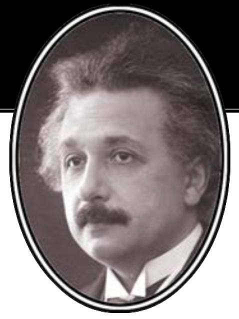
Einstein's Critique of the Equivalence Principle

How does Einstein's Original 1907 Equivalence Principle Differ from the Modern Equivalence Principles?

Hans Mühlen

2025



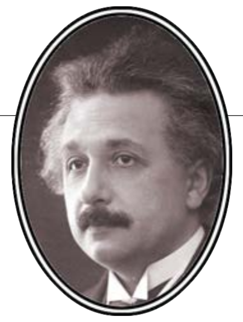
This talk is based on a true story

Some of the arguments have been changed for dramatic purposes

BEFORE I START…

Today I want to talk about
the early development phase of
the theory of general relativity

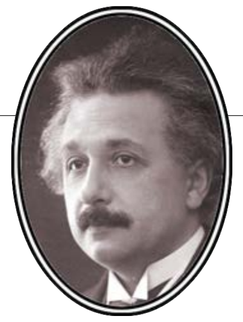
…but first I need to ask…



QUESTION TO THE AUDIENCE

Who of you have *any* experience with general relativity (GR)?

(Taken a course in GR,
taught a course in GR,
used GR in your research…)

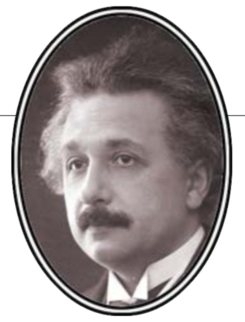


Don't worry —

The only theoretical tools I will allow myself to use in this talk are the same that were available to the young Einstein **in 1907**:

- **Newtonian mechanics** (**NM**),
- the new mechanics of **special relativity** (**SR**),
- **Newton's theory of gravity** (**NG**), and
- the **Maxwell-Lorentz theory of electromagnetism** (**EM**).

[*It's very tempting to mix in arguments from the future* ***theory of general relativity*** (**GR**), *but we must try to resist that temptation.*]

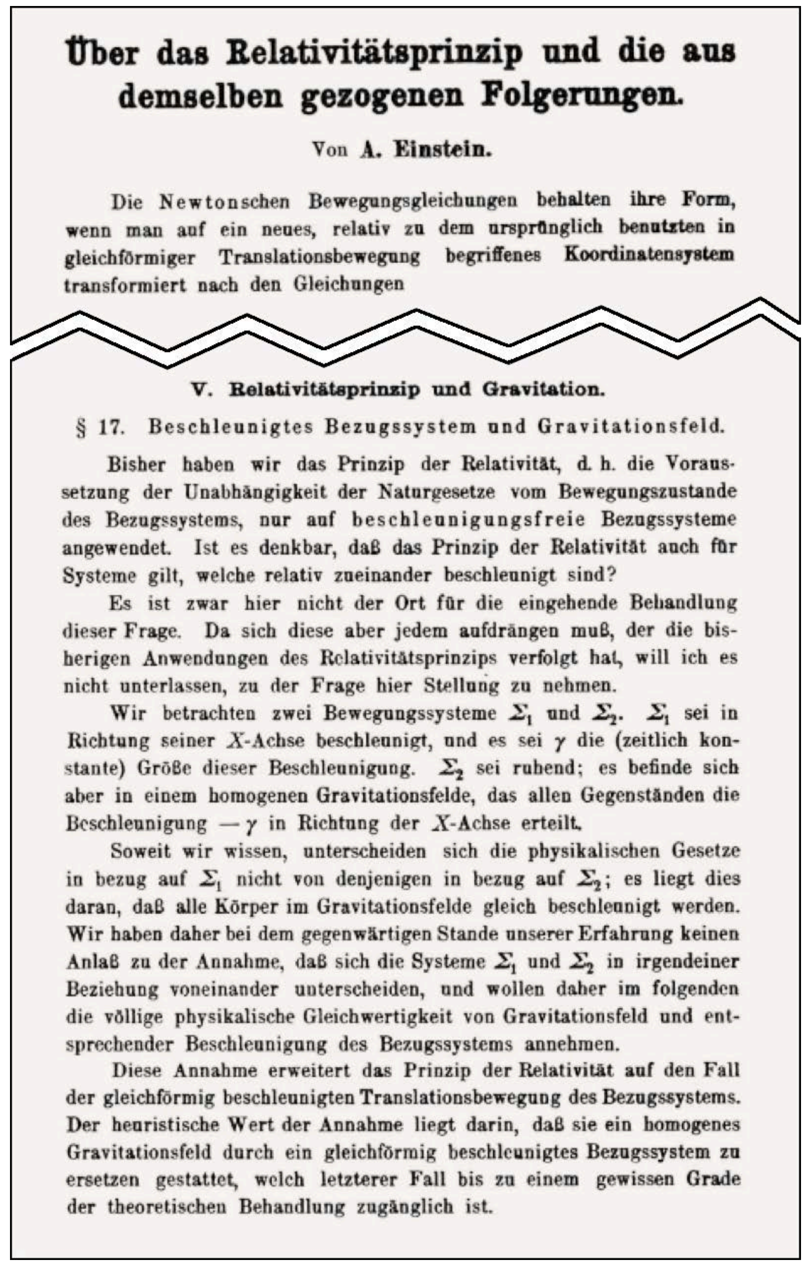


THIS TALK

It took Einstein eight long years to find the theory of general relativity:

It's a fascinating but complex story…

Submitted
4 December
1907



Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen.

Von **A. Einstein.**

Die Newtonschen Bewegungsgleichungen behalten ihre Form, wenn man auf ein neues, relativ zu dem ursprünglich benutzten in gleichförmiger Translationsbewegung begriffenes Koordinatensystem transformiert nach den Gleichungen

V. Relativitätsprinzip und Gravitation.

§ 17. Beschleunigtes Bezugssystem und Gravitationsfeld.

Bisher haben wir das Prinzip der Relativität, d. h. die Voraussetzung der Unabhängigkeit der Naturgesetze vom Bewegungszustande des Bezugssystems, nur auf beschleunigungsfreie Bezugssysteme angewendet. Ist es denkbar, daß das Prinzip der Relativität auch für Systeme gilt, welche relativ zueinander beschleunigt sind?

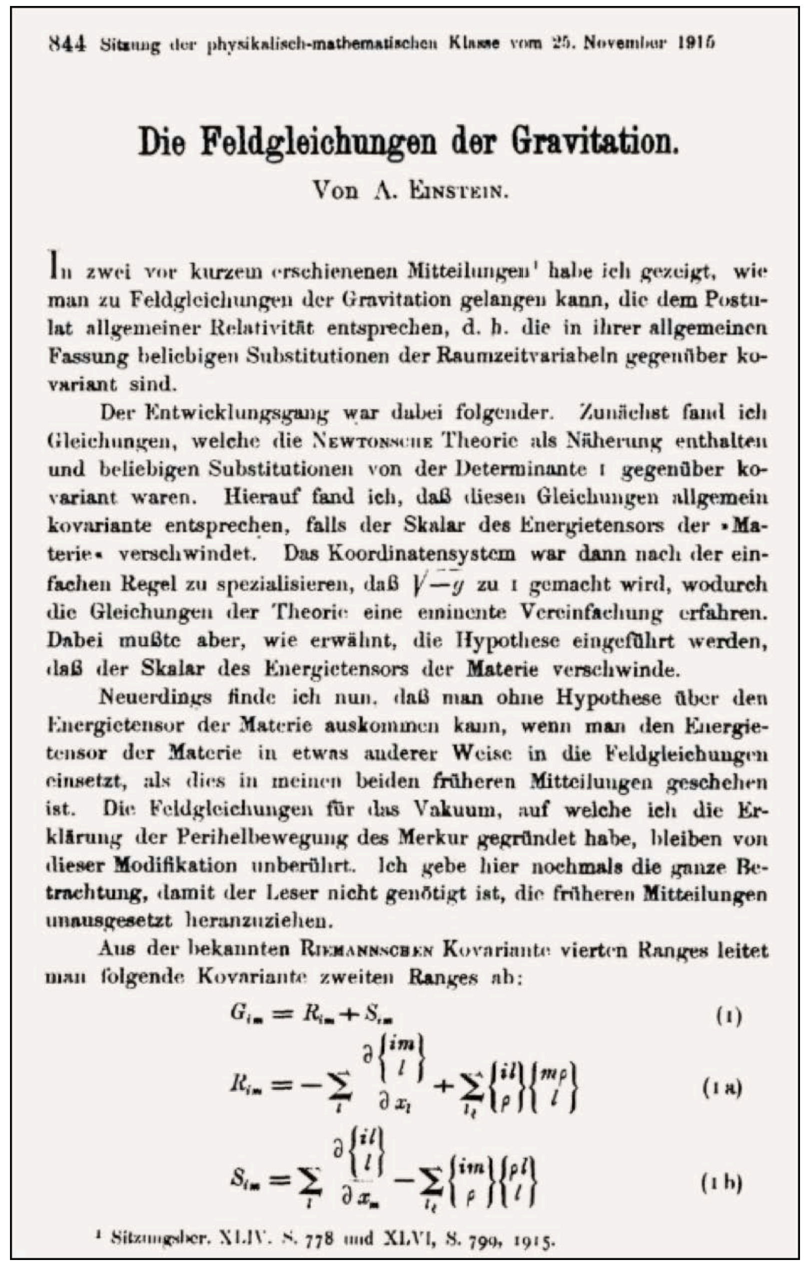
Es ist zwar hier nicht der Ort für die eingehende Behandlung dieser Frage. Da sich diese aber jedem aufdrängen muß, der die bisherigen Anwendungen des Relativitätsprinzips verfolgt hat, will ich es nicht unterlassen, zu der Frage hier Stellung zu nehmen.

Wir betrachten zwei Bewegungssysteme Σ_1 und Σ_2. Σ_1 sei in Richtung seiner X-Achse beschleunigt, und es sei γ die (zeitlich konstante) Größe dieser Beschleunigung. Σ_2 sei ruhend; es befinde sich aber in einem homogenen Gravitationsfelde, das allen Gegenständen die Beschleunigung $-\gamma$ in Richtung der X-Achse erteilt.

Soweit wir wissen, unterscheiden sich die physikalischen Gesetze in bezug auf Σ_1 nicht von denjenigen in bezug auf Σ_2; es liegt dies daran, daß alle Körper im Gravitationsfelde gleich beschleunigt werden. Wir haben daher bei dem gegenwärtigen Stande unserer Erfahrung keinen Anlaß zu der Annahme, daß sich die Systeme Σ_1 und Σ_2 in irgendeiner Beziehung voneinander unterscheiden, und wollen daher im folgenden die völlige physikalische Gleichwertigkeit von Gravitationsfeld und entsprechender Beschleunigung des Bezugssystems annehmen.

Diese Annahme erweitert das Prinzip der Relativität auf den Fall der gleichförmig beschleunigten Translationsbewegung des Bezugssystems. Der heuristische Wert der Annahme liegt darin, daß sie ein homogenes Gravitationsfeld durch ein gleichförmig beschleunigtes Bezugssystem zu ersetzen gestattet, welch letzterer Fall bis zu einem gewissen Grade der theoretischen Behandlung zugänglich ist.

Submitted
25 November
1915



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \tag{1}$$

$$R_{im} = -\sum_{l} \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \tag{1a}$$

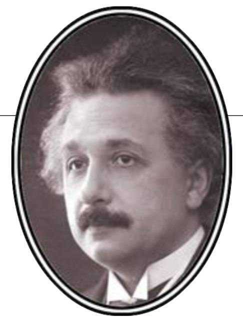
$$S_{im} = \sum_{l} \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \tag{1b}$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915.

1905 **1907** 1909 1911 1913 **1915** 1917 1919 1921 1923

Albert Einstein, "*On the relativity principle and the conclusions drawn from it*" (1907)

Albert Einstein, "*The Field Equations of Gravitation*" (1915)

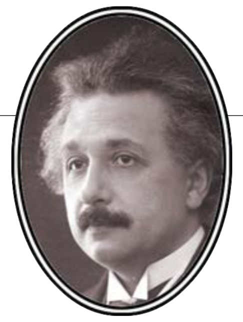


THIS TALK

I have chosen to focus on only a small part of this long and multifaceted story:

the role of the Equivalence Principle
in Einstein's search for general relativity

In particular I want to see **how the formulation and understanding of Einstein's original ideas have changed** over the past century.



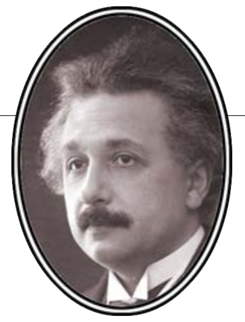
THIS TALK

The standard reference to the historical development and meaning of Einstein's Equivalence Principle is

John D. Norton
What was Einstein's Principle of Equivalence?
Studies in History and Philosophy of Science, **16** (1985) 203–246
`doi.org/10.1016/0039-3681(85)90002-0`

More works by Norton on the history of relativity can be found at

`sites.pitt.edu/~jdnorton`



A WORD OF WARNING

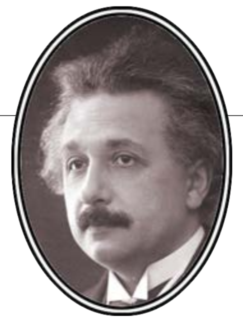
As many of you may know, …

in modern texts on general relativity one would typically distinguish between **a "weak" and a "strong" version of "the Equivalence Principle"**.

However, analysis of Einstein's original texts shows that…

the modern "weak" and "strong" Equivalence Principles are very different from the idea that Einstein proposed back in 1907 and called "the Equivalence Principle".

So to avoid confusion…



A WORD OF WARNING

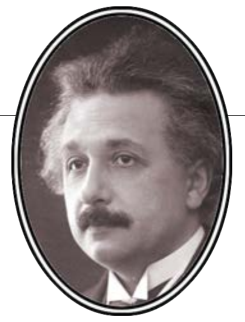
…so to avoid confusion, I will often use the German phrase

ÄQUIVALENZPRINZIP

to denote the **original 1907 idea of Einstein**, and the English phrase

EQUIVALENCE PRINCIPLE

for the various **modern principles**.



