

09:00 hrs

23/11/2016

S4WNNDH

EXAMS OFFICE
USE ONLY

University of the Witwatersrand, Johannesburg

Course or topic No(s)

COMS2015

Course or topic name(s)

Paper Number & title

Analysis of Algorithms

Examination to be
held during the months(s) of

November 2016

Year of Study

2nd

Degrees/Diplomas for which
this course is prescribed

BSc, BEconSc, HDipCS

Faculty/ies presenting
candidates

Science; Commerce

Internal examiner(s)
and telephone extension
number(s)

Pravesh Ranchod

External examiner(s)

Prof. S. Gruner

Special materials required

None

Time allowance

Course nos	COMS2015	Hours	2
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Instructions to candidates

There are 60 marks available. See first page of
question paper for more instructions

Instructions

- This is a closed book exam.
- This exam lasts for 2 hours.
- There are 4 pages excluding the cover page.
- There are 60 marks available in total.
- 60 marks = 100%
- The number of marks available for a question is given in square brackets at the end of the question.
- You may answer the questions in any order, and you may answer all questions.
- **Answer the question!** Writing an answer that is true but irrelevant to the question asked will waste your time and receive zero marks.
- If you are not sure about something, **put up your hand and wait for an invigilator to come to you.**

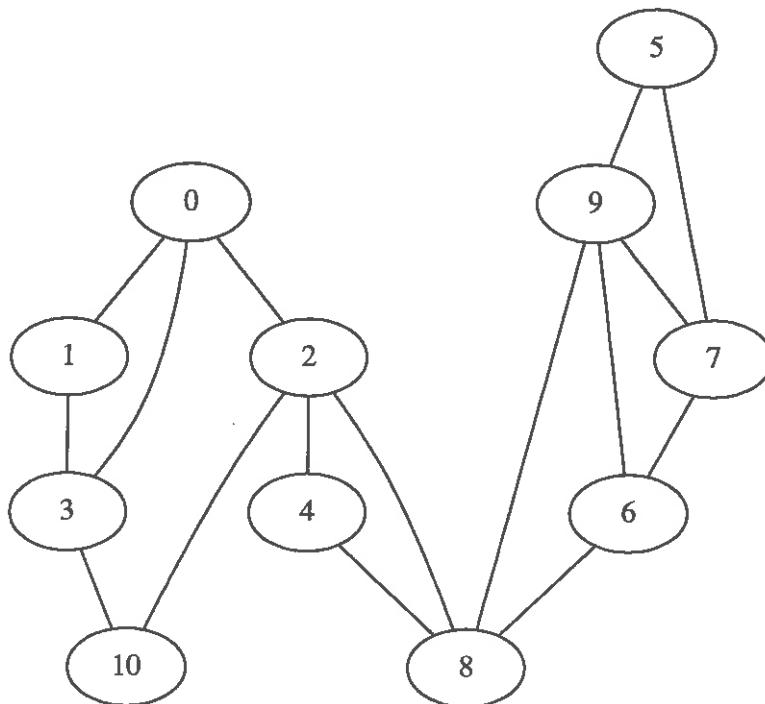


Figure 1: Graph for question 1c and 1d

Graphs**[14 marks]**

1. (a) Prove that every edge in a tree is a bridge. [2 marks]
- (b) What is the chromatic number of a bipartite graph? [2 marks]
- (c) Draw the depth first search tree of the graph in Figure 1, rooted at vertex 2. [5 marks]
- (d) Using the depth first search tree constructed above, find the articulation points of the graph in Figure 1 using the algorithm presented in the course. For each vertex drawn above, indicate the back number and the dfs number of the vertex when the algorithm is complete. [5 marks]

Graph Colouring**[10 Marks]**

2. A dinner party is being held for old musicians. The host thinks that people who used to be in a band together should not sit at the same table or they will end up arguing with each other. Most old musicians have been in multiple bands. The people involved are Freddie, Kurt, Marilyn, James, Peter, Phil and Joe.

- Freddie, Kurt and Marilyn used to be members of “Back Slash”

- Freddie, James and Peter used to be members of “ALL CAPS”
- Phil and James used to be members of “Transmission Control Protocol”
- Joe, Phil and Marilyn used to be members of “Convex Hull”
- Kurt, Phil and Peter used to be members of “F10”

Use graph colouring to determine how to seat these people. State which people will be seated together, and the order in which the assignments were made.

[10 marks]

(Hint:Construct a graph with the musicians as vertices. Decide what criteria to use for creating edges)

Stable Marriage [11 marks]

3. (a) Under what condition is a marriage considered to be unstable? [3 marks]
- (b) Provide pseudocode for the Gale Shapley algorithm to find stable marriages [6 marks]
- (c) For efficiency, implementations of the Gale Shapley algorithm make use of preference arrays and ranking arrays to represent the same information. Give an example of an operation that is more efficient to perform using a ranking array rather than a preference array. [2 marks]

Cheap Translation [9 Marks]

4. You are the manager of the translation staff at Parliament. You have on your staff a number of translators, each of whom can translate between two languages, in either direction. You can also translate through one or more intermediate languages (eg. You can translate from English into French by translating from English into Spanish, and then from Spanish into French). However, every translator must be paid for that particular translation. You have the following translations available:

Translator	Language 1	Language 2	Cost
1	English	isiXhosa	11
2	English	Afrikaans	2
3	English	Sesotho	100
4	English	Sepedi	2
5	Sesotho	isiXhosa	4
6	Sesotho	Tshivenda	80
7	Sesotho	Tswana	1
8	Sesotho	Sepedi	3
9	Sepedi	Tswana	14
10	Sepedi	Afrikaans	38
11	Afrikaans	Tswana	55
12	Tshivenda	Tswana	3

Find the cheapest way of converting from English to every other language. Show your workings, making sure to show changes to your parent, cost and marked arrays.

[9 marks]

Algorithm Analysis**[12 marks]**

```

5. public void doSomething(int[] inputs, int key) {
    for (int i=0; i<inputs.length; i++){
        if (inputs[i]!=key){
            for (int j=0; j<inputs.length; j++){
                System.out.println(i + " : "+inputs[j]);
            }
        } else {
            break;
        }
    }
}

```

- (a) What is the best case complexity of the `doSomething` method given above and under what circumstances does it occur? Support your answer. [3 marks]
- (b) What is worst case complexity of the `doSomething` method given above and under what circumstances does it occur? Support your answer. [3 marks]

6. Indicate whether each of the following statements is true or false. Prove your answer.

- | | |
|---|-----------|
| (a) Every function f in $O(n)$ is also in $O(n^2)$ | [2 marks] |
| (b) Every function f in $\Omega(n^3)$ is also in $O(n^4)$ | [2 marks] |
| (c) $2n^2 + 5n + 3 \in \Theta(n^2)$ | [2 marks] |

Hats**[4 marks]**

7. In a small village in the middle of nowhere, three innocent prisoners are sitting in a jail. One day, the cruel jailer takes them out and places them in a line on three chairs, in such a way that man C can see both man A and man B, man B can see only man A, and man A can see none of the other men. The jailer shows them 5 hats, 2 of which are black and 3 of which are white. After this, he blindfolds the men, places one hat on each of their heads, and removes the blindfolds again.

The jailer tells his three prisoners that if one of them is able to determine the color of his hat within one minute, all of them are released. Otherwise, they will all be jailed for life. None of the prisoners can see his hat, and all are intelligent. After 59 seconds, man A shouts out the correct color of his hat, and they're all saved.

What is the colour of man A's hat, and how does he know? [4 marks]

End of question paper

