$$f(x) = rac{1}{\sqrt{(2\pi)^k |\Sigma|}} e^{-rac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)},$$

where  $x, \mu \in \mathbb{R}^k$ ,  $\Sigma$  is a k-by-k positive definite matrix and  $|\Sigma|$  is its determinant. Show that  $\int_{\mathbb{R}^k} f(x) \, dx = 1$ .

For  $Z_{k*k}$  is symmetric and positive definite, there exists an invertible matrix A 5, t,  $Z_{-}=AA^{T}$ , moreover  $|Z_{1}|=(\det A)^{2}$ Now, we define the new standale  $y=A^{T}(x-u) \Rightarrow x=u+Ay$ Moreover, we have the Jacobian determinant of this transformation is a  $dx=|\det A|dy=|Tz_{1}|dy$ 

2° Subsitituting X=4+Ay Tinto our exponent: (X-4) = y T AT (AAT) Ay = y Ty = 114112.

By2 1 Narok121 Jakexp(=1 11412) I detail dy

Therefore,  $\int_{\mathbb{R}^{k}} f(x) dx = \int_{-\infty}^{\infty} f(x)$ 

By Caculus 1 F

- 2. Let A, B be n-by-n matrices and x be a n-by-1 vector.
  - (a) Show that  $\frac{\partial}{\partial A} \mathrm{trace}(AB) = B^T$ .
  - (b) Show that  $x^T A x = \operatorname{trace}(x x^T A)$ .

(C) Derive the maximum likelihood estimators for a multivariate Gaussian.

(A)

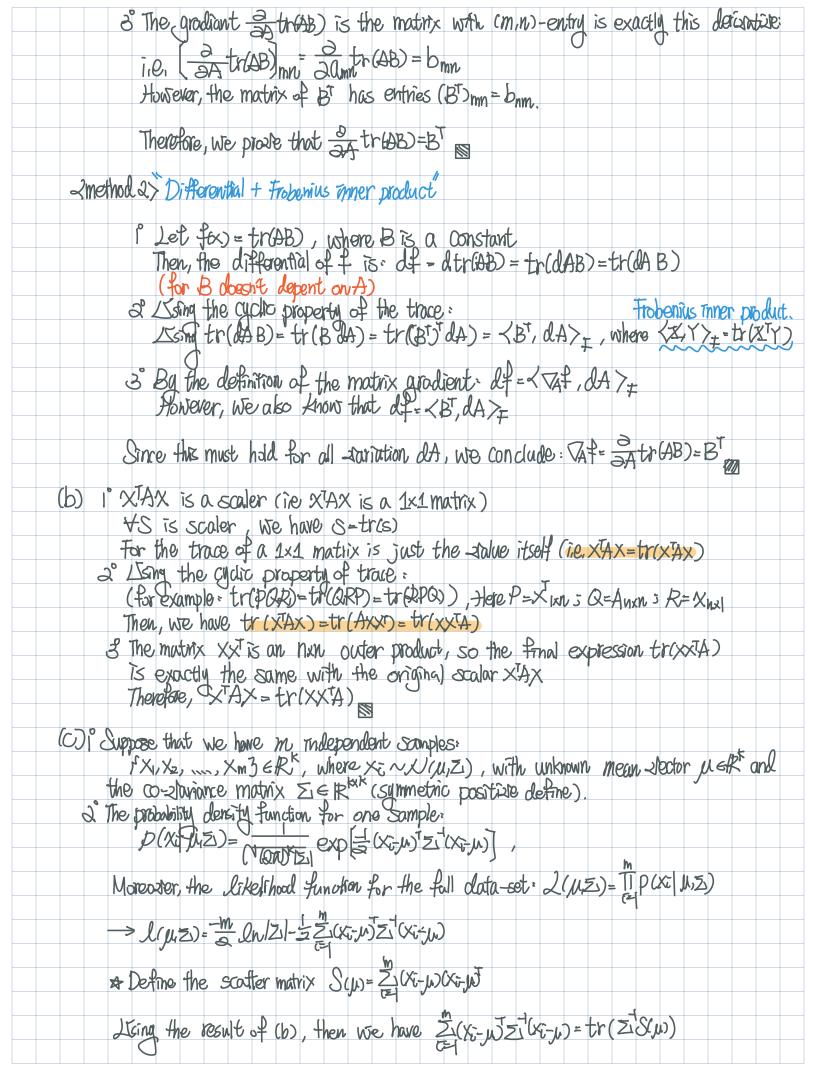
Lipfy 1° Suppose that A=[aij] = B=[bij], then n n

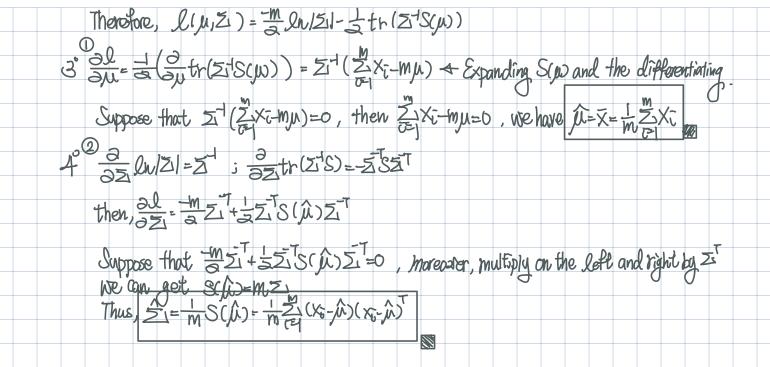
(AB); = Z=aikbki. Therefore, tr(AB) = Z=Z=aikbki

a We take the partial derivative for ann.

We can see that the only term which depend on ann is when i=mik=n.

Thus, the derivative = 2 tr(AB) = bmn





## 3. Unanswered Questions

There are unanswered questions from the lecture, and there are likely more questions we haven't covered.

(3: If there is a muticlass classification, how do we find the boundary?