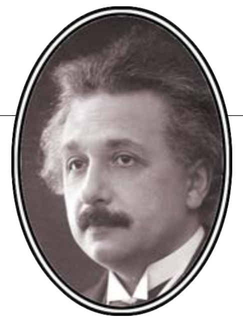
THIS TALK — OUTLINE

PART 1

1. A statement of **the two modern versions** of the (weak and strong) Equivalence Principles
2. **Einstein's formulation** of the original *Äquivalenz-prinzip* of 1907
3. **How Einstein made use of** the 1907 *Äquivalenz-prinzip* in his search for a relativistic theory of gravity

PART 2

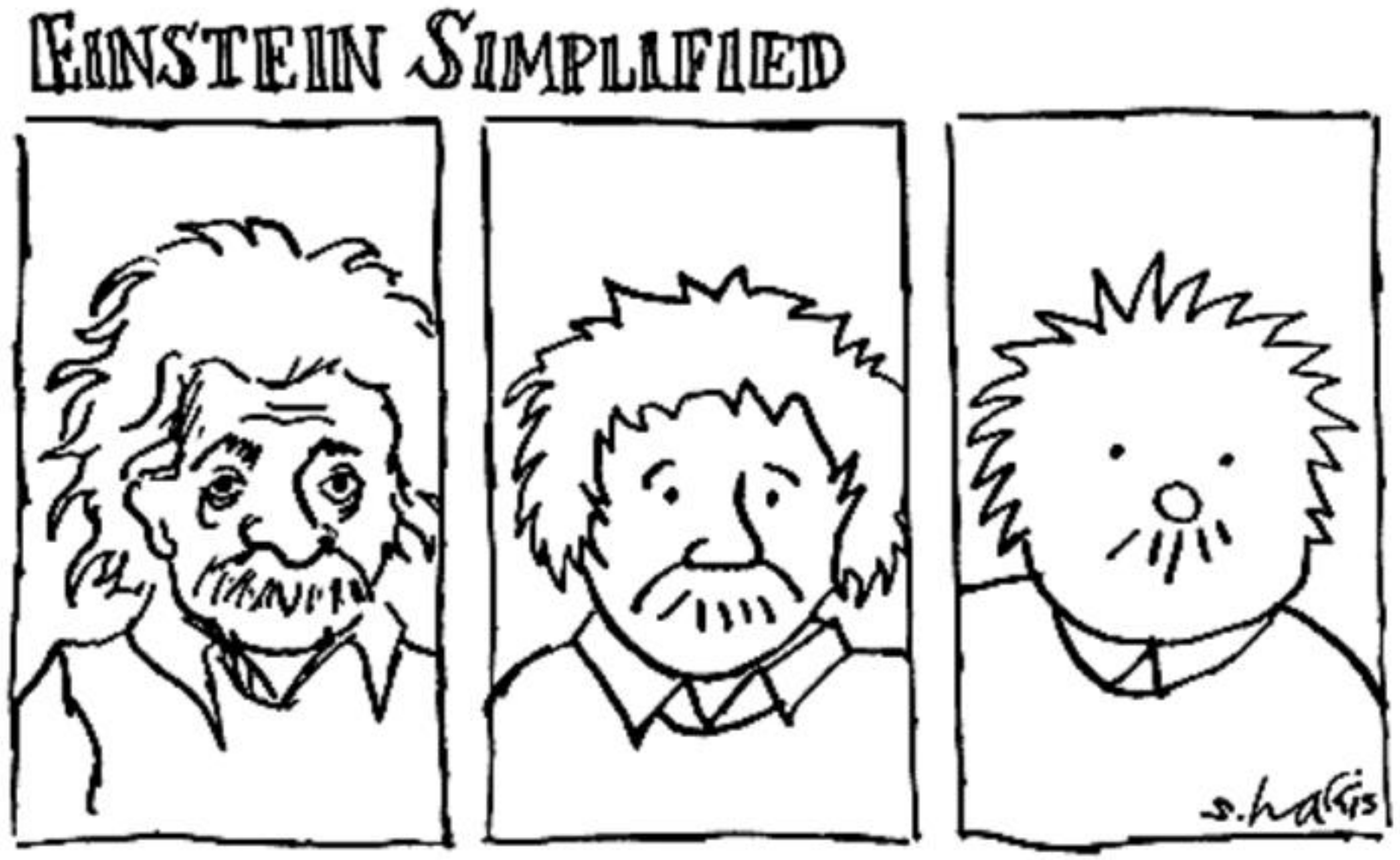
4. **The "generative" and "eliminative" observers** in different statements of the *Äquivalenzprinzip*
5. **Attempts to generalise the *Äquivalenzprinzip*:** the Infinitesimal Equivalence Principle
6. **Einstein's critique** of the generalised principles



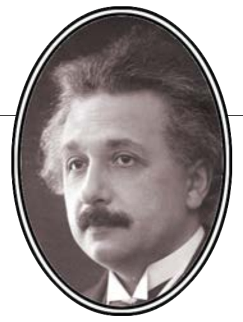
ANOTHER WORD OF WARNING

For the benefit of time **I have had to simplify the story** of what motivated Einstein to formulate his Equivalence Principle. So…

my presentation will not be historically accurate!

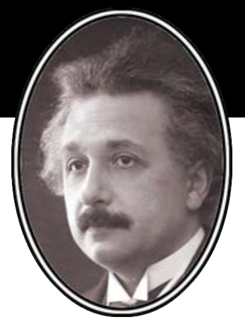


Cartoon by **Sidney Harris**



PART 1

EINSTEIN’S *ÄQUIVALENZPRINZIP*



QUESTION TO THE AUDIENCE

Who can help me give a
(roughly correct) **statement**
of the modern
"Weak Equivalence Principle",
and of the
"Strong Equivalence Principle"?

THE MODERN EQUIVALENCE PRINCIPLES

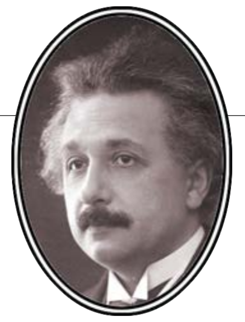
In the literature you can find MANY different variants of the definitions of WEP and SEP. These will suffice for now:

Weak Equivalence Principle (WEP)

The acceleration of a freely falling body is independent of its mass.

Strong Equivalence Principle (SEP)

In an infinitesimal spacetime region, for a freely falling observer, the laws of physics reduce to their special relativistic form.



THE MODERN EQUIVALENCE PRINCIPLES

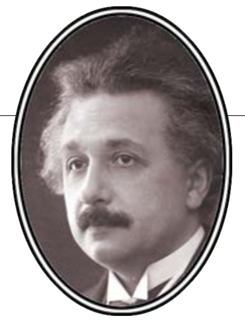
The "Equivalence Principle" has been **severely criticized** by many authors over the years:

> **I have never been able to understand this Principle** [*of Equivalence*]. Does it mean that the signature of the space-time metric is +2 [...]? **If so, it is important, but hardly a Principle** [...] Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? **If so, it is false.**

J. L. Synge, "Relativity: The General Theory" (1960)

> Despite the pivotal role played at the dawn of general relativity, **the "principle of equivalence" is now regarded as a rather vague heuristic statement**, perhaps useful for teaching purposes, but surely **unable to challenge the contemporary approach to gravity**, based on sharp and neat mathematical axioms and precision-test experiments.

Eolo Di Casola, Stefano Liberati, Sebastiano Sonego,
"Weak Equivalence Principle for Self-Gravitating Bodies" (2014)



THE MODERN EQUIVALENCE PRINCIPLES

The “Equivalence Principle” has been **severely criticized** by many authors over the years:

> Various statements identified with the principle of equivalence **are not acceptable** because they are either not generally valid or are simply the definition of an inertial coordinate system.

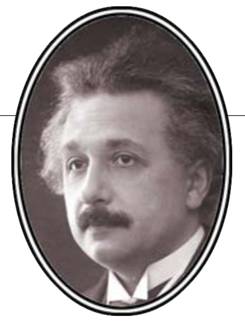
Fritz Rohrlich, “The Principle of Equivalence” (1963)

> In contrast with early statements by Einstein and others on the axiomatic basis of the theory of general relativity, **the *principle of equivalence* is not one of its underlying axioms**.

Mendel Sachs, “On the Logical Status of Equivalence Principles…” (1976)

> [*Einstein’s*] formulation [*of the principle of equivalence*] is **deeply flawed** when taken literally.

E. Poisson, C. M. Will, “Gravity. Newtonian, Post-Newtonian, Relativistic” (2014)



THE MODERN EQUIVALENCE PRINCIPLES

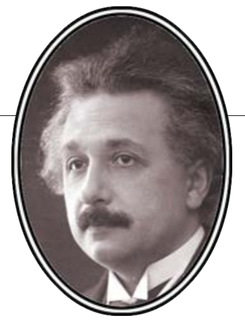
But then there are other authors who **claim that the "Equivalence Principle" is indispensable**, and part of the foundations of general relativity:

> **At the foundations** of Einstein's geometrodynamics [=*the theory of general relativity*] and of its geometrical structure is one of the best-tested principles in the whole field of physics: the equivalence principle.

Ignazio Ciufolini, John A. Wheeler, "Gravitation and Inertia" (1995)

> The principle of equivalence has historically played an important role in the development of gravitation theory. [...] In 1907, Einstein used the principle as a basic element in his development of general relativity (GR). **We now regard the principle of equivalence as the foundation**, not of Newtonian gravity or of GR, but of the broader idea that spacetime is curved.

C. M. Will, "The Confrontation between General Relativity and Experiment" (2014)



THE MODERN EQUIVALENCE PRINCIPLES

This fundamental disagreement between relativists on the status of the principle has led to **a vast "improved Equivalence Principles" literature**, with no signs of convergence so far:

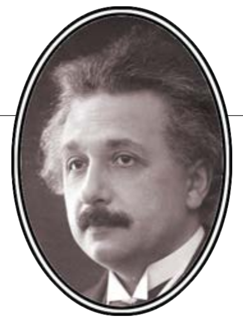
> The strict adoption of [*Einstein's*] principle has led to **a pointless literature of apparent paradoxes, debates and conundrum.**

Eric Poisson, Clifford M. Will, "Gravity. Newtonian, Post-Newtonian, Relativistic" (2014)

A selection of proposed "clarifications" of the Equivalence Principle:

*Weak Equivalence Principle (**WEP**), Universality of Free Fall (**UFF**), Equality of Inertial and Gravitational Mass, Gravitational Weak Equivalence Principle (**GWEP**), Newton's Equivalence Principle (**NEP**), Galilei's Equivalence Principle, Newstein Equivalence Principle…*

*Strong Equivalence Principle (**SEP**), Einstein Equivalence Principle (**EEP**), Medium Strong Equivalence Principle (**MSEP**), Very Strong Equivalence Principle (**VSEP**), Strict Local Equivalence Principle (**SLEP**), $\forall\delta\exists\varepsilon$ Equivalence Principle (**DEEP**):, Punctual Equivalence Principle (**PEP**), Local Lorentz Invariance (**LLI**), Local Position Invariance (**LPI**), Comma-Goes-to-Semicolon Rule…*



So what was Einstein’s original *Äquivalenzprinzip?*

We have to go back to 1907, when several attempts at finding a (special) relativistic theory of gravity had failed.

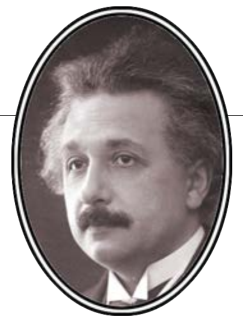
Then **Einstein thought he had found an argument** which promised to reveal some of the properties of a more fundamental **relativistic theory of gravity**.

So what was Einstein's original *Äquivalenzprinzip?*

Einstein formulated the argument **in the context of Newtonian physics**, and in **3D space** (rather than spacetime).

This is entirely textbook material, so many of you will have no problem recognising it.

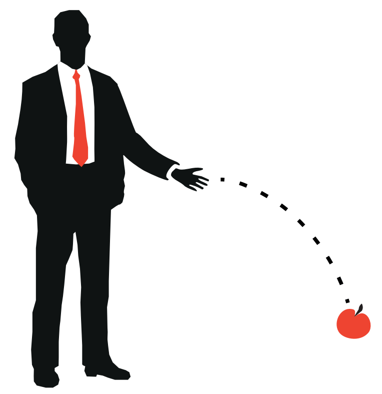
Only the final step — **how Einstein actually made use of his idea to find the new theory** — might be new to you.



Step 1

In an otherwise empty region of Newtonian 3D space there is an observer performing **mechanical experiments:**

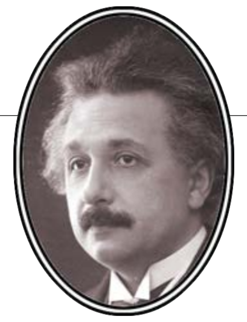
Observation



When the observer throws a ball it **accelerates** along a **curved path**.

How would the observer explain* the observed behaviour of the thrown ball?

