Ch 24 Credit Risk

Credit risk arises from the possibility that borrowers and counterparties in derivatives transactions may default.

Credit Ratings

- ratings agencies provide ratings describing creditworthiness of corporate bonds
- high credit rating means almost no chance of default

Historical Default Probabilities

- for IG bonds, probability of default in a year tends to be an increasing function of time
  – because issuer is initially creditworthy, but financial health could decline over time
- for HY bonds, probability of default is often a decreasing function of time
  – for these bonds, the next year or two may be critical
  – the longer the issuer survives, the greater the chance its financial health improves

Hazard Rates

unconditional default probability - probability of default during a particular year

Average cumulative default rate (%)

Year 1	Year 2	Year 3
16.448	27.867	36.908

Probability of default during third year is $36.908 - 27.867 = 9.041\%$
Probability bond survives until end of year 2 is $100 - 27.867 = 72.133\%$
Probability of default during third year conditional on no earlier default is

$$0.09041/0.72133 = 12.53\%$$

The 12.53% is a conditional probability for a 1-year time period.

The **hazard rate** $\lambda(t)$ at time t is defined so that $\lambda(t)\Delta t$ is the probability of default between time t and $t + \Delta t$ conditional on no earlier default.

Let $V(t)$ be the cumulative probability of the company surviving to time t.
The conditional probability of default between time t and $t + \Delta t$ is

$$\frac{V(t) - V(t + \Delta t)}{V(t)}$$

This equals $\lambda(t)\Delta t$ by definition, so it follows that

$$V(t + \Delta t) - V(t) = -\lambda(t)V(t)\Delta t$$

$$\frac{dV(t)}{dt} = -\lambda(t)V(t)$$

$$V(t) = e^{-\int_0^t \lambda(\tau)d\tau}$$

Let $Q(t)$ be probability of default by time t, so $Q(t) = 1 - V(t)$, which gives

$$Q(t) = 1 - e^{-\int_0^t \lambda(\tau)d\tau}$$

$$Q(t) = 1 - e^{-\bar{\lambda}(t)t}$$

where $\bar{\lambda}(t)$ is the average hazard rate between time 0 and time t.

This is also called the *default intensity*.

Recovery Rates

When a company goes bankrupt, those owed money by the company file claims against the assets of the company. Creditors may agree to a partial payment of their claims. Assets may be sold by a liquidator and proceeds used to meet claims as much as possible. Some claims have priority over other claims.

recovery rate - bond's market value a few days after default as a percentage of its face value

Historical average recovery rates

Class	*Average recovery rate (%)*
Senior secured bond	51.6
Senior unsecured bond	37.0
Senior subordinated bond	30.9
Subordinated bond	31.5
Junior subordinated bond	24.7

The Dependence of Recovery Rates on Default Rates

Mortgages

- average recovery rate on mortgages is negatively related to the mortgage default rate
- as mortgage default rate increases, foreclosure leads to increased supply
- leads to decline in house prices

- results in decline in recovery rates

Corporate Bonds

- average recovery rate has a similar negative dependence on default rates
- when bond default rate is low, economic conditions are usually good
  – so average recovery rate on bonds that do default can be high
- when bond default rate is high, economic conditions are usually poor
  – so average recovery rate on defaulting bonds may be low

Result of the negative dependence is that poor results of a bad year are emphasized, because high default rate is usually accompanied by low recovery rate.

Estimating Default Probabilities from Bond Yield Spreads

A bond's yield spread is the excess of the promised yield on the bond over the risk-free rate.

Let bond yield spread for a T-year bond be $s(T)$ per annum. This means the average loss rate on the bond between time 0 and time T should be approximately $s(T)$ per annum.

Let average hazard rate during this time be $\bar{\lambda}(t)$.
Another expression for the average loss rate is $\bar{\lambda}(T)(1-R)$ where R is recovery rate.

So it is approximately true that

$$\bar{\lambda}(T)(1-R) = s(T)$$

$$\bar{\lambda}(T) = \frac{s(T)}{1-R}$$

Matching Bond Prices

A more precise calculation involves choosing hazard rates so that they match bond prices.

