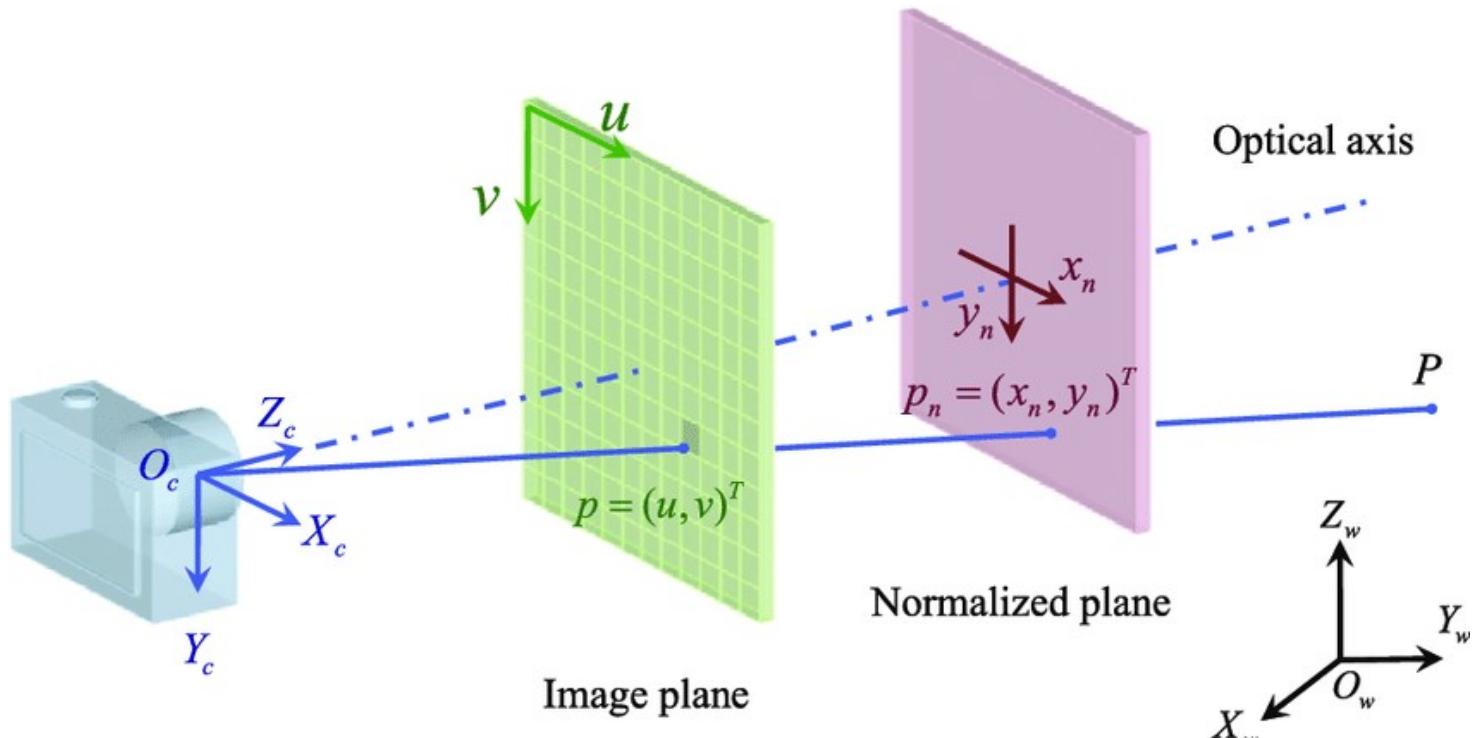
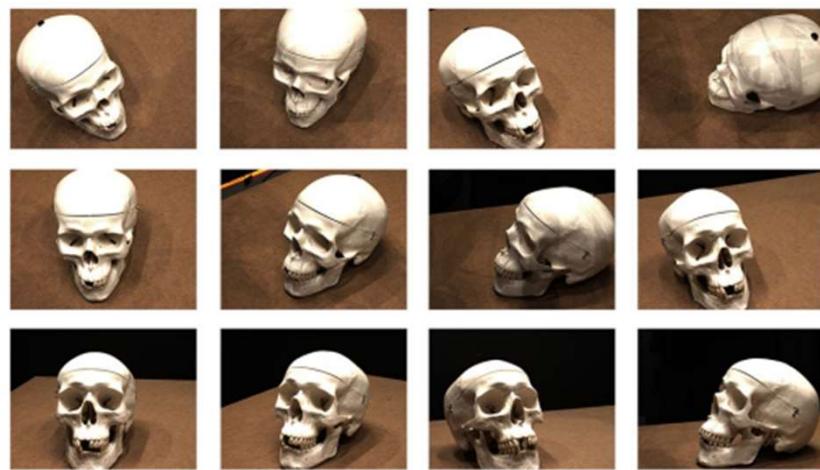


