \text{ B}/(d/s)^2,$$

$$p_{21} = 0.001 \text{ B}/(d/s)^2, \quad p_{22} = -0.001 \text{ B}/(d/s)^2, \quad p_{23} = 0.001 \text{ B}/(d/s)^2,$$

$$p_{31} = 0.001 \text{ B}/(d/s)^2, \quad p_{32} = 0.001 \text{ B}/(d/s)^2, \quad p_{13} = -0.001 \text{ B}/(d/s)^2.$$

In order that rank $\boldsymbol{\omega}_{n\times 7} = 7$, we set the projections of angular rates of the table as

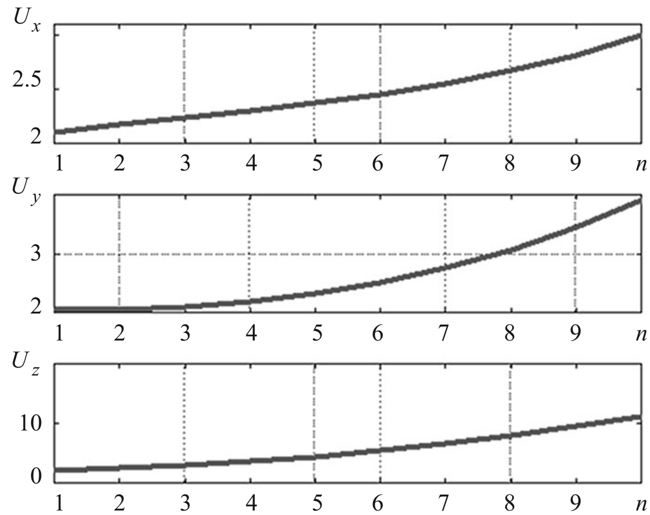

Fig. 4

$$\omega_\xi = \omega_i, \quad \omega_{yi} = e^{-\omega_i}, \quad \omega_{zi} = \omega_i^{1/3}.$$

The angular rate ω_i is varied from 0 to 100 °/sec at an increment of 10 °/sec (n = 10 values total).

Figure 4 shows the calculated output signals of the gyroscope unit. This matrix equation (4.3) is solved by the least-squares method:

$$\hat{\mathbf{X}}_4 = \begin{bmatrix} 2 & 2 & 2 \\ 0.01 & -0.01 & 0.03 \\ 0.01 & 0.1 & -0.01 \\ -0.02 & 0.01 & 0.1 \\ -0.001 & 0.001 & 0.001 \\ 0.001 & 0.001 & -0.001 \\ 0.001 & -0.001 & 0.001 \end{bmatrix}.$$

Thus, this calibration method allows us to determine the zero signals of the gyros and the elements of the matrices $\mathbf{M}_{1\omega}$ and $\mathbf{M}_{2\omega}$.

Conclusions.

1. A calibration method for an accelerometer and gyroscope units that involves a turn of the IMU through a finite angle or rotation of the IMU around the vector of finite turn has been proposed.

2. To solve the calibration equation, it is necessary to make the rank of the matrix of the calibration equation equal to its order or number of columns.

3. Mathematical simulation has validated the method.

REFERENCES

1. V. V. Avrutov, "Scalar calibration of gyroscope and accelerometer units," *Visnyk NTUU "KPI," Ser. Pryladobuduv.*, 40, 10–17 (2010).
2. Ya. S. Bugrov and S. M. Nikol'skii, *Higher Mathematics. Elements of Linear Algebra and Analytic Geometry* [in Russian], Nauka, Moscow (1984).
3. S. V. Golovach, "Experimental study of the characteristics of a laser gyroscope," *Visnyk NTUU "KPI," Ser. Pryladobuduv.*, 40, 33–38 (2014).