The Risk-Free Rate

- benchmark risk-free rate used by bond traders is usually a Treasury rate
- Treasury rates are too low to be useful proxies for risk-free rates
- CDS spreads can be used to imply risk-free rates by comparing bond yields to CDS spreads
- implied risk-free rate is often close to the corresponding LIBOR/swap rate

Asset Swap Spreads

- LIBOR/swap rate is often used as the risk-free benchmark for credit calculations
- asset swap spreads provide a useful direct estimate of the spread of bond yields over the LIBOR/swap

curve

Present value of the asset swap spread is the amount by which the price of the corporate bond is exceeded by the price of a similar risk-free bond where the risk-free rate is assumed to be given by the LIBOR/swap curve.

Comparison of Default Probability Estimates

- default probabilities estimated from historical data are usually lower than those derived from bond yields

Rating	*Historical hazard rate*	*Hazard rate from bonds*	*Ratio*	*Difference*
Aaa	0.04	0.60	17.0	0.56
Aa	0.09	0.73	8.2	0.64
A	0.21	1.15	5.5	0.94
Baa	0.42	2.13	5.0	1.71
Ba	2.27	4.67	2.1	2.50
B	5.67	8.02	1.4	2.35
Caa and lower	12.50	18.39	1.5	5.89

- ratio of historical rate to bond price derived rate is high for IG company bonds
- difference between the two rates tends to increase as credit rating declines

Real-World vs Risk-Neutral Probabilities

- default probabilities implied from credit spreads are risk-neutral estimates
  – can be used to calculate expected cash flows in a risk-neutral world with credit risk
- default probabilities calculated from historical data are real-world (*physical*) probabilities
  – lower than risk-neutral probabilities

Why the difference?

- corporate bonds are relatively illiquid, so returns are higher than they otherwise would be
- traders may be allowing for depression scenarios worse than anything in the historical data
- bonds do not default independently of each other
  – gives rise to systematic risk (cannot be diversified away)
  – traders earn excess expected return for bearing the risk

credit contagion - default by one company has a ripple effect resulting in defaults by others

Bonds also have nonsystematic (idiosyncratic) risk which is not so easy to diversify away. This is because bond returns are highly skewed with limited upside. For example, a bond may have a 99% change of a 7% return and a 1% change of -60% return. The first outcome corresponds to no default and the second to default. This type of risk is difficult to diversify away in the way that nonsystematic risk from stocks can be diversified away. It would require thousands of different bonds.

In practice bond portfolios are far from fully diversified. So traders may earn extra return for bearing nonsystematic risk as well as bearing systematic risk.

Which Default Probability Estimate Should Be Used?

Use risk-neutral default probabilities when

- valuing credit derivatives
- estimating the impact of default risk on the pricing of instruments

Use real-world default probabilities when

- carrying out scenario analyses to calculate potential future losses from defaults

Using Equity Prices to Estimate Default Probabilities

Estimating real-world probability of default relies on company's credit rating.
However, credit ratings are revised relatively infrequently.
Equity prices can provide more up-to-date information for estimating default probabilities.

Suppose a firm has one zero-coupon bond outstanding and the bond matures at time T.

V_0 : Value of company assets today
V_T : Value of company assets at time T
E_0 : Value of company equity today
E_T : Value of company equity at time T
D : Debt repayment due at time T
σ_V : Volatility of assets (assumed constant)
σ_E : Instantaneous volatility of equity

If $V_T < D$ then it is rational for the company to default on the debt at time T.
Then the value of equity is zero.

If $V_T > D$ then the company should make the debt repayment at time T.
Then the value of equity is $V_T - D$.

The model gives the value of firm equity at time T as

$$E_T = \max(V_T - D, 0)$$

This shows that the equity is a call option on the value of the assets with a strike price equal to the repayment required on the debt.

The Black-Scholes formula gives the value of the equity today as

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

The value of the debt today is $V_0 - E_0$.

The risk-neutral probability the company will default on the debt is $N(-d_2)$.

From Ito's lemma

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0$$

The above two equations provide two conditions that must be satisfied by V_0 and σ_V.
Solve the simultaneous equations for these two variables, which can then determine $N(-d_2)$.