* *using only **Newtonian** physics!*



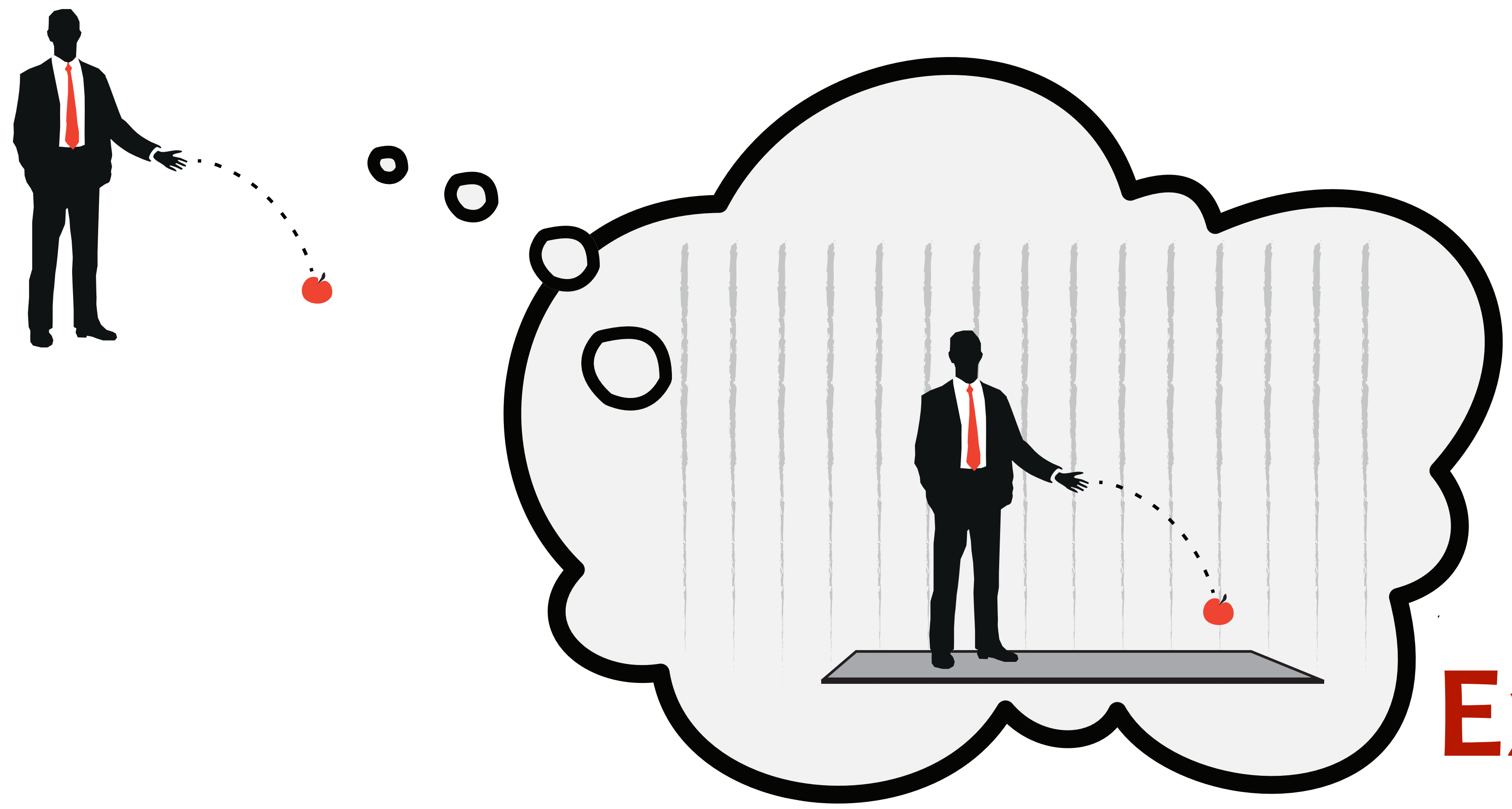
The observer might think that he is **in uniformly accelerated motion** in a region of space **without any gravitational field**…

Explanation 1.

Equations of motion for body thrown by the
observer in uniform acceleration $\boldsymbol{a_z = a}$
(in coordinates adapted to the rest frame of the observer):

$$\frac{d^2x^\mu}{dt^2} = \frac{d^2y^\mu}{dt^2} = 0, \quad \frac{d^2z^\mu}{dt^2} = -a$$ … **a parabola**

… or the observer could claim to be **at rest in a homogeneous gravitational field.**



Explanation 2.

Equations of motion for body thrown by the **observer at rest in a homogeneous gravitational field with potential** g (in coordinates adapted to the rest frame of the observer):

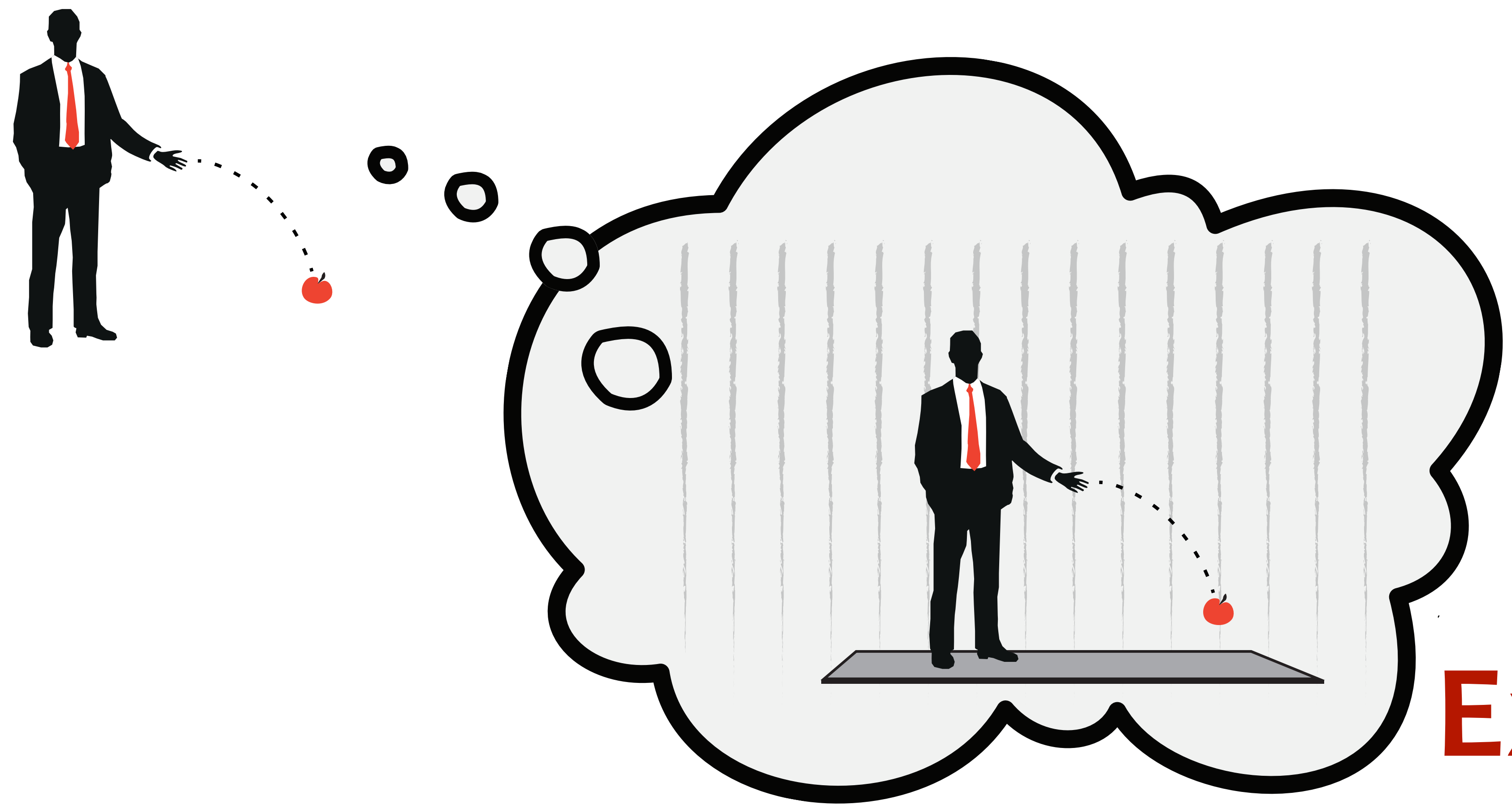
$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2}=\frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2}=0, \quad \boldsymbol{m_i}\frac{\mathbf{d^2}\boldsymbol{z^\mu}}{\mathbf{d}\boldsymbol{t^2}}=-\boldsymbol{m_g g}$$

But if $\boldsymbol{m_i=m_g}$ we get:

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2}=\frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2}=0, \quad \frac{\mathbf{d^2}\boldsymbol{z^\mu}}{\mathbf{d}\boldsymbol{t^2}}=-\boldsymbol{g}$$

… **a parabola**

… or the observer could claim to be **at rest in a homogeneous gravitational field.**



Explanation 2.

Note that *this* result only obtains if we *combine* the **theoretical formalisms** of **Newtonian mechanics** and **Newtonian gravity** with the **empirical result** $\boldsymbol{m_i = m_g}$

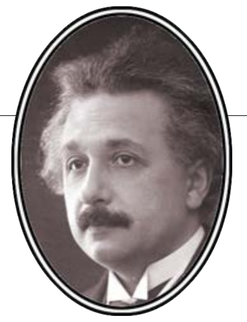
Equations of motion for body thrown by the **observer at rest in a homogeneous gravitational field with potential g** (in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2} = 0, \quad m_i\frac{\mathrm{d}^2z^\mu}{\mathrm{d}t^2} = -m_g g$$

But if $m_i = m_g$ we get:

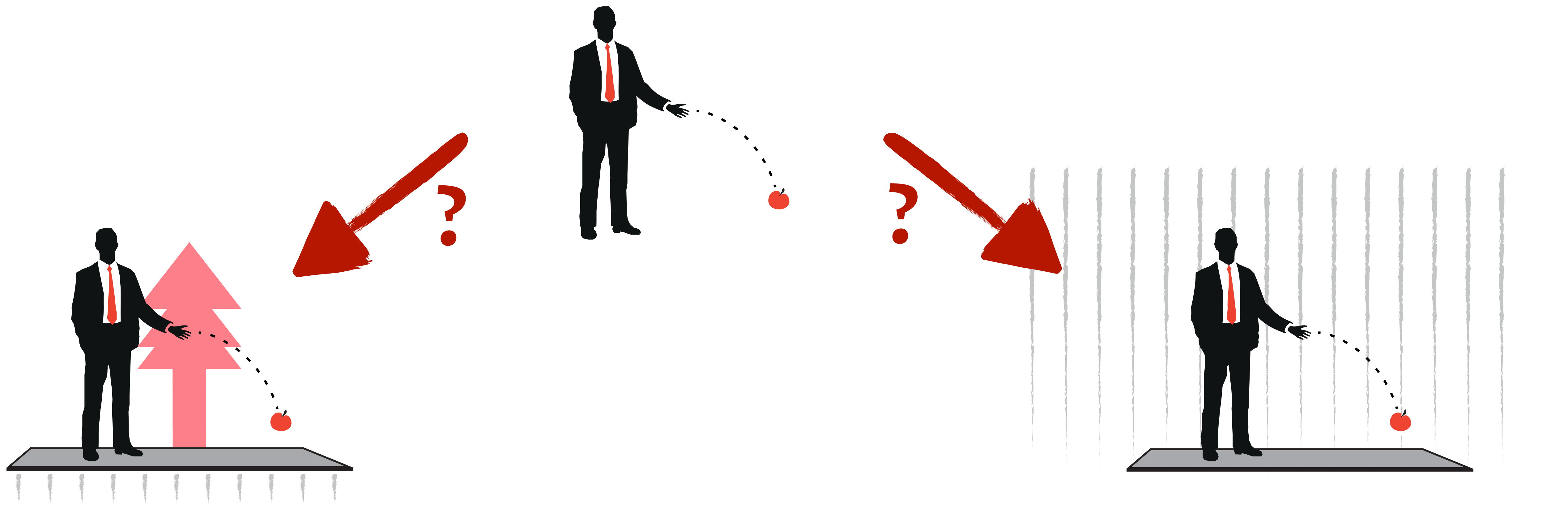
$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2} = 0, \quad \frac{\mathrm{d}^2z^\mu}{\mathrm{d}t^2} = -g$$

… a parabola



EINSTEIN'S EQUIVALENT NEWTONIAN OBSERVERS

Even though the observed effect is the same, the two explanations are of a **very different nature**:



In this case the observer is only **accelerating.**

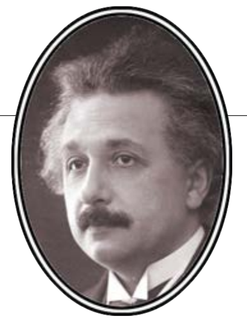
The observed curved path is a **kinematic effect** (acceleration).

The path of the body is due to its inertial motion, which is **determined only by the properties of space**.

In this case the observer is only **in a gravitational field.**

The observed curved path is a **dynamical effect** (gravitational force).

The path of the body is due to its response to an external gravitational field, which is **determined by the distribution of matter**.



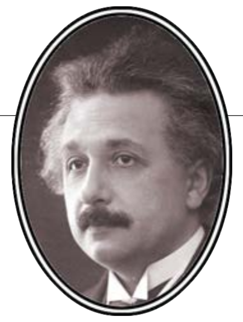
ASIDE: THE “EINSTEIN ELEVATOR”

Question from the audience:

*“What about **elevators**?*

There are always elevators involved in discussions of the Equivalence Principle.

*We demand to see the actual **Einstein elevator**!”*

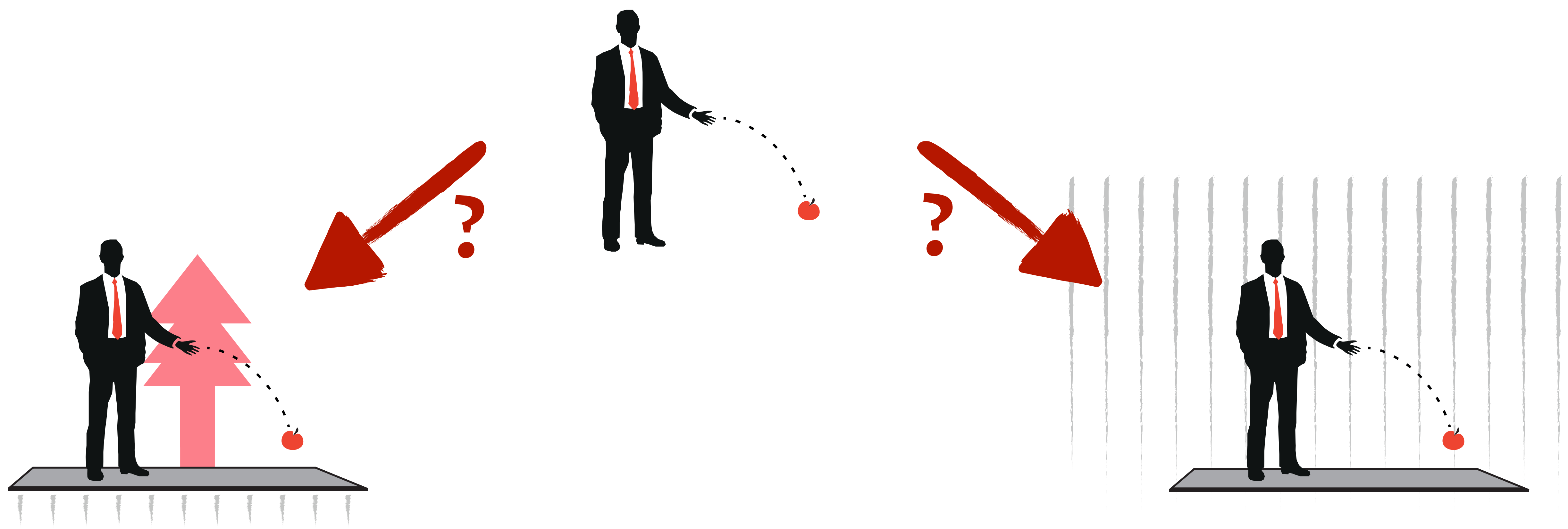


ASIDE: THE "EINSTEIN ELEVATOR"

Here is how Einstein describes the situation:

> **An observer enclosed in a box** can in no way decide whether the box is at rest in a static gravitational field, or whether it is in accelerated motion, maintained by forces acting on the box, in a space that is free of gravitational fields.

