2013 IEEE 10th International Symposium on Biomedical Imaging:
From Nano to Macro
San Francisco, CA, USA, April 7-11, 2013

MOTION COMPENSATED COMPRESSED SENSING DYNAMIC MRI WITH LOW RANK PATCH-BASED RESIDUAL RECONSTRUCTION

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ABSTRACT

One of the technical challenges of the predication/residual encoding in motion estimated and compensated k-t FOCUSS is that after the prediction, the energy of the residual measurement is significantly reduced compared to the original measurement. This implies that the residual measurement should be judiciously used to recover important geometric features rather than background noise during residual encoding stage. To address this, this paper proposes a novel patch-based residual encoding scheme to exploit geometric similarity in the residual images. In particular, this paper is interested in patch-based low rank constraint from similarity patches [1] since rank structures are relatively less sensitive to global intensity changes but easier to capture edges and etc. To address the nonconvexity and non-smoothess of the rank penalty, a concave-convex procedure is proposed. Experimental results show that the proposed algorithm clearly reconstructs important anatomic structures in cardiac cine image and provides significant performance improvement compared to the existing motion compensated k-t FOCUSS.

***Index Terms*—** compressed sensing, motion compensation, dynamic cardiac imaging, k-t FOCUSS, patch, CCCP, low-rank, thresholding, noise reduction

1. INTRODUCTION

MR imaging is an inherently slow imaging modality since it is designed to acquire 2-D (or 3-D) k-space data through 1-D free induction decay or echo signals. This limitation is very critical especially for high resolution dynamic cardiac imaging.

To address this issue, many researchers have recently applied compressed sensing (CS) approaches for dynamic imaging applications [2]. Compressed sensing (CS) tells us that accurate reconstruction is possible as long as nonzero support is sparse and sampling basis are incoherent [3]. For example, as dynamic images can be effectively sparsified in transform domain thanks to temporal redundancy, k-t FOCUSS [4] imposes sparsity constraint in a transform domain. Several sparsifying transform have been proposed in k-t FOCUSS framework. When a high resolution reference frame is available, motion estimation and compensation (ME/MC) is quite effective among various sparsifying transforms [4]. However, one of the limitations of ME/MC is that the energy of the residual measurement after prediction is significantly reduced compared to the original measurement.

Recently, patch-based signal processing algorithms that exploit self-similarity within images have been investigated quite extensively among MR community. For example, Ravishankar and Bresler applied a dictionary learning algorithm for static MRI reconstruction [5], whereas Akçakaya *et al* applied BM3D collaborative filtering for cardiac MR application [6]. To incorporate nonlocal means algorithm, Yang *et al* proposed a variational framework in compressed sensing MR reconstruction [7]. Extending the idea of calibration free parallel imaging using low-rank properties, Trzasko *et al* recently introduced a patch based generalization called CLEAR(calibration-free locally low-rank encouraging reconstruction) for calibration free parallel imaging applications[8]. Also, Suyash et at. suggested the low-rank modeling for spatio-temporal dictionary learing for dynamic MR imaging[9].

By extending the prior works, this paper is interested in exploiting the self-similarities in temporal direction. In particular, this paper is interested in patch-based low rank constraint from similarity patches [1] since rank structures are relatively less sensitive to global intensity changes but easier to capture edges and etc. To achieve the goal, the proposed algorithm provides a two step hierarchical approach. More specifically, we perform a novel patch-based non-local ME/MC using a diastole phase reconstruction as a reference, after which the patch-based low rank penalty is applied for residual encoding. This is because a novel patch-based non-local motion compensation scheme can effectively remove the coherent aliasing artifact and recover most of the dominant signal components, after which the patch based low rank penalty effectively capture geometric similarities such as edges and boundaries in residual image from aliasing free residual signals. In addition, unlike the existing patch-based

Grant sponsor: Korea Science and Engineering Foundation, Grant number 2011-0000353. We would like to thank Dr. Daniel Kim at Univ. of Utah for providing the data set.

processing for MR, we employ a non-convex rank proxy proposed in [10] due to its excellent performance. To deal with non-smooth and non-convex regularization terms, we propose a novel globally convergent concave-convex procedure (CCCP)[11]. The resulting algorithm is a special case of majorize-minimize (MM) procedure, and does not require any additional memory for storing Lagrangian parameters, which makes the algorithm suitable for memory bandwidth limited parallel implementation. Furthermore, each subprogram has a closed form solution, which can be easily accelerated.

