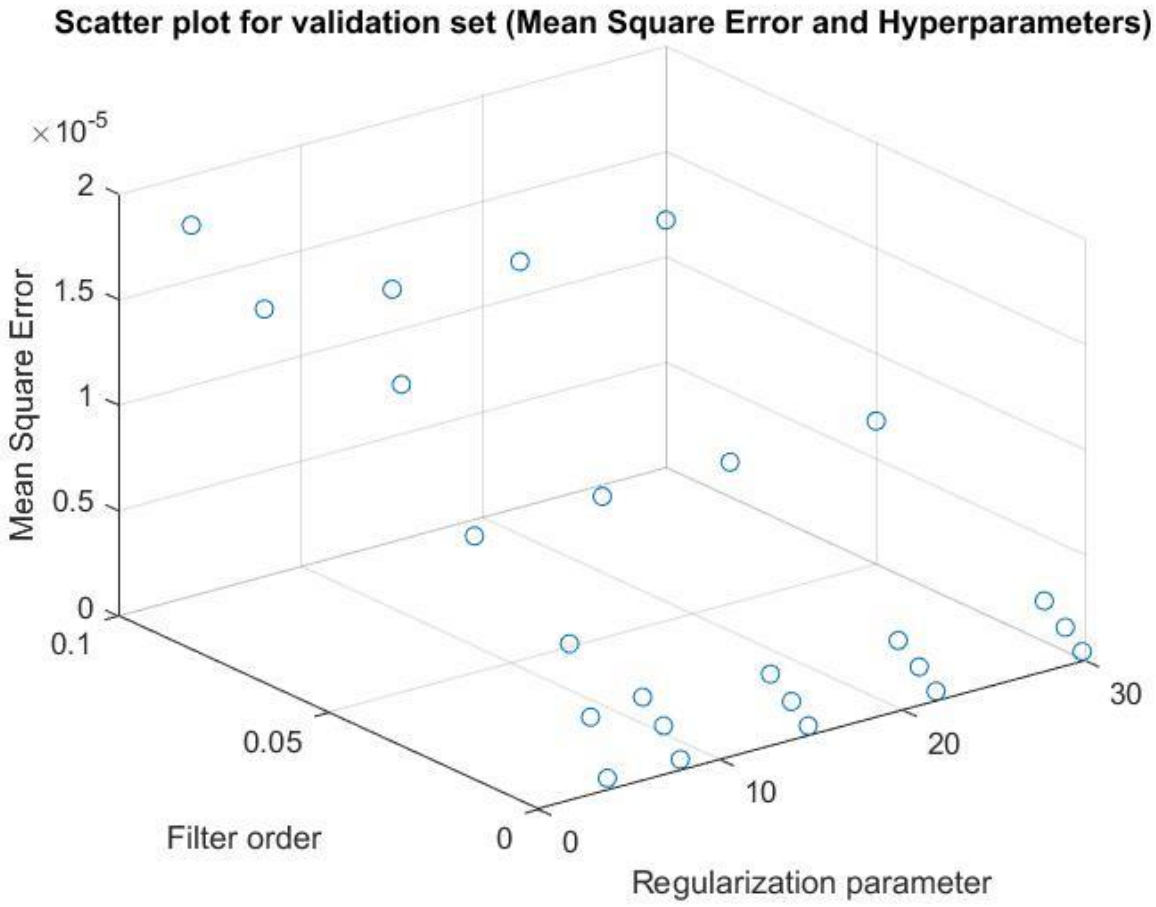
Question 1 : Select the hyper parameters for this adaptive filter, namely, filter order and regularization parameter (using validate.mat). The idea is to experiment several different values in training and select the one that provides the best mean square error in the validation set. Present a plot of the validation set in the 2-d space of filter order and regularization parameter.

Solution : . The training set contains 3000 data points and first we have to construct a matrix where lag values are stored since our X in equation (inv(XX')X'Y)) to determine optimal parameters is dependent on lag values and the number of lag values will be the order of our filter. Some initial values of X can't be computed since we don't have the past values for them so instead of manipulating the data given, I will not take them. For example: for a filter order of 3 I will not take first 3 values.

