CONCORDIA UNIVERSITY

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING (ECE)

$\mathcal{ETMA}$: An Efficient Tool for Event Trees Modeling and Analysis

Case Study: Protective Fault Trip Circuit

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Date: September 26, 2019

1 TRIP CIRCUIT ANALYSIS

The power grid consists of one generator, 9 circuit breakers (CB), 4 bus bars (BB), 2 transmission lines (TL), 2 loads, 2 (on step up and one step down) transformers (Trans), 2 trip circuits (TC) with 1 relay (R) and 1 current transformer (CT), as shown in Fig. 1. During normal operation, all CBs are in a closed position. If a fault (F) occurs on TL_1, a primary current (I_p) spike rises to about 20 times from a normal current level. Then, the CT detects that there is a fault in TL_1 and the secondary current (I_s) also rises with the same ratio simultaneously. Consequently, the relay coil increases the magnetic field and attracts the relay contacts, which are connected to the two separated trip circuits 1 and 2. Each trip circuit is provided with a battery. So, when the relay contact closes, it becomes a closed loop. Finally, the magnetic field produced by the trip coils 1 and 2 will push CB_1 and CB_2 to open and isolate TL_1. If all components of the trip circuit work correctly, then the fault becomes isolated and the grid is safe. If not, then the grid is in a risk situation of a blackout and back-up decisions should be made. In this paper, we study the ET-based probabilistic analysis of all scenarios of failure and success that can occur in the trip circuit.

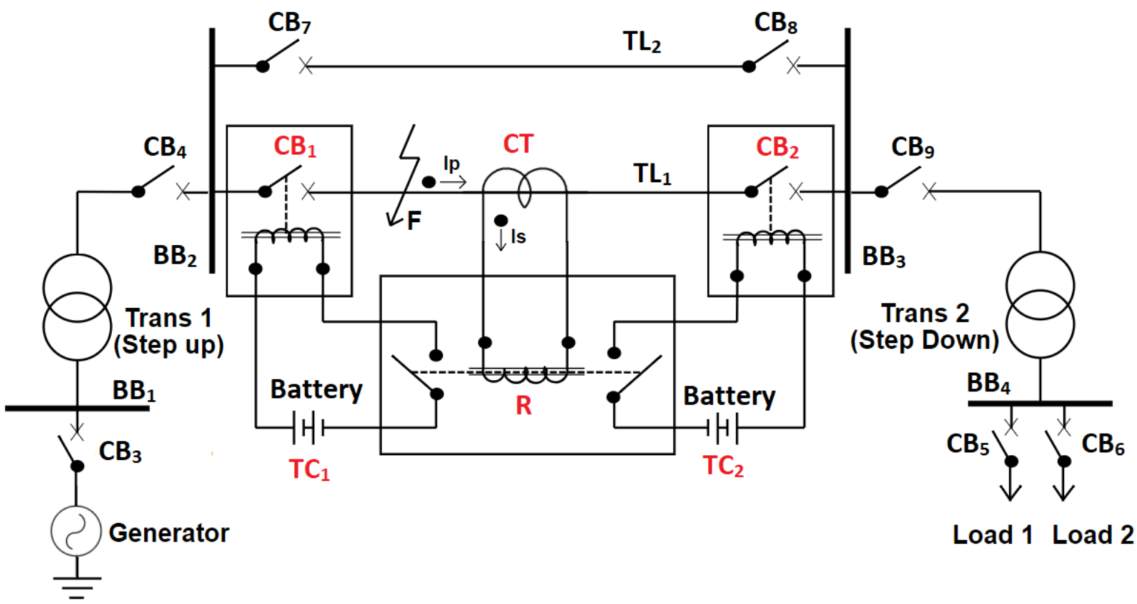

Figure 1: Single line diagram of a trip circuit in a power grid

We start the ET analysis of the trip circuit in $\mathcal{ETMA}$ by first generating a complete ET model. Then, we delete the unnecessary nodes and branches to obtain a reduced ET that models the actual behavior of the trip circuit. Afterwards, we estimate the probabilities of different events that can occur in the trip circuit, for instance, the probability of both breakers CB_1 and CB_2 failing. Following are the steps required to conduct the trip circuit ET analysis in $\mathcal{ETMA}$:

1.1 Complete ET Generation

We enter the details of the trip circuit components consisting of one CT, one R, two TCs (TC_1 and TC_2) and two CBs (CB_1 and CB_2) and each having two operational states, i.e., operating or failing, as shown in the following figures:

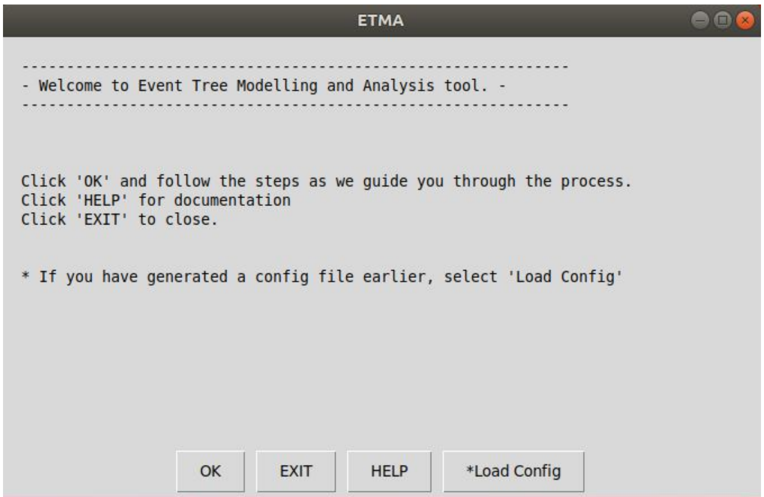

Figure 2: $\mathcal{ETMA}$: Welcome Page

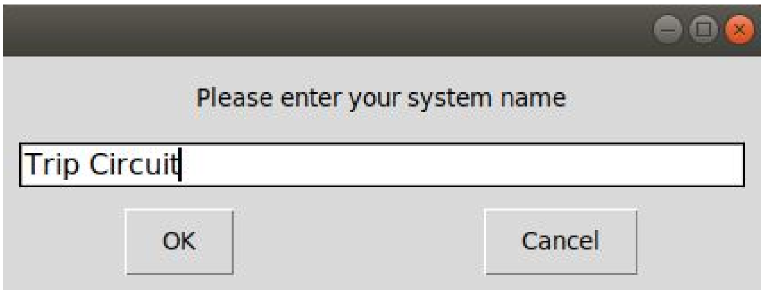

Figure 3: $\mathcal{ETMA}$: Enter system name

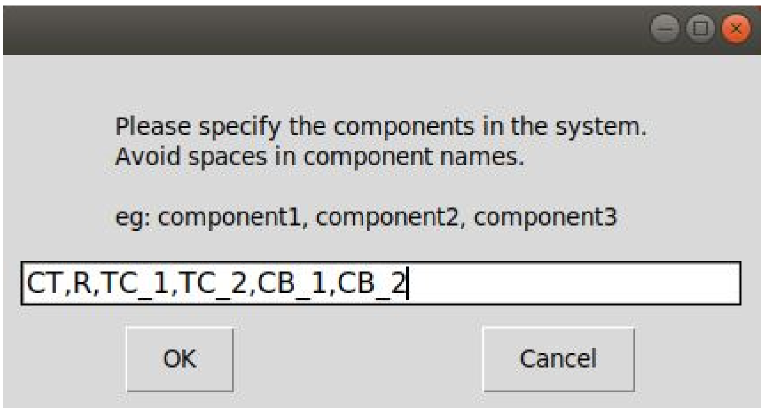

Figure 4: $\mathcal{ETMA}$: Enter system components

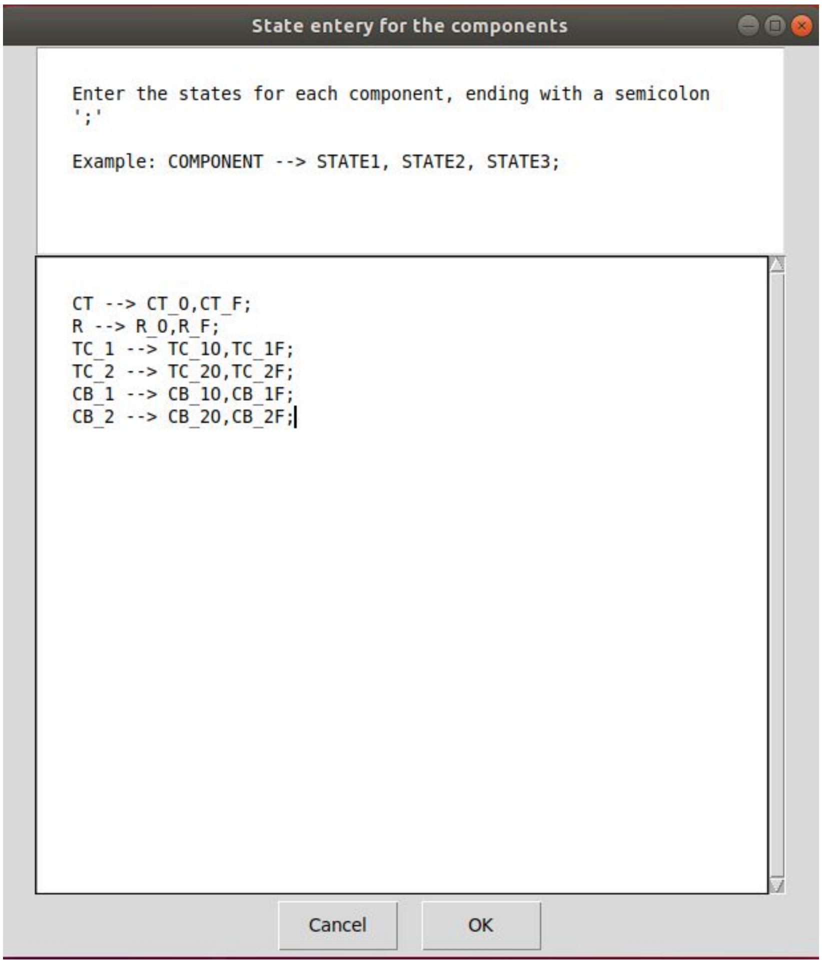

Figure 5: $\mathcal{ETMA}$: Enter system components states