Albert Einstein, Marcel Grossmann, "Outline of a Generalized Theory of Relativity and of a Theory of Gravitation" (1913)

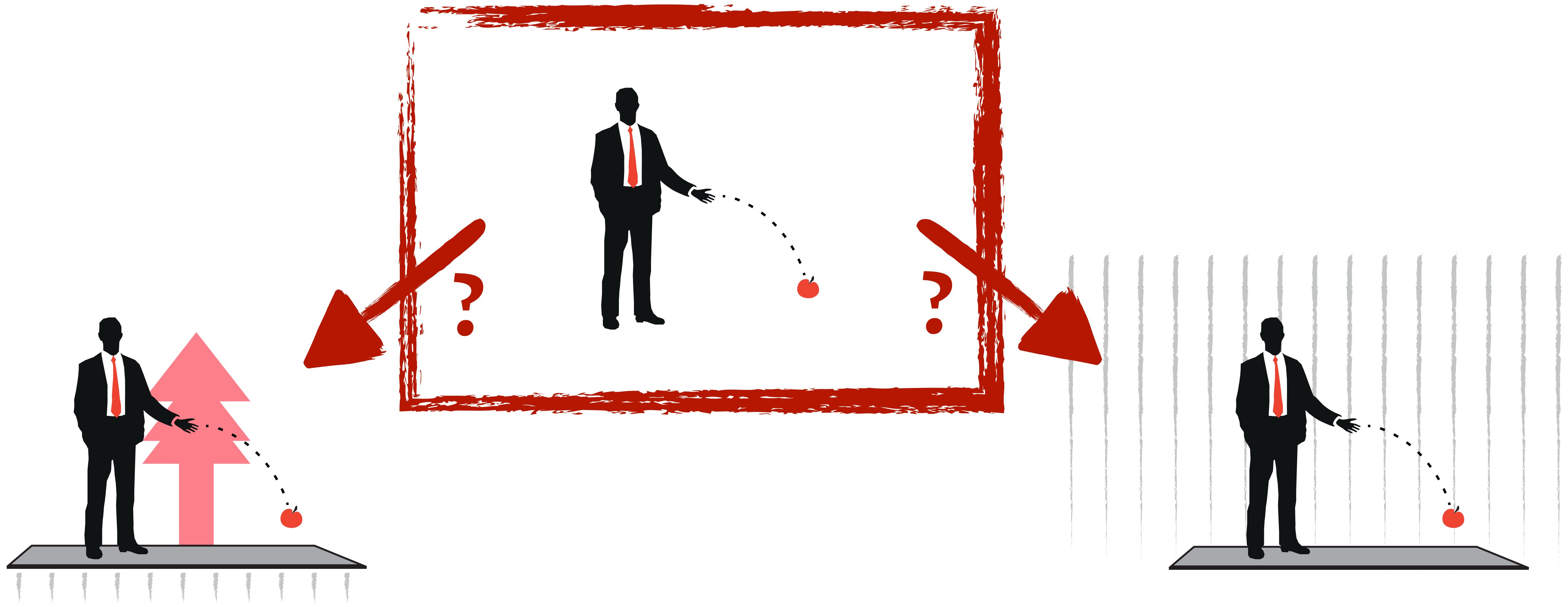


Explanation 1. **Explanation 2.**

Here is how Einstein describes the situation:

> **An observer enclosed in a box** can in no way decide…

But why does the observer have to be **"enclosed in a box"**?



ASIDE: THE “EINSTEIN ELEVATOR”

Here is how Einstein describes the pair of equivalent observers:

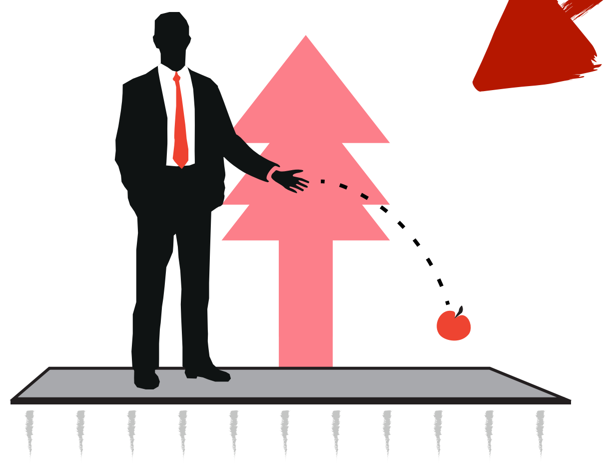
> **An observer enclosed in a box** can in no way decide…

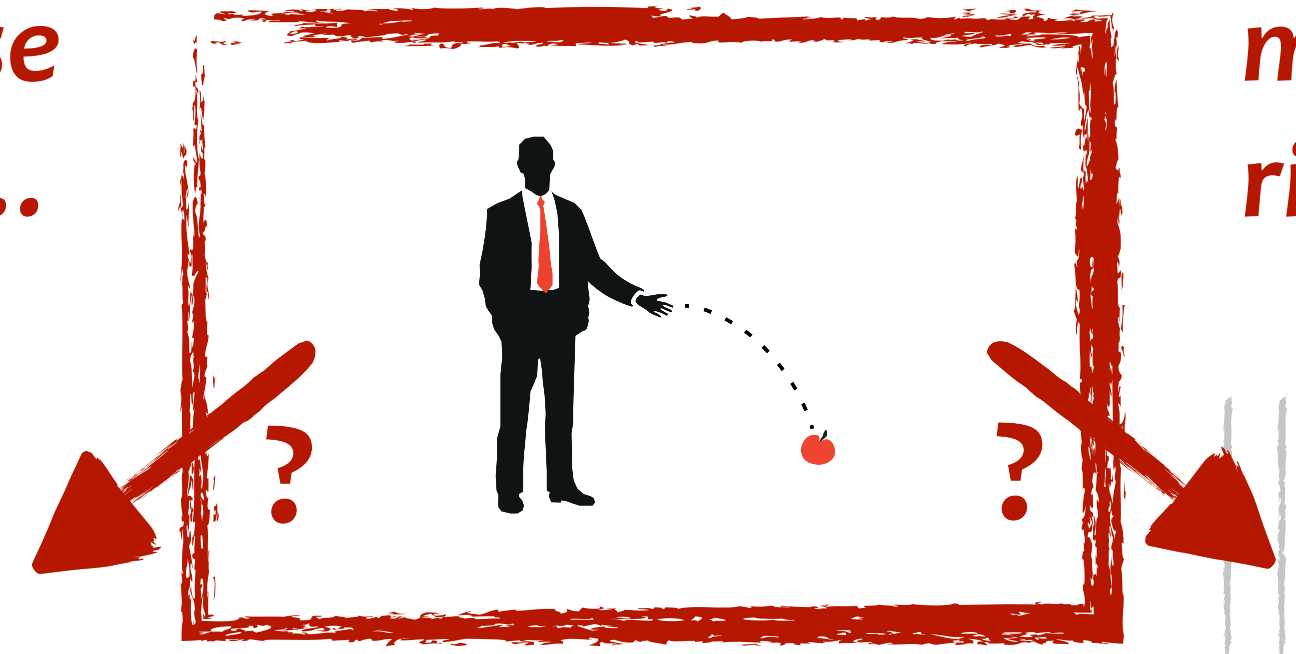
But why does the observer have to be **“enclosed in a box”**?

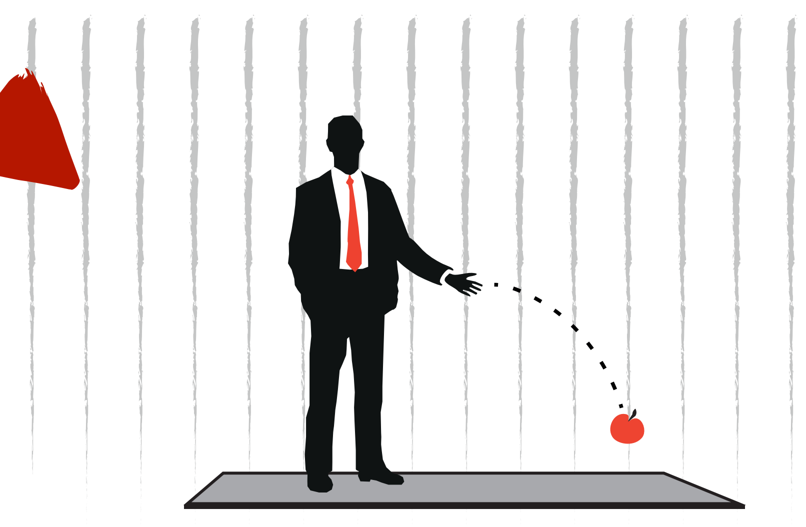
This is of course to prevent the observer…

…from checking for any device that may cause an acceleration…

…or from checking for any massive bodies that could give rise to a gravitational field.







ASIDE: THE "EINSTEIN ELEVATOR"

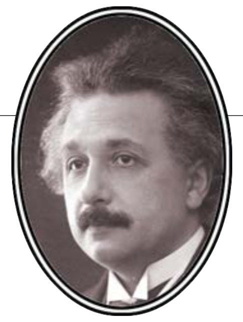
Two physicists, A and B, **awake from a narcotic sleep** and notice that **they are in a closed box that has nontransparent walls** and is equipped with all their instruments. They have no idea where the box is situated and whether it moves...

Albert Einstein, "On the Present State of the Problem of Gravitation" (1913)

Let us imagine **a spacious chest resembling a room** with an observer inside who is equipped with apparatus...

To the middle of the lid of the chest is fixed externally a hook with rope attached, and now **a 'being' (what kind of a being is immaterial to us)** begins pulling at this with a constant force...

Albert Einstein, "Relativity. The Special and the General Theory. A Popular Exposition" (1917)



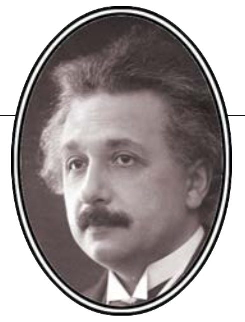
ASIDE: THE "EINSTEIN ELEVATOR"

So **there is no mention of "elevators"** in Einstein's writings from this time. Not even of "lifts".

ASIDE: THE "EINSTEIN ELEVATOR"

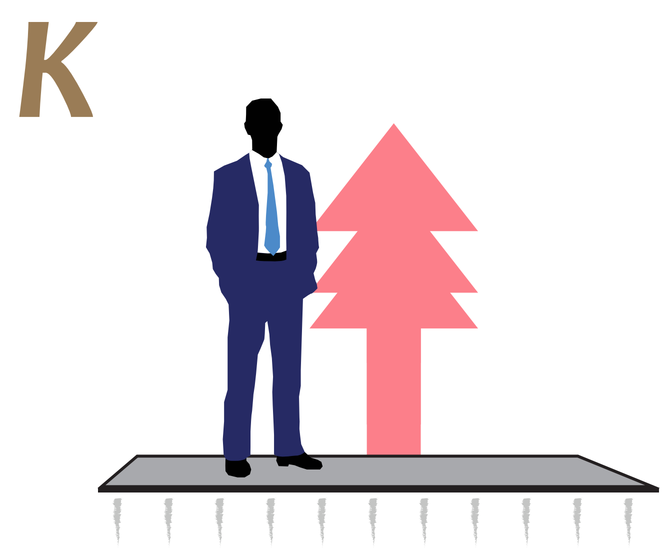
So **there is no mention of "elevators"** in Einstein's writings from this time. Not even of "lifts".

(What made Einstein daydream about drugging experimentalists and putting them in a box together with their equipment, and then have the box dragged into outer space by an alien life form, remains an unsolved problem in Einstein studies.)



Step 2

The next step is to think about the situations in the two explanations as actually being **two different observers**, ***K*** and ***K'***, in different parts of the universe:



*an observer **uniformly accelerated in empty space***

*an observer **at rest in a homogeneous gravitational field.***

EINSTEIN’S EQUIVALENT NEWTONIAN OBSERVERS

We have established that **the laws of Newtonian mechanics** (together with the observed equality of inertial and gravitational mass) **make exactly the same predictions** for any mechanical experiments the observers ***K*** and ***K’*** may make.