三维视觉中的相机模型

Camera Basics for 3D Vision



3D Vision: 3D & 2D Conversion



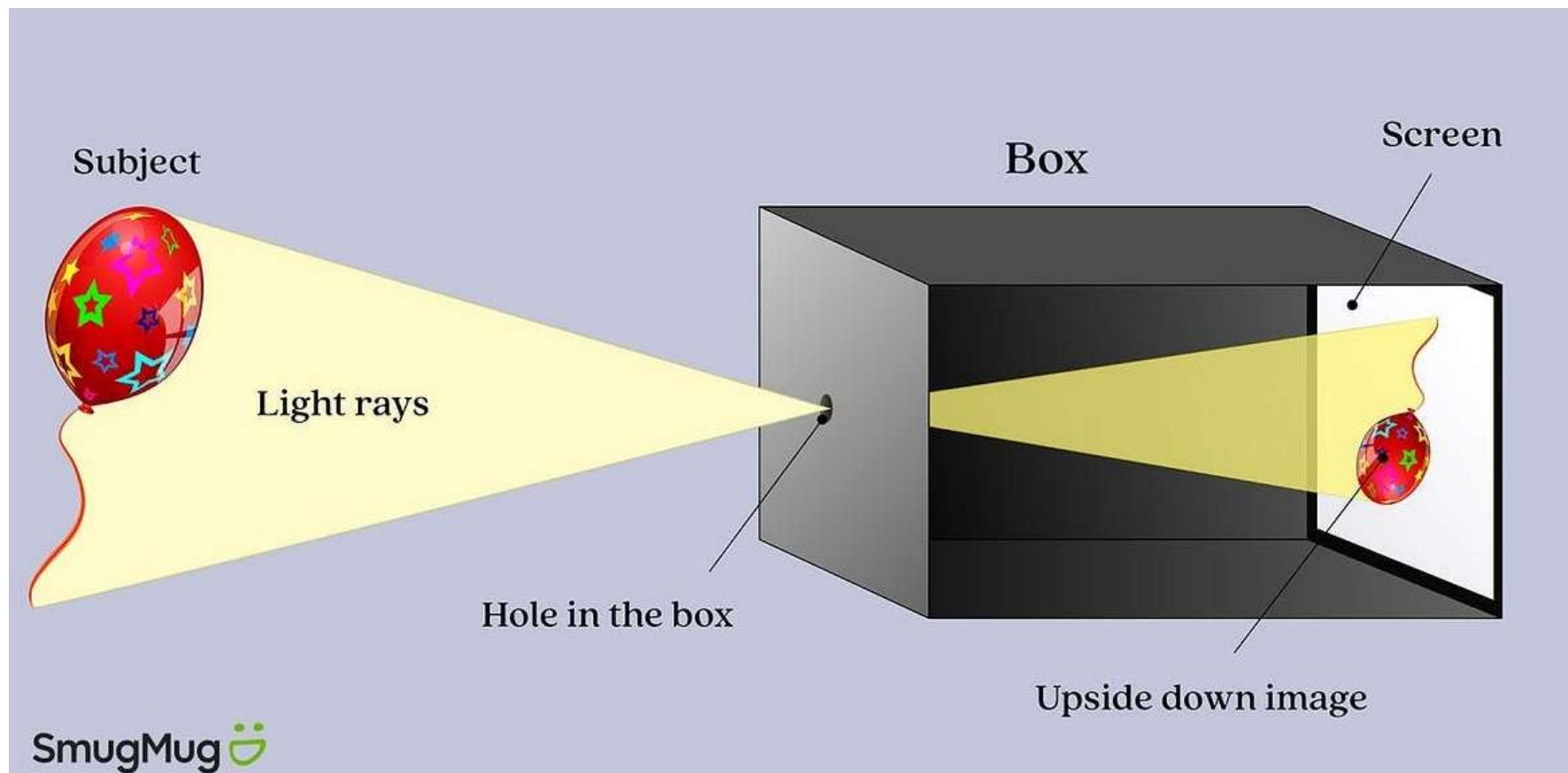
Rendering
↔
Reconstruction



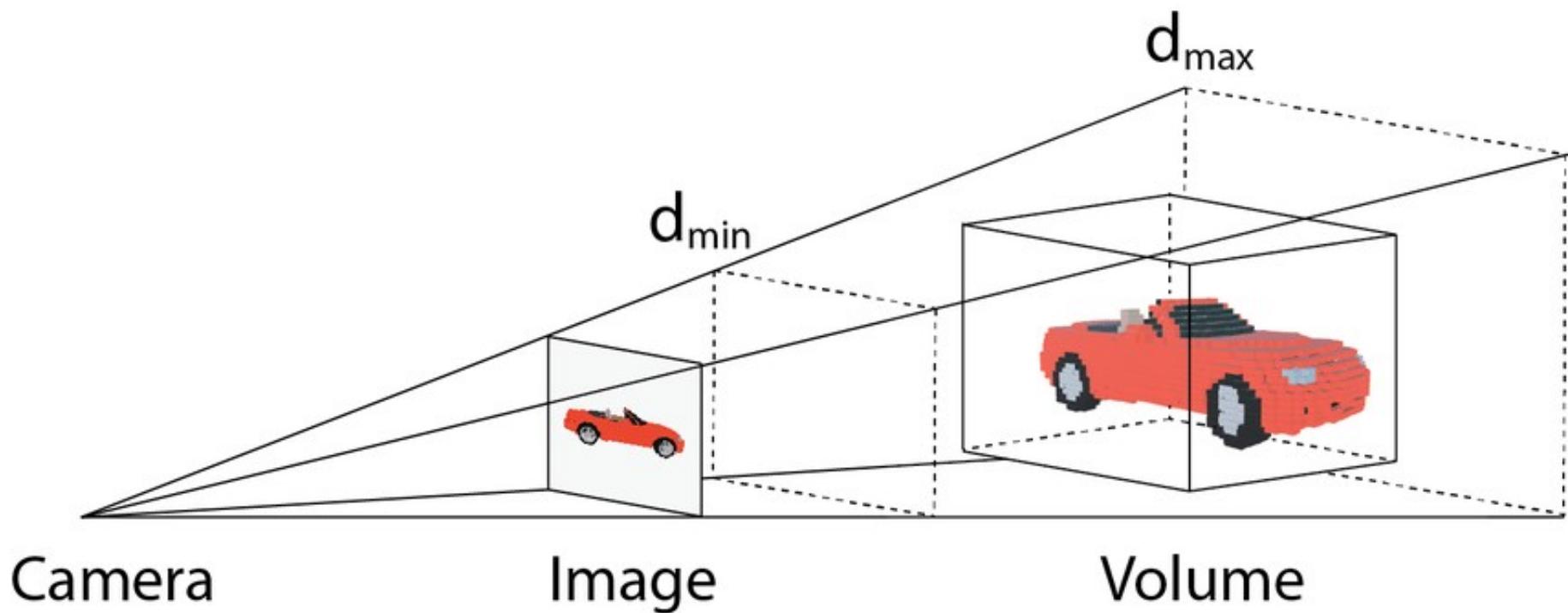
Camera: Bridge between 3D & 2D



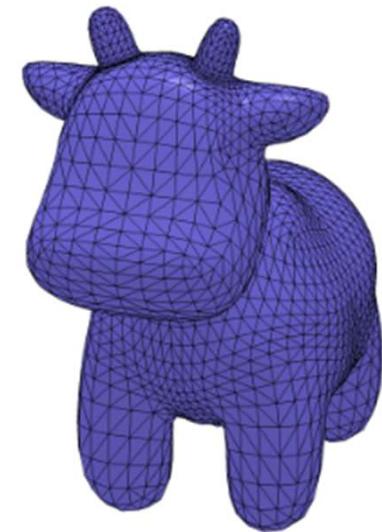
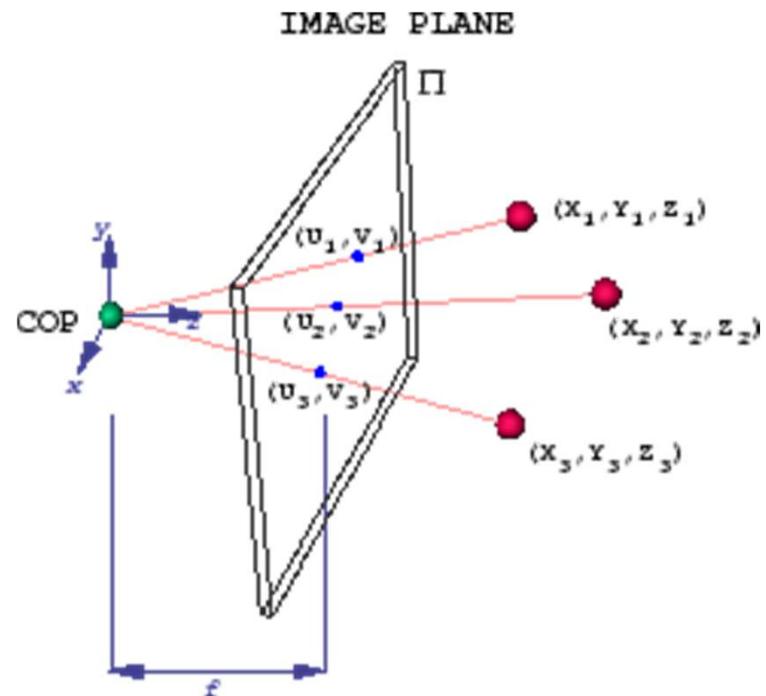
Pinhole Camera



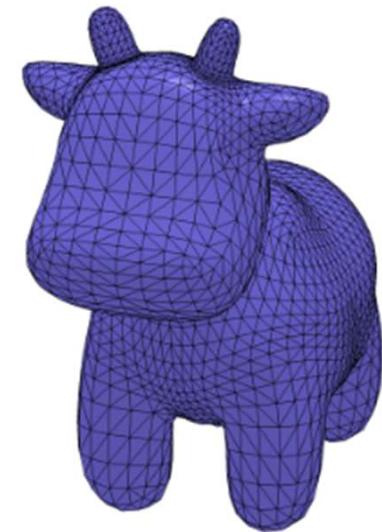
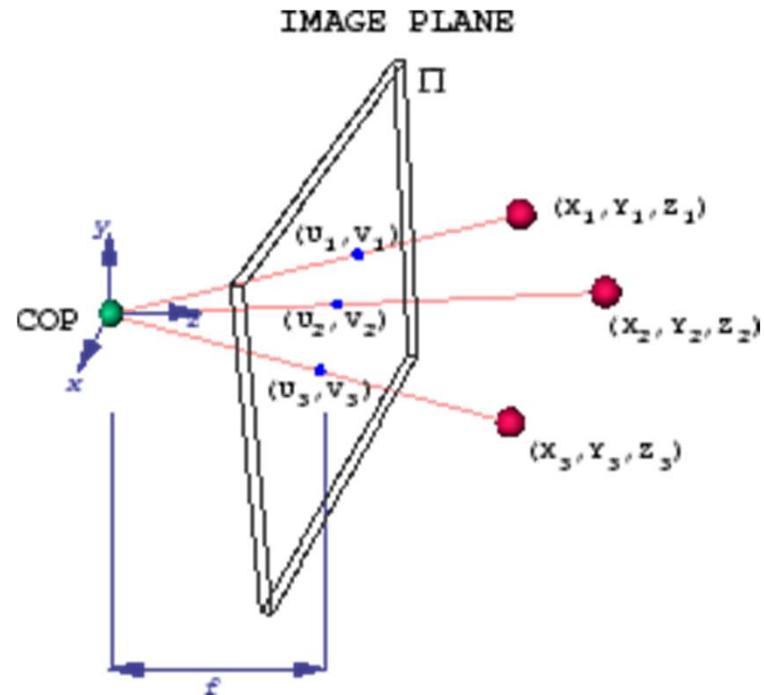
Digital Camera Now: Perspective Camera



Bridge between 3D and 2D

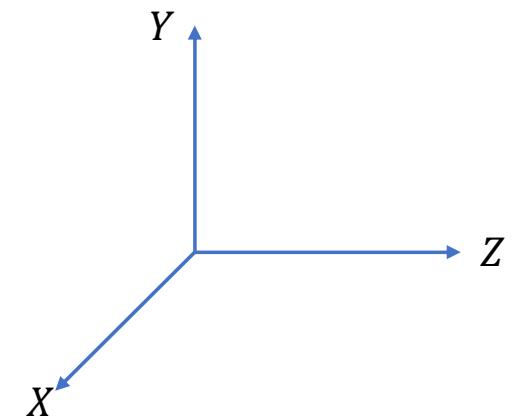
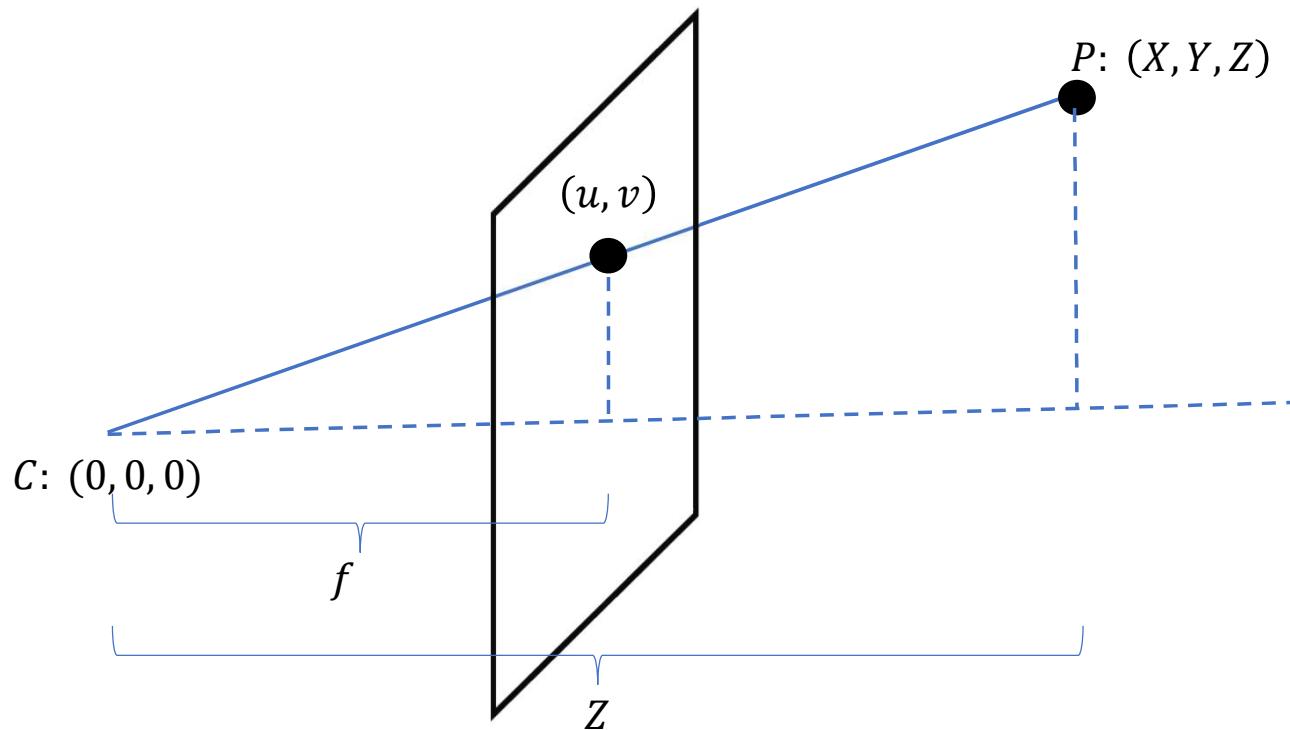


