
References

- [1] Lars V. Ahlfors. *Complex Analysis*. McGraw-Hill, New York, 1979.
- [2] Karol Borsuk and Wanda Szmielew. *Foundations of Geometry*. North Holland Publishing Co., Amsterdam, 1960.
- [3] John B. Conway. *Functions of One Complex Variable*. Springer-Verlag, New York, second edition, 1973.
- [4] H. S. M. Coxeter. *Introduction to Geometry*. John Wiley and Sons, New York, second edition, 1961.
- [5] H. S. M. Coxeter. The Non-Euclidean Symmetry of Escher's Picture Circle Limit III. *Leonardo*, 12:19–25, 1979.
- [6] H. S. M. Coxeter. *Non-Euclidean Geometry*. The Mathematical Association of America, Washington, D.C., sixth edition, 1998.
- [7] William Dunham. *Journey Through Genius*. Penguin Books, New York, 1991.
- [8] Richard L. Faber. *Foundations of Euclidean and Non-Euclidean Geometry*. Marcel-Dekker, Inc., New York, 1983.
- [9] W. T. Fishback. *Projective and Euclidean Geometry*. John Wiley and Sons, Inc., New York, 1969.
- [10] G. H. Hardy. *A Mathematician's Apology*. Cambridge University Press, London, 2012 (reissue).
- [11] Robin Hartshorne. *Geometry: Euclid and Beyond*. Springer-Verlag, New York, 2000.
- [12] Robin Hartshorne. *Foundations of Projective Geometry*. Ishi Press International, Bronx, NY, 2009.
- [13] David Hilbert. *Foundations of Geometry*. Open Court Press, LaSalle, Illinois, 1971.

- [14] David Hilbert and S. Cohn-Vossen. *Geometry and the Imagination*. Chelsea Publishing Co., New York, 1952.
- [15] Einar Hille. *Analytic Function Theory, Volume I*. Blaisdell Publishing, New York, 1959.
- [16] Edmund Landau. *Foundations of Analysis*. AMS Chelsea Publishing Co., Providence, Rhode Island, 2001.
- [17] Norman Levinson and Raymond M. Redheffer. *Complex Variables*. Holden-Day, San Francisco, 1970.
- [18] Eric W. Weisstein. Conic section. Web page. <http://mathworld.wolfram.com/ConicSection.html>.
- [19] Harold E. Wolfe. *Non-Euclidean Geometry*. Henry Holt and Co., New York, 1945.

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