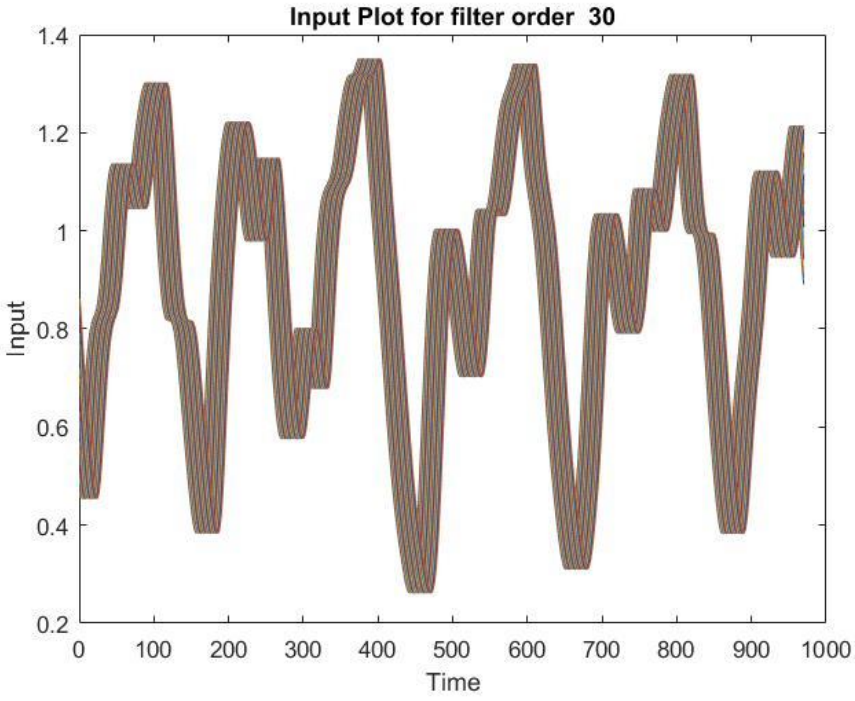
The order of the filters which I selected are: 4,8,15,22,30 and the regularization parameter values which I selected are : 0.001,0.005,0.01,0.05,0.1. For each filter order I find the optimal parameters from my training data. After this I found the regularization parameter and filter order for which the Mean-Square error is minimum. This determination is done using validation data. From the fig.1 it can be seen that the minimum Mean Square error occurred at filter order=30 and regularization parameter = 0.001.

