Ch 26 Exotic Options

exotic options - nonstandard products created by financial engineers

Why?

- generally more profitable than plain vanilla products
- may meet a genuine hedging need in the market
- tax, accounting, legal, or regulatory reasons
- designed to reflect a view on potential future movements in particular market variables

Packages

A **package** is a portfolio consisting of standard European calls, standard European puts, forward contracts, cash, and the underlying asset itself.

Examples of packages

- bull spreads
- bear spreads
- butterfly spreads
- calendar spreads
- straddles
- strangles
- etc

Any derivative can be converted into a zero-cost product by deferring payment until maturity.

Nonstandard American Options

Standard American option

- can exercise at any time during the life of the option
- exercise price is always the same

Nonstandard features

- early exercise restricted to certain dates (*Bermuda option*)
- early exercise may be allowed during only part of the life of the option
  – initial lock out period
- strike price may change during life of option

Warrants issued by companies on their own stock often have some or all of these features.

Can be valued using a binomial tree.

Gap Options

A gap call option is a European call option that pays off $S_T - K_1$ when $S_T > K_2$.
A gap put option is a European put option that pays off $K_1 - S_T$ when $S_T < K_2$.

The difference between a gap call and a regular call with strike price K_2 is that the payoff when $S_T > K_2$ is increased or decreased by $K_2 - K_1$ (depending on whether this is positive or negative).

Can be valued by a small modification to the Black-Scholes formula.

The price is greater than the price for a regular call option with strike price K_2 by

$$(K_2 - K_1)e^{-rT}N(d_2)$$

This is because the probability of exercise is $N(d_2)$ and the payoff when exercising the gap option is greater than that of the regular option by $K_2 - K_1$.

Forward Start Options

Forward start options are options that will start at some time in the future.

Employee stock options can be viewed as forward start options. This is because the company commits to granting at-the-money options to employees in the future.

Consider a forward start at-the-money European call option that will start at time T_1 and mature at time T_2. Suppose asset price is S_0 at time zero and S_1 at time T_1.

Recall that the value of an at-the-money call option on an asset is proportional to asset price.
The value of the forward start option at time T_1 is therefore cS_1/S_0, where c is the value at time zero of an at-the-money option that lasts for $T_2 - T_1$.

Risk-neutral valuation gives the value of the forward start option at time zero as

$$e^{-rT_1}\hat{E}\left[c\frac{S_1}{S_0}\right]$$

where $\hat{E}$ denotes expected value in a risk-neutral world.

Since c and S_0 are known and $\hat{E}[S_1] = S_0 e^{(r-q)T_1}$ the value of the forward start option is

$$f = ce^{-qT_1}$$

For a non-dividend-paying stock, $q = 0$, so the value of the forward start option is exactly the same as the value of a regular at-the-money option with the same life as the forward start option.

Cliquet Options

A cliquet option is a series of call or put options with rules for determining the strike price.

Suppose the reset dates are $\tau, 2\tau, \ldots, (n-1)\tau$, with $n\tau$ being the end of the cliquet's life.

Then the first option with strike price K lasts between times 0 and τ.
The second option provides payoff at time 2τ with strike price equal to asset value at time τ.
The third option provides payoff at time 3τ with strike price equal to asset value at time 2τ.
And so on.

This is a regular option plus $n-1$ forward start options.

Some cliquet options are more complicated

- may have upper and lower limits on the total payoff over the whole period
- may terminate at the end of a period if asset price is in a certain range

Monte Carlo simulation is often the best approach for valuation.

Compound Options

Compound options are options on options.

Four main types of compound options

- call on a call
- put on a call
- call on a put
- put on a put

Compound options have two strike prices and two exercise dates.

Example

1. Buyer buys a compound option (call on a call)
2. Buyer has the right to pay the first strike price K_1 at time T_1 to receive a call option
3. Buyer has the right to buy the underlying asset for the second strike price K_2 at time T_2

The option will be exercised at T_1 only if the value of the option on that date is greater than K_1.

With the usual assumption of geometric Brownian motion, European-style compound options can be valued analytically in terms of integrals of the bivariate normal distribution.

Chooser Options

A chooser option has the feature that, after a specified period of time, the holder can choose whether the option is a call or a put.

Suppose the time the choice is made is T_1. Then the value of the choose option at this time is

$$\max(c, p)$$

where c is the value of the call and p is the value of the put.

If both underlying options are European and have the same strike price, the choose option can be valued using put-call parity.

Suppose S_1 is the asset price at time T_1, K is the strike price, T_2 is the maturity of the options, and r is the risk-free interest rate.