Our results demonstrate that patch-based residual encoding can effectively recover residual signals arounds important anatomical structures rather than background noises present from measurements. Such effective use of measurements data makes the algorithm to provide outstanding performance compared to the existing one even at high acceleration.

The organization of this paper is as follows. Details of the proposed algorithm is presented in section 2. Section 3 shows the experimental results. Finally, section 4 gives concluding remarks.

2. THEORY

2.1. Problem Formulation

Let $\mathbf{y} \in \mathbb{C}^{mT}$ and $\mathbf{x} \in \mathbb{C}^{nT}$ denote a vectorized spatio-temporal k-space measurement and an unknown image, respectively. Here, m, n and T denote the number of k-space samples, pixels of image frame, and temporal frames, respectively. Then, a forward model for dynamic MR imaging problem is given by

$$\mathbf{y} = F\mathbf{x} + \mathbf{w} \tag{1}$$

and $\mathbf{w}$ denotes the measurement noise, and F denotes a spatio-temporal Fourier sensing matrix.

Suppose we are given a predictable component $\mathbf{m}_c$ of the unknown image such that we have

$$\mathbf{x} = \bar{\mathbf{m}} + \mathbf{d}\ , \tag{2}$$

where $\mathbf{d}$ denotes the unknown residual signal that needs to be estimated. Then, our imaging problem can be represented as

$$\mathcal{C}\left(\mathbf{d}\right) = \frac{1}{2}\|\mathbf{y} - F\bar{\mathbf{m}} - F\mathbf{d}\|_F^2 + \Psi(\mathbf{d}) \tag{3}$$

where $\Psi(\mathbf{d})$ denote a penalty terms for the unknown residual signal $\mathbf{d}$. In the following, we describe methods to estimate the prediction term $\bar{\mathbf{m}}$ and the residual term $\mathbf{d}$ using geometric similarities.

2.2. Prediction using Non-Local Motion Compensation

Recall that ME/MC is an essential step in video coding that uses motion vectors to exploit the temporal redundancies between frames. Jung et al [4] employed a overlapped block motion compensation (OBMC), where a multiple ME blocks are overlapped during compensation. By extending the OBMC, we are interested in minimizing the following *apodized* cost function for the image:

$$\bar{\mathbf{m}} = \arg\min_{\mathbf{m}} \sum_p \sum_{i \in \mathcal{N}_p} \exp\left(-\frac{\|\mathcal{R}_p\mathbf{m} - \mathcal{R}_i\mathbf{r}\|^2}{h}\right) \tag{4}$$

where $h > 0$ is a hyper parameter, $\mathcal{R}_p$ denotes an operator to extract the p-th patch, and $\mathcal{N}_p$ is the neighborhood index set to search the similarity patch, which is defined such that the l_2 norm difference from the p-th patch is less than some predefined threshold value. In Eq. (4), we can easily find the fixed point equation for $\mathcal{R}_p\mathbf{m}_c$ by calculating the derivative with respect to $\mathbf{m}_c$. Furthermore, if we first initialize $\bar{M} = \bar{M}^{(0)}$, where $\bar{M}^{(0)}$ is obtained using, for example, k-t FOCUSS [4] , then the resulting fixed point update equation is given by

$$\mathcal{R}_p\mathbf{m} = \frac{\sum_{i\in\mathcal{N}_p} \mathcal{R}_i\mathbf{r}_c \exp\left(-\frac{\|\mathcal{R}_p\mathbf{m}^{(0)} - \mathcal{R}_i\mathbf{r}\|^2}{h}\right)}{\sum_{i\in\mathcal{N}_p} \exp\left(-\frac{\|\mathcal{R}_p\mathbf{m}^{(0)} - \mathcal{R}_i\mathbf{r}\|^2}{h}\right)}\ , \tag{5}$$

where $\mathbf{r}_c$ represents c-th coil's reference frame. As we allow the overlapping of the patches, the overlapped pixel values are averaged after the fixed point iteration.

2.3. Residual Encoding using Patch-based Low-Rank Penalty

In this subsection, we exploit the geometric similarity in residual images to judiciously use residual measurement $\mathbf{y} - F\bar{\mathbf{m}}$ during residual encoding phase.