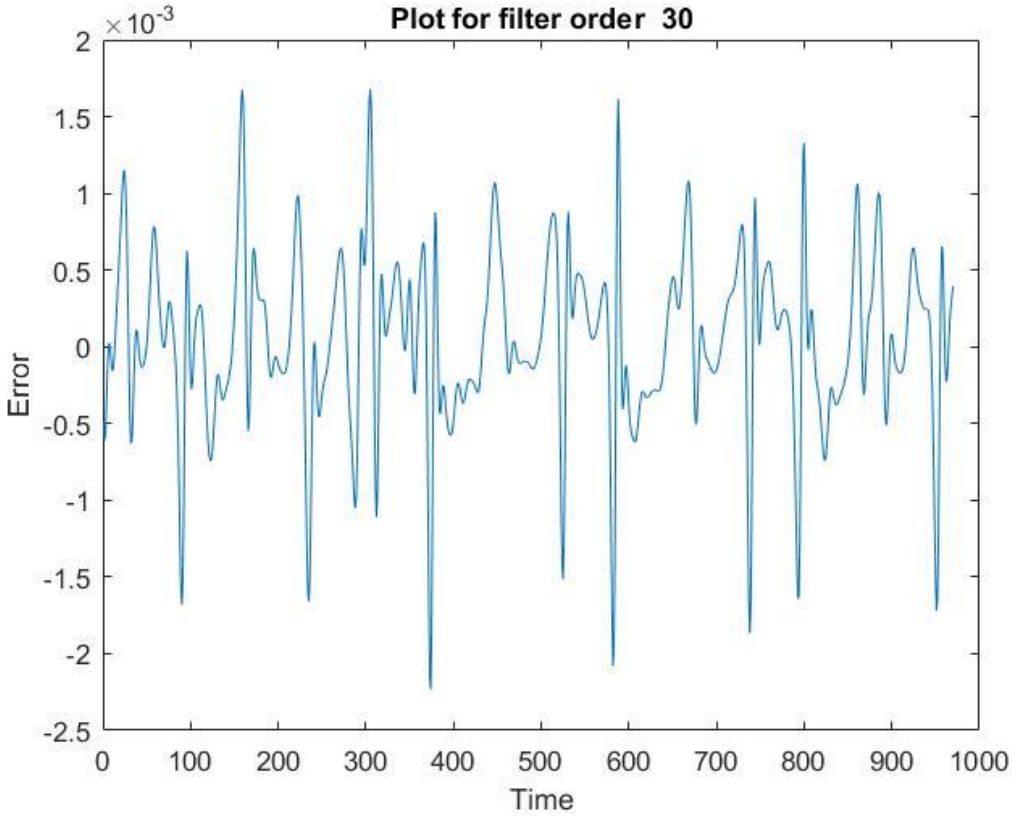


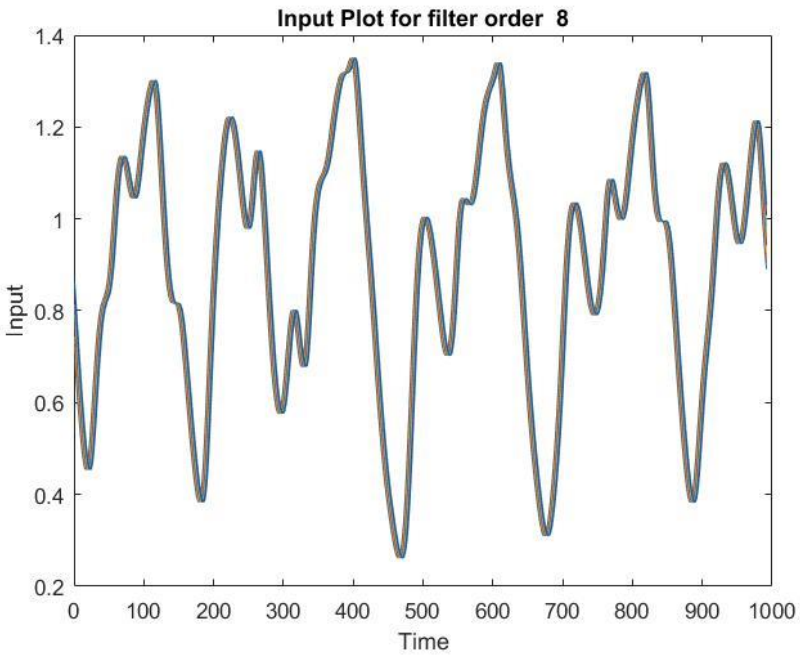
Question 2 : Plot both the filter input and the prediction error over time (using the test set test.mat) for fixed weights and filter order of 4, 8, 30, along with the best value you determined in 2.1, to see if the validation was correct. Why is the error not constant across time?

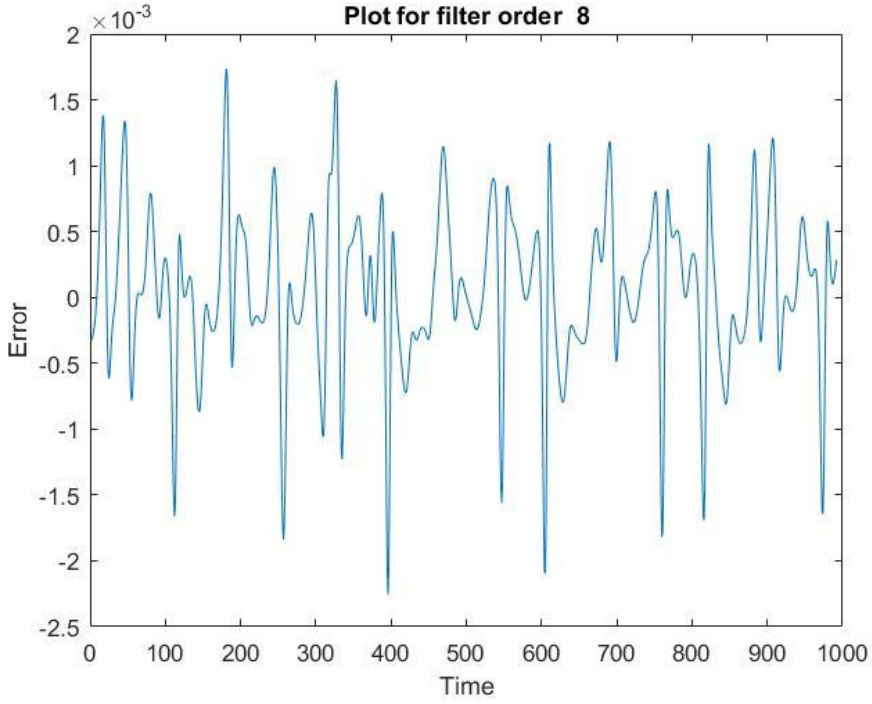
Solution: Since I get the minimum MSE at order =30 and regularization parameter = 0.001.
In the plots below I have given two plots for each order: 1) Input vs time 2) Error vs time
The plot for input vs time has different width because for different order of filter we are taking different features.
Observations:

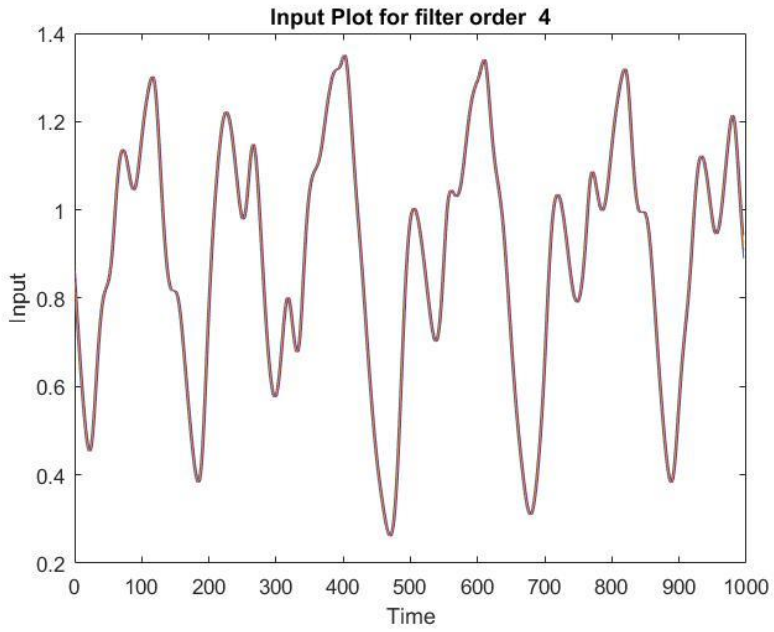
1) Error is minimum for the filter order and regularization parameter I determined using validation set. Hence my result is correct.
2) Error is normally distributed i.e it is symmetric and follows a bell curve. When any output is a combination of different independent instances then the we will have a normal distribution. This is the reason why error is not constant because it is also dependent on our calculated output which is the sum of independent instances (our different lag values for different order of filter).

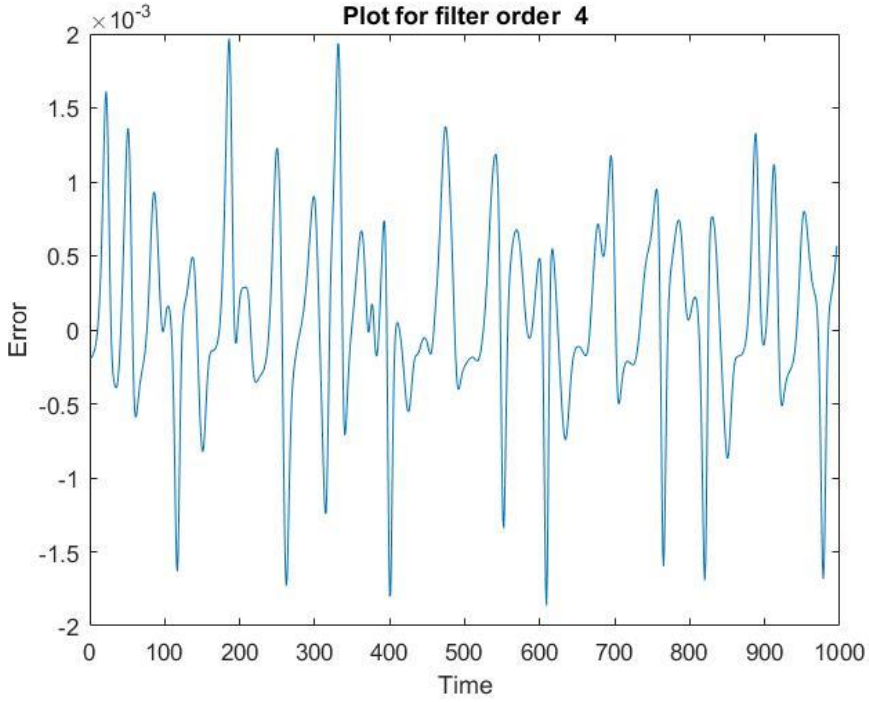

Input Plot for filter order 30
Input
Time
1.4
1.2
1
0.8
0.6
0.4
0.2
0
100
200
300
400
500
600
700
800
900
1000


Plot for filter order 30
×10-3
Error
Time
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
100
200
300
400
500
600
700
800
900
1000


Input Plot for filter order 8
Input
Time
1.4
1.2
1
0.8
0.6
0.4
0.2
0
100
200
300
400
500
600
700
800
900
1000