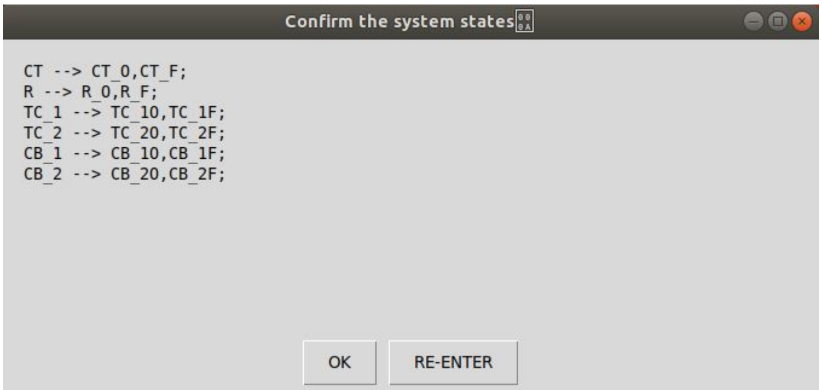

Figure 6: $\mathcal{ETMA}$: Confirm system components states

The entered details are sufficient for $\mathcal{ETMA}$'s function to automatically generate the complete graph ET model as shown in Fig. 7. This model shows the whole possible scenarios of failure and success for the trip circuit components states.

