

# Ch 25 Credit Derivatives

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Credit derivatives are contracts where the payoff depends on the creditworthiness of one or more companies or countries.

- allow companies to trade credit risks
- mostly used by banks (buyers) and insurance companies (sellers)

single-name: credit default swap

multi-name: collateralized debt obligation

## Credit Default Swaps

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- most popular credit derivative
- contract that provides insurance against risk of default by a particular company

reference entity - the particular company

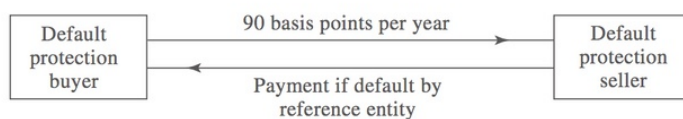
credit event - default by the company

The buyer obtains the right to sell bonds issued by the company for their face value when a credit event occurs. The seller is obligated to buy the bonds for their face value.

notional principal - total face value of the bonds that can be sold

The buyer makes periodic payments to the seller until end of life of CDS or until credit event.

The settlement in event of default involves either physical delivery of bonds or cash payment.



CDS spread - total amount paid per year as a percent of the notional principal

- 90 bps in the diagram above

A key aspect of a CDS contract is the definition of a credit event (default)

- failure to make a payment as it becomes due
- restructuring of debt
- bankruptcy

## Credit Default Swaps and Bond Yields

A CDS can be used to hedge a position in a corporate bond.

Suppose investor buys a 5-year corporate bond yielding 7% per year for its face value and at the same time enters into a 5-year CDS to buy protection against the bond issuer defaulting.

Let the CDS spread be 200 bps (2%).

The effect of the CDS is to convert the corporate bond to a risk-free bond (approximately).

If the bond issuer does not default, the investor earns 5% per year.

If the bond issuer does default, the investor earns 5% up to the time of the default. Then the investor can exchange the bond for its face value, which can then be invested at the risk-free rate for the remainder of the 5 years.

So the spread of the yield on an  $n$ -year bond issued by a company over the risk-free rate should equal the company's  $n$ -year CDS spread. If it is more, the investor can profit by buying the bond and buying protection. If it is less, the investor can profit by shorting the bond and selling CDS protection.

$$\text{CDS-bond basis} = \text{CDS spread} - \text{Bond yield spread}$$

The bond yield spread is calculated using the LIBOR/swap rate as the risk-free rate.

The arbitrage argument above suggests that the CDS-bond basis should be close to zero.

In practice it is positive or negative during various time periods.

### **The Cheapest-to-Deliver Bond**

Recall that the recovery rate on a bond is defined as the value of the bond immediately after default as a percent of face value.

So a payoff from a CDS is

$$L(1 - R)$$

where  $L$  is the notional principal and  $R$  is the recovery rate.

A CDS specifies that a number of different bonds can be delivered in the event of a default.

The bonds typically have the same seniority, but they may not sell for the same percentage of face value immediately after a default. This gives the holder of a CDS a cheapest-to-deliver bond option. An auction process is used to determine the value of the cheapest-to-deliver bond.

This then determines the payoff to the buyer of protection.

## **Valuation of Credit Default Swaps**

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The CDS spread for a particular reference entity can be calculated from default probability estimates.

### **Marking to Market a CDS**

Like most other swaps, a CDS is marked to market daily. It may have a positive or negative value.

### **Estimating Default Probabilities**

The default probabilities used to value a CDS should be risk-neutral default probabilities, not real-world default probabilities. Risk-neutral default probabilities can be estimated from bond prices or asset swaps. Alternatively they can be implied from CDS quotes. This is similar to implying volatilities in options markets by using prices of actively traded options to value other options.

### **Binary Credit Default Swaps**

Structured similarly to a regular CDS except that the payoff is a fixed dollar amount.

### **How Important is the Recovery Rate?**

Estimating default probabilities requires an estimate of the recovery rate. However, if the same recovery rate is used for estimating risk-neutral default probabilities and valuing a CDS, the value of the CDS is not very sensitive to the recovery rate. This is because the implied probabilities of default are proportional to  $1/(1 - R)$  and the payoffs from a CDS are proportional to  $1 - R$ .

This does not apply to valuation of binary CDS. Implied probabilities of default are still proportional to  $1/(1 - R)$  but the payoffs are independent of  $R$ .

A CDS spread for a plain vanilla CDS and a CDS spread for a binary CDS can be used together to estimate both the recovery rate and the default probability.

## **Credit Indices**

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There are indices to track credit default swap spreads.

