Ch 34 Energy and Commodity Derivatives

Commodity prices often exhibit mean reversion (similar to interest rates).
They are also sometimes subject to jumps.

Some derivatives depend on variables with no systematic risk (weather, insurance).
The expected value of temperature at some location can be assumed to be the same in a risk-neutral world and in the real world. So historical data is more useful for valuing these derivatives.

Agricultural Commodities

Include products that are grown (or created from products that are grown)

- corn
- wheat
- soybeans
- cocoa
- coffee
- sugar
- cotton
- frozen orange juice

Include products related to livestock

- cattle
- hogs
- pork bellies

The price is determined by supply and demand.

stocks-to-use-ratio - ratio of year-end inventory to the year's usage

- typically between 20% and 40%
- has an impact on price volatility
- as ratio becomes lower, commodity price becomes more sensitive to supply changes
  – volatility increases

Mean reversion

- as prices decline, farmers find it less attractive to produce the commodity
  – supply decreases, creating upward pressure on price
- as prices increase, farmers are more likely to devote resources to producing the commodity
  – supply increases, creating downward pressure on the price

Prices of agricultural commodities tend to be seasonal

- storage is expensive
- there is limit to the length of time for which a product can be stored
- weather plays a key role in determining the price
  – frosts and hurricanes can decimate crop, etc
- volatility of the price of a commodity that is grown tends to be highest at pre-harvest times
  – declines when the size of the crop is known
- price process liable to exhibit jumps due to weather during the growing season

Metals

Includes metals such as

- gold
- silver
- platinum
- palladium
- copper
- tin
- lead
- zinc
- nickel
- aluminum

Metals have different characteristics from agricultural commodities

- prices are unaffected by weather and are not seasonal
- extracted from ground
- divisible
- relatively easy to store
- some are used almost entirely in the manufacture of goods (copper)
  – classified as consumption assets
- some are held purely for investment as well as for consumption (gold and silver)
  – classified as investment assets

Inventory levels determine short-term price volatility.
One potential source of supply is recycling.

Mean reversion

- investment asset metals are not usually assumed to follow mean-reverting processes
  – mean reversion would give rise to an arbitrage opportunity for the investor
- consumption asset metals may exhibit mean reversion
  – as price increases, it is likely to become less attractive to use the metal in some processes

– as price increases, it is more economically viable to extract from difficult locations
– so there is a downward pressure on the price

Energy Products

A wide range of energy derivatives trade in both OTC market and on exchanges.

Three examples of energy products

- oil
- natural gas
- electricity

All three follow mean reverting processes

- as price of energy source rises, it is likely to be consumed less and produced more
  – creates downward pressure on prices
- as price of energy source falls, it is likely to be consumed more and produced less
  – creates upward pressure on prices

Crude Oil

- largest commodity market in the world
- many grades of crude oil, reflecting variations in gravity and sulfur content
- refined into products such as gasoline, heating oil, fuel oil, kerosene
- contracts settled both by cash and by physical delivery

Natural Gas

- supplier is not necessarily the same as the producer
  – due to deregulation and elimination of monopolies
- suppliers faced with the problem of meeting daily demand
- seller is usually responsible for moving the gas through pipelines to the specified location
- popular source of energy for heating buildings
- also used to produce electricity which is used for air conditioning
- demand is seasonal and dependent on weather

Electricity

- cannot be easily stored
- air conditioning is a major use of electricity
- demand and price is much greater in the summer months

There is a contract in electricity and natural gas markets called the **swing option** also known as the *take-and-pay option*. A minimum and maximum for the amount of power that must be purchased at a certain price by the option holder is specified for each day during a month and for the month in total. The option holder can change (swing) the rate at which the power is purchased during the month. There is usually a limit on the total number of changes that can be made.

Modeling Commodity Prices

Expected future price of the commodity in the traditional risk-neutral world is the futures price.

A Simple Process

Assume the expected growth rate in the commodity is dependent solely on time and the volatility of the commodity price is constant.

The risk-neutral process for the commodity price S has the form

$$\frac{dS}{S} = \mu(t)\,dt + \sigma\,dz$$

and

$$F(t) = \hat{E}[S(t)] = S(0)e^{\int_0^t \mu(\tau)\,d\tau}$$

where $F(t)$ is the futures price for a contract with maturity t and $\hat{E}$ denotes expected value in a risk-neutral world.

It follows that

$$\ln F(t) = \ln S(0) + \int_0^t \mu(\tau)\,d\tau$$

Differentiate both sides with respect to time

$$\mu(t) = \frac{\partial}{\partial t}[\ln F(t)]$$

Mean Reversion

Most commodity prices follow mean-reverting processes.

A more realistic risk-neutral process followed by the commodity price S is

$$d\ln S = [\theta(t) - a\ln S]\,dt + \sigma\,dz$$

This incorporates mean reversion and is analogous to the lognormal process assumed for the short-term interest rate.

Note that it can also be written

$$\frac{dS}{S} = [\theta^*(t) - a \ln S]\, dt + \sigma\, dz$$

The equivalence can be shown using Ito's lemma when $\theta^*(t) = \theta(t) + \frac{1}{2}\sigma^2$.