Extensions of the above model

- assume a default occurs whenever the value of the assets falls below a barrier level
- allow payments on debt instruments to be required at more than one time

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Accuracy of the model

- this model produces a good ranking of default probabilities (risk-neutral or real-world)
- underlying assumption is that the ranking of risk-neutral default probabilities of different companies is the same as the ranking of their real-world default probabilities

Credit Risk in Derivatives Transactions

- bilaterally cleared derivatives transactions use a Master Agreement
- Master Agreement defines what happens under an *event of default*

event of default

- fail to make payments as required
- fail to post collateral as required
- declare bankruptcy

The other side has the right to terminate all outstanding transactions.

Two circumstances that will likely to lead to a loss for the nondefaulting party:

1. Total value of transactions to the nondefaulting party is positive and greater than the collateral posted by the defaulting party. The nondefaulting party is then an unsecured creditor for the uncollateralized value of the transactions.
2. Total value of the transactions to the defaulting party is positive and the collateral posted by the nondefaulting party is greater than this value. The nondefaulting party is then an unsecured creditor for the return of the excess collateral it has posted.

CVA and DVA

credit value adjustment - present value of expected cost to bank of a default by counterparty
debt value adjustment - present value of cost to counterparty of a default by bank

The possibility of the bank defaulting is a benefit to the bank

- it means the bank will not have to make required payments on its derivatives
- so DVA is a benefit to the bank and a cost to the counterparty

The no-default value f_{nd} is the value of outstanding transactions assuming neither side defaults.
The value to the bank when possible defaults are taken into account is

$$f_{\mathrm{nd}} - \mathrm{CVA} + \mathrm{DVA}$$

Suppose the life of the longest outstanding derivative is T years.
The interval between time 0 and time T is divided into N subintervals.

$$\mathrm{CVA} = \sum_{i=1}^{N} q_i v_i \qquad \mathrm{DVA} = \sum_{i=1}^{N} q_i^* v_i^*$$

q_i : risk-neutral probability of counterparty defaulting during ith interval
v_i : present value of expected loss to bank if counterparty defaults at midpoint of ith interval

q_i^* : risk-neutral probability of bank defaulting during ith interval
v_i^* : present value of expected loss to counterparty if bank defaults at midpoint of ith interval

Banks have one CVA and one DVA for each counterparty. The values change as market variables values change, counterparty credit spreads change, and bank credit spreads change. The risks in CVA and DVA are managed in the same way as risks in other derivatives, such as Greek letter calculations, scenario analyses, etc.

Credit Risk Mitigation

One way to reduce credit risk in bilaterally cleared transactions is netting

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Netting Example

Suppose a bank has three uncollateralized transactions: +10, +30, -25

No Netting: The bank's exposure is 10 + 30 + 0 = 40
W/ Netting: The bank's exposure is 15
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Another way to reduce credit risk is via collateral agreements

- can be either cash (which earns interest) or marketable securities
- derivatives transactions receive favorable treatment in the event of default
  – nondefaulting party is entitled to keep any collateral posted by the other side

Another way to reduce credit risk is *downgrade trigger*

- if credit rating on counterparty falls below a certain level, the bank has the option to close out all outstanding derivatives transactions at market value
- does not provide protection against a large fall in a counterparty's credit rating
- only work well if little use is made of them
  – if a company has many downgrade triggers with its counterparties, they are likely to provide little protection to those counterparties

Default Correlation

default correlation - tendency for two companies to default at about the same time

- means credit risk cannot be completely diversified away
- major reason why risk-neutral default probabilities are greater than real-world default probabilities
- important in determination of probability distributions for default losses from a portfolio of exposures to different counterparties

Two types of default correlation models

1. Reduced form models
2. Structural models

Reduced form models

- assume hazard rates for different companies follow stochastic processes and are correlated with macroeconomic variables
- when hazard rate for company A is high, there is a tendency for hazard rate for company B to be high
- this induces a default correlation between the two companies

Advantages

- mathematically attractive
- reflect tendency for economic cycles to generate default correlations

Disadvantages

- range of default correlations that can be achieved is limited
  – even perfect correlation of hazard rates results in low probability of default
  – however two companies may warrant a high default correlation

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Structural models

- a company defaults if the value of its assets is below a certain level
- assume the stochastic process followed by company A assets is correlated with the stochastic process followed by company B assets

Advantages

- correlation can be made as high as desired

Disadvantages

- liable to be computationally slow

The Gaussian Copula Model for Time to Default

Gaussian copula model

- assumes all companies will default eventually
- attempts to quantify the correlation between the probability distributions of the times to default for two or more companies

Let t_1 be time to default of company 1 and t_2 be time to default of company 2.
If probability distributions of t_1 and t_2 are normal, then the joint probability distribution of t_1 and t_2 is bivariate normal.