Bridge between 3D and 2D

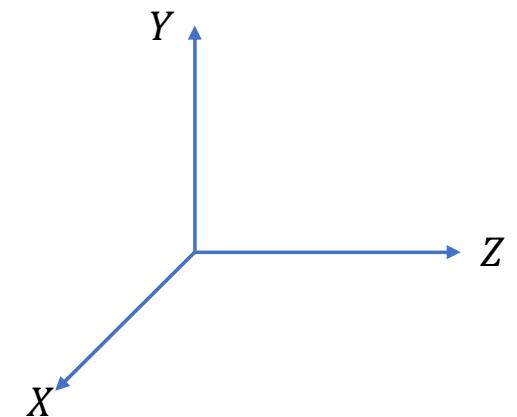
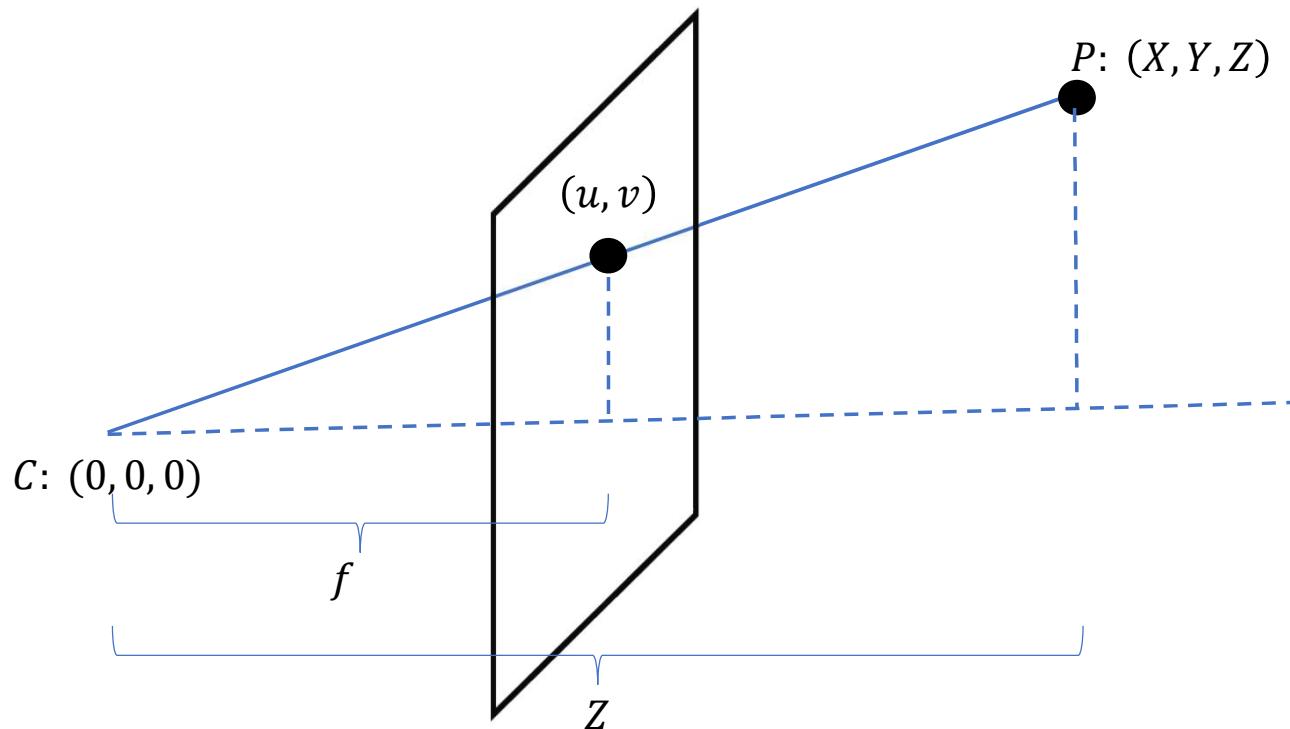


$$\begin{aligned}\Pi: R^3 &\rightarrow R^2 \\ (X, Y, Z) &\rightarrow (u, v)\end{aligned}$$

Bridge between 3D and 2D



Bridge between 3D and 2D



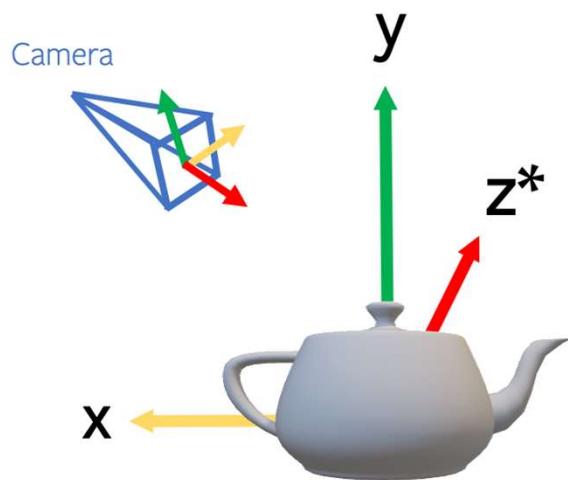
$$(X, Y, Z) \rightarrow (u, v)$$

$$u = \frac{f}{Z} * X$$

$$v = \frac{f}{Z} * Y$$

Coordinate Systems (in 3D)

World space

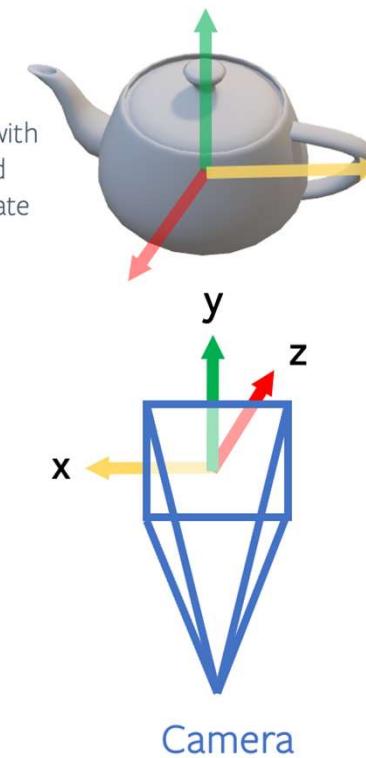


World-to-camera
transform

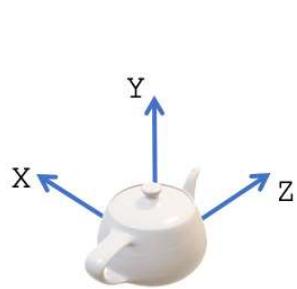
Camera-to-world
transform

View space

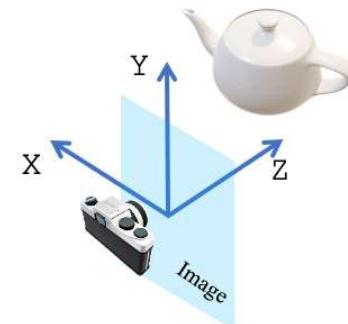
(The view from the camera)



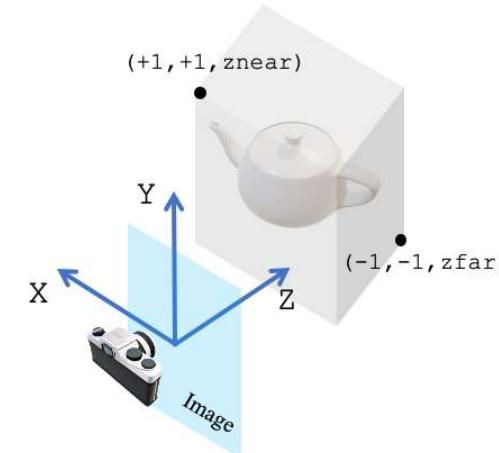
Coordinate Systems: Whole Pipeline



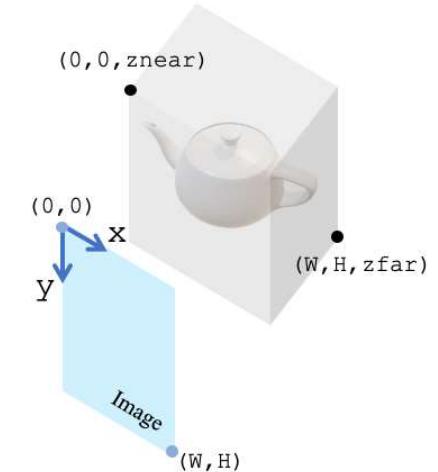
R, T
↻



World
Coordinate System

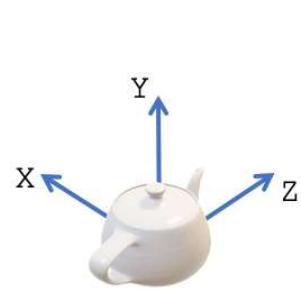


NDC
Coordinate System

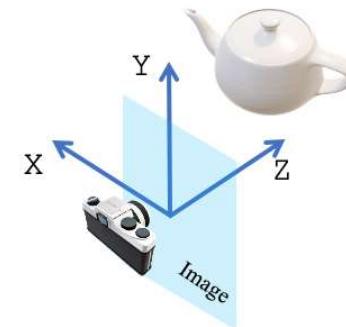


Screen
Coordinate System

Coordinate Systems: Whole Pipeline

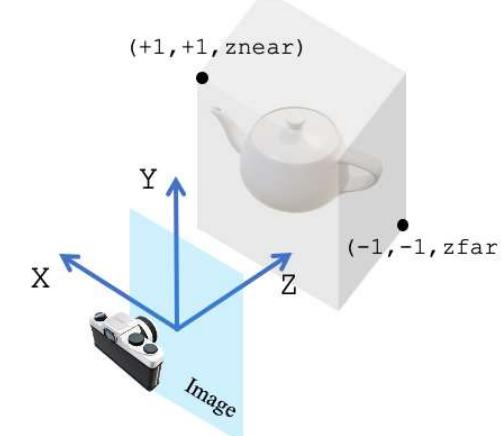


R, T
↻

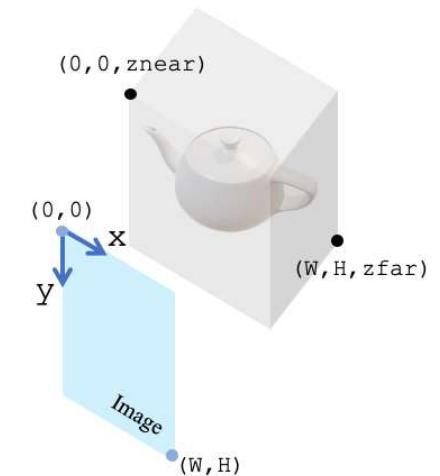


World
Coordinate System

Camera View
Coordinate System



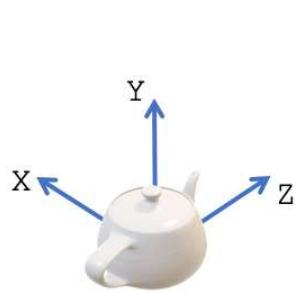
NDC
Coordinate System



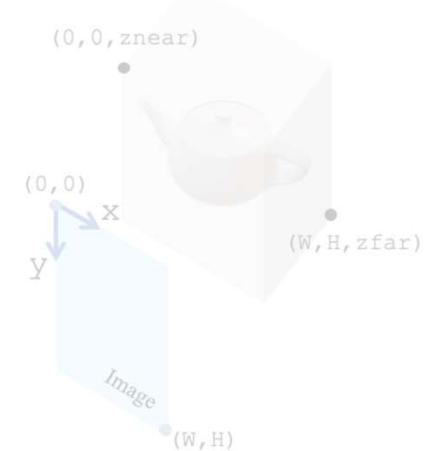
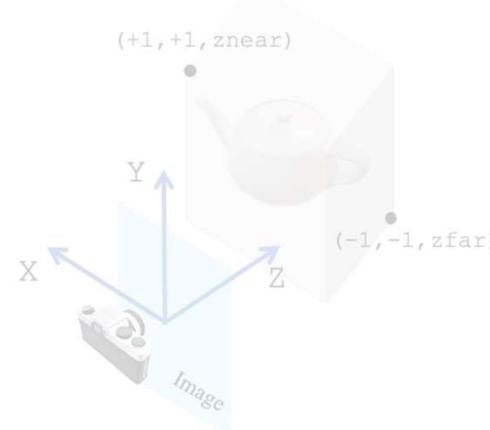
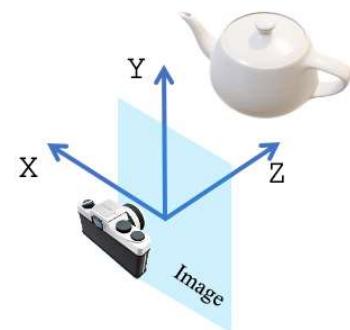
Screen
Coordinate System

How we transform each coordinate space to the next?