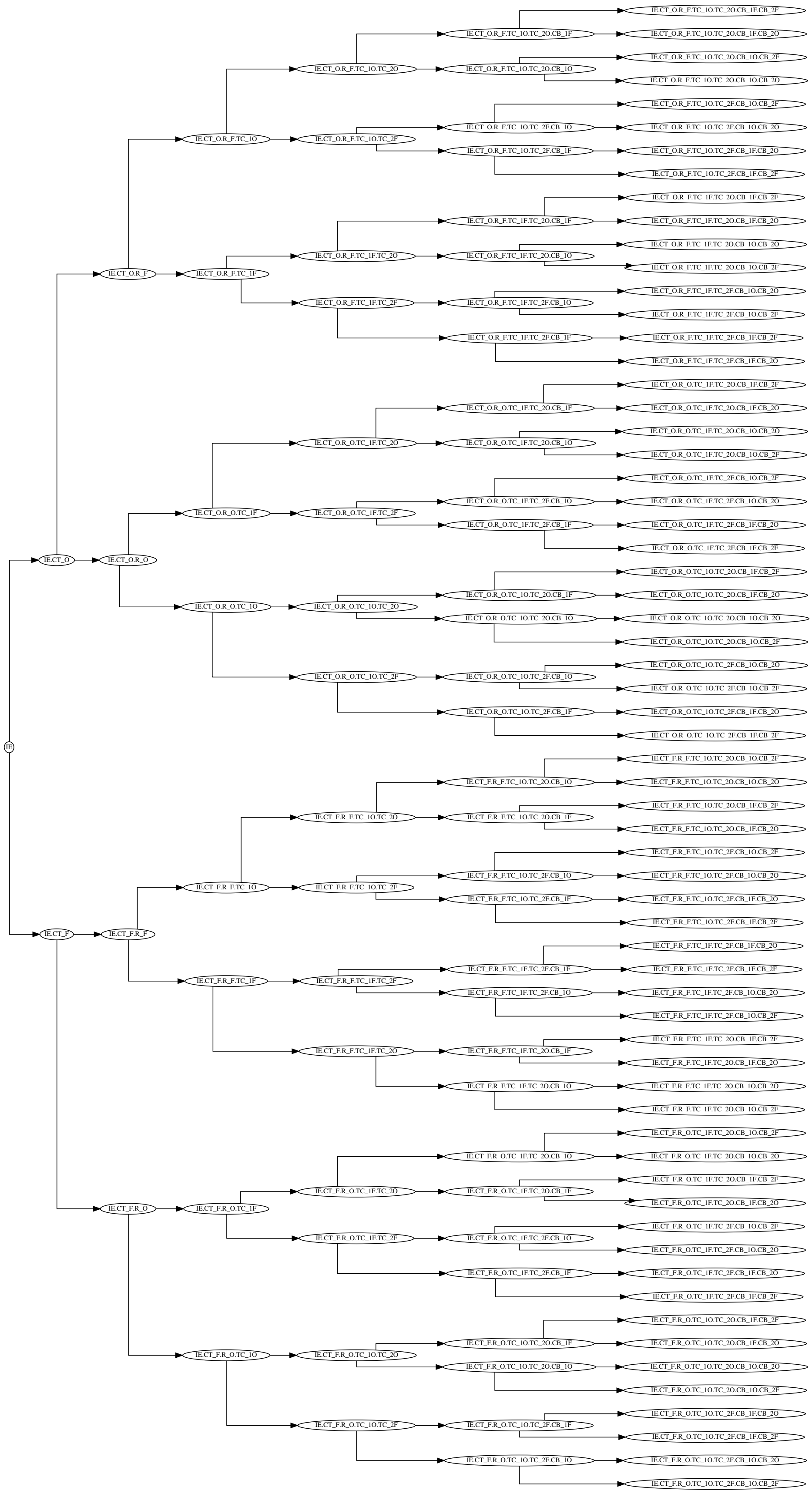

Figure 7: $\mathcal{ETMA}$: Trip circuit complete ET model

$\mathcal{ETMA}$ also automatically produces a complete event outcome space (64 paths from 0 to 63) from the complete ET model as:

Path 0 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 1 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 2 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 3 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 4 = $[CT_O, R_O, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 5 = $[CT_O, R_O, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 6 = $[CT_O, R_O, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 7 = $[CT_O, R_O, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2F}]$
Path 8 = $[CT_O, R_O, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 9 = $[CT_O, R_O, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 10 = $[CT_O, R_O, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 11 = $[CT_O, R_O, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 12 = $[CT_O, R_O, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 13 = $[CT_O, R_O, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 14 = $[CT_O, R_O, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 15 = $[CT_O, R_O, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2F}]$
Path 16 = $[CT_O, R_F, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 17 = $[CT_O, R_F, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 18 = $[CT_O, R_F, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 19 = $[CT_O, R_F, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 20 = $[CT_O, R_F, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 21 = $[CT_O, R_F, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 22 = $[CT_O, R_F, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 23 = $[CT_O, R_F, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2F}]$
Path 24 = $[CT_O, R_F, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 25 = $[CT_O, R_F, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 26 = $[CT_O, R_F, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 27 = $[CT_O, R_F, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 28 = $[CT_O, R_F, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 29 = $[CT_O, R_F, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 30 = $[CT_O, R_F, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 31 = $[CT_O, R_F, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2F}]$
Path 32 = $[CT_F, R_O, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 33 = $[CT_F, R_O, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 34 = $[CT_F, R_O, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 35 = $[CT_F, R_O, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 36 = $[CT_F, R_O, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 37 = $[CT_F, R_O, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 38 = $[CT_F, R_O, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 39 = $[CT_F, R_O, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2F}]$
Path 40 = $[CT_F, R_O, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 41 = $[CT_F, R_O, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 42 = $[CT_F, R_O, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2O}]$

