Ch 11 Properties of Stock Options

Factors Affecting Option Prices

1. Current stock price, S_0
2. Strike price, K
3. Time to expiration, T
4. Volatility of stock price, σ
5. Risk-free interest rate, r
6. Dividends that are expected to be paid, D

Effect on price of a stock option of *increasing* one variable while keeping all others fixed.

Variable	*European call*	*European put*	*American call*	*American put*
S_0	$+$	$-$	$+$	$-$
K	$-$	$+$	$-$	$+$
T	$?$	$?$	$+$	$+$
σ	$+$	$+$	$+$	$+$
r	$+$	$-$	$+$	$-$
D	$-$	$+$	$-$	$+$

Stock Price and Strike Price

Call option payoff is amount by which stock price exceeds strike price.

- more valuable as stock price increases
- less valuable as strike price increases

Put option payoff is amount by which strike price exceeds stock price.

- less valuable as stock price increases
- more valuable as strike price increases

Time to Expiration

American option owners have more exercise opportunities if time to expiration date is longer.

- long-life options must be at least as valuable as short-life option
- likely to be more valuable

European options usually become more valuable as time to expiration increases, but not always.

Volatility

volatility - measure of uncertainty of future stock price movements

Increasing volatility increases the change a stock will do very well or very poorly.
For owners of stock, the outcomes offset each other.
For owners of options, this is not the case.

- owner of a call benefits from price increase but has limited downside risk if price decreases – most owner can lose is the price of the option
- owner of a put benefits from price decrease but has limited downside risk if price increases – most owner can lose is the price of the option

Increasing volatility increases value of both calls and puts.

Risk-Free Interest Rate

Increasing interest rates in the economy has two effects:

1. Expected return required by investors increases
2. Present value of future cash flows received by option holders decreases

The two effects combine to increase value of call options and decrease value of put options.

Note that interest rates usually do not change alone.
Rising or falling interest rates tend to cause stock prices to decrease or increase.

Amount of Future Dividends

Dividends reduce the stock price on the ex-dividend date.

- bad for call options
- good for put options

Assumptions and Notation

Assumptions

1. No transaction costs
2. All trading profits subject to same tax rate
3. Borrowing and lending are possible at the risk-free interest rate

Notation

S_0 : Current stock price
K : Strike price
T : Time to expiration
S_T : Stock price on expiration date
r : Risk-free interest rate (nominal)

C : Value of American call option to buy one share
P : Value of American put option to sell one share
c : Value of European call option to buy one share
p : Value of European put option to sell one share

Upper and Lower Bounds for Option Prices

Option prices that are out-of-bounds result in profitable arbitrage opportunities.

Upper Bounds

Call option gives holder right to buy one share of a stock for a certain price.
The option can never be worth more than the stock. Stock price is upper bound to option price.

$$c \leq S_0 \text{ and } C \leq S_0$$

American put option gives holder right to sell one share of a stock for strike price K.
The option can never be worth more than K.

$$P \leq K$$

For European put options, the option at maturity cannot be worth more than K.
The option can never be worth more than the present value of K today.

$$p \leq Ke^{-rT}$$

If the conditions do not hold, arbitrageurs can make a riskless profit by selling the option and buying the stock (for calls) or investing proceeds at the risk-free rate (for puts).

Lower Bound for European Calls on Non-Dividend-Paying Stocks

European call option lower bound for non-dividend-paying stock is

$$c \geq \max(S_0 - Ke^{-rT}, 0)$$

It can be derived by considering two portfolios:

1. one European call option plus a zero-coupon bond that provides payoff K at time T
2. one share of the stock

The first portfolio has value $\max(S_T, K)$ at time T.

The second portfolio has value S_T at time T.

This means the first portfolio is always worth as much as or more than the second portfolio, both at the option's maturity and today.

The zero-coupon bond is worth Ke^{-rT} today, so $c + Ke^{-rT} \geq S_0$, or $c \geq S_0 - Ke^{-rT}$.
Noting the call option value can never be negative ($c \geq 0$) gives the final result.

Lower Bound for European Puts on Non-Dividend-Paying Stocks

European put option lower bound for non-dividend-paying stock is

$$p \geq \max(Ke^{-rT} - S_0, 0)$$

It can be derived by considering two portfolios:

1. one European put option plus one share of the stock
2. a zero-coupon bond that provides payoff K at time T

The first portfolio has value $\max(S_T, K)$ at time T.
The second portfolio has value K at time T.

