

# Quantifying Urban Expansion In Estonia



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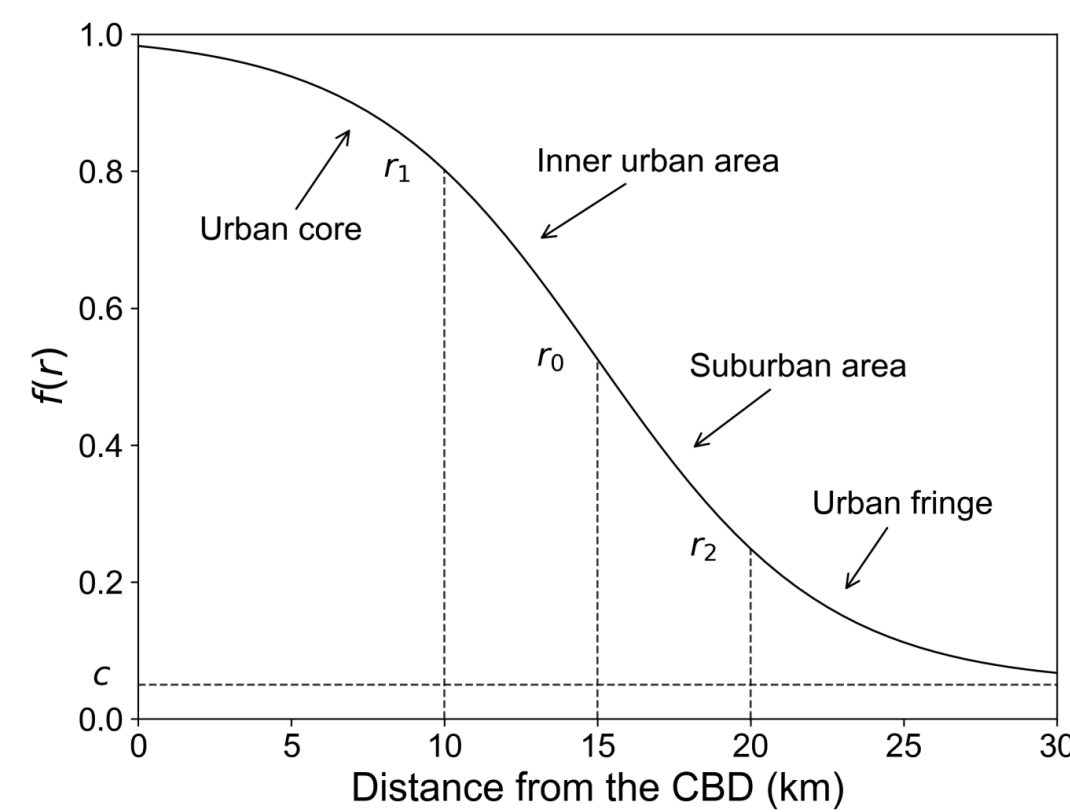
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## Introduction

- Attempts to mathematically model urban growth and form have been a part of urban geography ever since its beginnings. Many models are based on the spatiotemporal change of population density in cities.
- Due to the mobility of the modern city-dwellers, however, population data may not always accurately describe the complex dynamics of today's cities. In recent years, the increase in the quality and availability of satellite images has made it possible to use remote sensing combined with GIS software as a new alternative for modelling urban expansion.
- The aim of this study** was to evaluate the applicability of one such model, originally developed in China, in the context of Estonian cities. The model proposed an urban land density (ULD) function (*Fig. 1*), which describes ULD change in cities with a high accuracy, and provides derived indicators to measure the compactness and urban expansion of cities (*Fig. 2*).