Path 43 = $[CT_F, R_O, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 44 = $[CT_F, R_O, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 45 = $[CT_F, R_O, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 46 = $[CT_F, R_O, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 47 = $[CT_F, R_O, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2F}]$
Path 48 = $[CT_F, R_F, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 49 = $[CT_F, R_F, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 50 = $[CT_F, R_F, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 51 = $[CT_F, R_F, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 52 = $[CT_F, R_F, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 53 = $[CT_F, R_F, TC_{1O}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 54 = $[CT_F, R_F, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 55 = $[CT_F, R_F, TC_{1O}, TC_{2F}, CB_{1F}, CB_{2F}]$
Path 56 = $[CT_F, R_F, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 57 = $[CT_F, R_F, TC_{1F}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 58 = $[CT_F, R_F, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 59 = $[CT_F, R_F, TC_{1F}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 60 = $[CT_F, R_F, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2O}]$
Path 61 = $[CT_F, R_F, TC_{1F}, TC_{2F}, CB_{1O}, CB_{2F}]$
Path 62 = $[CT_F, R_F, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2O}]$
Path 63 = $[CT_F, R_F, TC_{1F}, TC_{2F}, CB_{1F}, CB_{2F}]$

1.2 ET Reduction Process

If the user desires to take into consideration the complete ET model generated in *Step* 1, then $\mathcal{ETMA}$ provides a bypassing option for *Step* 2 (i.e, ET reduction process), as shown in Fig. 8.

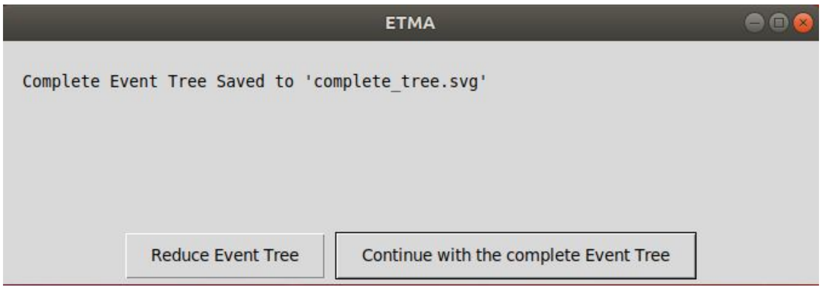

Figure 8: $\mathcal{ETMA}$: Continue with complete or reduced ET model

In our case, to model the exact logical behavior of the trip circuit system, we need to delete the irrelevant branches and nodes from the complete ET model, as shown in Fig. 9 and Fig. 10, respectively. For instance, consider the paths from 32 to 63, if the CT fails then the likelihood or probability of occurrence of these paths are equal to the probability of CT failure only, regardless of the status of other components. So, in $\mathcal{ETMA}$, we deleted the branches $[CT_F, R_O]$ and $[CT_F, R_F]$.

