Ch 33 Swaps Revisited

Interest rate swap valuation methodology: “assume forward rates are realized”

Steps for valuing plain vanilla LIBOR-for-fixed interest rate swaps

1. Calculate the swap’s net cash flow assuming LIBOR rates in the future equal the forward rates calculated from instruments trading in the market today
2. Set the value of the swap equal to the present value of the net cash flows

Variations on the Vanilla Deal

Many interest rate swaps involve minor variations to the plain vanilla swap

- notional principal can change in a predetermined way over time
  – *step-up swaps* - notional principal is an increasing function of time
  – *amortizing swaps* - notional principal is a decreasing function of time
- the principal can be different on the two sides of a swap
- the frequency of payments can be different on the two sides of a swap
- the floating reference rate is not always LIBOR
  – in some swaps it is the commercial paper (CP) rate or the OIS rate
  – *basis swap* - a floating-floating interest rate swap with different reference rates

The variations above do not affect the valuation methodology.

Compounding Swaps

Another variation on the plain vanilla swap is a **compounding swap**

- interest is compounded forward instead of being paid on each payment date

The “assume forward rates are realized” approach can be used to deal with the fixed side of the swap because the payment that will be made at maturity is known with certainty.

The approach can also be used for the floating side because there exist a series of forward rate agreements (FRAs) where the floating-rate cash flows are exchanged for the values they would have if each floating rate equaled the corresponding forward rate.

Currency Swaps

Currency swaps

- enable an interest rate exposure in one currency to be swapped for an interest rate exposure in another currency
- two principals specified, one in each currency
- the principals are exchanged at both the beginning and the end of the life of the swap

Cross-currency interest rate swaps

- floating rate in one currency is exchanged for a fixed rate in another currency

Floating-for-floating and cross-currency interest rate swaps can be valued using the “assume forward rates are realized” rule. Future LIBOR rates in each currency are assumed to equal today’s forward rates. This enables the cash flows in the currencies to be determined.

More Complex Swaps

These swaps cannot use the rule “assume forward rates will be realized”.
Assume that an adjusted forward rate (not the actual forward rate) is realized.

LIBOR-in-Arrears Swap

A plain interest rate swap is designed so that the floating rate of interest on one payment date is paid on the next payment date.

An alternative is a **LIBOR-in-arrears swap**

- floating rate paid on a payment date equals the rate observed on the payment date itself

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Suppose the reset dates in the swap are t_i for $i = 0, 1, \ldots, n$ with $\tau_i = t_{i+1} - t_i$.

Let R_i be the LIBOR rate for the period between t_i and t_{i+1}.
Let F_i be the forward value of R_i.
Let σ_i be the volatility of this forward rate.

In a LIBOR-in-arrears swap the payment on the floating side at time t_i is based on R_i rather than R_{i-1}.

A convexity adjustment to the forward rate is necessary when the payment is valued.

The valuation should be based on the assumption that the floating rate paid is

$$F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1 + F_i \tau_i}$$

and not F_i.

CMS and CMT Swaps

A constant maturity swap (CMS) is an interest rate swap where floating rate equals the swap rate for a swap with a certain life.

Example: floating payments on a CMS swap might be made every 6 months at a rate equal to the 5-year swap rate.

Usually there is a lag so that the payment on a particular payment date is equal to the swap rate observed on the previous payment date.

Suppose rates are set at times $t_0, t_1, t_2, \ldots$ and payments are made at times $t_1, t_2, t_3, \ldots$ and the notional principal is L.

The floating payment at time t_{i+1} is

$$\tau_i L S_i$$

where $\tau_i = t_{i+1} - t_i$ and S_i is the swap rate at time t_i.

Let y_i be the forward value of the swap rate S_i. An adjustment to the forward swap rate is needed to value the payment at time t_{i+1}. So the realized swap rate is assumed to be

$$y_i - \frac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + F_i \tau_i}$$

rather than y_i.

In this equation $\sigma_{y,i}$ is the volatility of the forward swap rate, F_i is the current forward interest rate between times t_i and t_{i+1}, $\sigma_{F,i}$ is the volatility of this forward rate, and ρ_i is the correlation between the forward swap rate and the forward interest rate.