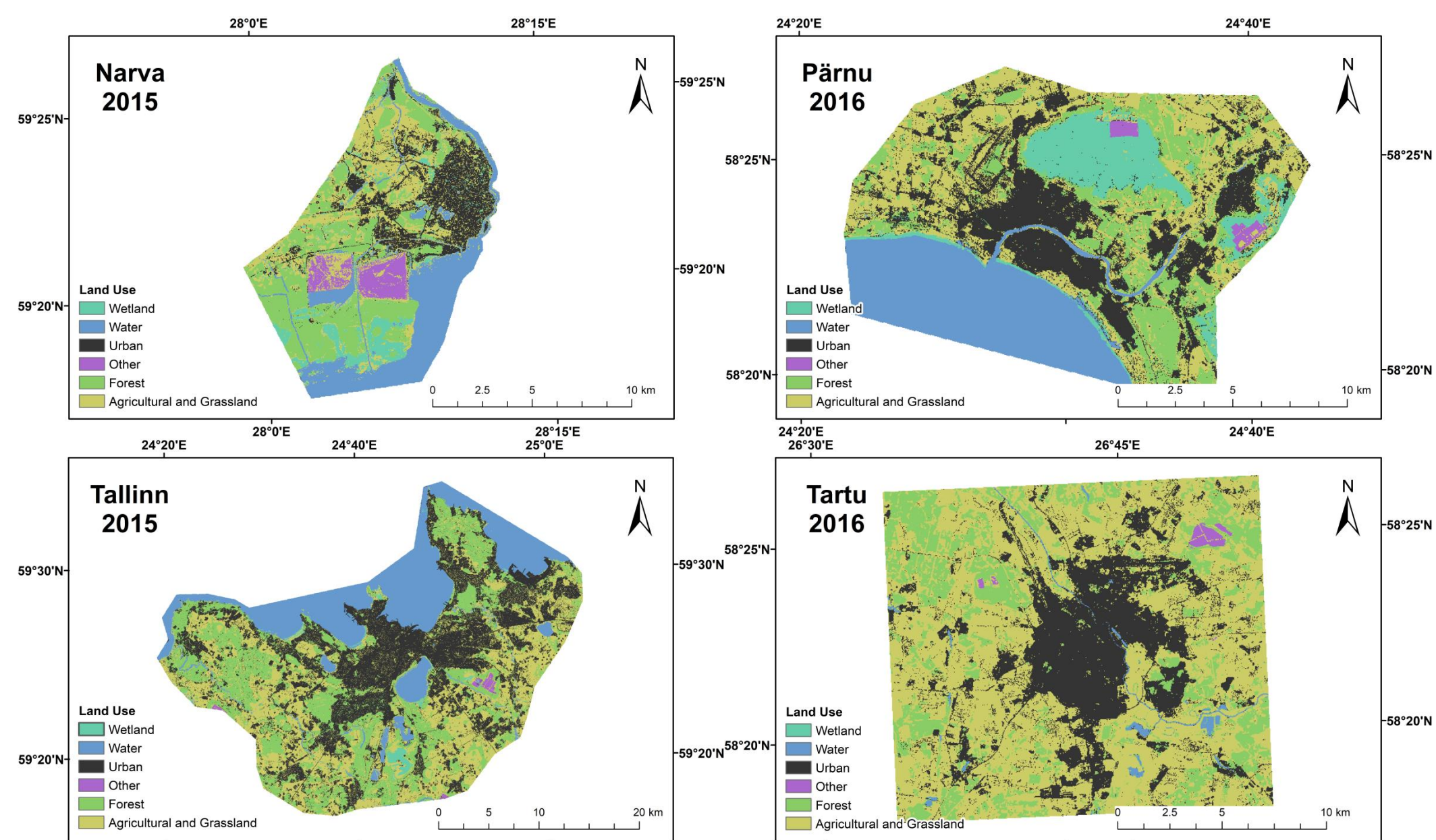
*Fig. 1. Graph of the function showing the sigmoidal decline of ULD.*