However, the probability distributions in reality are not normal.

Transform t_1 and t_2 into new variables x_1 and x_2

$$x_1 = N^{-1}[Q_1(t_1)] \qquad x_2 = N^{-1}[Q_2(t_2)]$$

where Q_1 and Q_2 are cumulative probability distributions for t_1 and t_2,
and N^{-1} is the inverse of the cumulative normal distribution.

By construction, x_1 and x_2 have normal distributions with mean zero and standard deviation 1.
The model assumes the join distribution of x_1 and x_2 is bivariate normal.
The assumption is referred to as using a **Gaussian copula**.

The default correlation between t_1 and t_2 is measured as the correlation between x_1 and x_2. This is referred to as the **copula correlation**.

The Gaussian copula model can be extended to many companies. It is a useful way of representing the correlation structure between variables that are not normally distributed. The assumption is that after the variables are transformed they are multivariate normal.

A Factor-Based Correlation Structure

A one-factor model can avoid defining a different correlation between every pair of companies in the Gaussian copula model.

The assumption is that

$$x_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

where F is a common factor affecting defaults for all companies and Z_i is a factor affecting only company i. The variable F and the variables Z_i have independent standard normal distributions. The a_i are constant parameters between -1 and $+1$. The correlation between x_i and x_j is $a_i a_j$.

Suppose the probability company i will default by a particular time T is $Q_i(T)$.
Under the Gaussian copula model, a default happens by time T when $N(x_i) < Q_i(T)$, or equivalently, $x_i < N^{-1}[Q_i(T)]$.

Using the equation above, this condition becomes

$$a_i F + \sqrt{1 - a_i^2} Z_i < N^{-1}[Q_i(T)]$$

or equivalently

$$Z_i < \frac{N^{-1}[Q_i(T)] - a_i F}{\sqrt{1 - a_i^2}}$$

The probability of default conditional on the value of the factor F is therefore

$$Q_i(T \mid F) = N\left(\frac{N^{-1}[Q_i(T)] - a_i F}{\sqrt{1 - a_i^2}}\right)$$

A particular case of the one-factor Gaussian copula model is where the probability distributions of default are the same for all i and correlation between x_i and x_j is the same for all i and j.

Let $Q_i(T) = Q(T)$ for all i and let the common correlation be ρ, so that $a_i = \sqrt{\rho}$ for all i.

$$Q(T \mid F) = N\left(\frac{N^{-1}[Q(T)] - \sqrt{\rho} F}{\sqrt{1 - \rho}}\right)$$

Credit VaR

Consider a bank with a large portfolio of similar loans. Assume the probability of default is the same for each loan and the correlation between each pair of loans is the same.

The Gaussian copula model above is approximately equal to the percentage of defaults by time T as a function of F. The factor F has a standard normal distribution. There is $X\%$ certainty that its value will be greater than $N^{-1}(1-X) = -N^{-1}(X)$.

Therefore there is $X\%$ certainty the percentage of losses over T years on a large portfolio will be less than $V(X,T)$, where

$$V(X,T) = N\left(\frac{N^{-1}[Q(T)] + \sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}}\right)$$

As before, $Q(T)$ is the probability of default by time T and ρ is the copula correlation.

A rough estimate of the credit VaR for confidence level $X\%$ and time horizon T is therefore

$$L(1-R)V(X,T)$$

where L is the size of the loan portfolio and R is the recovery rate.

CreditMetrics

- procedure for calculating credit VaR
- involves estimating a probability distribution of credit losses by carrying out a Monte Carlo simulation of the credit rating changes of all counterparties

Advantages

- incorporates credit losses from both credit downgrades and defaults
- incorporates impact of credit mitigation techniques

Disadvantages

- computationally time intensive

The procedure does not assume credit rating changes of companies are independent. It uses a Gaussian copula model to construct a joint probability distribution of rating changes. The copula correlation is based on correlations between equity returns.