More specifically, for the current block $\mathbf{v}_{p1} = \mathcal{R}_{p1}\mathbf{d}$ with pixel numbers B, we search similarity patches $\{\mathbf{v}_{pq}\}_{q=2}^{Q_p}$ within the 3D neighborhood to construct a matrix

$$V_p = [\mathcal{R}_{p1}\mathbf{d},\ \mathcal{R}_{p2}\mathbf{d},\ \cdots,\ \mathcal{R}_{pQ_p}\mathbf{d}] \in \mathbb{R}^{B\times Q_p}. \tag{6}$$

Then the constraint term can be written using the following rank penalty:

$$\Psi(\mathbf{d}) = \lambda \sum_p \mathrm{Rank}(V_p). \tag{7}$$

One of the popular convex relaxation for the rank function is given by the nuclear norm [1]. However, as it has been shown that concave penalty outperforms that convex nuclear norm [10], we use the following rank prior [10]

$$\|V_p\|_\nu = \sum_{k=1}^{\mathrm{Rank}(V_p)} h_{\mu,\nu}\left(\sigma_k(V_p)\right), \quad 0 < \nu \le 1. \tag{8}$$

where the generalized Huber function $h_{\mu,\nu}(t)$ is defined as

$$h_{\mu,\nu}(t) = \left\{ \begin{array}{ll} |t|^2/2\mu, & \text{if } |t| < \mu^{1/(2-\nu)} \\ |t|^\nu/\nu - \delta & \text{if } |t| \ge \mu^{1/(2-\nu)} \end{array} \right. \tag{9}$$

and $\delta = (1/\nu - 1/2)\mu^{\nu/(2-\nu)}$ to make the function continuous. Then, the resulting regularization for the residual is given by

$$\Psi(\mathbf{d}) = \lambda \sum_p \|V_p(\mathbf{d})\|_\nu . \tag{10}$$

For the generalized Huber function in Eq. (9), even though it is not convex by itself, it is easy to show that $|t|^2/\mu - h_{\mu,\nu}(t)$ is strictly convex. Therefore, the Legendre-Fenchel transform tells us that there exist $g_{\mu,\nu}$ such that

$$h_{\mu,\nu}(t) = \min_s \{|s-t|^2/\mu + g_{\mu,\nu}(s)\} . \tag{11}$$

The corresponding rank penalty for a matrix V is given by

$$\begin{aligned} \|V\|_{h_{\mu,\nu}} &= \sum_{k=1} h_{\mu,\nu}(\sigma_k(V)) \\ &= \min_W \left\{ \frac{1}{\mu}\|V - W\|_F^2 + \|W\|_{g_{\mu,\nu}} \right\} \end{aligned} \tag{12}$$

where

$$\|W\|_{g_{\mu,\nu}} = \sum_{k=1} g_{\mu,\nu}(\sigma_k(V)). \tag{13}$$

Using Eq. (12), the resulting cost function is given by

$$\begin{aligned} \mathcal{C}\,(W, \mathbf{d}) = \sum_{c=1}^{C} \|\mathbf{y}_c - F\bar{\mathbf{m}}_c - F\mathbf{d}_c\|^2 \\ + \lambda \sum_p \min_W \left\{ \frac{1}{\mu}\|V_p(\mathbf{d}) - W_p\|_F^2 + \|W_p\|_{g_{\mu,\nu}} \right\} \end{aligned} \tag{14}$$

One of the main advantages of the proposed CCCP framework is that each subproblem has close form solutions. First, note that the problem can be decomposed into individual patches. Therefore, for a given estimate of $V_p^{(k)}$, we need to solve the following minimization problem:

$$W_p^{(k+1)} = \arg\min_W \left\{ \frac{1}{\mu}\|V_p^{(k)} - W\|_F^2 + \|W\|_{g_{\mu,\nu}} \right\} \tag{15}$$

Even though $g_{\mu,\nu}(s)$ in Eq. (13) is not convex for $\nu < 1$ and does not have close form expression, there exist a close form expression for the minimizer of Eq. (11) given as [12, 10]

$$\begin{aligned} \mathrm{shrink}_\nu(t,\mu) := \arg\min_s \{|s-t|^2/\mu + g_{\mu,\nu}(s)\} = \\ \max\{0, |t| - \mu|t|^{\nu-1}\} t/|t| . \end{aligned} \tag{16}$$

