

$$r = \ln \tau_{\max}(\theta)$$