1)  $f(r) = \frac{1-c}{1+e^{\frac{r-r_1}{r_2-r_1}}}$  + c 2)  $r_1 = \frac{D}{2} \left( \frac{1.316957}{\alpha} + 1 \right)$   $r_2 = \frac{D}{2} \left( \frac{1.316957}{\alpha} - 1 \right)$  3)  $k_s = \frac{0.57735(1-c)\alpha}{1.316957D}$   $k_p = \frac{r_2-r_1}{D}$  4)  $S_r = \frac{\delta r_2}{\delta r_1} = \frac{(r_2^i - r_2^{i-1})r_1^{i-1}}{r_2^{i-1}(r_1^i - r_1^{i-1})}$

*Fig. 2. 1) ULD function, where  $r$  is the distance to the CBD,  $e$  is Euler's number,  $\alpha$  is the parameter controlling the curve,  $c$  is the ULD in the urban fringe and  $D$  is the boundary between the urban area and its hinterland. 2) Equations for the radii of the urban core ( $r_1$ ) and suburban area ( $r_2$ ), which denote the locations where the rate of decrease changes the fastest. 3) Compactness indicators derived from the function. 4) Equation for the degree of urban sprawl, where  $r_1^i$  and  $r_2^i$  are the radii of the urban core and the suburbs at time  $i$  and  $r_1^{i-1}$  and  $r_2^{i-1}$  are the radii at a previous time-point.*