If $V_p^{(k)}$ has a singular value decomposition $V_p^{(k)} = L\Sigma U^H$, then the closed form solution for Eq. (15) is given by

$$W_p^{(k+1)} = L\, \mathrm{shrink}_\nu(\Sigma, \mu)\, U^H . \tag{17}$$

Next step is to find a closed form expression for the minimization of Eq. (14) with respect to $\mathbf{d}_c$. This can be done easily using conjugate gradient method. We are aware that Trzasko *et al* [8] proposed a FISTA based algorithm for similar patch-based rank penalty for calibration free parallel imaging. To apply FISTA[13], we need to apply patch-based SVD for each iteration of gradient step. In our CCCP framework, we nearly solve $\mathbf{d}$ using conjugate gradient step, after which SVD is applied. This makes algorithm converges much faster than FISTA.

3. EXPERIMENTAL RESULTS

Two different *in vivo* data sets are used to verify the algorithm performance. They were acquired from cartesian and radial trajectories, respectively. Retrospective downsampling from fully sampled data set was used to compare the results with the fully sampled reconstruction. Cartesian data was acquired using a 3 T whole-body MRI scanner. The acquisition sequence was bSSFP and prospective cardiac gating was used with following parameters : FOV 300 $\times$ 200 mm^2, TE/TR 1.37/2.7 ms, matrix size 128 $\times$ 128, and the acceleration factor was 7. Radial data was acquired using electrocordiogram-triggered segmented bSSFP pulse sequence from a healthy volunteer at 3T on a whole-body MR scanner with following parameters : FOV 200 $\times$ 300 mm^2, TR/TE 3.4/1.7 ms, matrix size was 192 $\times$ 384, 32 cardiac phases and 8 views per segment. Acceleration factor was 8.

In Fig. 1, we have compared k-t FOCUSS, k-t FOCUSS with ME/MC, and the proposed algorithm. Both of cartesian (a)(b) and radial (c)(d) results confirmed that the proposed method outperforms the other algorithms. Note that k-t FOCUSS is prone to blurring near cardiac wall boundaries as also confirmed from temporal reconstruction profile (IV). For residual reconstruction, existing k-t FOCUSS with ME/MC (II) contained significant amount of background noises, whereas the proposed residual encoding (III) removes most of them and retains only geometrically meaningful features along cardiac wall boundaries. Due to the noise, the reconstruction results using k-t FOCUSS (IV) and k-t FOCUSS with ME/MC (V) reconstruction still contain the aliasing artifacts, where the results using the proposed method (VI) are nearly aliasing free. Temporal profiles of the proposed reconstruction images (VI of (b)(d)) revealed accurate cardiac wall motions in cross-sectional segment. Arrows in temporal profiles indicate some specific positions where the proposed algorithm shows significantly better reconstruction performance.

4. CONCLUSION

In this paper, an improved motion compensated k-t FOCUSS algorithm using non-local motion compensation and patch-based residual encoding was proposed. In prediction step, a high resolution reference frame is generated by averaging