World Space to Camera (View) Space



R, T
↻



Transformation between two orthogonal system

if I have a Orthogonal coordinate system with normalized vector x,y,z and origin o_1 .

and another orthogonal coordinate system with normalized vector i,j,k and origin o_2

please formulate the transformation between the two space

- **Coordinate System 1:**

- Basis vectors: $\mathbf{x}, \mathbf{y}, \mathbf{z}$
- Origin: \mathbf{o}_1
- Point in this system: \mathbf{p}_1



- **Coordinate System 2:**

- Basis vectors: $\mathbf{i}, \mathbf{j}, \mathbf{k}$
- Origin: \mathbf{o}_2
- Point in this system: \mathbf{p}_2

- **Coordinate System S_1 :**
 - Orthonormal basis vectors: $\mathbf{x}, \mathbf{y}, \mathbf{z}$.
 - Origin: \mathbf{o}_1 .
 - Coordinates of point P : (α, β, γ) .
- **Coordinate System S_2 :**
 - Orthonormal basis vectors: $\mathbf{i}, \mathbf{j}, \mathbf{k}$.
 - Origin: \mathbf{o}_2 .
 - Coordinates of point P : $(\alpha', \beta', \gamma')$.

- In S_1 :

$$\mathbf{P} = \mathbf{o}_1 + \alpha\mathbf{x} + \beta\mathbf{y} + \gamma\mathbf{z}$$

- In S_2 :

$$\mathbf{P} = \mathbf{o}_2 + \alpha'\mathbf{i} + \beta'\mathbf{j} + \gamma'\mathbf{k}$$

2. Equate the Two Expressions of \mathbf{P}

Since both expressions represent the same point in space, we can set them equal to each other:

$$\mathbf{o}_1 + \alpha\mathbf{x} + \beta\mathbf{y} + \gamma\mathbf{z} = \mathbf{o}_2 + \alpha'\mathbf{i} + \beta'\mathbf{j} + \gamma'\mathbf{k}$$

3. Rearrange the Equation

Bring all terms to one side:

$$\alpha\mathbf{x} + \beta\mathbf{y} + \gamma\mathbf{z} - \alpha'\mathbf{i} - \beta'\mathbf{j} - \gamma'\mathbf{k} = \mathbf{o}_2 - \mathbf{o}_1$$

4. Express $\mathbf{i}, \mathbf{j}, \mathbf{k}$ in Terms of $\mathbf{x}, \mathbf{y}, \mathbf{z}$

Since S_1 and S_2 are related by rotation, the basis vectors of S_2 can be expressed in terms of those of S_1 :

$$\mathbf{i} = r_{11}\mathbf{x} + r_{12}\mathbf{y} + r_{13}\mathbf{z}$$

$$\mathbf{j} = r_{21}\mathbf{x} + r_{22}\mathbf{y} + r_{23}\mathbf{z}$$

$$\mathbf{k} = r_{31}\mathbf{x} + r_{32}\mathbf{y} + r_{33}\mathbf{z}$$

Here, $\mathbf{R} = [r_{ij}]$ is the rotation matrix from S_1 to S_2 .

5. Substitute $\mathbf{i}, \mathbf{j}, \mathbf{k}$ into the Equation

Substitute the expressions of $\mathbf{i}, \mathbf{j}, \mathbf{k}$ into the equation:

$$\begin{aligned}\alpha\mathbf{x} + \beta\mathbf{y} + \gamma\mathbf{z} - \alpha'(r_{11}\mathbf{x} + r_{12}\mathbf{y} + r_{13}\mathbf{z}) \\ - \beta'(r_{21}\mathbf{x} + r_{22}\mathbf{y} + r_{23}\mathbf{z}) - \gamma'(r_{31}\mathbf{x} + r_{32}\mathbf{y} + r_{33}\mathbf{z}) = \mathbf{o}_2 - \mathbf{o}_1\end{aligned}$$

6. Collect Like Terms

Group the terms with common basis vectors:

$$[\alpha - \alpha'r_{11} - \beta'r_{21} - \gamma'r_{31}] \mathbf{x} + [\beta - \alpha'r_{12} - \beta'r_{22} - \gamma'r_{32}] \mathbf{y} + [\gamma - \alpha'r_{13} - \beta'r_{23} - \gamma'r_{33}] \mathbf{z} = \mathbf{o}_2 - \mathbf{o}_1$$

7. Write Component Equations

Since $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are linearly independent, their coefficients must equal the components of $\mathbf{o}_2 - \mathbf{o}_1$ in S_1 :

$$\begin{cases} \alpha - \alpha' r_{11} - \beta' r_{21} - \gamma' r_{31} = (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{x} \\ \beta - \alpha' r_{12} - \beta' r_{22} - \gamma' r_{32} = (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{y} \\ \gamma - \alpha' r_{13} - \beta' r_{23} - \gamma' r_{33} = (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{z} \end{cases}$$

8. Express α', β', γ' in Terms of α, β, γ

Rewriting the equations:

$$\begin{cases} \alpha' r_{11} + \beta' r_{21} + \gamma' r_{31} = \alpha - (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{x} \\ \alpha' r_{12} + \beta' r_{22} + \gamma' r_{32} = \beta - (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{y} \\ \alpha' r_{13} + \beta' r_{23} + \gamma' r_{33} = \gamma - (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{z} \end{cases}$$

These can be written in matrix form:

$$\begin{pmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{pmatrix} \begin{pmatrix} \alpha' \\ \beta' \\ \gamma' \end{pmatrix} = \begin{pmatrix} \alpha - t_x \\ \beta - t_y \\ \gamma - t_z \end{pmatrix}$$

where $t_x = (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{x}$, $t_y = (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{y}$, $t_z = (\mathbf{o}_2 - \mathbf{o}_1) \cdot \mathbf{z}$.

Multiply both sides by \mathbf{R} (since $\mathbf{R}\mathbf{R}^T = \mathbf{I}$ for an orthogonal matrix):

$$\begin{pmatrix} \alpha' \\ \beta' \\ \gamma' \end{pmatrix} = \mathbf{R} \begin{pmatrix} \alpha - t_x \\ \beta - t_y \\ \gamma - t_z \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} t_\alpha \\ t_\beta \\ t_\gamma \end{pmatrix} = \mathbf{R} \begin{pmatrix} -t_x \\ -t_y \\ -t_z \end{pmatrix}$$

12. Final Transformation Formula

Thus, the transformation between the coordinates in S_1 and S_2 is:

$$\boxed{\begin{pmatrix} \alpha' \\ \beta' \\ \gamma' \end{pmatrix} = \mathbf{R} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} + \mathbf{t}}$$

Implement (Represent) the 3D Transform

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

Implement (Represent) the 3D Transform

$$\begin{matrix} X_c & X_w \\ Y_c = \begin{pmatrix} R_{33} & T_{13} \\ 0_{13} & 1 \end{pmatrix} Y_w \\ Z_c & Z_w \\ 1 & 1 \end{matrix}$$

What if we want to optimize (backward)?

$$\begin{matrix} X_c & X_w \\ Y_c & = \begin{pmatrix} R_{33} & T_{13} \\ 0_{13} & 1 \end{pmatrix} Y_w \\ Z_c & Z_w \\ 1 & 1 \end{matrix}$$

A blue arrow points vertically upwards from the term $R_{33} + \delta_{33}$ to the element R_{33} in the matrix.

What if we want to optimize (backward)?

$$\begin{matrix} X_c \\ Y_c \\ Z_c \\ 1 \end{matrix} = \begin{pmatrix} R_{33} & T_{13} \\ 0_{13} & 1 \end{pmatrix} \begin{matrix} X_w \\ Y_w \\ Z_w \\ 1 \end{matrix}$$

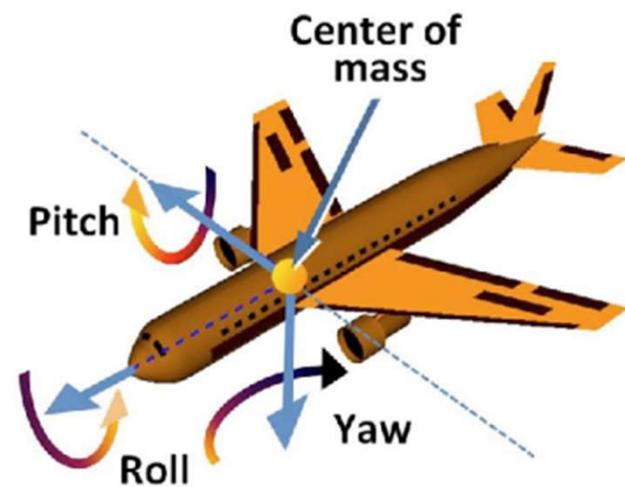
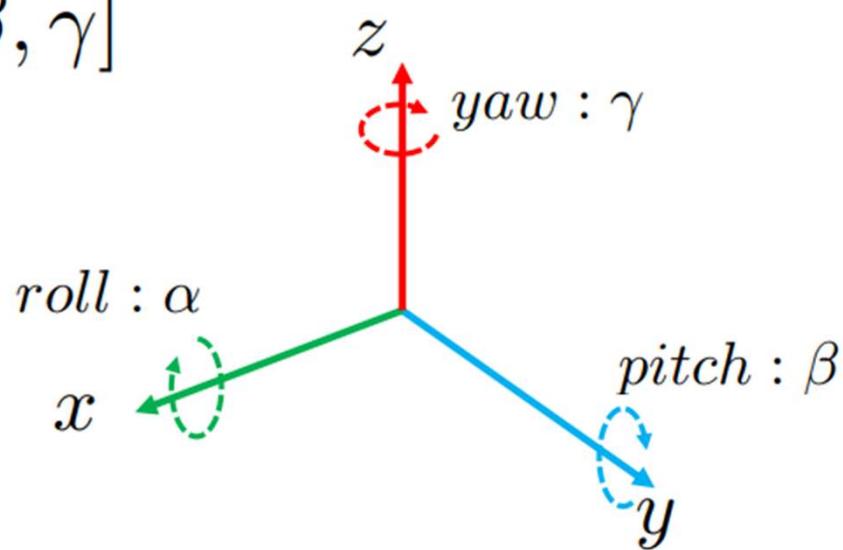
$$R_{33} + \delta_{33}$$

very likely not a rotation matrix

Euler Angles

Three elemental rotations sequentially applied on each axes.