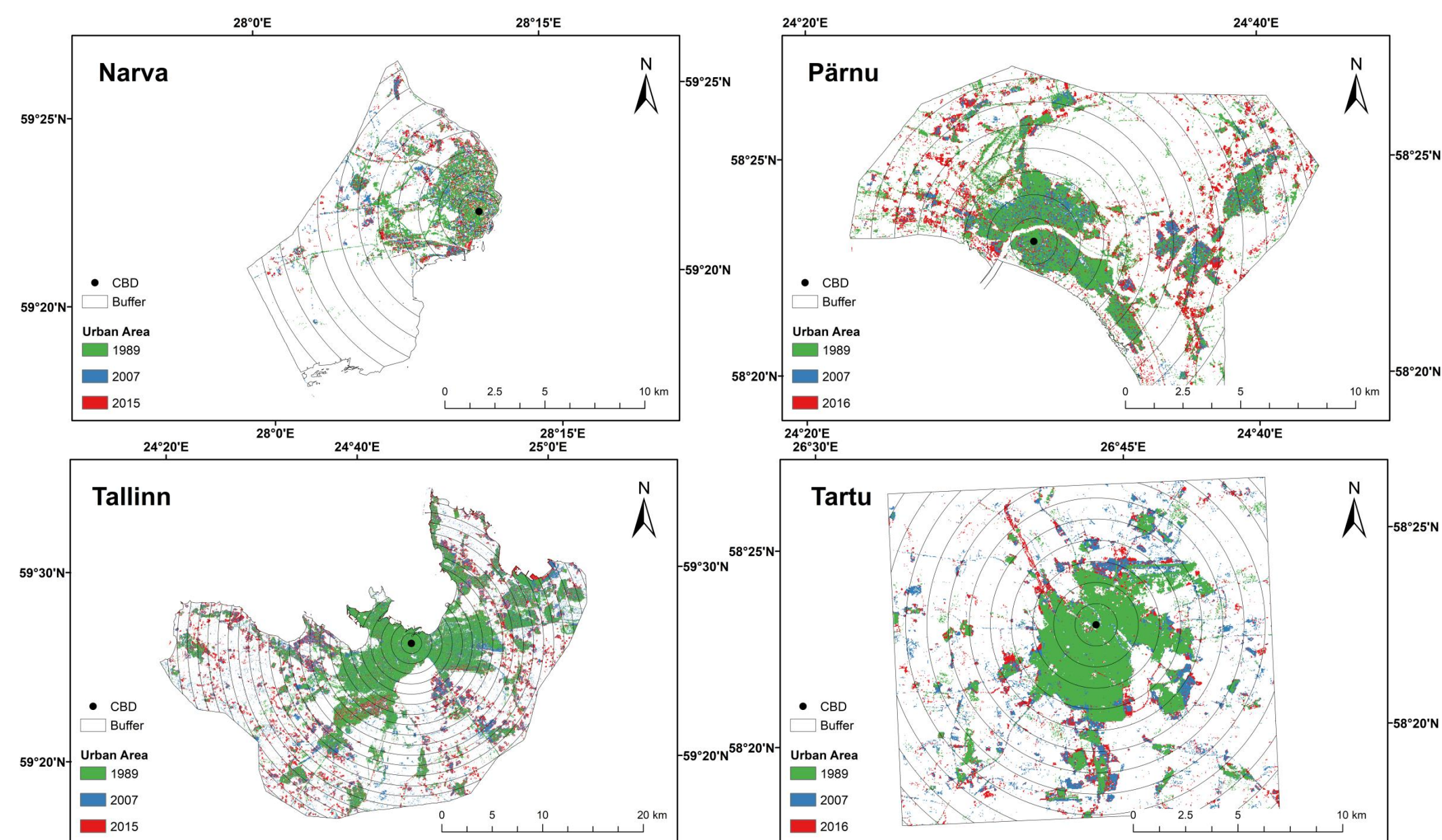
## Data & Methods

- Land use rasters (*Fig. 3*) derived from Landsat 30 m resolution images
- Four Estonian cities and three time-points (1989, 2007 & 2015/16)



*Fig. 3. An example of land use rasters used in this study.*

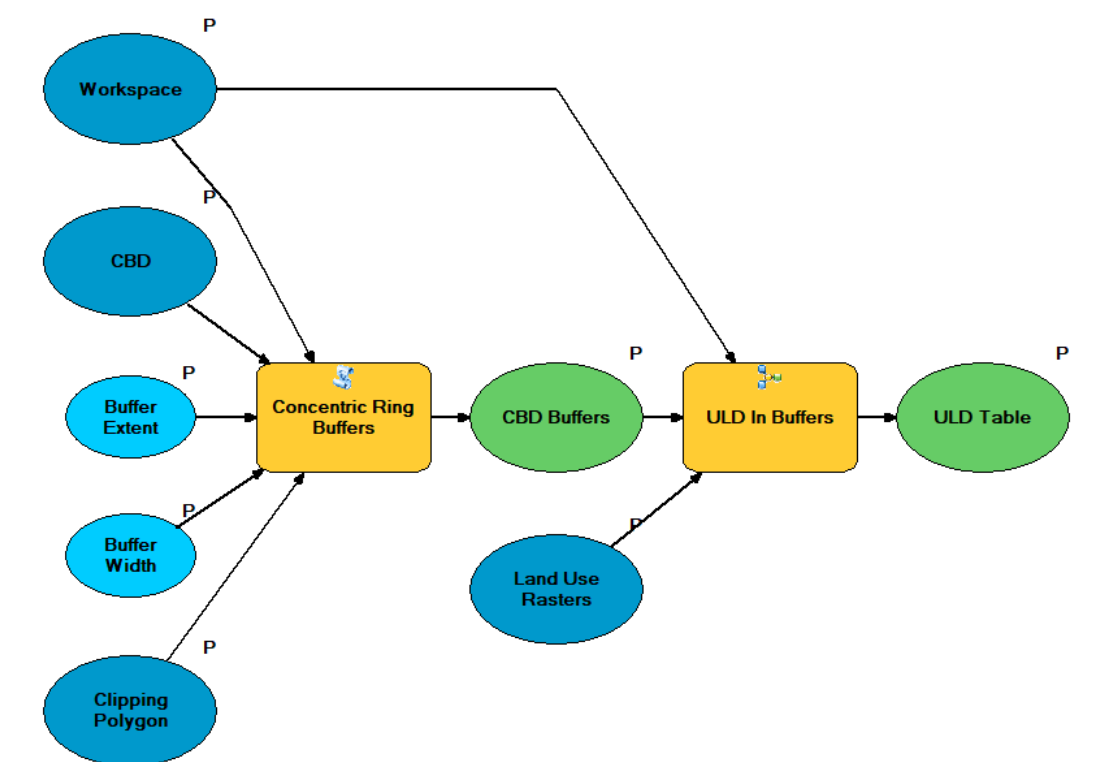
- ArcPy functions of the ArcGIS Spatial Analyst extension provided with the ESRI Student License enabled to create tools for the concentric ring partitioning of the cities (*Fig. 4*) and for calculating the ULD in the buffers.
- An automated workflow was created in Model Builder (*Fig. 5*).