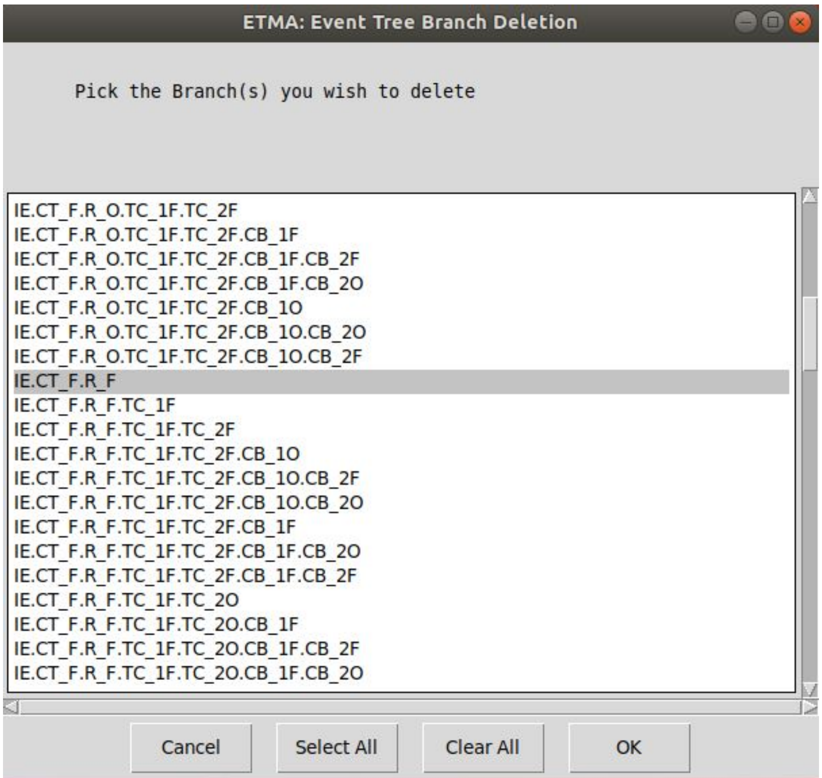

Figure 9: $\mathcal{ETMA}$: Deletion of branches

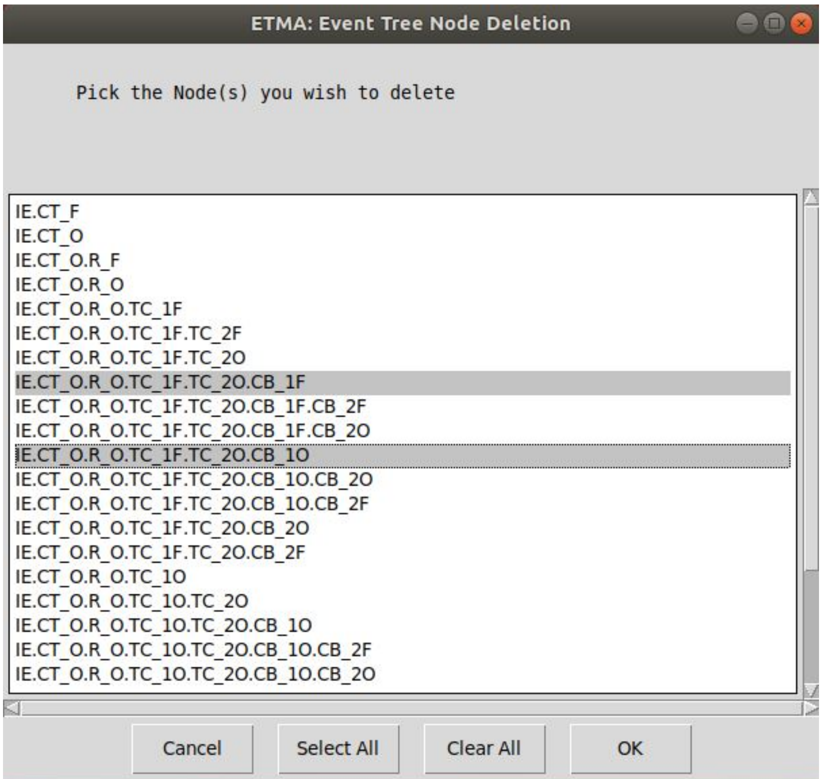

Figure 10: $\mathcal{ETMA}$: Deletion of nodes

The entered details are sufficient for $\mathcal{ETMA}$'s function to generate the reduced graph ET model as shown in Fig. 11.

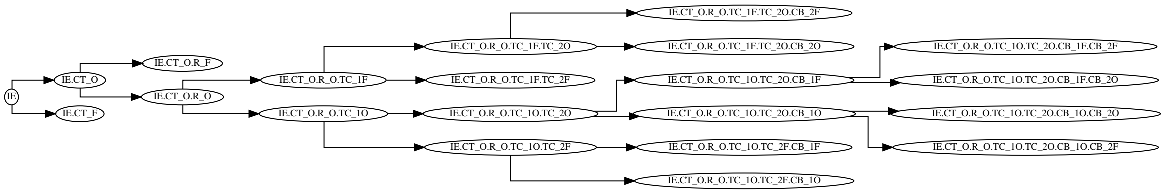

Figure 11: $\mathcal{ETMA}$: Trip circuit reduced ET model

The reduced event outcome space (11 paths from 0 to 10) produced from the reduced ET model is as:

Path 0 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2O}]$
Path 1 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1O}, CB_{2F}]$
Path 2 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2O}]$
Path 3 = $[CT_O, R_O, TC_{1O}, TC_{2O}, CB_{1F}, CB_{2F}]$
Path 4 = $[CT_O, R_O, TC_{1O}, TC_{2F}, CB_{1O}]$
Path 5 = $[CT_O, R_O, TC_{1O}, TC_{2F}, CB_{1F}]$
Path 6 = $[CT_O, R_O, TC_{1F}, TC_{2O}, CB_{2O}]$
Path 7 = $[CT_O, R_O, TC_{1F}, TC_{2O}, CB_{2F}]$
Path 8 = $[CT_O, R_O, TC_{1F}, TC_{2F}]$
Path 9 = $[CT_O, R_F]$
Path 10 = $[CT_F]$

1.3 Probability Evaluation

To estimate the probability of events associated with the trip circuit components, we assign probability values to each operational state of the components, as shown in Fig. 12 and Table 1. Assume that the times to failure of the trip circuit components are exponentially distribution with failure rate λ and time index t. Then the unreliability function or the probability of failure can be computed as:

$$F(t) = \mathcal{P}(X \leq t) = 1 - e^{-\lambda t} \tag{1}$$

where X is a time-to-failure random variable. Similarly, the reliability of a component can be estimated by taking the complement of unreliability function with respect to the probability space as:

$$R(t) = \mathcal{P}(X > t) = 1 - F(t) \tag{2}$$

Table 1: Trip circuit probability of components states

Component	λ (f/yr)	Prob. of Failure (%) After 6 Months	Prob. of Success (%) After 6 Months
CT	0.06	CT_F (3%)	CT_O (97%)
R	0.04	R_F (2%)	R_O (98%)
TC_1	0.08	TC_{1F} (4%)	TC_{1O} (96%)
TC_2	0.08	TC_{2F} (4%)	TC_{2O} (96%)
CB_1	0.06	CB_{1F} (3%)	CB_{1O} (97%)
CB_2	0.06	CB_{2F} (3%)	CB_{2O} (97%)