Put-call parity implies that

$$\begin{aligned}\max(c, p) &= \max(c, c + Ke^{-r(T_2-T_1)} - S_1 e^{-q(T_2-T_1)}) \\ &= c + e^{-q(T_2-T_1)} \max(0, Ke^{-(r-q)(T_2-T_1)} - S_1)\end{aligned}$$

This shows that the choose option is a package consisting of:

1. A call option with strike price K and maturity T_2
2. An amount $e^{-q(T_2-T_1)}$ of put options with strike price $Ke^{-(r-q)(T_2-T_1)}$ and maturity T_1

More complex chooser options can be defined where the call and the put do not have the same strike price and time to maturity. Then they are not packages and have features similar to compound options.

Barrier Options

Barrier options are options where the payoff depends on whether the underlying asset's price reaches a certain level during a certain period of time.

These are attractive to some because they are less expensive than the corresponding regular options.

Barrier options can be either *knock-out options* or *knock-in-options*

- knock-out option ceases to exist when underlying asset price reaches a certain barrier
- knock-in options comes into existence only when underlying asset price reaches a barrier

A *down-and-out call* is one type of knock-out option

- regular call option that ceases to exist if asset price reacher a certain barrier level H
- the barrier level is below the initial asset price

A *down-and-in call* is the corresponding knock-in option

- regular call option that comes into existence only if asset price reaches the barrier level
- the barrier level is below the initial asset price

An *up-and-out call* is a type of knock-out option

- regular call option that ceases to exist if asset price reaches a barrier level H
- the barrier level is above the initial asset price

An *up-and-in call* is the corresponding knock-in option

- regular call option that comes into existence only if asset price reaches the barrier level
- the barrier level is above the initial asset price

Put barrier options are defined similarly to call barrier options.

All valuations make the usual assumption that the probability distribution for the asset price at a future time is lognormal. Analytical formulas must be adjusted depending on the frequency at which the asset price S is observed (continuously vs discretely).

Barrier options often have different properties from regular options

- sometimes vega is negative (consider up-and-out call with asset price close to barrier)
  - as volatility increases, probability the barrier will be hit increases
  - so a volatility increase can cause the price of the barrier option to decrease

Disadvantages

- a spike in asset price can cause the option to be knocked in or out

Alternative: *Parisian option*

- asset price has to be above or below barrier for a period of time for the option to be knocked in or out
- the time period requirement may be either continuous or aggregate
- more difficult to value than regular barrier options
  - use Monte Carlo simulation and binomial trees

Binary Options

Binary options are options with discontinuous payoffs.

An example is a *cash-or-nothing call*

- pays off nothing if asset price ends up below the strike price at time T
- pays a fixed amount Q if asset price ends up above the strike price at time T

In a risk-neutral world, the probability of the asset price being above the strike price at the maturity of an option is $N(d_2)$. So the value of the cash-or-nothing call option is

$$f = Qe^{-rT}N(d_2)$$

Another example is a *cash-or-nothing put*

- pays off Q if asset price is below strike price
- pays off nothing is asset price is above strike price

$$f = Qe^{-rT}N(-d_2)$$

Another type of binary option is an *asset-or-nothing call*

- pays off nothing if the underlying asset price ends up below the strike price
- pays off the asset price if it ends up above the strike price

$$f = S_0e^{-qT}N(d_1)$$

The corresponding put option is the *asset-or-nothing put*

- pays off nothing if the underlying asset price ends up above the strike price
- pays off the asset price if it ends up below the strike price

$$f = S_0e^{-qT}N(-d_1)$$

A regular European call option is equivalent to the combination:

1. long position in an asset-or-nothing call
2. short position in a cash-or-nothing where the cash payoff equals the strike price

A regular European put option is equivalent to the combination:

1. long position in a cash-or-nothing put where the cash payoff equals the strike price
2. short position in an asset-or-nothing

Lookback Options

The payoffs from lookback options depend on the maximum or minimum asset price reached during the life of the option.

One example is the *floating lookback call*

- payoff is the amount the final asset price exceeds minimum asset price during option life
- allows holder to buy underlying asset at lowest price achieved during the life of the option

The corresponding option is the *floating lookback put*

- payoff is the amount the maximum asset price during option life exceeds final asset price
- allows holder to sell underlying asset at highest price achieved during the life of the option

There are analytical valuation formulas for floating lookback options.

Another example is the fixed lookback option, in which a strike price is specified.