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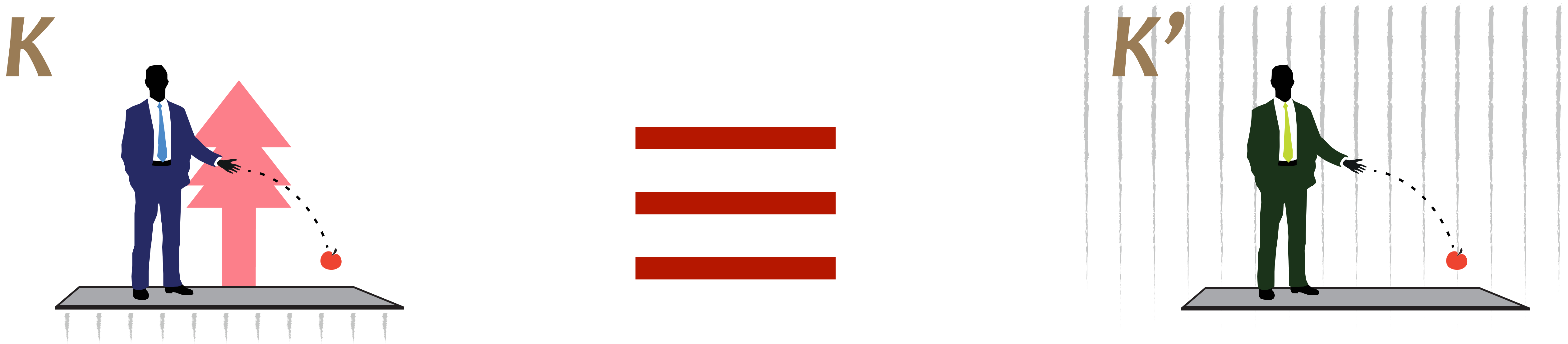

Einstein used to formulate this by saying that:

the two observers *K* and *K’* are
“equivalent”

EINSTEIN'S EQUIVALENT NEWTONIAN OBSERVERS

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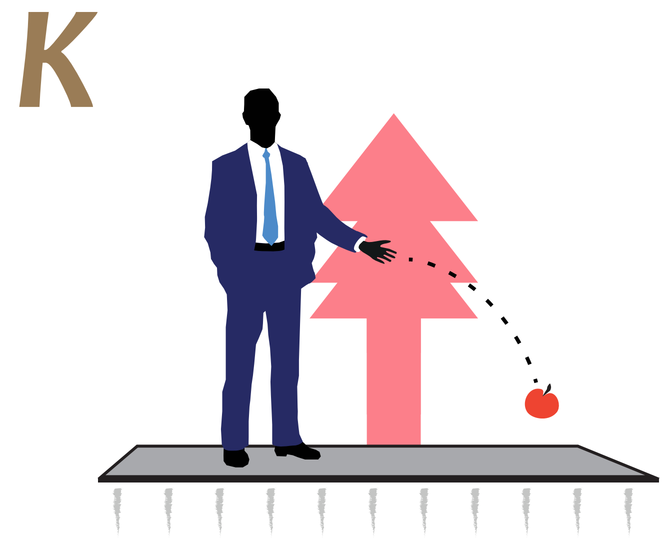

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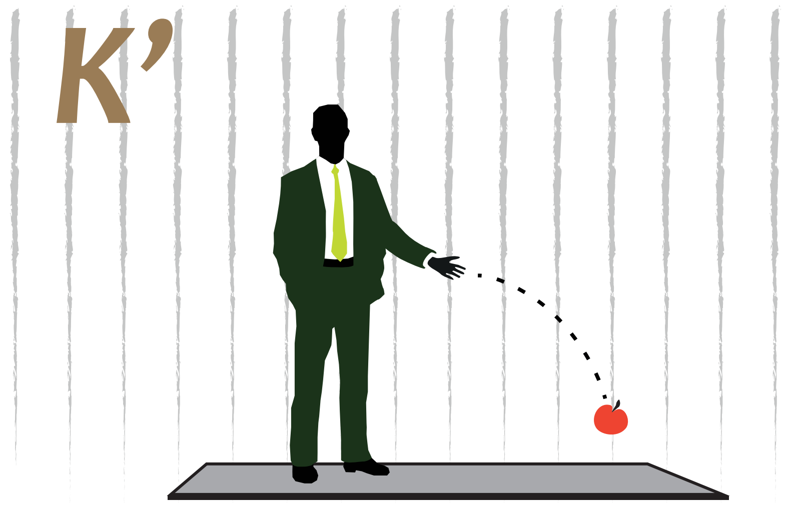
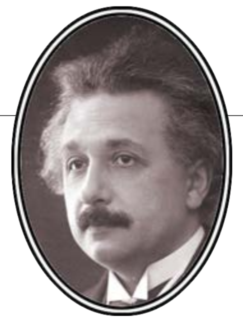


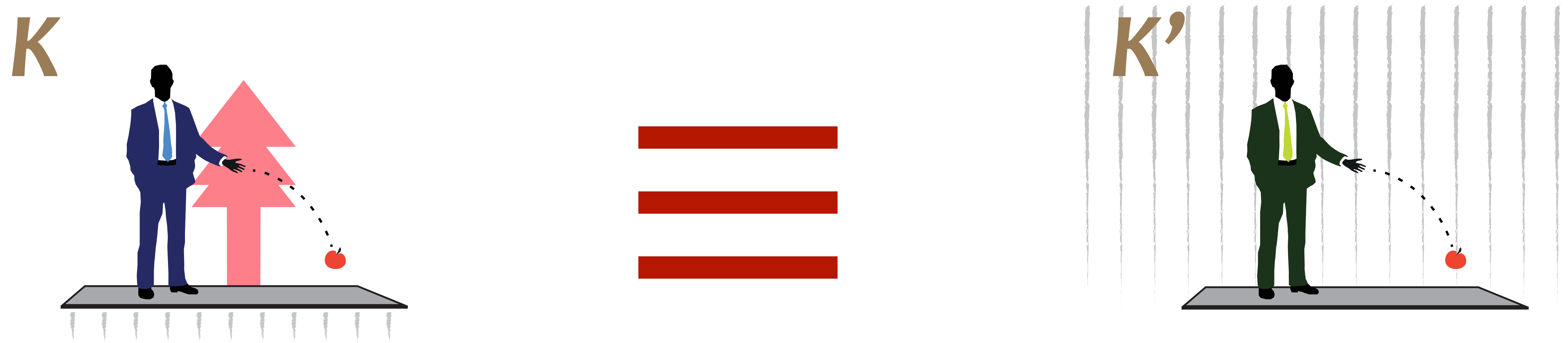
Einstein used to formulate this by saying that **the two observers *K* and *K'* are equivalent**. And here it is important to add that the **equivalence has only been established**

with respect to the laws of *Newtonian mechanics and Newtonian gravity*



EINSTEIN'S EQUIVALENT NEWTONIAN OBSERVERS

Or put differently: following Einstein we say that two observers ***K*** and ***K'*** are **equivalent with respect to a certain set of laws** if they:
(1) make the **same kinematical predictions** for a physical process, but
(2) may give **different dynamical explanations** for the process.

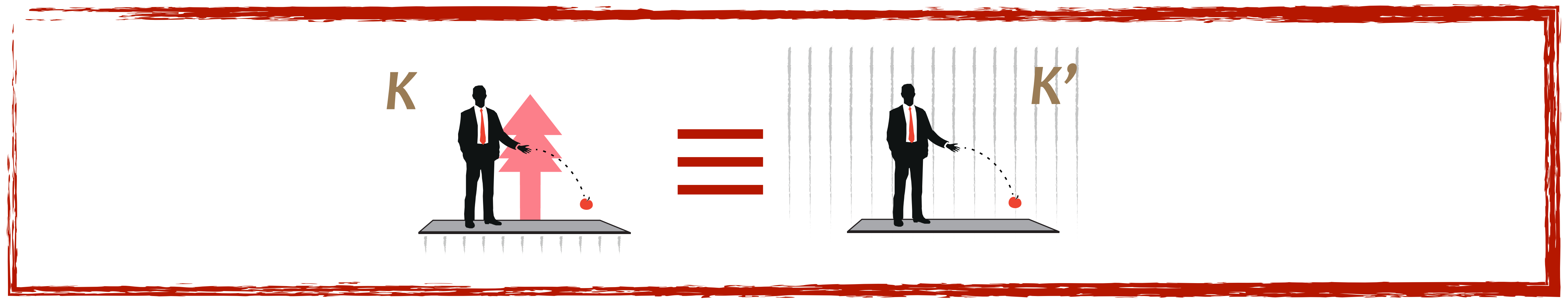


prediction	curve of ball is a parabola	curve of ball is a parabola
explanation	ball is inertial, observer is uniformly accelerated	ball falls freely in homogeneous gravitational field

PART 1 — THE ARGUMENT SO FAR…

1. THEOREM

Analysing **Newtonian physics,** and assuming that inertial and gravitational masses are equal, we have found two special observers that are **equivalent with respect to the laws of Newtonian physics**



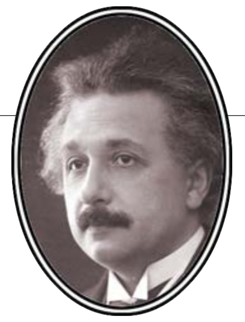
Newtonian physics

QUIZ *

What is it that is “equivalent”
in Einstein’s *Äquivalenzprinzip*?

---

* *Discuss amongst yourselves…*



Step 3

Now let's make things a bit more interesting!

We will give the two observers ***K*** and ***K'*** **equipment to perform** **non-mechanical** **experiments.**

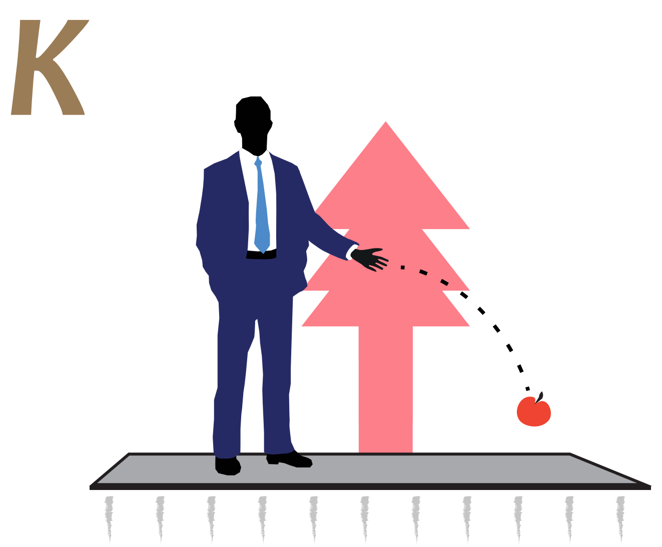
In particular: experiments involving an **electromagnetic field.**

Remember that ***Newtonian gravity does not affect electro-magnetic fields***!

TROUBLE IN PARADISE

We will get different predictions for the path of a light ray (in the rest frame of each observer) depending on whether or not there is a gravitational field present:

mechanical experiment



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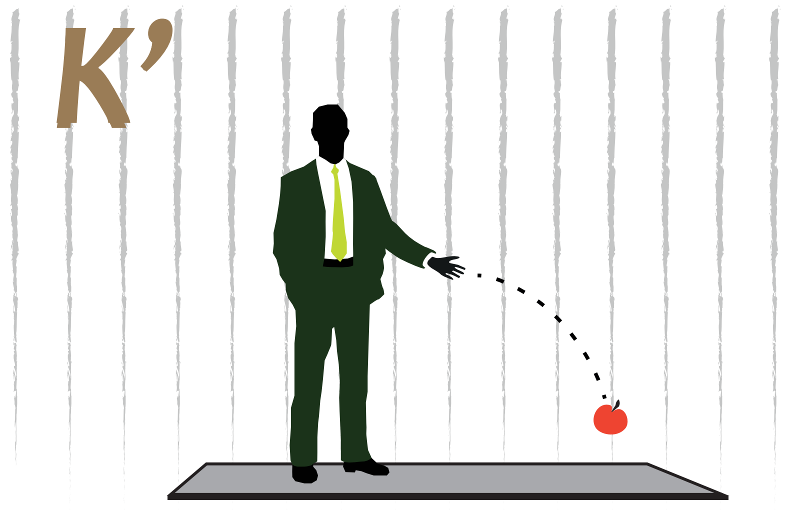
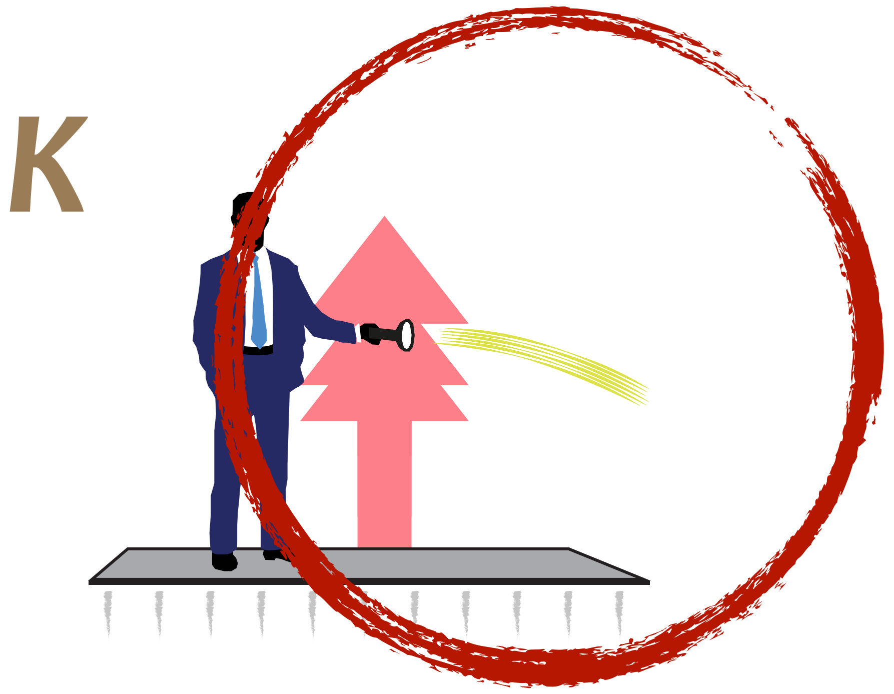


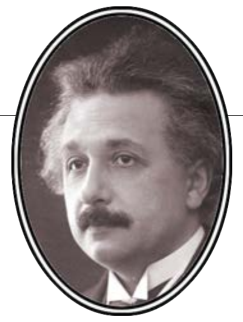
both observers get the **same** result (they are **equivalent** with respect to the laws of **mechanics**)

experiment with **light**



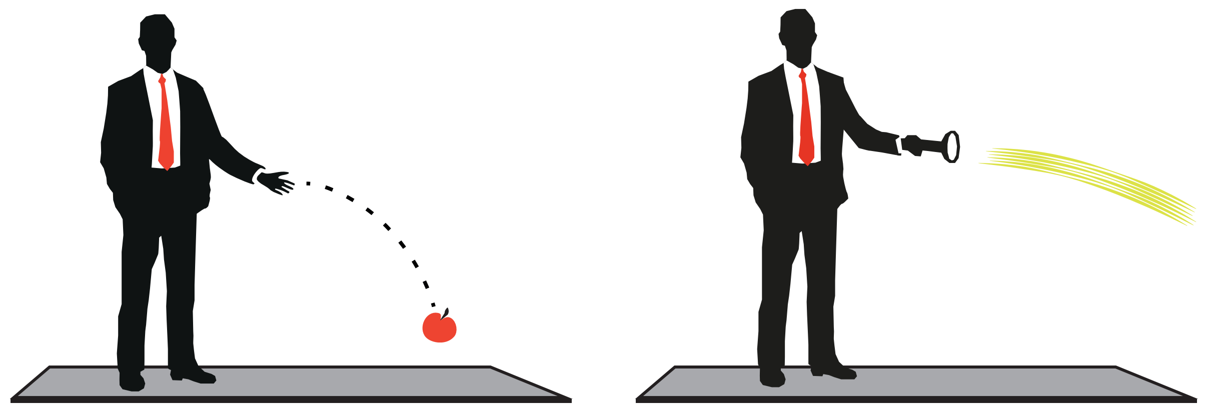
$\not\equiv$

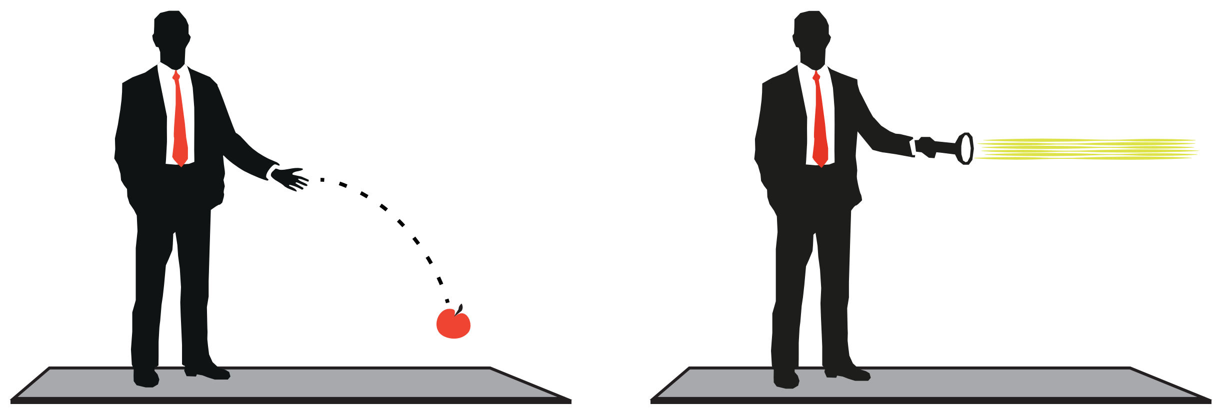
the observers get **different** results (they are **not equivalent** with respect to the laws of **electrodynamics**)