Two standard portfolios used by index providers:

1. CDX NA IG, a portfolio of 125 investment grade companies in North America
2. iTraxx Europe, a portfolio of 125 investment grade names in Europe

The index is the average of the CDS spreads on the companies in the underlying portfolio. The portfolios are updated twice a year.

## **The Use of Fixed Coupons**

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For each underlying and each maturity, a coupon and a recovery rate are specified.

The price is calculated from the quoted spread using the following procedure:

1. Assume four payments per year, made in arrears
2. Imply a hazard rate from the quoted spread
3. Calculate a duration  $D$  for the CDS payments
4. The price  $P$  is given by

$$P = 100 - 100 \times D \text{ times}(s - c)$$

where  $s$  is the spread and  $c$  is the coupon expressed in decimal form

## CDS Forwards and Options

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### Forward CDS

- obligation to buy or sell a particular CDS on a particular reference entity at a particular future time  $T$
- if the reference entity defaults before time  $T$ , the forward contract ceases to exist

### CDS option

- option to buy or sell a particular CDS on a particular reference entity at a particular future time  $T$
- option will be exercised depending on the value of the CDS spread at option maturity
- cost of option is paid up front
- if the reference entity defaults before time  $T$ , the option contract ceases to exist

## Basket Credit Default Swaps

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basket CDS - there are a number of reference entities

add-up basket CDS - provides a payoff when any of the reference entities defaults

first-to-default CDS - provides a payoff only when the first default occurs

second-to-default CDS - provides a payoff only when the second default occurs

$k$ th-to-default CDS - provides a payoff only when the  $k$ th default occurs

Payoffs are calculated in the same way as for a regular CDS.

After the relevant default occurs, there is a settlement.

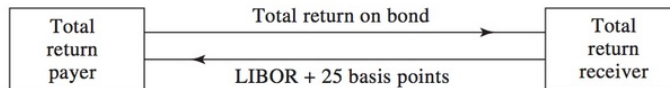
The swap then terminates and there are no further payments by either party.

## Total Return Swaps

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total return swap - agreement to exchange the total return on a bond for LIBOR plus a spread

- total returns includes coupons, interest, and gain/loss on the asset over the life of the swap



Consider a total return swap that is a 5-year agreement with notional principal \$100m to exchange the total return on a corporate bond for LIBOR plus 25 bps.

- on coupon dates the payer pays the coupons earned on an investment of \$100m on the bond
- the receiver pays interest at a rate of LIBOR plus 25 bps on a principal of \$100m
- at the end of the life of the swap there is a payment reflecting the change in value of the bond
- if there is a default on the bond, the swap is terminated, and the receiver makes a final payment equal to the excess of \$100m over the market value of the bond

If the payer owns the bond, the total return swap allows it to pass the credit risk to the receiver.

If the payer does not own the bond, the total return swap allows it to take a short position in the bond.

The spread over LIBOR received by the payer is compensation for bearing the risk that the receiver will default. The payer loses money if the receiver defaults at a time when the bond's price has declined. So the spread depends on the credit quality of the receiver, the credit quality of the bond issuer, and the correlation between the two.

## Collateralized Debt Obligations

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collateralized debt obligation - an ABS where the underlying assets are bonds

### Synthetic CDOs

A CDO created from a bond portfolio is a **cash CDO**.

A long position in a corporate bond has a similar risk to a short position in a CDS when the reference entity in the CDS is the company issuing the bond.

This led to the **synthetic CDO**.

The originator of the synthetic CDO chooses a portfolio of companies and a maturity.

It sells CDS protection on each company with the CDS maturities equal to the overall maturity.

The synthetic CDO principal is the sum of the notional principals of the underlying CDSs.

The originator has cash inflows equal to the CDS spreads.

The originator has cash outflows when companies in the portfolio default.

Cash inflows and outflows are distributed to tranches.

### Example of Tranche Setup

1. Equity tranche responsible for payouts on CDSs until they reach 5% of the synthetic CDO principal.  
Earns a spread of 1000 bps per year on outstanding tranche principal.

2. Mezzanine tranche responsible for payouts in excess of 5% up to 20% of the synthetic CDO principal. Earns a spread of 100 bps per year on the outstanding tranche principal.
3. Senior tranche responsible for payouts in excess of 20%. Earns a spread of 10 bps per year on the outstanding tranche principal.

Cash CDOs require an initial investment by the tranche holders (to finance underlying bonds).

Synthetic CDOs do not require an initial investment. In practice the holders of synthetic CDOs are often required to post the initial tranche principal as collateral. When a tranche becomes responsible for a payoff on a CDS, the money is taken out of the collateral. The collateral account earns interest at LIBOR.