$$[\alpha, \beta, \gamma]$$



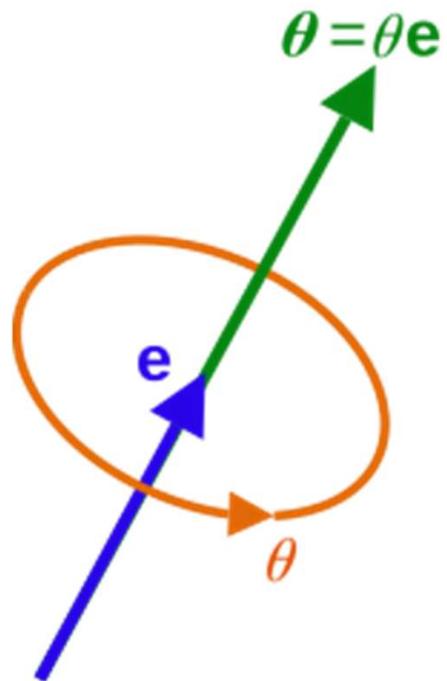
Euler Angles

$$R = R_z(\gamma) R_y(\beta) R_x(\alpha)$$

$$\begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

Order Matters

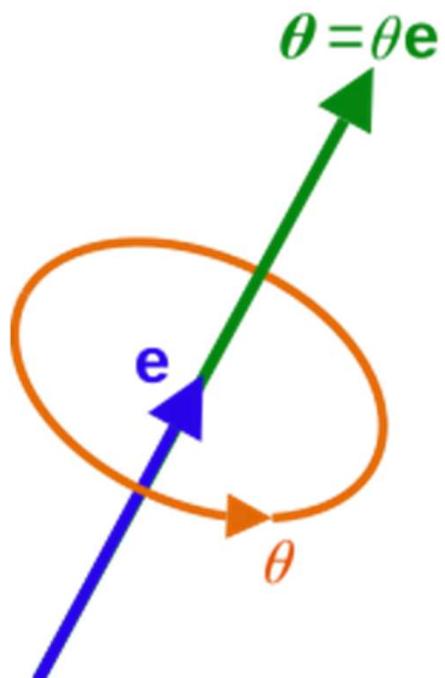
Axis-angle



Axis-angle

Rotation Matrix:

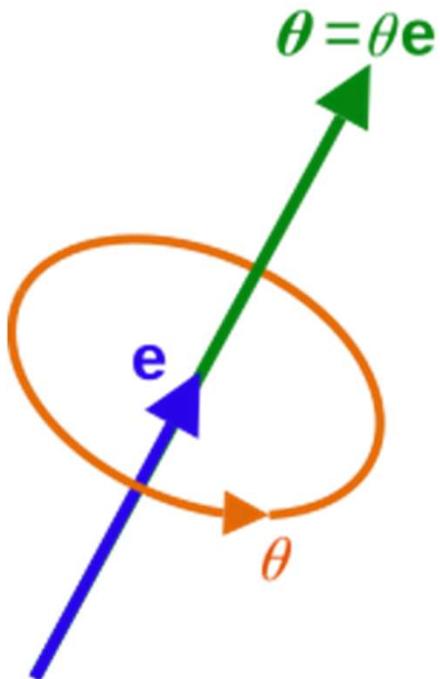
$$\mathbf{R} = \begin{bmatrix} \cos \theta + e_x^2(1 - \cos \theta) & e_x e_y (1 - \cos \theta) - e_z \sin \theta & e_x e_z (1 - \cos \theta) + e_y \sin \theta \\ e_y e_x (1 - \cos \theta) + e_z \sin \theta & \cos \theta + e_y^2 (1 - \cos \theta) & e_y e_z (1 - \cos \theta) - e_x \sin \theta \\ e_z e_x (1 - \cos \theta) - e_y \sin \theta & e_z e_y (1 - \cos \theta) + e_x \sin \theta & \cos \theta + e_z^2 (1 - \cos \theta) \end{bmatrix}$$



Axis-angle

Rotation Matrix:

$$\mathbf{R} = \begin{bmatrix} \cos \theta + e_x^2(1 - \cos \theta) & e_x e_y (1 - \cos \theta) - e_z \sin \theta & e_x e_z (1 - \cos \theta) + e_y \sin \theta \\ e_y e_x (1 - \cos \theta) + e_z \sin \theta & \cos \theta + e_y^2(1 - \cos \theta) & e_y e_z (1 - \cos \theta) - e_x \sin \theta \\ e_z e_x (1 - \cos \theta) - e_y \sin \theta & e_z e_y (1 - \cos \theta) + e_x \sin \theta & \cos \theta + e_z^2(1 - \cos \theta) \end{bmatrix}$$



3. Skew-Symmetric Matrix (\mathbf{K}):

$$\mathbf{K} = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix}$$

4. Rodrigues' Rotation Formula:

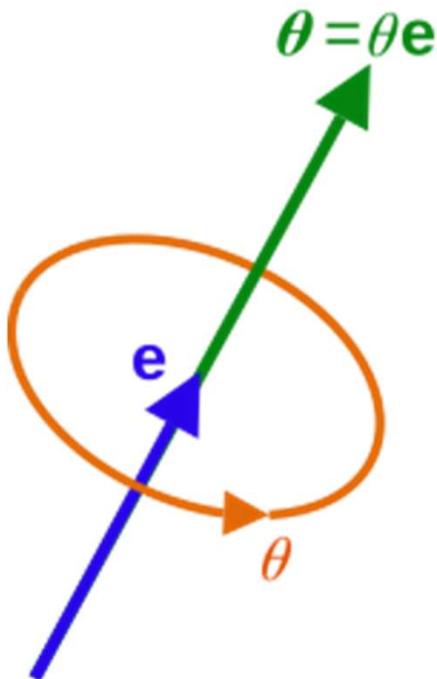
$$\mathbf{R} = \mathbf{I} \cos \theta + (1 - \cos \theta)(\mathbf{e} \otimes \mathbf{e}) + \mathbf{K} \sin \theta$$

This formula constructs the rotation matrix by combining the identity matrix, the outer product of the axis vector with itself, and the skew-symmetric matrix derived from the axis vector, all scaled appropriately by functions of the rotation angle θ .

Axis-angle

Rotation Matrix:

$$\mathbf{R} = \begin{bmatrix} \cos \theta + e_x^2(1 - \cos \theta) & e_x e_y (1 - \cos \theta) - e_z \sin \theta & e_x e_z (1 - \cos \theta) + e_y \sin \theta \\ e_y e_x (1 - \cos \theta) + e_z \sin \theta & \cos \theta + e_y^2(1 - \cos \theta) & e_y e_z (1 - \cos \theta) - e_x \sin \theta \\ e_z e_x (1 - \cos \theta) - e_y \sin \theta & e_z e_y (1 - \cos \theta) + e_x \sin \theta & \cos \theta + e_z^2(1 - \cos \theta) \end{bmatrix}$$



3. Skew-Symmetric Matrix (\mathbf{K}):

$$\mathbf{K} = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix}$$

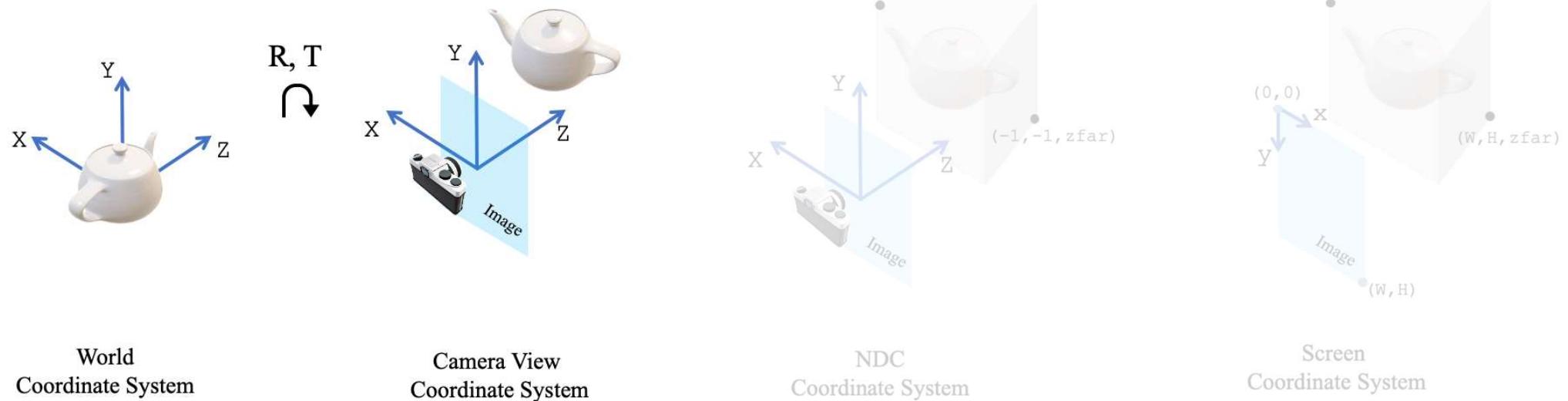
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$$\mathbf{R} = \mathbf{I} \cos \theta + (1 - \cos \theta)(\mathbf{e} \otimes \mathbf{e}) + \mathbf{K} \sin \theta$$

This formula constructs the rotation matrix by combining the identity matrix, the outer product of the axis vector with itself, and the skew-symmetric matrix derived from the axis vector, all scaled appropriately by functions of the rotation angle θ .

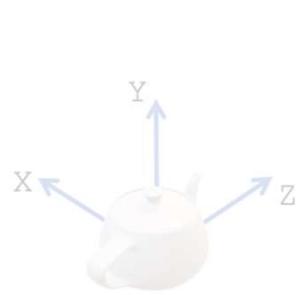
<https://chatgpt.com/share/672231db-ed18-8005-8b31-8ef9655a1b60>

World Space to Camera (View) Space

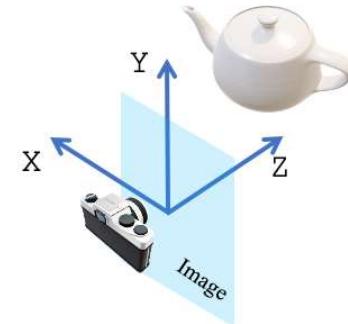


3d Rotation (euler angles/axis-angle, etc...) and Translation

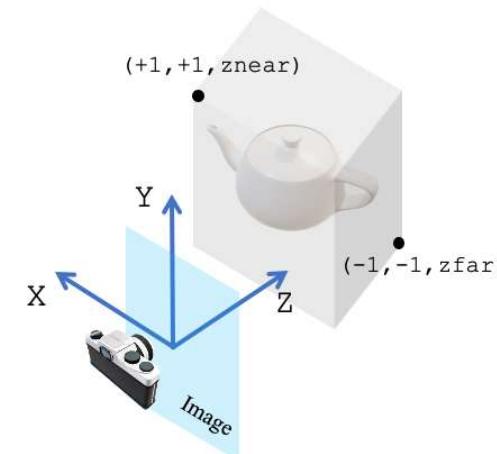
Camera Space to Image Space



R, T

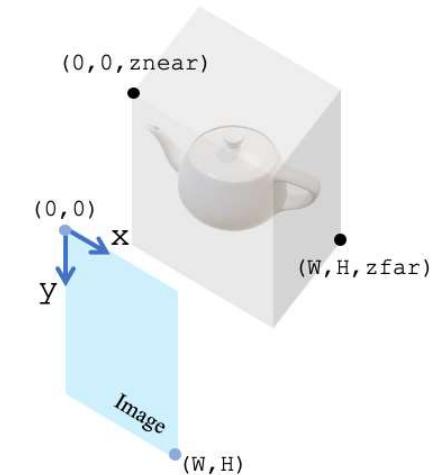


World
Coordinate System



Camera View
Coordinate System

NDC
Coordinate System



Screen
Coordinate System



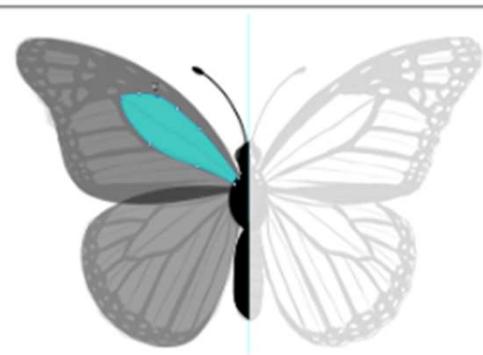
Projection Transformation

GAMES101: 现代计算机图形学入门

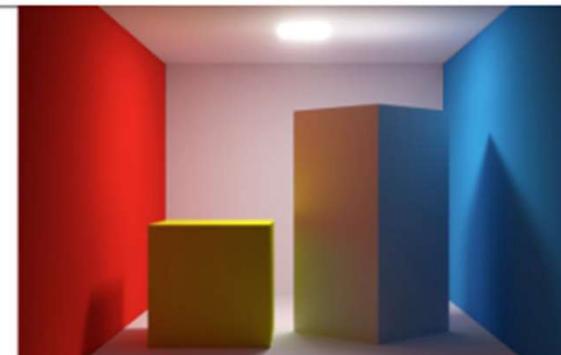
<https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html>