TROUBLE IN PARADISE

We now realise that by using a combination of mechanical and non-mechanical (optical) experiments, **we can unambiguously determine whether or not there is a (Newtonian) gravitational field** in the region of space where the observers are:





Observation

if both mechanical experiments and experiments with light give **the same outcome**

Explanation

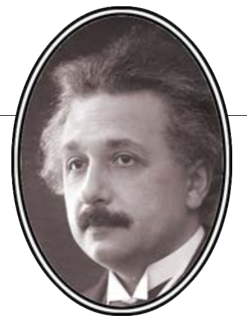
— then **the observer is accelerated in a region without gravitational field**

Observation

if mechanical experiments and experiments with light give **different outcomes**

Explanation

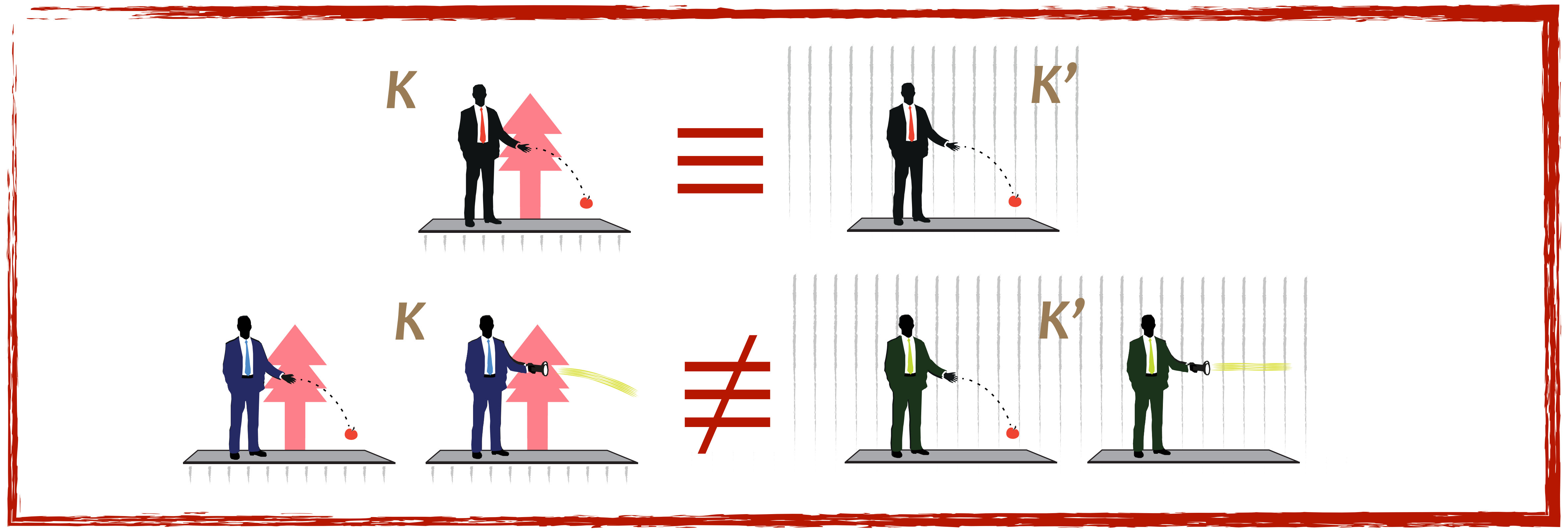
— then **the observer is at rest in a gravitational field**



PART 1 — THE ARGUMENT SO FAR…

1. THEOREM

Analysing **Newtonian physics**, and assuming that inertial and gravitational masses are equal, we have found two special observers that are equivalent w.r.t the laws of Newtonian physics, but *not* equivalent w.r.t other laws, e.g. those of **electrodynamics**.



Newtonian physics

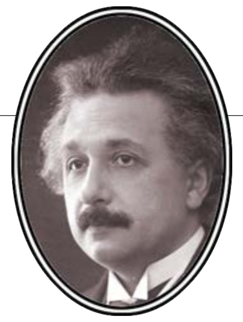
Classical physics

PARADISE REGAINED — THE *ÄQUIVALENZPRINZIP*

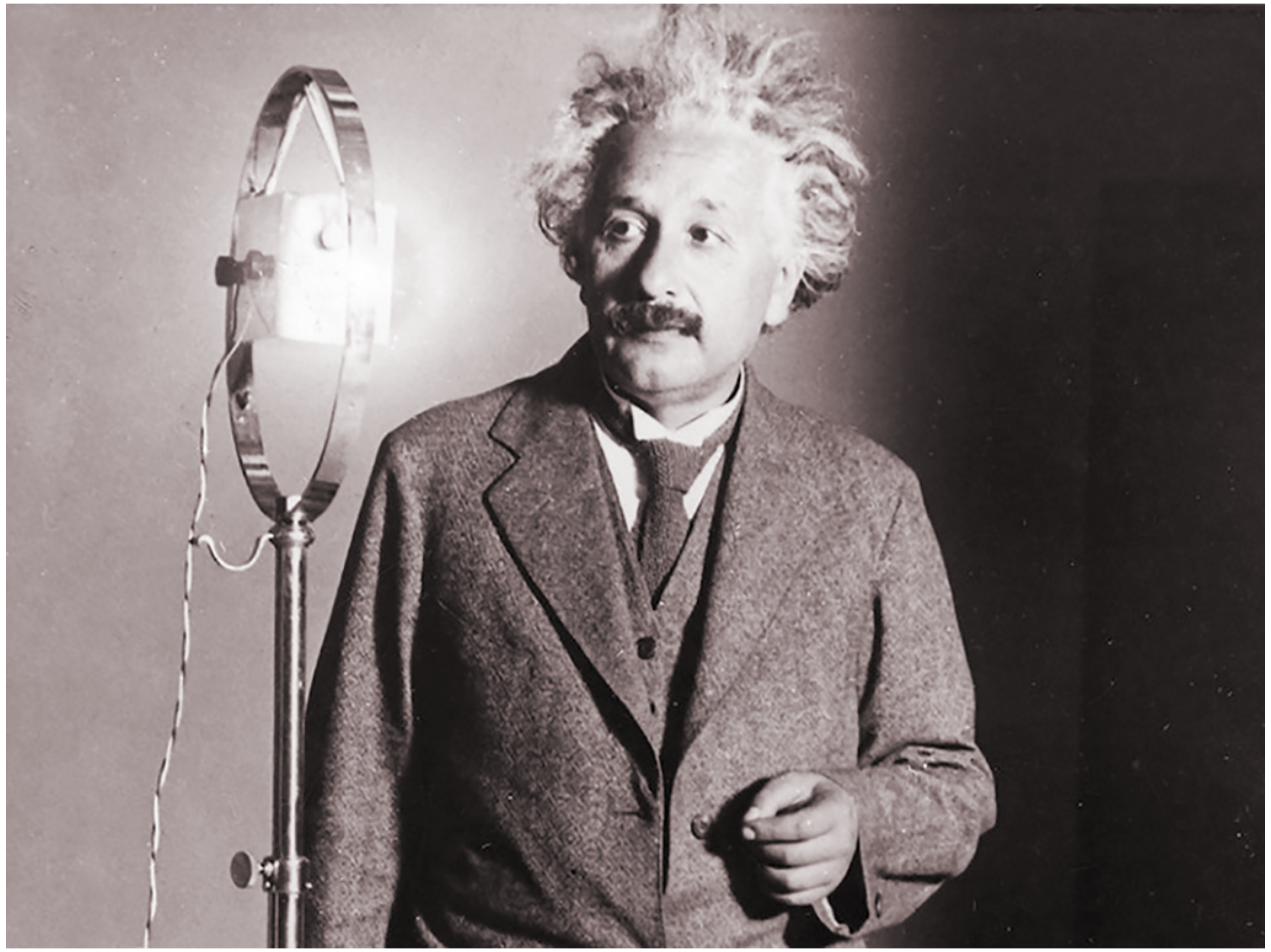
So far everything I have talked about is completely uncontroversial, and belongs to **standard Newtonian theory.**

The only addition has been the **empirically supported** assumption of numerical equality of inertial and gravitational mass.

And **the only input from Einstein** has been the terminology of "**equivalent observers** (with respect to a particular theory)".



What Einstein does next
is quite unexpected…



PARADISE REGAINED — THE *ÄQUIVALENZPRINZIP*

Physicists are now (1907) **looking for a relativistic theory of gravity.**

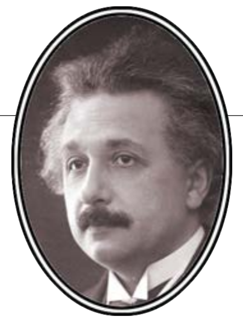
The standard SR guiding principle that

“all relativistic theories must be Lorentz invariant”

has so far **not helped anyone** to find such a theory of gravity.

What we need is **a *new* guiding principle**!

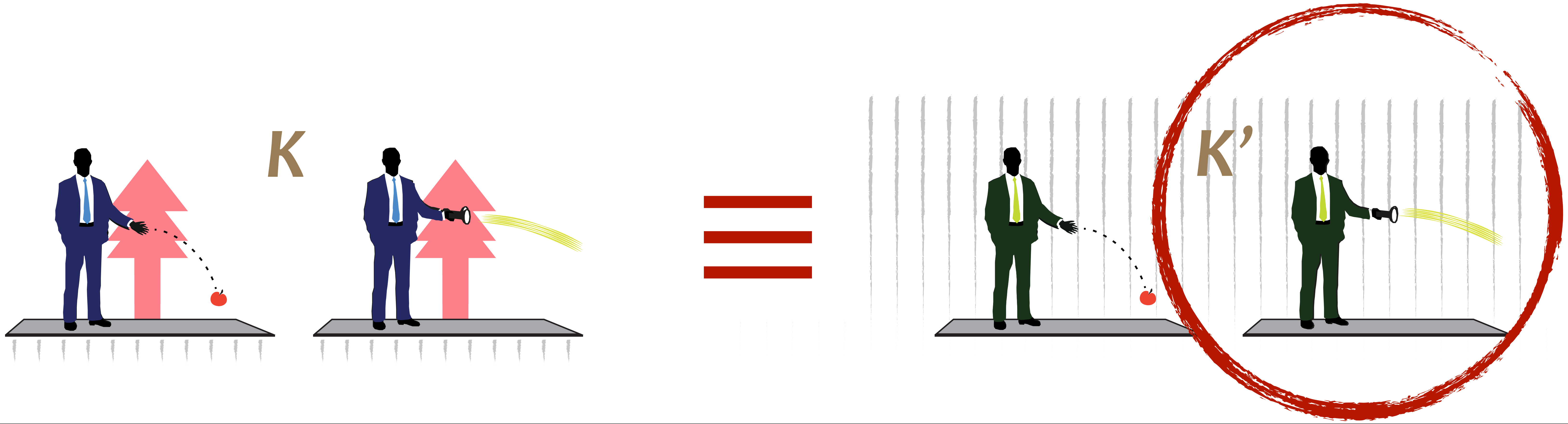
Einstein now proposes we try out the following guiding principle, and then see where it leads us:



Step 4

ÄQUIVALENZPRINZIP:

A uniformly accelerated observer in empty space and an observer at rest in a homogeneous gravitational field are **equivalent** with respect to ***all* laws of physics.**



The relativistic theory must be such that this is true

PARADISE REGAINED — THE *ÄQUIVALENZPRINZIP*

The idea of postulating this equivalence between the two special observers was introduced already in 1907, but **the principle got its name *Äquivalenzprinzip* only later, in 1912**.

Here is a typical formulation of the principle:

Theorem in Newtonian physics

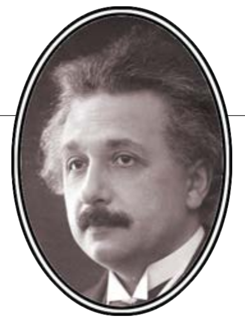
> In a **homogeneous gravitational field** all motions take place in the same way as in the absence of a gravitational field in relation to a **uniformly accelerated coordinate system.**

New **postulate**

New **guiding principle**?

> **If this principle held good for any events whatever** (the "principle of equivalence") [. . .] we were to reach **a natural theory of the gravitational fields.**

Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)



PARADISE REGAINED — THE ÄQUIVALENZPRINZIP

Another Einstein quote clarifies the logic of the "*Äquivalenz-prinzip*" argument even further:

Theorem in Newtonian physics

As long as we confine ourselves to purely mechanical processes within the range of validity of Newton's mechanics, **we can be sure of the equivalence of the systems *K* and *K*'.**

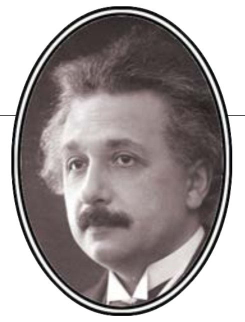
New **postulate**

However, for our conception [*of equivalence*] to acquire deeper significance, **the systems *K* and *K*' must be equivalent with respect to all physical processes**, *i.e.*, the natural laws with respect to *K* must coincide completely with those with respect to *K*'.