### **Standard Portfolios and Single-Tranche Trading**

In synthetic CDOs, tranche holders sell protection to the originator, who in turn sells protection on CDSs to other market participants. However, trading of a tranche without the underlying portfolio of short CDS positions being created is possible. This is called **single-tranche trading**.

Two parties to a trade

1. buyer of protection on a tranche
2. seller of protection on the tranche

The portfolio of short CDS positions is used as a reference point to define cash flows between the two sides. But it is not created. The buyer pays the tranche spread to the seller. The seller pays amounts to the buyer that correspond to those losses on the reference portfolio of CDSs that the tranche is responsible for.

## **Role of Correlation in a Basket CDS and CDO**

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The cost of protection in a  $k$ th-to-default CDS or a tranche of a CDO is highly dependent on default correlation.

When default correlation between reference entities is zero, a first-to-default CDS is valuable but as tenth-to-default CDS is worth almost nothing.

When the default correlation is perfect, the probability of one or more defaults equals the probability of ten or more defaults. Consider the extreme situation where all reference entities are the same. Then either they all default or none of them default.

The valuation of tranches in a synthetic CDO is similarly dependent on default correlation. If the correlation is low, the equity tranche is very risky and the senior tranche is very safe. As correlation increases, the junior tranches become less risky and the senior tranches become more risky. When default correlation is perfect and recovery rate is zero, the tranches are all equally risky.

## **Valuation of a Synthetic CDO**

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Suppose payment dates on a synthetic CDO tranche are at times  $\tau_1, \tau_2, \dots, \tau_m$ , and  $\tau_0 = 0$ .

Let  $E_j$  be the expected tranche principal at time  $\tau_j$ .

Let  $v(t)$  be the present value of \$1 receive at time  $\tau$ .

Let  $s$  be the spread per year on a particular tranche.

The present value of the expected regular spread payments on the CDO is  $sA$ , where

$$A = \sum_{j=1}^m (\tau_j - \tau_{j-1}) E_j v(\tau_j)$$

The expected loss between times  $\tau_{j-1}$  and  $\tau_j$  is  $E_{j-1} - E_j$ .

Assume the expected loss occurs at the midpoint of the interval.

The present value of the expected payoffs on the CDO tranche is

$$C = \sum_{j=1}^m (E_{j-1} - E_j) v(0.5\tau_{j-1} + 0.5\tau_j)$$

The accrual payment due on the losses is given by  $sB$ , where

$$B = \sum_{j=1}^m 0.5(\tau_j - \tau_{j-1})(E_{j-1} - E_j) v(0.5\tau_{j-1} + 0.5\tau_j)$$

The value of the tranche to the buyer of protection is

$$C - sA - sB$$

The breakeven spread occurs when present value of payments equals present value of payoffs

$$C = sA + sB$$

so the breakeven spread is

$$s = \frac{C}{A + B}$$

## Alternatives to the Standard Market Model

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The standard market model is the one-factor Gaussian copula model.

However there are a number of alternatives.

### Heterogeneous Model

The standard market model is homogeneous. The time-to-default probability distributions are assumed to be the same for all companies and the copula correlations for any pair of companies are the same. The assumption can be relaxed to create a heterogeneous model.

### Other Copulas

The one-factor Gaussian copula model is a particular model of the correlation between times to default. There are other one-factor copula models, such as the Student  $t$  copula, the Clayton copula, Archimedean copula, and Marshall-Olkin copula.

Another approach is to increase the number of factors in the model.

### Random Factor Loadings

The copula correlation  $\rho$  can be a function of  $F$ . In general,  $\rho$  increases as  $F$  decreases. This means that when the default rate is high ( $F$  is low), the default correlation is also high.

### The Implied Copula Model

The copula can be implied from market quotes. The simplest version assumes that a certain average hazard rate applies to all companies in a portfolio over the life of a CDO. That rate has a probability distribution that can be implied from the pricing of tranches.

### Dynamic Models

The previous models are static models. They model the average default environment over the life of a CDO. Dynamic models are different from static models in that they attempt to model the evolution of the loss on a portfolio through time.

#### 1. Structural Models

- the stochastic processes for asset prices of many companies are modeled simultaneously
- when asset price of a company reaches a barrier, there is a default
- the processes followed by the assets are correlated
- must be implemented with Monte Carlo simulation, so calibration is difficult

#### 2. Reduced Form Models

- the hazard rates of companies are modeled
- it is necessary to assume that there are jumps in the hazard rates, in order to build in a realistic amount of correlation

#### 3. Top Down Models

- the total loss on a portfolio is modeled directly
- these models do not consider what happens to individual companies