光栅化成像 [实时软阴影 (NVIDIA)]



几何表示 [Adobe Photoshop 中的钢笔工具]



光线传播理论 [光子映射 (Jensen)]



动画与模拟 [流体模拟 (Muller)]

Credit to Lingqi Yan

Projection Transformation

- Projection in Computer Graphics
 - 3D to 2D
 - Orthographic projection
 - Perspective projection

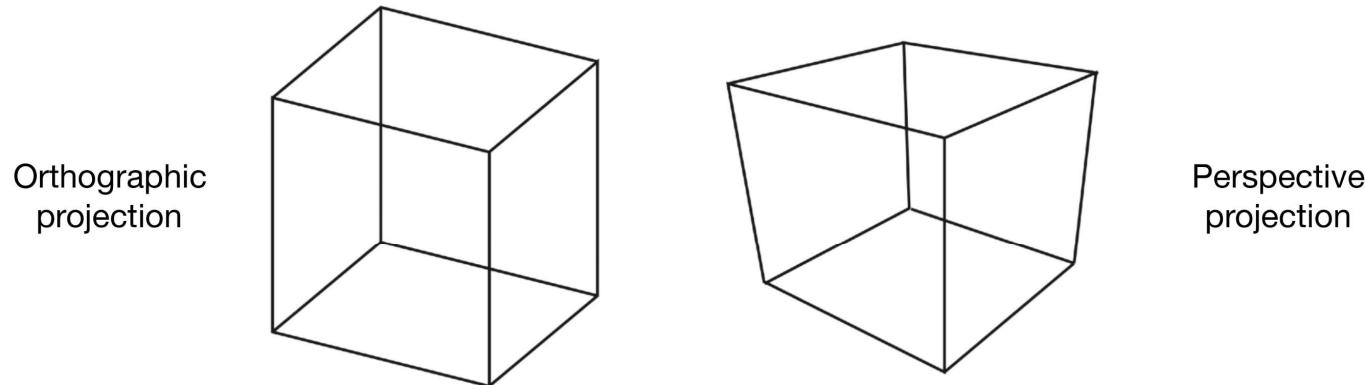
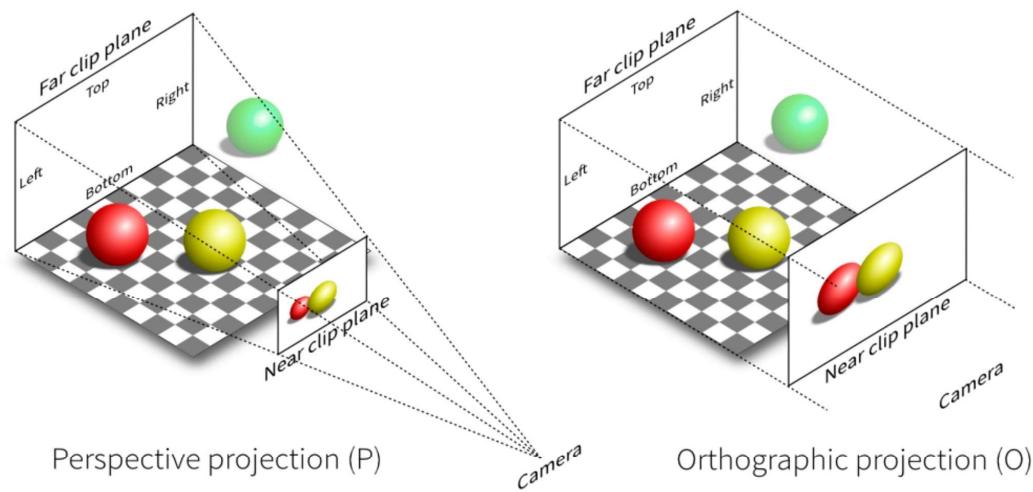


Fig. 7.1 from *Fundamentals of Computer Graphics, 4th Edition*

Projection Transformation

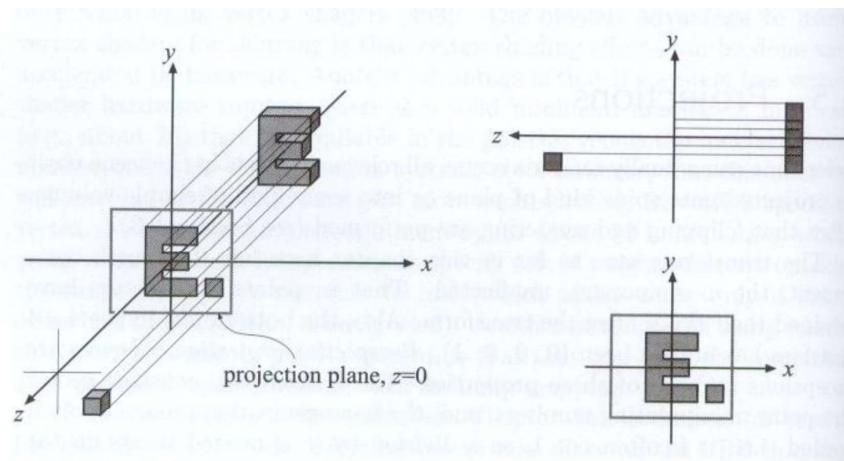
- Perspective projection vs. orthographic projection



<https://stackoverflow.com/questions/36573283/from-perspective-picture-to-orthographic-picture>

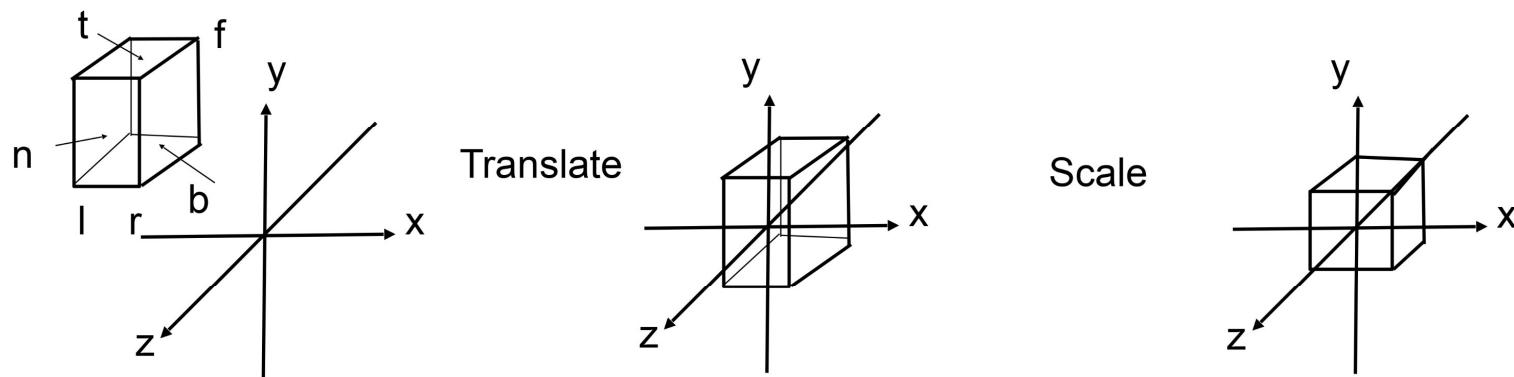
Orthographic Projection

- A simple way of understanding
 - Camera located at origin, looking at -Z, up at Y (looks familiar?)
 - Drop Z coordinate
 - Translate and scale the resulting rectangle to $[-1, 1]^2$



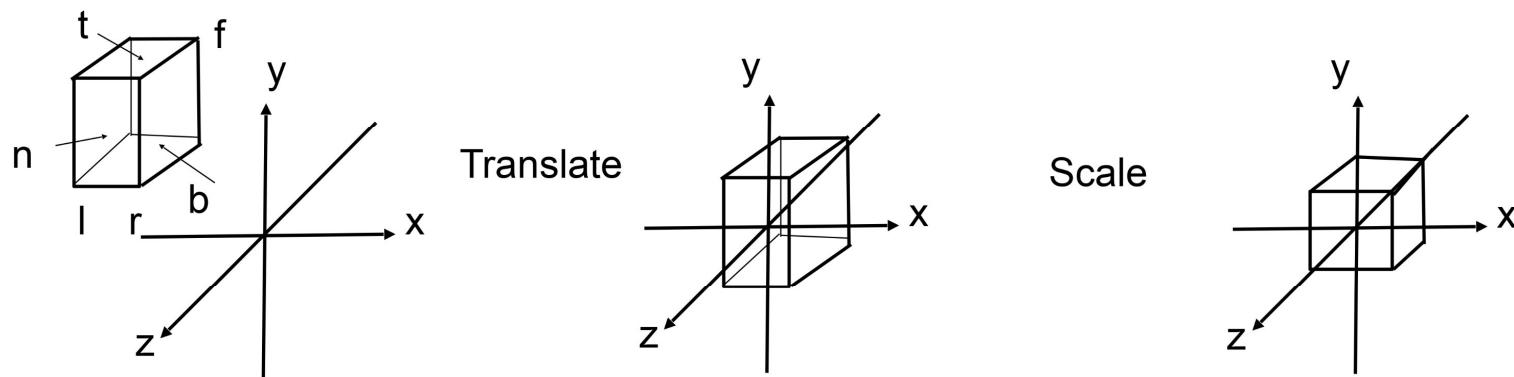
Orthographic Projection

- In general
 - We want to map a cuboid $[l, r] \times [b, t] \times [\mathbf{f}, \mathbf{n}]$ to the “canonical (正则、规范、标准)” cube $[-1, 1]^3$