*Fig. 4. The concentric ring partitioning of the cities.*

## Data & Methods (Cont.)

- The SciPy Python library was used for fitting the ULD values to the function.
- NumPy and Pandas libraries were used to calculate the compactness and urban sprawl indicators.

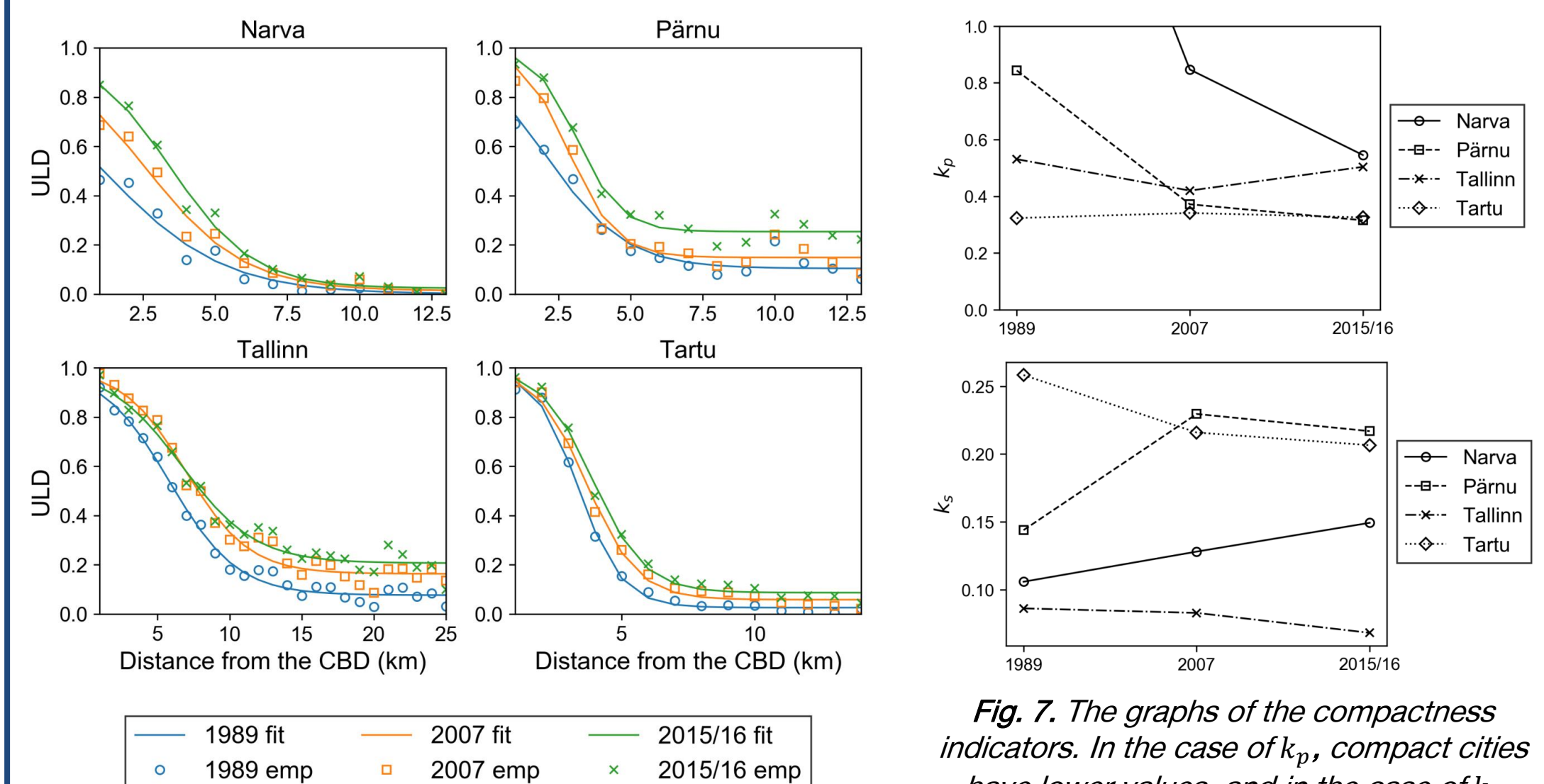


*Fig. 5. The workflow used for the ring buffer creation and ULD calculation.*

## Results

*Table 1. Optimized ULD function parameters and  $R^2$  values of the four cities at three time-points.*

City	1989				2007				2015/16			
	$\alpha$	$c$	$D$	$R^2$	$\alpha$	$c$	$D$	$R^2$	$\alpha$	$c$	$D$	$R^2$
Narva	0.546	0.000	2.263	<b>0.96</b>	1.555	0.013	5.257	<b>0.98</b>	2.414	0.023	6.926	<b>0.99</b>
Pärnu	1.561	0.104	4.263	<b>0.96</b>	3.541	0.148	5.759	<b>0.97</b>	4.177	0.253	6.304	<b>0.98</b>
Tallinn	2.481	0.077	11.662	<b>0.99</b>	3.136	0.163	13.894	<b>0.98</b>	2.164	0.207	13.353	<b>0.98</b>
Tartu	4.076	0.026	6.737	<b>0.99</b>	3.860	0.058	7.381	<b>0.99</b>	4.047	0.087	7.844	<b>0.99</b>



*Fig. 6. Comparison of the fitted and empirical urban land density graphs for the four cities.*

*Table 2. The growth rates of  $r_1$  and  $r_2$  ( $\delta r_1$  and  $\delta r_2$  respectively) and the  $S_r$  values of the cities for the two time periods observed in this study. Higher values ( $> 1$ ) indicate a tendency toward dispersed growth.*

City	1989–2007			2007–2015/16		
	$\delta r_2$	$\delta r_1$	$S_r$	$\delta r_2$	$\delta r_1$	$S_r$
Narva	–1.59	7.72	–0.21	0.20	5.69	0.04
Pärnu	0.01	5.80	0.00	0.35	1.38	0.26
Tallinn	2.57	11.55	0.22	0.71	–7.07	–0.10
Tartu	1.12	0.67	1.67	0.61	1.06	0.57

## Conclusions

- A sigmoidal decline of ULD from the CBD outward was observed in all four Estonian cities examined in this study. Therefore, the „Inverse-S shaped Rule“ does not depend on the size of the city.
- In compact cities, the ULD graphs showed a steeper decline than in cities that are more dispersed.
- Due to urban expansion, ULD in the hinterland increased over time and the boundary between urban area and its hinterland was pushed further away from the CBD.
- Some of the indicators derived from the model required a high enough ULD in order to work properly. For example, in cities with a very low ULD (Narva in 1989)  $k_p$  can produce unreliably high values.
- The  $k_s$  indicator was less dependent of ULD. However, it was biased against cities that have a higher  $D$  due to their size.
- $S_r$  produced reliable results only in Tartu, indicating dispersed growth in the earlier and infilling in the later time period. In other cities, the values were unreliably low or even negative.

## References

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