New **guiding principle**?

If we accept this assumption, **we obtain a principle that possesses great heuristic significance…**

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



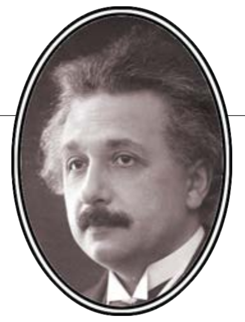
I CAN'T EMPHASIZE THIS STRONGLY ENOUGH!

- The equivalence (**with respect to mechanics**) of the observers ***K*** and ***K'*** **is just a theorem in Newtonian physics** (with additional empirical input about $m_i = m_g$).

This tells us nothing new about gravity!

- But Einstein wants to **go beyond Newtonian physics**! He **postulates** the equivalence of the observers **with respect to *all* laws of physics** (this is the *Äquivalenzprinzip*).

The hope is that this might give us a glimpse of the (unknown) relativistic theory of gravity.



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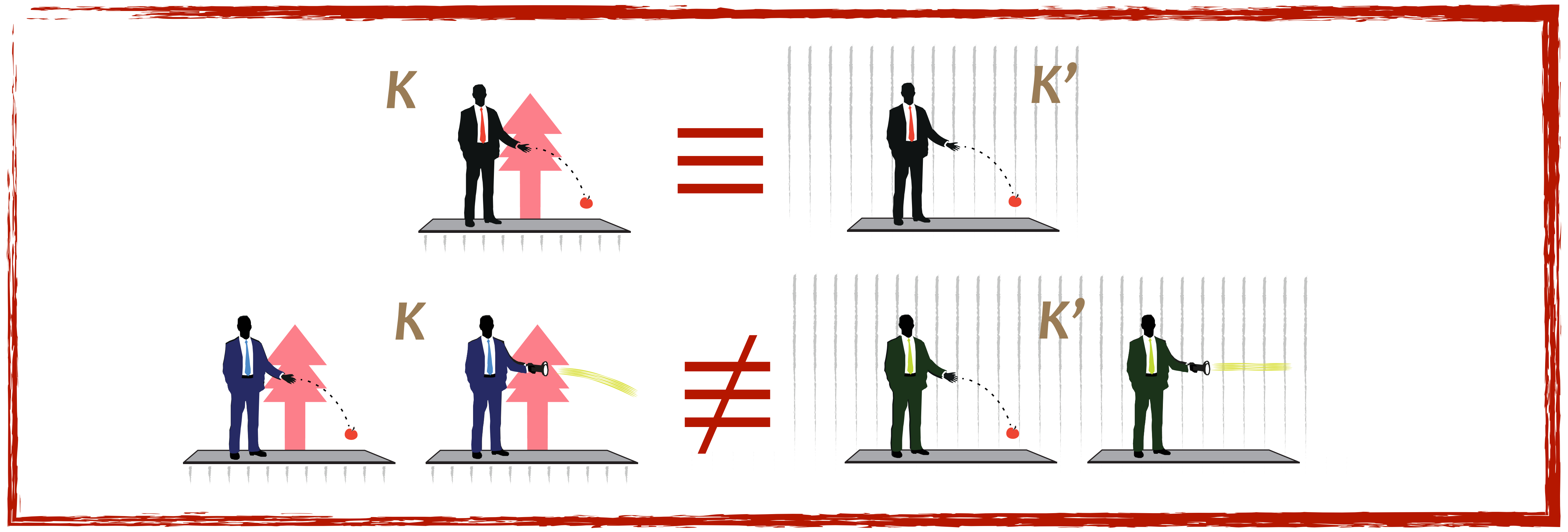
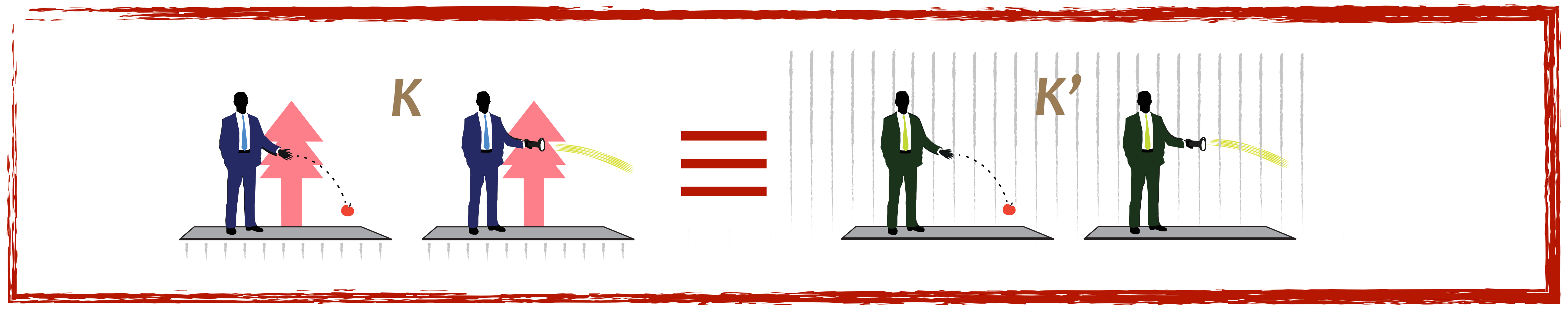


Newtonian physics

Classical physics

2. POSTULATE

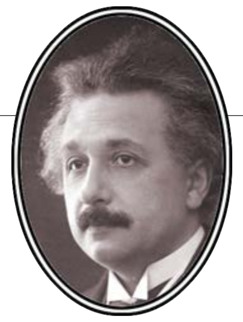
Äquivalenzprinzip. In **a relativistic theory of gravitation** the special observers ***K*** and ***K'*** must be equivalent with respect to **all laws of physics.**



Unknown relativistic theory of gravity

Step 5

So how did Einstein think that the ***Äquivalenzprinzip*** could help him get a glimpse of the new unknown relativistic theory of gravity?

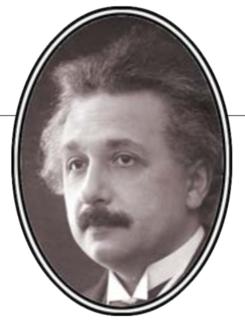


“…we obtain a principle that possesses **great heuristic significance**”

> **The heuristic value** of this assumption [*the Equivalence Principle*] rests on the fact that it permits the replacement of a homogeneous gravitational field by a uniformly accelerated reference system, **the latter case being to some extent accessible to theoretical treatment.**

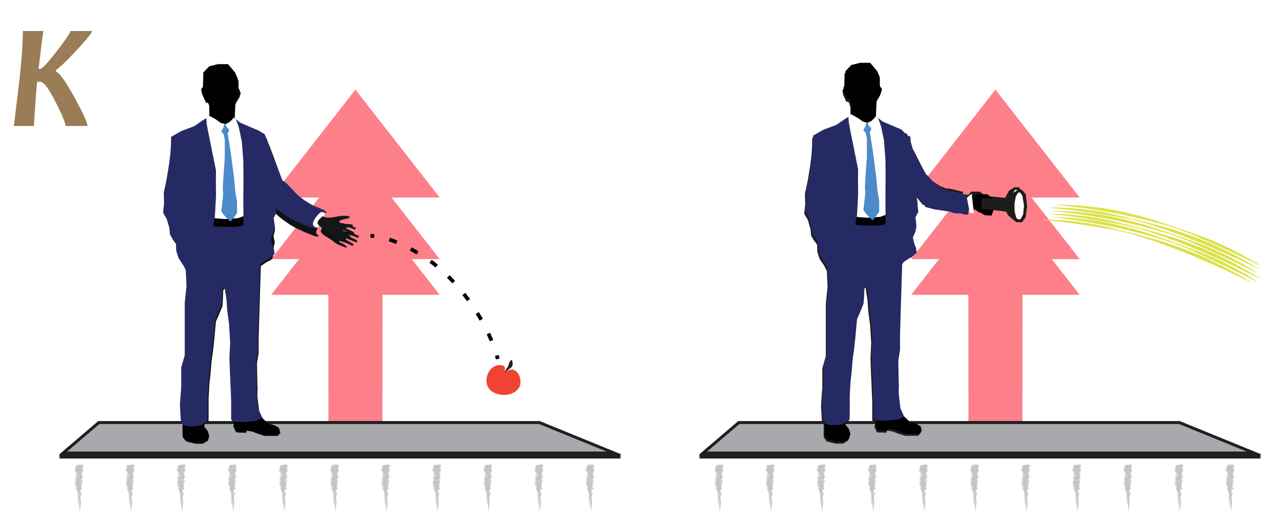
Albert Einstein, “On the relativity principle and the conclusions drawn from it” (1907)

But what does this mean?

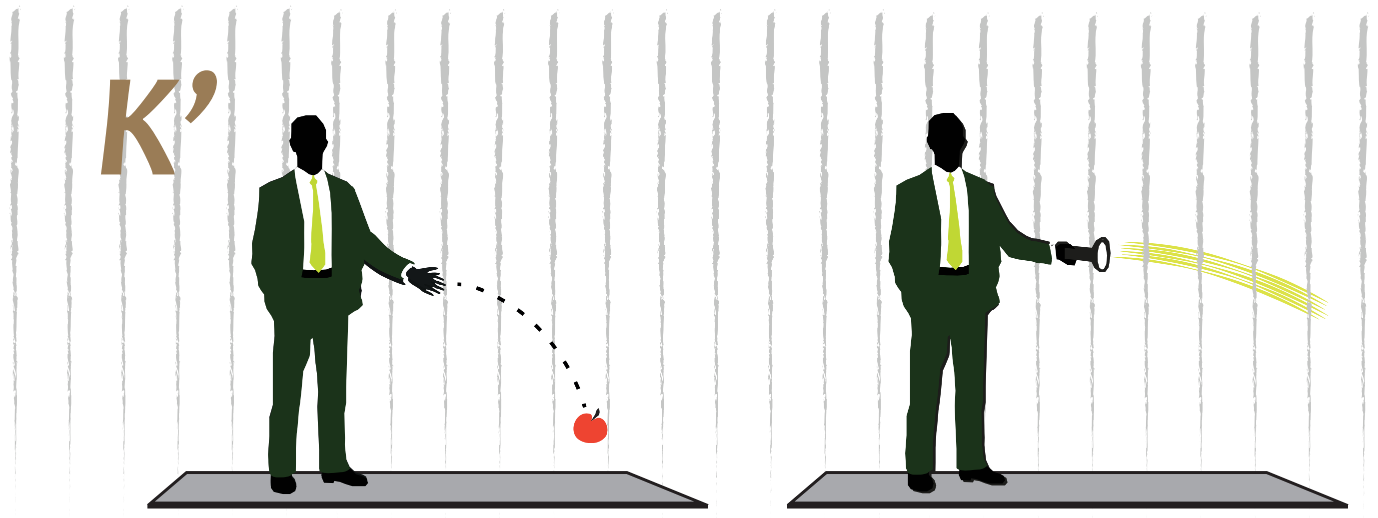


Let’s take a closer look at our pair of observers, and the description of their physical experiments (assuming the *Äquivalenzprinzip*).

Non-inertial observer
in gravity-free space



Inertial observer
in a gravitational field



THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

Let's take a closer look at our pair of observers, and the description of their physical experiments (assuming the *Äquivalenzprinzip*).

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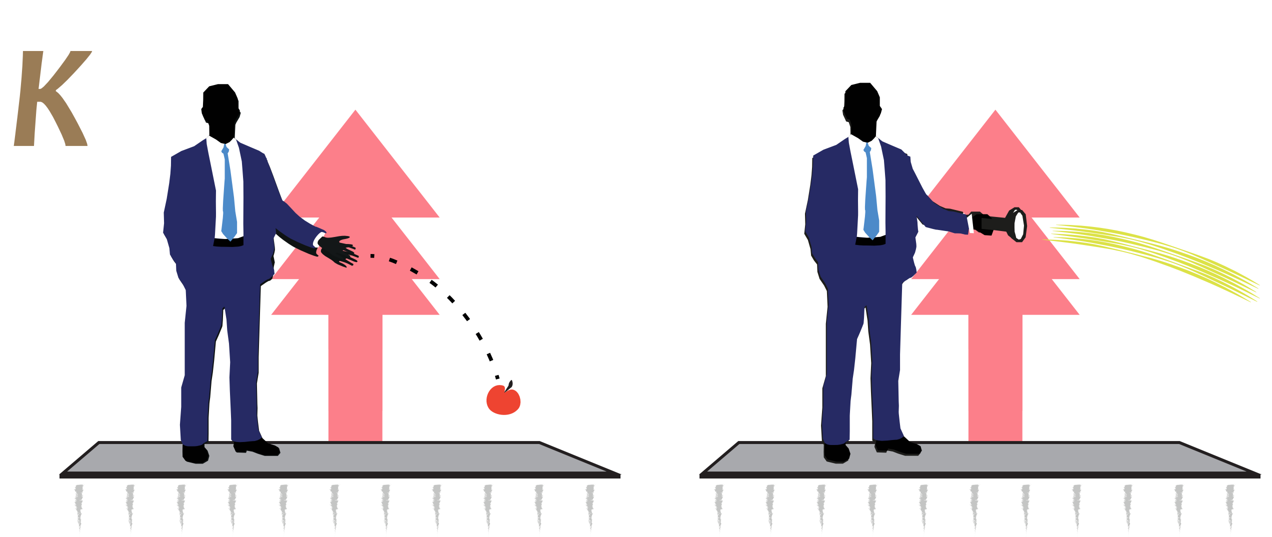
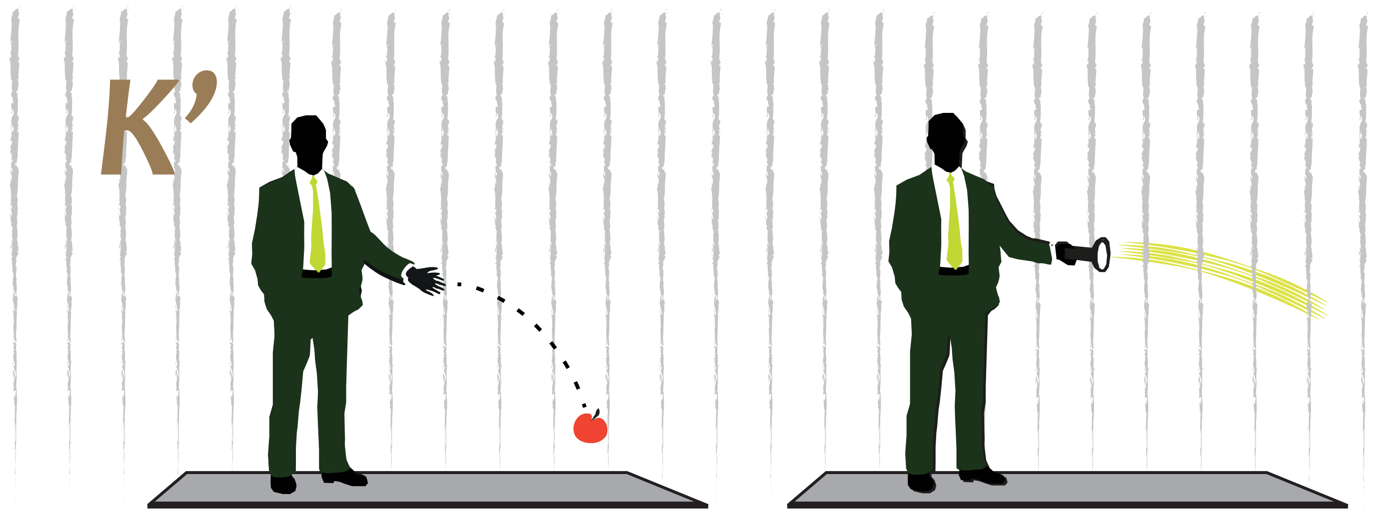
in gravity-free space