4. E. A. Izmailov, S. N. Lepe, A. V. Molchanov, and E. F. Polikovskii, "Scalar calibration and balancing method for strapdown inertial navigation systems," in: *Proc. 15th Int. Conf. on Integrated Navigation Systems* [in Russian], GNTs RF TsNII Elektropribor, Saint Petersburg (2008), pp. 145–154.
5. V. B. Larin and A. A. Tunik, "On inertial-navigation system without angular-rate sensors," *Int. Appl. Mech.*, **49**, No. 4, 488–499 (2013).
6. V. V. Meleshko and O. I. Nesterenko, *Strapdown Inertial Navigation Systems* [in Russian], Polimed-Servis, Kirovograd (2011).
7. O. A. Stepanov, I. B. Chelpanov, and A. V. Motorin, "Accuracy of estimating the constant component of the error of sensors and its relation to the Allan variance," in: *Proc. 22nd Int. Conf. on Integrated Navigation Systems* [in Russian], GNTs RF TsNII Elektropribor, Saint Petersburg (2015), pp. 485–491.
8. G. Artese and A. Trecroci, "Calibration of a low cost MEMS INS sensor for an integrated navigation system," in: *The Int. Archives of the Photogrammetry*, *Remote Sensing and Spatial Information Sciences*, XXXVII, Part B5, Beijing (2008), pp. 877–882.
9. V. Avrutov, S. Golovach, and T. Mazepa, "On scalar calibration of an inertial measurement unit," in: *Proc. 19th St. Petersburg Int. Conf. on Integrated Navigation Systems*, State Research Center (CSRI) Elektropribor, St. Petersburg, Russia (2012), pp. 117–121.
10. V. Avrutov, "Scalar diagnostics of the inertial measurement unit," *I. J. Intelligent Systems and Applications*, **11**, 1–9 (2015).
11. A. El-Rabbany and M. El-Diasty, "An efficient neural modal for denoising of MEMS-based inertial data," *The J. of Navigation*, **57**, 407–415 (2004).
12. J. Gaysse, "A low cost absolute position calculation system," in: *Proc. SICE-ICASE Int. Joint Conf.*, Korea, Busan (2006), pp. 5658–5661.
13. M. S. Grewal, V. D. Henderson, and R. S. Miysako, "Application of Kalman filtering to the calibration and alignment of inertial navigation systems," *IEEE Trans. on Automatic Control*, **36**, 3–13 (1991).
14. C. Hide, T. Moore, and M. Smith, "Adaptive Kalman filtering for low-cost INS/GPS," *The J. of Navigation*, **56**, 143–152 (2003).
15. A. Lawrence, *Modern Inertial Technology. Navigation*, *Guidance and Control*, Springer-Verlag, New York (1993).
16. E. Nebot and H. Durrant-Whyte, "Initial calibration and alignment of low cost inertial navigation units for land vehicle applications," *J. of Robotics Systems*, **16**, No. 2, 81–92 (1999).
17. N. Nikbakht, M. Mazlom, and A. Khayatian, "Evaluation of solid-state accelerometer for positioning of vehicle," in: *Proc. IEEE Int. Conf. on Industrial Technology*, Hong Kong (2005), pp. 729–733.
18. G. Pang and H. Liu, "Evaluation of a low-cost MEMS accelerometer for distance measurement," *The J. of Intelligent and Robotic Systems*, **30**, 249–265 (2001).
19. S. C. Shen, C. J. Chen, and H. J. Huang, "A new calibration method for low cost MEMS inertial sensor module," *J. of Marine Science and Technology*, **18**, No. 6, 819–824 (2010).
20. D. H. Titterton and J. L. Weston, "Strapdown inertial navigation technology," *IEE Radar*, *Sonar*, *Navigation and Avionics*, Ser. 17, 558 (2004).
21. H. Wang and W. Tian, "Modeling the random drift of micro-machined gyroscope with neural network," *Neural Processing Letters*, **22**, 235–247 (2005).
22. W. Wrigley, W. Hollister, and W. Denhard, *Gyroscopic Theory*, *Design and Instrumentation*, MIT Press, Cambridge (1969).