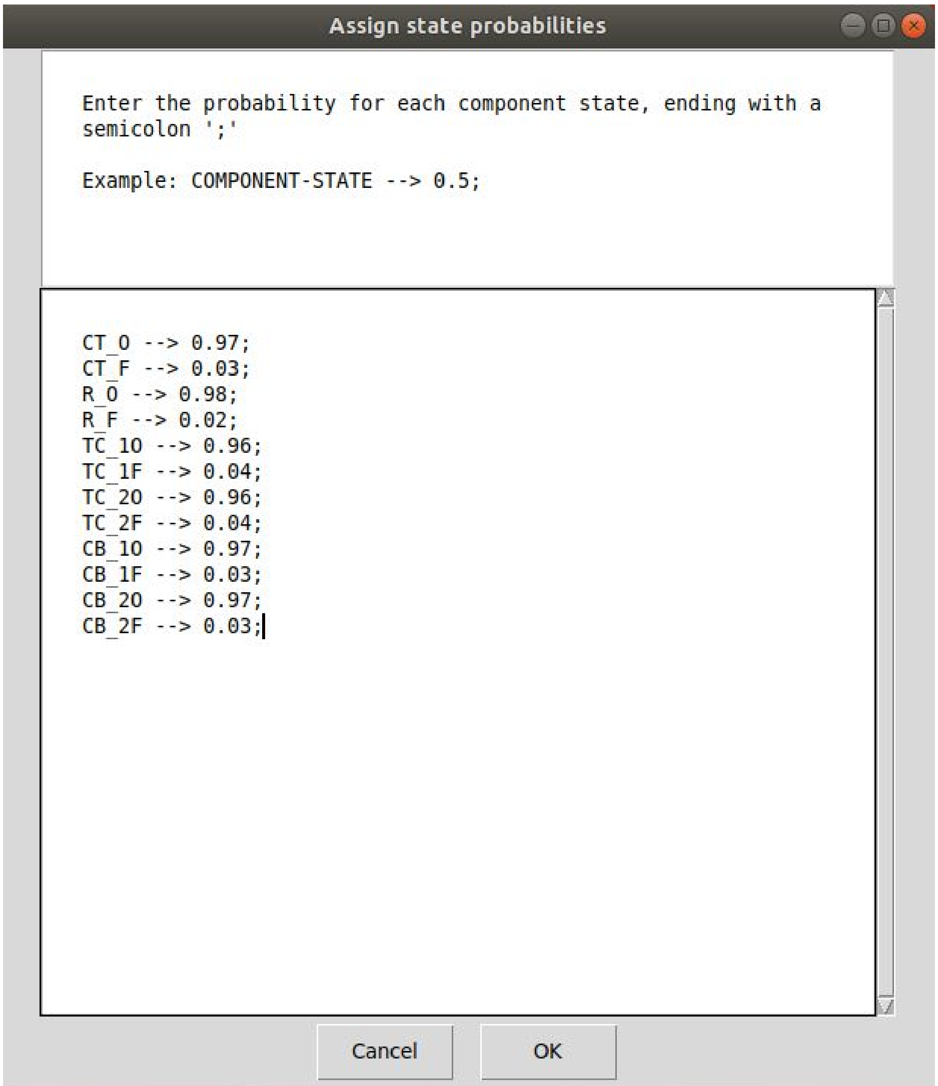

Figure 12: $\mathcal{ETMA}$: Enter the probability of components states

1.4 Partition Outcome Space

The partitioning of the outcome space is essential as we are only interested in the occurrence of certain events in an ET. Suppose, we are only focusing on the failure of CB_1, then paths 2, 3, and 5-10 are obtained as shwon in Fig. 13. Similarly, different sets of paths can be obtained by observing the behavior of the trip circuit components as:

- $\mathcal{P}$ (CB_1 Only Fails) = $\sum\mathcal{P}(2,3,5-10)$
- $\mathcal{P}$ (CB_1 Only Operates) = $\sum\mathcal{P}(0,1,4)$
- $\mathcal{P}$ (CB_2 Only Fails) = $\sum\mathcal{P}(1,3-5,7-10)$
- $\mathcal{P}$ (CB_2 Only Operates) = $\sum\mathcal{P}(0,2,6)$
- $\mathcal{P}$ (Both CB_1 and CB_2 Fail) = $\sum\mathcal{P}(3,5,7-10)$
- $\mathcal{P}$ (Both CB_1 and CB_2 Operate) = $\sum\mathcal{P}(0)$

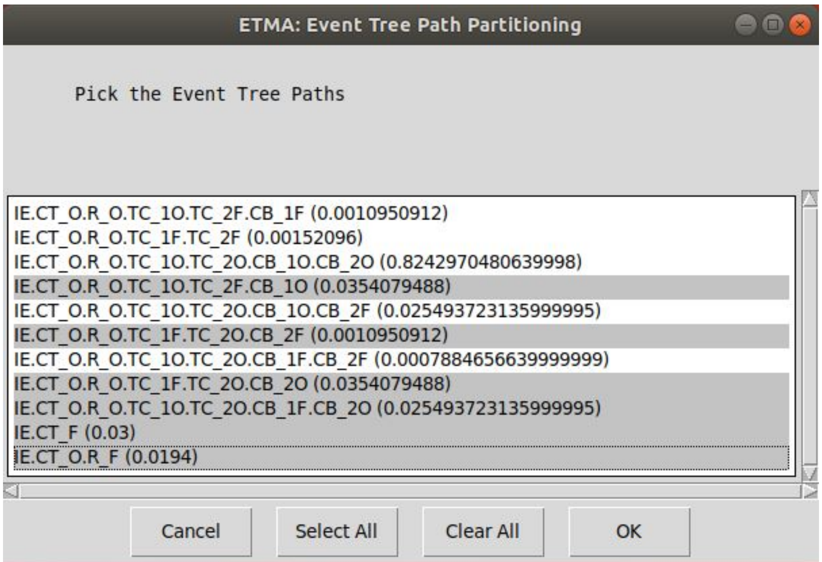

Figure 13: $\mathcal{ETMA}$: Partition Outcome Space

The probabilities of the different trip circuit events, which are calculated using $\mathcal{ETMA}$ are as follows:
$\mathcal{P}$ (Both CB_1 and CB_2 Fail) = 5.389960806400000%
$\mathcal{P}$ (Both CB_1 and CB_2 Operate) = 82.429704806399980%
$\mathcal{P}$ (CB_1 Only Fails) = 11.480127999999999%
$\mathcal{P}$ (CB_1 Only Operates) = 88.519871999999980%
$\mathcal{P}$ (CB_2 Only Fails) = 11.480127999999999%
$\mathcal{P}$ (CB_2 Only Operates) = 88.519871999999980%