One type is the *fixed lookback call*

- payoff is the same as a regular European call option except that final asset price is replaced by the maximum asset price during option life

The corresponding option is the *fixed lookback put*

- payoff is the same as a regular European put option except that final asset price is replaced by the minimum asset price during option life

Lookback options are appealing to investors but are expensive compared to regular options.
The value can be sensitive to the frequency with which asset price is observed.

Shout Options

A shout option is a European option where the holder can shout to the writer at one time.

At the end of option life, the holder receives either the usual payoff or the intrinsic value at the time of the shout, whichever is greater.

Suppose strike price is $50 and the holder of a call shouts when the price of the underlying is $60. If final asset price is less than $60, the holder receives a payoff of $10. If it is greater than $60, the holder receives the excess of the asset price over $50.

The payoff from the option is

$$\max(0, S_T - S_\tau) + (S_\tau - K)$$

where τ is the time of the shout, S_τ is the asset price at this time, K is the strike price, and S_T is the asset price at time T.

So the value at time τ if the holder shouts is the present value of $S_\tau - K$ (received at time T) plus the value of a European option with strike price S_τ.

A shout option is valued via binomial tree.

Asian Options

Asian options are options where the payoff depends on the average of the price of the underlying asset during the life of the option.

The payoff from an *average price call* is

$$\max(0, S_{\text{ave}} - K)$$

The payoff from an *average price put* is

$$\max(0, K - S_{\text{ave}})$$

These are less expensive than regular options, and they can be valued using similar formulas to those used for regular options, assuming that S_{ave} is lognormal. A popular approach is to fit a lognormal distribution to the first two moments of S_{ave} and use Black's model.

Another type of Asian option is an average strike option.

The payoff from an *average strike call* is

$$\max(0, S_T - S_{\text{ave}})$$

The payoff from an *average strike put* is

$$\max(0, S_{\text{ave}} - S_T)$$

Average strike options can guarantee that the average price paid for an asset in frequent trading over a period of time is not greater than the final price. They can also guarantee that the average price received for an asset in frequent trading over a period of time is not less than the final price. It can be valued as an option to exchange one asset for another when S_{ave} is assumed to be lognormal.

Options Involving Several Assets

Options involving two or more risky assets are *rainbow options*.

The most popular option involving several assets is a European *basket option*

- payoff is dependent on the value of a portfolio (basket) of assets
- assets are either individual stocks or stock indices or currencies
- can be valued via Monte Carlo simulation by assuming assets follow correlated geometric Brownian motion processes
- can also be valued by calculating the first two moments of the basket at maturity of the option in a risk-neutral world, assuming the value of the basket is lognormally distributed at that time, and using Black's model

Volatility and Variance Swaps

A volatility swap is an agreement to exchange the realized volatility of an asset between time 0 and time T for a prespecified fixed volatility.

The realized volatility from n daily observations on asset price from time 0 to time T is

$$\bar{\sigma} = \sqrt{\frac{252}{n-2} \sum_{i=1}^{n-1} \left[\ln\left(\frac{S_{i+1}}{S_i} \right) \right]^2}$$

where S_i is the ith observation on the asset price.

The payoff from the volatility swap at time T to the payer of the fixed volatility is

$$L_{\text{vol}}(\bar{\sigma} - \sigma_K)$$

where L_{vol} is the notional principal and σ_K is the fixed volatility.

A volatility swap has exposure only to volatility, whereas an option provides complex exposure to the asset price and volatility.

A variance swap is an agreement to exchange the realized variance rate $\bar{V}$ between time 0 and time T for a prespecified variance rate.

The variance rate is the square of the volatility, $\bar{V} = \bar{\sigma}^2$.

Variance swaps are easier to value than volatility swaps because the variance rate between time 0 and time T can be replicated using a portfolio of put and call options.

The payoff from a variance swap at time T to the payer of the fixed variance rate is

$$L_{\text{var}}(\bar{V} - V_K)$$

where L_{var} is the notional principal and V_K is the fixed variance rate.

Static Options Replication

Static options replication is a technique for hedging exotic options

- involves searching for a portfolio of actively traded options that replicates the exotic option
- shorting this position provides the hedge

Basic principle

- if two portfolios are worth the same on a certain boundary, they are also worth the same at all interior points of the boundary

To hedge a derivative, the portfolio that replicates its boundary conditions must be shorted.
The portfolio must be unwound when any part of the boundary is reached.

Static options replication has the advantage over delta hedging that it does not require frequent rebalancing. It can be used for a wide range of derivatives. There is a great deal of flexibility in choosing the boundary that is to be matched and the options that are to be used.