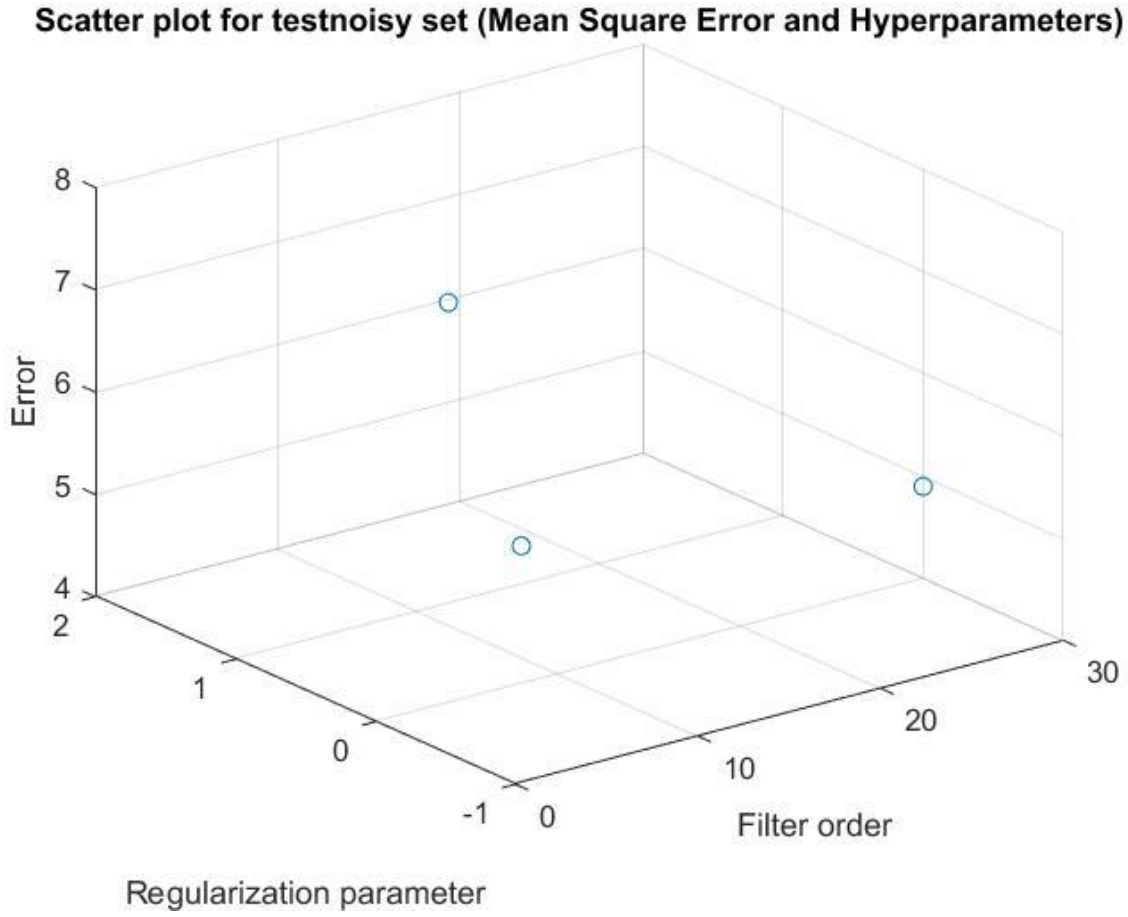

Plot for filter order 8
×10-3
Error
Time
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
100
200
300
400
500
600
700
800
900
1000


Input Plot for filter order 4
Input
Time
1.4
1.2
1
0.8
0.6
0.4
0.2
0
100
200
300
400
500
600
700
800
900
1000


Plot for filter order 4
×10-3
Error
Time
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
100
200
300
400
500
600
700
800
900
1000

1.3 (2 points) Quantify the performance in the MSE-sense of your algorithm using filter orders 4, 8 and 30 in the noisy data (testnoisy.mat). Which filter order works better for this noisy data? Explain the results.

Solution:The Scatter plot obtained for these three orders at regularization parameter = 0.001 is :



The observation is Mean square error is minimum at order = 30 and regularization parameter = 0.001 but the value has increased for testnoisy set.

Since we are using an FIR filter and FIR filter gives best performance for higher order. For this noisy test set we will use a higher order FIR filter. But if I would have been given a choice to select between FIR and IIR filter then I would have chosen IIR filter because it can eliminate noise and with less coefficients.

The value of Mean square error which I obtained for these three orders are:

Order	Regularization parameter	M.S.E
4	0.00100000000000000	7.90866568484954
8	0.00100000000000000	5.34311253655908
30	0.00100000000000000	4.89242199681275