This means the first portfolio is always worth as much as or more than the second portfolio, both at the option's maturity and today.

The zero-coupon bond is worth Ke^{-rT} today, so $p + S_0 \geq Ke^{-rT}$, or $p \geq Ke^{-rT} - S_0$.
Noting the put option value can never be negative ($p \geq 0$) gives the final result.

Put-Call Parity

Consider the following two portfolios (also seen in previous section):

1. one European call option plus a zero-coupon bond that provides payoff K at time T
2. one European put option plus one share of the stock

The call and put options have the same strike price K and the same time to maturity T.
The stock pays no dividends.

Portfolio values at time T

		$S_T > K$	$S_T < K$
Portfolio 1	Call Option	$S_T - K$	0
	Zero-Coupon Bond	K	K
	Total	S_T	K

Portfolio 2	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

The tables shows that both portfolios are worth $\max(S_T, K)$ at time T.
Because the options are European and cannot be exercised prior to T, they must also have the same value today.

$$c + Ke^{-rT} = p + S_0$$

The relationship is **put-call parity**.

American Options

Put-call parity holds only for European options.
However, when there are no dividends, it can be shown for American options that

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

This can be used to establish upper & lower bounds for the price of one option given the other.

American Calls on a Non-Dividend-Paying Stock

It is never optimal to early exercise an American call option on a non-dividend paying stock.

From previous, a European call option has lower bound

$$c \geq S_0 - Ke^{-rT}$$

An American call has all the exercise opportunities open to a European call, so $C \geq c$, and

$$C \geq S_0 - Ke^{-rT}$$

Given $r > 0$, it follows that $C > S_0 - K$ when $T > 0$. This means C is always greater than the option's intrinsic value prior to maturity, so it can never be optimal to exercise early.

There are two reasons to not early exercise an American call on a non-dividend-paying stock:

1. Insurance. The call option insures the holder against the stock price falling below the strike price. One the option is exercised, this insurance vanishes.
2. Time value of money. For the option holder, the later the strike price is paid out, the better.

Bounds

Since American calls are never exercised early when there are no dividends, the bounds are equivalent to those of European calls.

The lower and upper bounds are given by

$$\max(S_0 - Ke^{-rT}, 0) \text{ and } S_0$$

American Puts on a Non-Dividend-Paying Stock

It can be optimal to early exercise an American put option on a non-dividend-paying stock.
The put option should always be exercised early if it is sufficiently deep in the money.

Like a call option, a put option can be viewed as providing insurance. A put option held along with the stock insures the holder against the stock price falling below a certain level. However, it may be optimal for an investor to forego this insurance and exercise early in order to realize the strike price immediately.

The *early exercise* of a put option becomes more attractive as

- S_0 decreases
- r increases
- volatility decreases

Bounds

From previous, the lower and upper bounds for a European put option are given by

$$\max(Ke^{-rT} - S_0, 0) \leq p \leq Ke^{-rT}$$

An American put can be exercised at any time so the following condition must hold

$$P \geq \max(K - S_0, 0)$$

The resulting bounds for an American put option on a non-dividend-paying stock are

$$\max(K - S_0, 0) \leq P \leq K$$

Effect of Dividends

Assume that dividends paid during the life of the option are known.

Let D be the present value of the dividends during the life of the option.

Lower Bound for Calls and Puts

Consider the two portfolios:

1. one European call option plus an amount of cash equal to $D + Ke^{-rT}$
2. one share

$$c \geq \max(S_0 - D - Ke^{-rT}, 0)$$

Consider the two portfolios:

1. one European put option plus one share
2. an amount of cash equal to $D + Ke^{-rT}$

$$p \geq \max(D + Ke^{-rT} - S_0, 0)$$

Early Exercise

When dividends are expected, it may be optimal to exercise an American call early (immediately prior to an ex-dividend date). It is never optimal to exercise a call at other times.

Put-Call Parity

For European options

$$c + D + Ke^{-rT} = p + S_0$$

For American options

$$S_0 - D - K \leq C - P \leq S_0 - Ke^{-rT}$$