Ch 35 Real Options

Real Assets

- land
- buildings
- equipment

There are often options embedded in these investment opportunities

- option to expand the investment
- option to abandon the investment
- option to defer the investment
- etc

These options are difficult to value using traditional capital investment appraisal techniques.

real options - the approach to deal with this problem using option pricing theory

Capital Investment Appraisal

NPV - present value of expected future incremental cash flows

The discount rate is “risk-adjusted” to reflect the risk of the project

- risky project leads to a high discount rate
- less risky project leads to a low discount rate

Interpretation

- negative NPV indicates project will reduce value of company to shareholders
- positive NPV indicates project will increase shareholder wealth and should be undertaken

Calculating discount rate (capital asset pricing model)

1. Take a sample of companies whose main line of business is the same as that of the project
2. Calculate the betas of the companies and average them to obtain a proxy beta for project
3. Set the required rate of return equal to the risk-free rate plus the proxy beta times the excess return of the market portfolio over the risk-free rate

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Embedded options complicate the traditional NPV approach.
The options usually have different risk characteristics from the base project.

There is no easy way of estimating the risk-adjusted discount rates for cash flows when they arise from abandonment, expansion, and other options.

Another problem with the NPV approach lies in estimation of the discount rate for the base project. The companies used to estimate the proxy beta have expansion options and abandonment options of their own. Their betas reflect these options so they may not be appropriate for estimating the beta for the base project.

Extension of the Risk-Neutral Valuation Framework

Recall the market price of risk for a variable θ is

$$\lambda = \frac{\mu - r}{\sigma}$$

where r is the risk-free rate, μ is the return on a traded security dependent only on θ, and σ is the volatility. The market price of risk λ does not depend on the particular security chosen.

Suppose a real asset depends on several variables θ_i.
Let m_i and s_i be the expected growth rate and volatility of θ_i.

$$\frac{d\theta_i}{\theta_i} = m_i\,dt + s_i\,dz_i$$

where dz_i is a Wiener process.

Let λ_i be the market price of risk of θ_i.

Risk-neutral valuation can be extended to show any asset dependent on the θ_i can be valued by

1. Reducing expected growth rate of each θ_i from m_i to $m_i - \lambda_i s_i$
2. Discounting cash flows at the risk-free rate

Estimating the Market Price of Risk

The real options approach requires market price of risk parameters for all stochastic variables.
Historical data can be used to estimate market price of risk using the capital asset pricing model.

Consider an investment asset dependent solely on a particular variable

- Let μ be expected return of the investment asset
- Let σ be volatility of the return of the investment asset
- Let λ be market price of risk of the variable
- Let ρ be instantaneous correlation between the percentage changes in the variable and returns on a

broad index of stock market prices
- Let μ_m be expected return on broad index of stock market prices
- Let σ_m be volatility of return on the broad index of stock market prices
- Let r be short-term risk-free rate

The investment asset is dependent solely on the market variable, so the instantaneous correlation between its return and the broad index of stock market prices is also ρ.

From a continuous-time version of CAPM

$$\mu - r = \frac{\rho\sigma}{\sigma_m}(\mu_m - r)$$

From definition of market price of risk

$$\mu - r = \lambda\sigma$$

It follows that

$$\lambda = \frac{\rho}{\sigma_m}(\mu_m - r)$$

This equation can be used to estimate λ.

When there is no historical data for a variable, other similar variables can be used as proxies.

For some variables, it is not necessary to estimate the market price of risk because the process followed by a variable in a risk-neutral world can be estimated directly. For investment assets, the total return in a risk-neutral world is the risk-free rate.

Futures prices can be used to estimate the risk-neutral process for commodities.

Application to the Valuation of a Business

Traditional methods of business valuation (P/E multipliers, etc) do not work well for new businesses. Company earnings are typically negative during early years. The company must be valued by estimating future earnings and cash flows under different scenarios.

The real options approach can be useful in this situation

- develop a model relating future cash flows to variables
  – sales growth rates, variable costs as percent of sales, fixed costs, etc
- estimate a risk-neutral stochastic process for key variables
- carry out a Monte Carlo simulation to generate alternative scenarios for net cash flows per year in a risk-neutral world

- company value is the present value of the expected cash flow in each year using the risk-free rate for discounting

Evaluating Options in an Investment Opportunity

Most investment projects involve options.

1. Abandonment options

- option to sell or close down a project
- American put option on the project's value
- strike price is the liquidation value of the project less any closing-down costs
- mitigate impact of very poor investment outcomes and increase initial valuation of project

2. Expansion options

- option to make further investments and increase output if conditions are favorable
- American call option on value of additional capacity
- strike price is cost of creating this additional capacity discounted to time of option exercise

3. Contraction options

- option to reduce the scale of a project's operation
- American put option on the value of the lost capacity
- strike price is present value of future expenditures saved as seen at time of exercise

4. Options to defer

- American call option on the value of the project

5. Options to extend life

- European call option on the asset's future value
- extend life of asset by paying a fixed amount