A trinomial tree can be constructed for S to determine the value of $\theta(t)$
such that $F(t) = \hat{E}[S(t)]$.

Interpolation and Seasonality

With a large number of time steps, it is necessary to interpolate between futures prices to obtain a futures price at the end of each time step. When there is seasonality, the interpolation procedure should reflect this. The volatility of a commodity sometimes shows seasonality.

Jumps

Some commodities exhibit price jumps because of weather-related demand shocks (electricity, natural gas) or because of weather-related supply shocks (agriculture).

Jumps can be incorporated into the process for the spot price

$$d \ln S = [\theta(t) - a \ln S]\, dt + \sigma\, dz + dp$$

where dp is the Poisson process generating the percentage jumps.

Once the jump frequency and jump size probability distribution have been chosen, the average increase in the commodity price at a future time t that is a result of jumps can be calculated.

To determine $\theta(t)$ use the trinomial tree method with the futures price for maturity t reduced by the increase caused by jumps.

Monte Carlo simulation can be used to implement the model.

Other Models

More sophisticated models are sometimes used for oil prices.

Let y be the convenience yield.
The proportional drift of the spot price is $r - y$, where r is the short-term risk-free rate.

A natural process to assume for the spot price is

$$\frac{dS}{S} = (r - y)\, dt + \sigma_1\, dz_1$$

The convenience yield y can be modeled as a mean-reverting process

$$dy = k(\alpha - y)\,dt + \sigma_2\,dz_2$$

where k and α are constants and dz_2 is a Wiener process, which is correlated with dz_1.

Processes for gas and electricity prices with stochastic volatility have been proposed

$$\frac{dS}{S} = a(b - \ln S)\,dt + \sqrt{V}\,dz_1$$

$$dV = c(d - V)\,dt + e\sqrt{V}\,dz_2$$

where a, b, c, d, and e are constants, and dz_1 and dz_2 are correlated Wiener processes.

Weather Derivatives

The performance of many companies can be adversely affected by the weather.
These companies should consider hedging their weather risk.

HDD - heating degree days

$$\mathrm{HDD} = \max(0, 65 - A)$$

CCD - cooling degree days

$$\mathrm{CDD} = \max(0, A - 65)$$

where A is the average of the highest and lowest temperature during the day at a specified weather station measured in degrees Fahrenheit.

A typical OTC product is a forward or option contract providing a payoff dependent on the cumulative HDD or CDD during a month. Contracts are usually settled in cash.

Most weather derivatives are entered into by energy producers and energy consumers

- HDD is a measure of volume of energy required for heating during the day
- CDD is a measure of volume of energy required for cooling during the day

Weather futures and options also trade on exchanges.

Insurance Derivatives

Derivatives contracts used for hedging purposes have many of the same characteristics as insurance contracts. Both are designed to provide protection against adverse events.

Reinsurance

- used by insurance industry to hedge exposure to catastrophic (CAT) risks

CAT bond

- alternative to traditional reinsurance
- issued by subsidiary of insurance company
- pays a higher-than-normal interest rate
- bond holder agrees to provide excess-of-loss reinsurance contract
  – the interest or the principal or both can be used to meet claims, depending on the terms

Pricing Weather and Insurance Derivatives

Weather and insurance derivatives have no systematic risk (risk that is priced by the market).
So estimates made from historical data can be assumed to apply to the risk-neutral world.

Procedure

1. Use historical data to estimate the expected payoff
2. Discounting the estimated expected payoff at the risk-free rate

The uncertainty about how the underlying variables grow with time is itself uncertain

- uncertainty about February HDD in 4 years is usually only a little greater than the uncertainty about the February HDD in 1 year
- uncertainty about earthquake losses for a period starting in 4 years is usually only a little greater than the uncertainty about earthquake losses for a similar period starting in 1 year

In insurance there is no statistically significant correlation between the returns from CAT bonds and stock market returns. This confirms there is no systematic risk and that valuations can be based on the historical data collected by insurance companies.

CAT bonds give a high probability of an above-normal rate of interest and a low probability of a big loss. However the expected return is higher than the return on risk-free investments. But the risk in CAT bonds can be completely diversified away in a large portfolio, in theory. So CAT bonds have the potential to improve risk-return trade-offs.

How an Energy Producer Can Hedge Risks

Two components to the risks facing an energy producer

1. Risk associated with the market price for the energy (price risk)
2. Risk associated with the amount of energy that will be bought (volume risk)

Hedging

- price risk can be hedged using energy derivative contracts.
- volume risk can be hedged using weather derivatives.

Procedure

- Let Y be profit for a month
- Let P by average energy prices for the month
- Let T be relevant temperature variable (HDD or CDD) for the month

An energy producer can use historical data to obtain a best-fit linear regression relationship

$$Y = a + bP + cT + \epsilon$$

where ϵ is the error term.

The energy producer can then hedge risks for the month by

- taking a position $-b$ in energy forwards or futures
- taking a position $-c$ in weather forwards or futures

The relationship can also be used to analyze the effectiveness of alternative option strategies.