Also $G_i(x)$ is the price at time t_i of a bond as a function of its yield x. The bond pays coupons at rate y_i and has the same life and payment frequency as the swap from which the CMS rate is calculated. $G_i'(x)$ and $G_i''(x)$ are the first and second partial derivatives of G_i with respect to x.

The volatilities $\sigma_{y,i}$ can be implied from swaptions. The volatilities $\sigma_{F,i}$ can be implied from caplet prices. The correlation ρ_i can be estimated from historical data.

The equation involves a convexity adjustment and a timing adjustment.

The term

$$-\frac{1}{2}y_i^2\sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)}$$

is an adjustment based on the assumption that the swap rate S_i leads to only one payment at time t_i rather than to an annuity of payments.

The term

$$-\frac{y_i\tau_i F_i\rho_i\sigma_{y,i}\sigma_{F,i}t_i}{1+F_i\tau_i}$$

is an adjustment for the fact that the payment calculated from S_i is made at time t_{i+1} rather than t_i.

A constant maturity Treasury swap (CMT) works similarly to a CMS swap except that the floating rate is the yield on a Treasury bond with a specified life.

The analysis of a CMT swap is the same as that for a CMS swap with S_i defined as the par yield on a Treasury bond with the specified life.

Differential Swaps

A **differential swap** or *diff swap* is an interest rate swap where a floating interest rate is observed in one currency and applied to a principal in another currency.

Suppose that the LIBOR rate for the period between t_i and t_{i+1} in currency Y is applied to a principal in currency X with the payment taking place at time t_{i+1}.

Let V_i be the forward interest rate between t_i and t_{i+1} in currency Y.
Let W_i be the forward exchange rate for a contract with maturity t_{i+1}.

If the LIBOR rate in currency Y were applied to a principal in currency Y, the cash flow at time t_{i+1} would be valued on the assumption that the LIBOR rate at time t_i equals V_i.

A quanto adjustment is needed when it is applied to a principal in currency X.

Value the cash flow on the assumption that the LIBOR rate equals

$$V_i + V_i\rho_i\sigma_{W,i}\sigma_V t_i$$

where σ_V is the volatility of V_i, $\sigma_{W,i}$ is the volatility of W_i, and ρ_i is the correlation between V_i and W_i.

Equity Swaps

Equity swaps

- one party promises to pay the return on an equity index on a notional principal
- other party promises to pay a fixed or floating return on a notional principal

Goal

- enable fund managers to increase or reduce exposure to an index without buying and selling stock
- convenient way of packaging a series of forward contracts on an index to meet the needs of the market

Valuation

- between payment dates the equity cash flow and the LIBOR cash flow at the next payment date must be valued
- the LIBOR cash flow was fixed at the last reset date so it can be valued easily
- the value of the equity cash flow is

$$LE/E_0$$

  where L is the principal, E is the current value of the equity index, and E_0 is its value at the last payment date

Swaps with Embedded Options

Accrual Swaps

Accrual swaps are swaps where the interest on one side accrues only when the floating reference rate is within a certain range. Sometimes the range is fixed and sometimes it is reset periodically.

Cancelable Swap

A cancelable swap is a plain vanilla interest rate swap where one side has the option to terminate on one or more payment dates. Terminating a swap is the same as entering into the offsetting (opposite) swap.

Cancelable Compounding Swaps

Sometimes compounding swaps can be terminated on specified payment dates. On termination, the floating-rate payer pays the compounded value of the floating amounts up to the time of termination and the fixed-rate payer pays the compounded value of the fixed payments up to the time of termination.

Other Swaps

Index amortizing rate swap (indexed principal swap)

- the principal reduces in a way dependent on the level of interest rates

- the lower the interest rates, the greater the reduction in principal

Commodity swaps

- energy derivatives

Other swaps

- asset swaps
- total return swaps
- credit default swaps
- volatility swaps and variance swaps

Bizarre Deals

Some swaps have payoffs that are calculated in bizarre ways.