Orthographic Projection

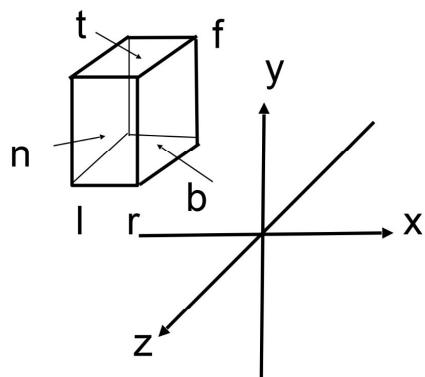
- Slightly different orders (to the “simple way”)
 - Center cuboid by translating
 - Scale into “canonical” cube



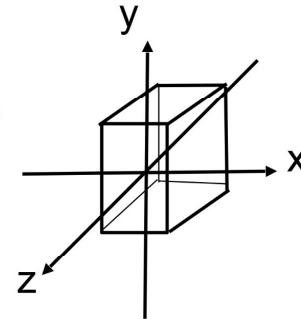
Orthographic Projection

- Transformation matrix?
 - Translate (**center** to origin) **first**, then scale (length/width/height to **2**)

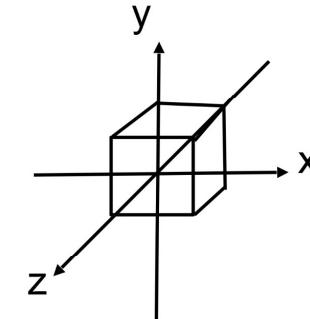
$$M_{ortho} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Translate

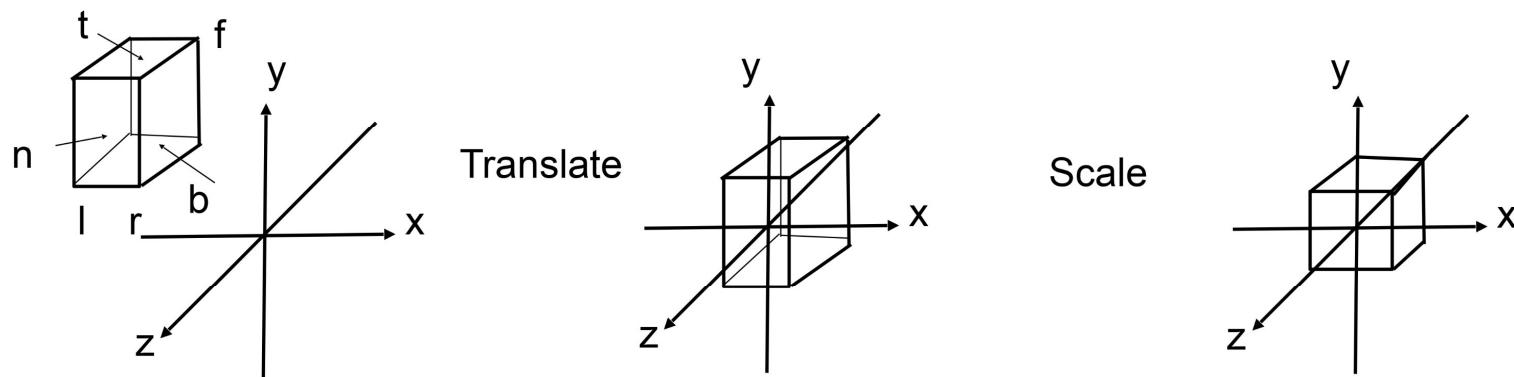


Scale



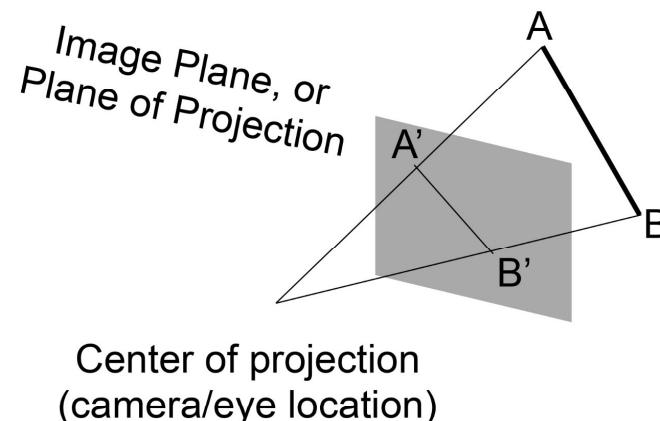
Orthographic Projection

- Caveat
 - Looking at / along -Z is making near and far not intuitive ($n > f$)
 - FYI: that's why OpenGL (a Graphics API) uses left hand coords.



Perspective Projection

- Most common in Computer Graphics, art, visual system
- Further objects are smaller
- Parallel lines not parallel; converge to single point



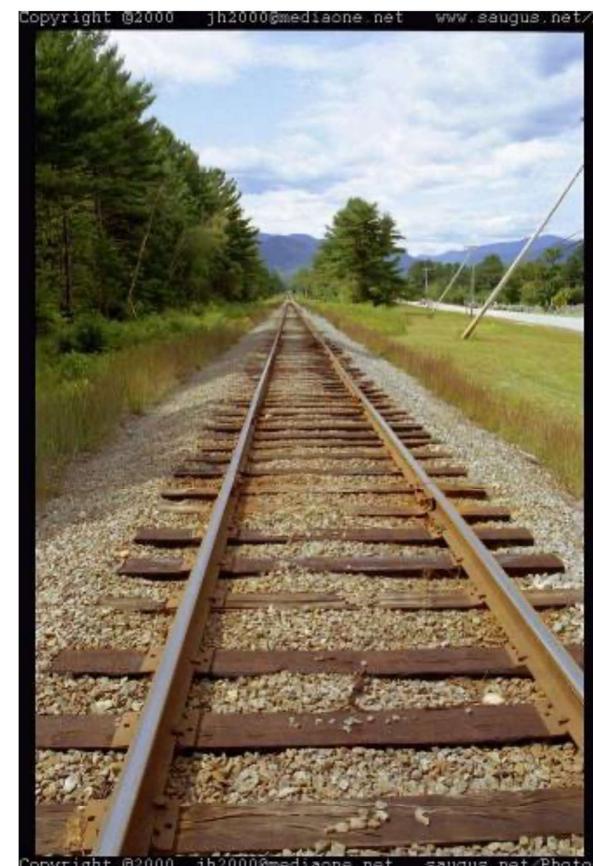
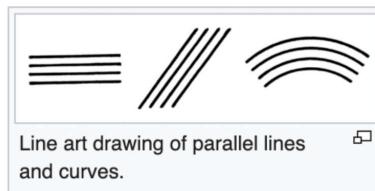
Perspective Projection

- Euclid was wrong??!!

In [geometry](#), **parallel** lines are [lines in a plane](#) which do not meet; that is, two lines in a plane that do not [intersect](#) or [touch](#) each other at any point are said to be parallel. By extension, a line and a plane, or two planes, in [three-dimensional Euclidean space](#) that do not share a point are said to be parallel. However, two lines in three-dimensional space which do not meet must be in a common plane to be considered parallel; otherwise they are called [skew lines](#). Parallel planes are planes in the same three-dimensional space that never meet.

Parallel lines are the subject of [Euclid's parallel postulate](#).^[1] Parallelism is primarily a property of [affine geometries](#) and [Euclidean geometry](#) is a special instance of this type of geometry. In some other geometries, such as [hyperbolic geometry](#), lines can have analogous properties that are referred to as parallelism.

[https://en.wikipedia.org/wiki/Parallel_\(geometry\)](https://en.wikipedia.org/wiki/Parallel_(geometry))



Perspective Projection

- Before we move on
- Recall: property of homogeneous coordinates
 - $(x, y, z, 1), (kx, ky, kz, k \neq 0)$, **$(xz, yz, z^2, z \neq 0)$** all represent the same point (x, y, z) in 3D
 - e.g. $(1, 0, 0, 1)$ and $(2, 0, 0, 2)$ both represent $(1, 0, 0)$
- Simple, but useful

Perspective Projection

- How to do perspective projection
 - First “squish” the frustum into a cuboid ($n \rightarrow n, f \rightarrow f$) ($M_{\text{persp} \rightarrow \text{ortho}}$)
 - Do orthographic projection (M_{ortho} , already known!)

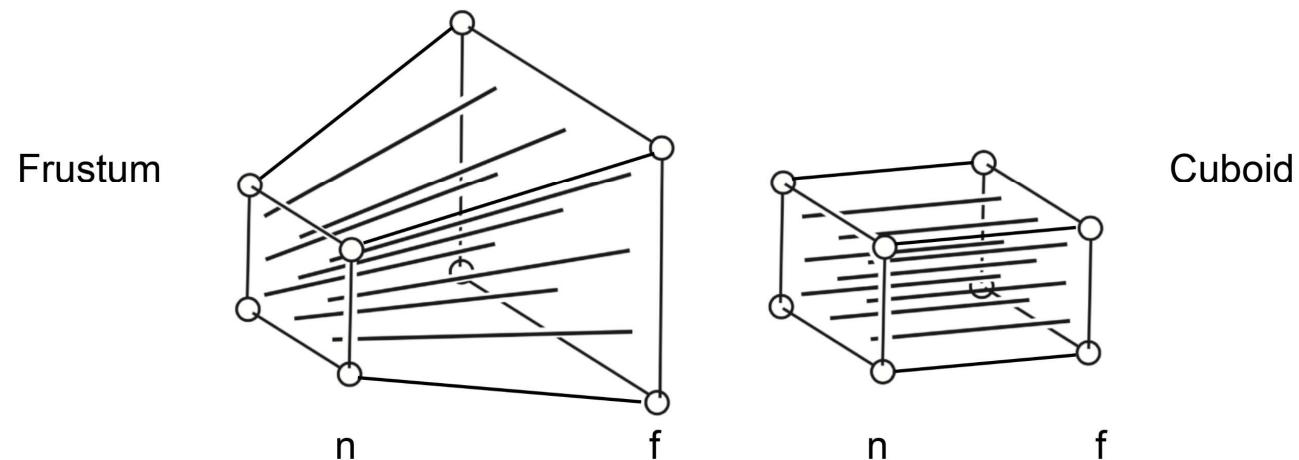
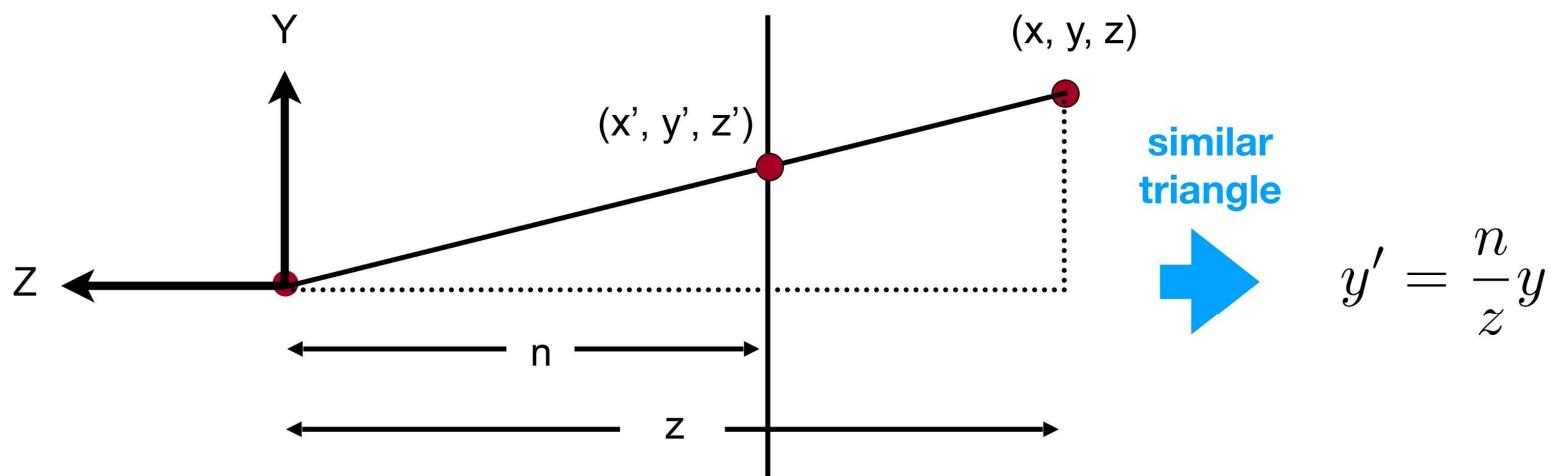