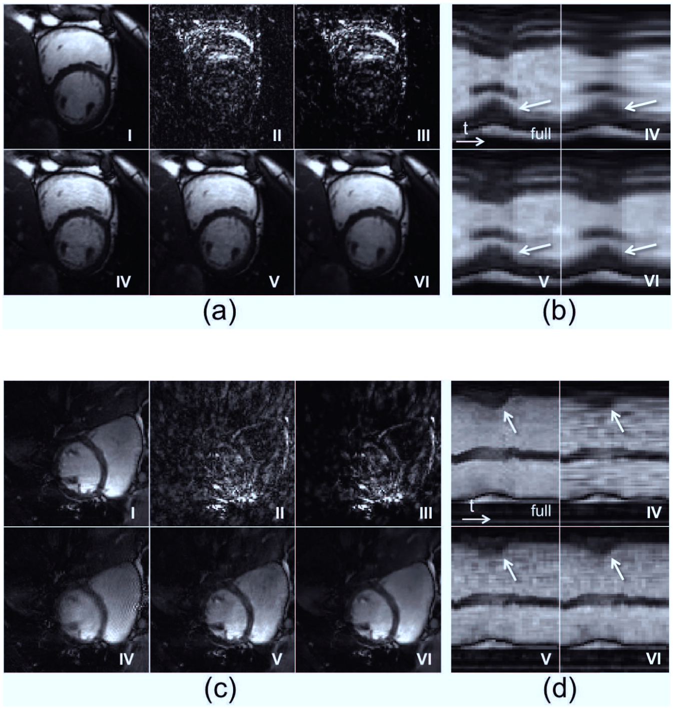

Fig. 1. Reconstruction images of the cartesian (a)(b) and radial (c)(d) data with acceleration factor of 7 and 8. (a)(c) represent ROI and (b)(d) represent temporal profile along horizontal cross-sectional view. Each sub-index represents the result of : I. ground truth, II. residual of k-t FOCUSS with ME/MC, III. residual of the proposed algorithm, IV. k-t FOCUSS, V. k-t FOCUSS with ME/MC, VI. proposed algorithm

images during diastole phase, and a NLMC improves the prediction accuracy by performing weighted averages of similarity patches. Our NLMC improves the accuracy improvement of prediction images and makes residual signals sparser. To judiciously use the residual signals during residual encoding, a residual encoding scheme using patch-based nonconvex low-rank penalty was proposed. To deal with the nonconvex and non-smooth penalty, a concave-convex procedure has been proposed. Extensive experimental results show that the proposed algorithm clearly reconstructs the important cardiac structures and improved over the conventional motion-compensated k-t FOCUSS.

5. REFERENCES

[1] E.J. Candès and B. Recht, "Exact matrix completion via convex optimization," *Foundations of Computational mathematics*, vol. 9, no. 6, pp. 717–772, 2009.

[2] M. Lustig, J.M. Santos, D.L. Donoho, and J.M. Pauly, "kt SPARSE: High frame rate dynamic mri exploiting spatio-temporal sparsity," in *Proceedings of the 13th Annual Meeting of ISMRM, Seattle*, 2006, p. 2420.

[3] D. L. Donoho, "Compressed sensing," *IEEE Trans. on Information Theory*, vol. 52, no. 4, pp. 1289–1306, April 2006.

[4] H. Jung, K. Sung, K.S. Nayak, E.Y. Kim, and J.C. Ye, "k-t FOCUSS: A general compressed sensing framework for high resolution dynamic MRI," *Magnetic Resonance in Medicine*, vol. 61, no. 1, pp. 103–116, 2009.

[5] S. Ravishankar and Y. Bresler, "MR image reconstruction from highly undersampled k-space data by dictionary learning," *IEEE Trans. on Medical Imaging*, vol. 30, no. 5, pp. 1028–1041, 2011.

[6] M. Akçakaya, TA Basha, B. Goddu, LA Goepfert, KV Kissinger, V. Tarokh, WJ Manning, and R. Nezafat, "Low-dimensional-structure self-learning and thresholding (LOST) : Regularization beyond compressed sensing for MRI reconstruction.," *Magnetic Resonance in Medicine*, vol. 66, no. 3, pp. 756–767, 2011.

[7] M. Jacob Z. Yang, "Robust non-local regularization framework for motion compensated dynamic imaging without explicit motion estimation," in *IEEE International Symposium on Biomedical Imaging (ISBI), Barcelona*, 2005.

[8] J.D. Trzasko and A. Manduca, "Calibrationless parallel MRI using CLEAR," in *IEEE Signals, Systems and Computers (ASILOMAR)*, 2011, pp. 75–79.

[9] S.P. Awate and E.V.R. DiBella, "Spatiotemporal dictionary learning for undersampled dynamic MRI reconstruction via joint frame-based and dictionary-based sparsity," in *IEEE International Symposium on Biomedical Imaging (ISBI), Barcelona*, 2012, pp. 318–321.

[10] R. Chartrand, "Nonconvex splitting for regularized low-rank+ sparse decomposition," *Los Alamos National Laboratory Report: LA-UR-11-11298*, 2012.

[11] A.L. Yuille and A. Rangarajan, "The concave-convex procedure," *Neural Computation*, vol. 15, no. 4, pp. 915–936, 2003.

[12] R. Chartrand and V. Staneva, "Restricted isometry properties and nonconvex compressive sensing," *Inverse Problems*, vol. 24, no. 3, pp. 035020, 2008.

[13] A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm with application to wavelet-based image deblurring," in *IEEE International Conference on Acoustics, Speech and Signal Processing, 2009*, pp. 693–696.