Fig. 7.13 from *Fundamentals of Computer Graphics, 4th Edition*

Perspective Projection

- In order to find a transformation
 - Recall the key idea: Find the relationship between transformed points (x', y', z') and the original points (x, y, z)



Perspective Projection

- In order to find a transformation
 - Find the relationship between transformed points (x' , y' , z') and the original points (x , y , z)

$$y' = \frac{n}{z}y \quad x' = \frac{n}{z}x \text{ (similar to } y')$$

- In homogeneous coordinates,

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} nx/z \\ ny/z \\ \text{unknown} \\ 1 \end{pmatrix} \stackrel{\text{mult. by } z}{=} \begin{pmatrix} nx \\ ny \\ \text{still unknown} \\ z \end{pmatrix}$$

Perspective Projection

- So the “squish” (persp to ortho) projection does this

$$M_{\text{persp} \rightarrow \text{ortho}}^{(4 \times 4)} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ \text{unknown} \\ z \end{pmatrix}$$

- Already good enough to figure out part of $M_{\text{persp} \rightarrow \text{ortho}}$

$$M_{\text{persp} \rightarrow \text{ortho}} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

WHY?

Perspective Projection

- How to figure out the third row of $M_{persp \rightarrow ortho}$

- Any information that we can use?

$$M_{persp \rightarrow ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

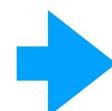
- Observation: the third row is responsible for z'
 - Any point on the near plane will not change
 - Any point's z on the far plane will not change

Perspective Projection

- Any point on the near plane will not change

$$M_{persp \rightarrow ortho}^{(4 \times 4)} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ \text{unknown} \\ z \end{pmatrix}$$

replace
z with n


$$\Rightarrow \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ n^2 \\ n \end{pmatrix}$$

- So the third row must be of the form (0 0 A B)

$$(0 \quad 0 \quad A \quad B) \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = n^2$$

**n² has nothing
to do with x and y**

Perspective Projection

- What do we have now?

$$(0 \ 0 \ A \ B) \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = n^2 \quad \rightarrow \quad An + B = n^2$$

- Any point's z on the far plane will not change

$$\begin{pmatrix} 0 \\ 0 \\ f \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 0 \\ f \\ 1 \end{pmatrix} == \begin{pmatrix} 0 \\ 0 \\ f^2 \\ f \end{pmatrix} \quad \rightarrow \quad Af + B = f^2$$

Perspective Projection

- Solve for A and B

$$An + B = n^2$$

$$Af + B = f^2$$

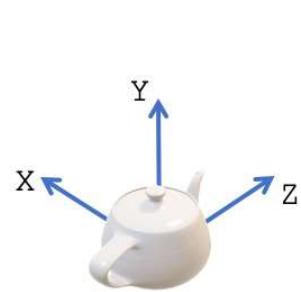


$$A = n + f$$

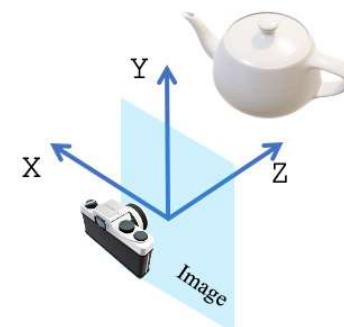
$$B = -nf$$

- Finally, every entry in $M_{\text{persp} \rightarrow \text{ortho}}$ is known!
- What's next?
 - Do orthographic projection (M_{ortho}) to finish
 - $M_{\text{persp}} = M_{\text{ortho}} M_{\text{persp} \rightarrow \text{ortho}}$

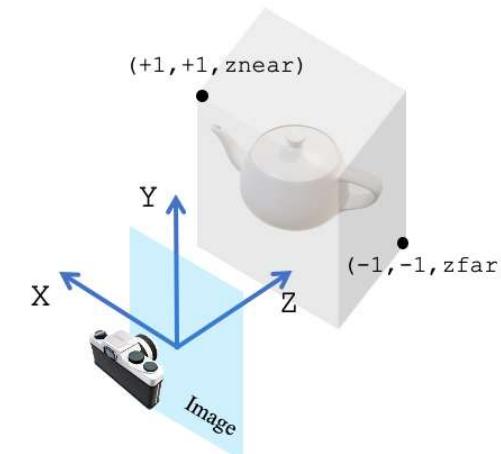
Coordinate Systems: Whole Pipeline



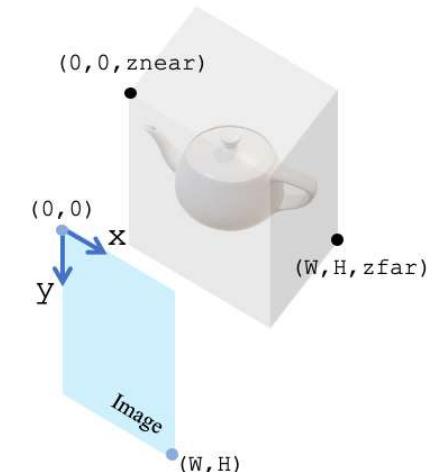
R, T
↻



World
Coordinate System



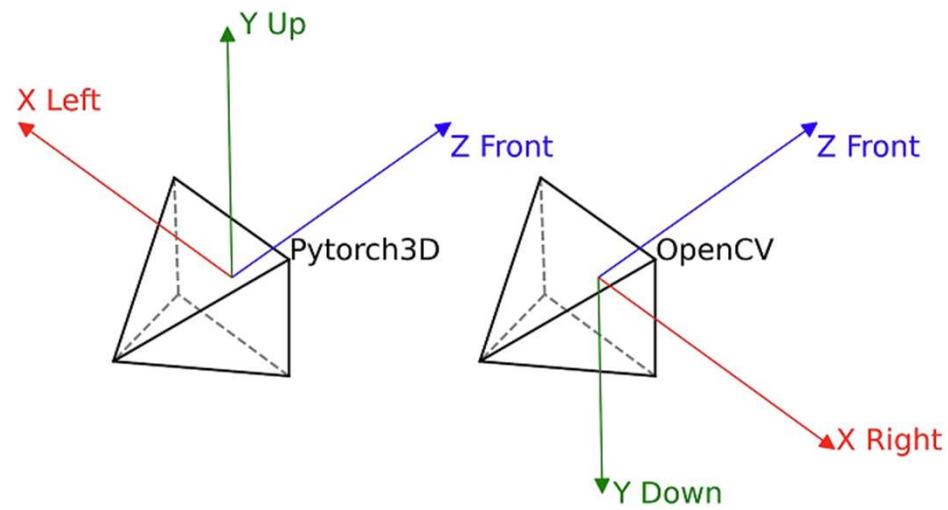
NDC
Coordinate System



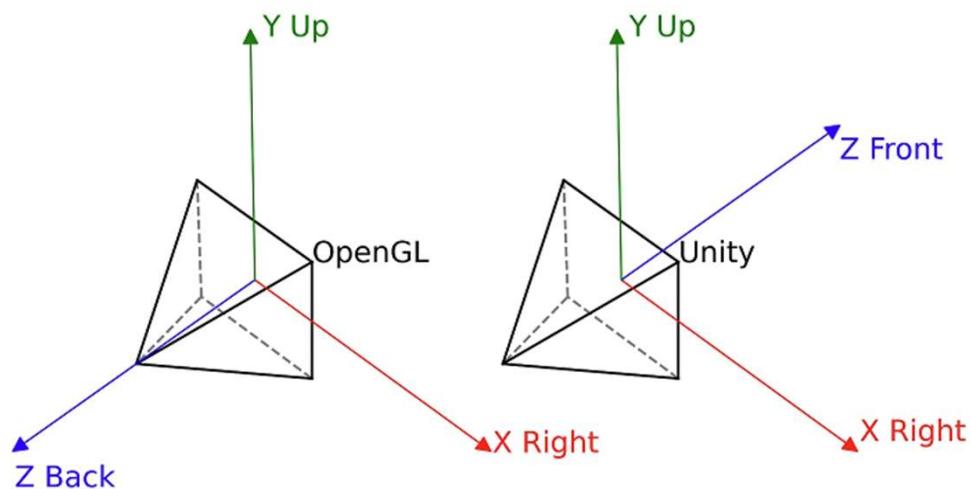
Screen
Coordinate System

All Homogeneous Matrix

Different Coordinates with Different Libs



How to Convert
between each other?



Revisit Rendering & Reconstruction (3D Vision)

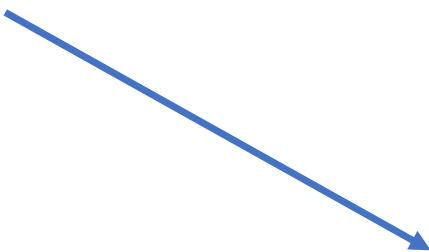
$$F(3D) = 2D$$

World space to camera space

Camera space projecting to (normalized space to) image space

Revisit Rendering & Reconstruction (3D Vision)

$$R(3D, \text{light}, \text{material}) = 2D$$

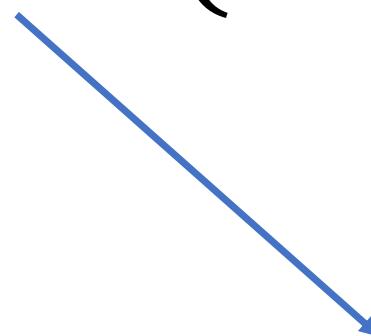


Rendering (next course)

- How to represent 3D
- How exactly the rendering function is

Revisit Rendering & Reconstruction (3D Vision)

$$F^{-1}(2D) = 3D$$



Reconstruction (Many remaining courses)



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谢谢观看！