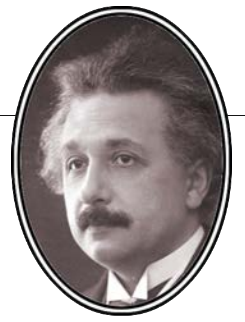


The observed curved path is a
kinematic effect
(acceleration)

Inertial observer
in a gravitational field



The observed curved path is a
dynamical effect
(gravitational force)



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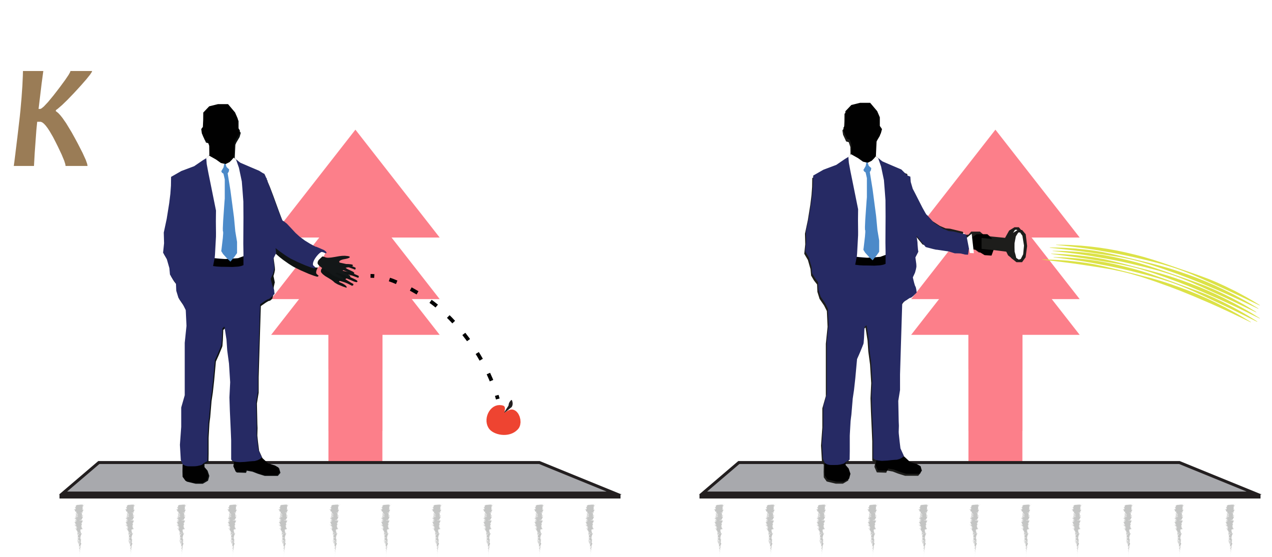
in gravity-free space

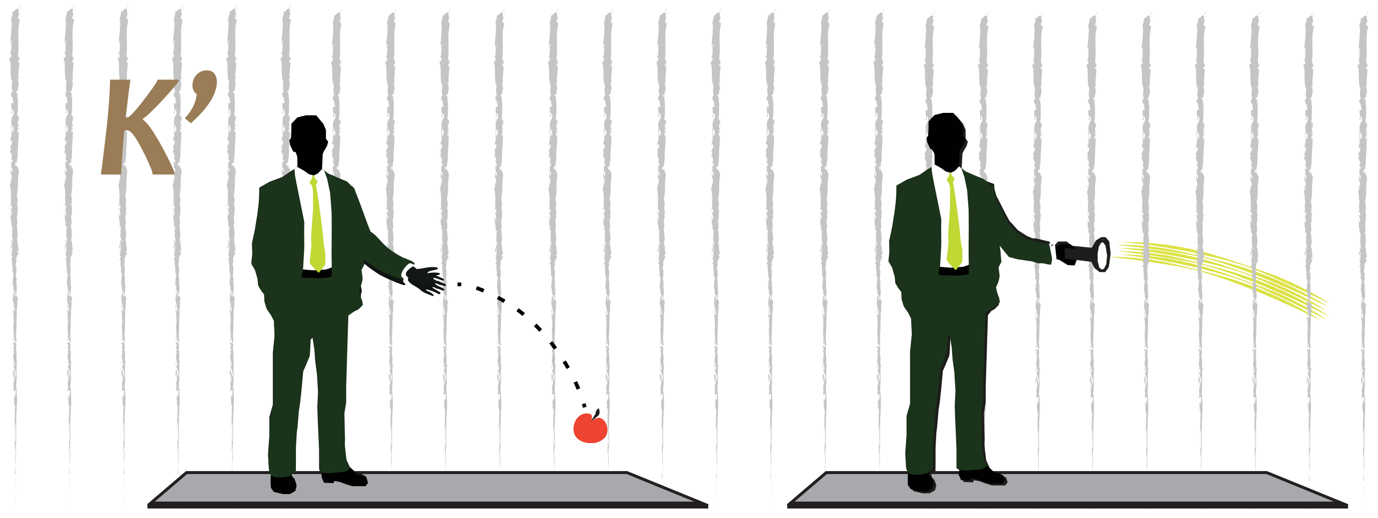


We know how to calculate **kinematic** effects

We can do this, *e.g.*, in **Newtonian physics** or in **special relativity**.

Kinematic effects are "**to some extent accessible to theoretical treatment**".

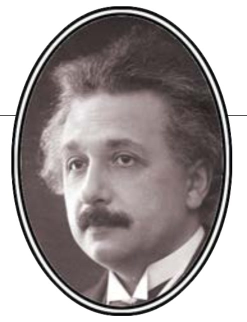
Inertial observer
in a gravitational field



We do not know how to calculate **dynamical** effects

This is because **the theory of *relativistic* gravitation is still *unknown***.

We need help to find the laws of *relativistic* gravitation!



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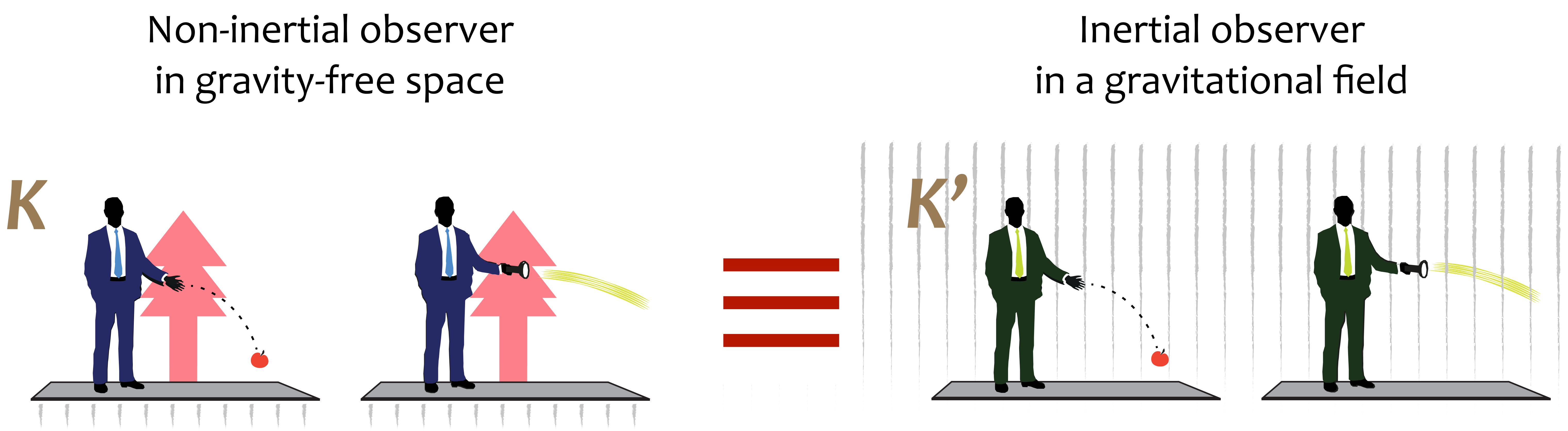
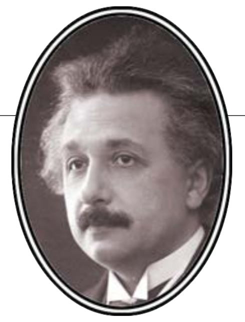
Let's take a closer look at our pair of observers, and the description of their physical experiments (assuming the *Äquivalenzprinzip*).



But we know — **by assumption** (this is the *Äquivalenzprinzip*) — that at least **these two special observers *K* and *K'* are *equivalent*** in the unknown theory of gravitation.

That is: **the laws of physics** (including the unknown laws of relativistic gravitation) **must give exactly the same physical predictions in both frames of reference.**

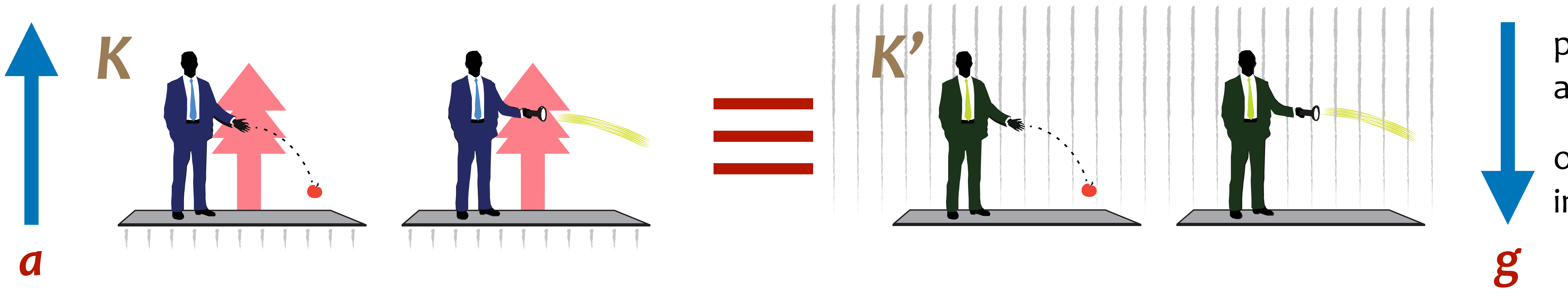


THE HEURISTIC POWER OF THE ÄQUIVALENZPRINZIP

This is how it could work (*in reality it is a bit more complicated…*)

observer is accelerated,

physical systems are inertial



physical systems are accelerated,

observer is inertial

1. **Perform calculations using** **known laws** of (gravity-free) physics, transformed to a frame of reference which is uniformly accelerated with acceleration ***a***.

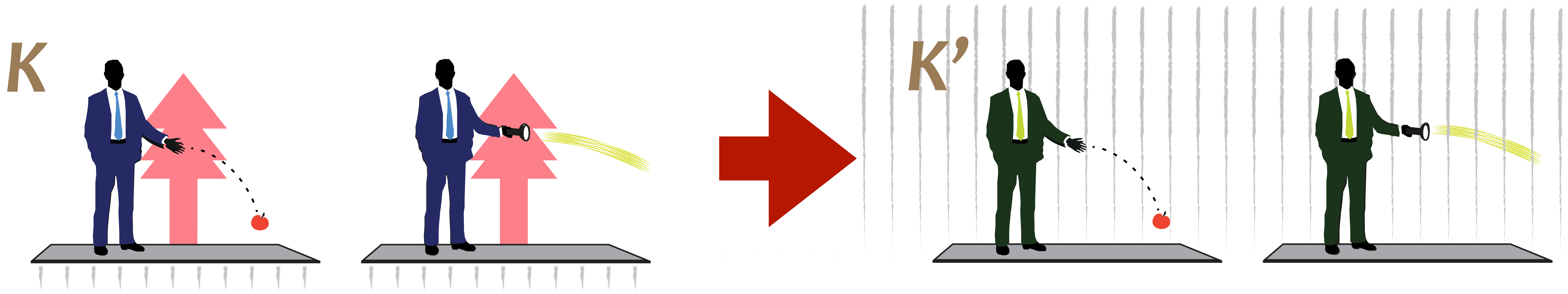
2. **Translate** the calculated results to an inertial frame in a homogeneous gravitational field by **replacing** the **kinematic acceleration** ***a*** with the **gravitational acceleration –g** in all relevant laws of physics.

3. **This gives us a clue** about the way gravity in the **unknown theory** will influence physical processes.

THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

An observation that will take us one step closer to **relativistic** gravity:

known physics

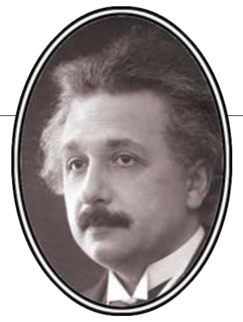


unknown physics

The **equivalence** of the two observers was **established** using **Newtonian physics**: we needed a theoretical context which had both kinematics and gravitational dynamics.

But note that now **all actual calculations** will be made **only on the kinematic side of the equivalence.**

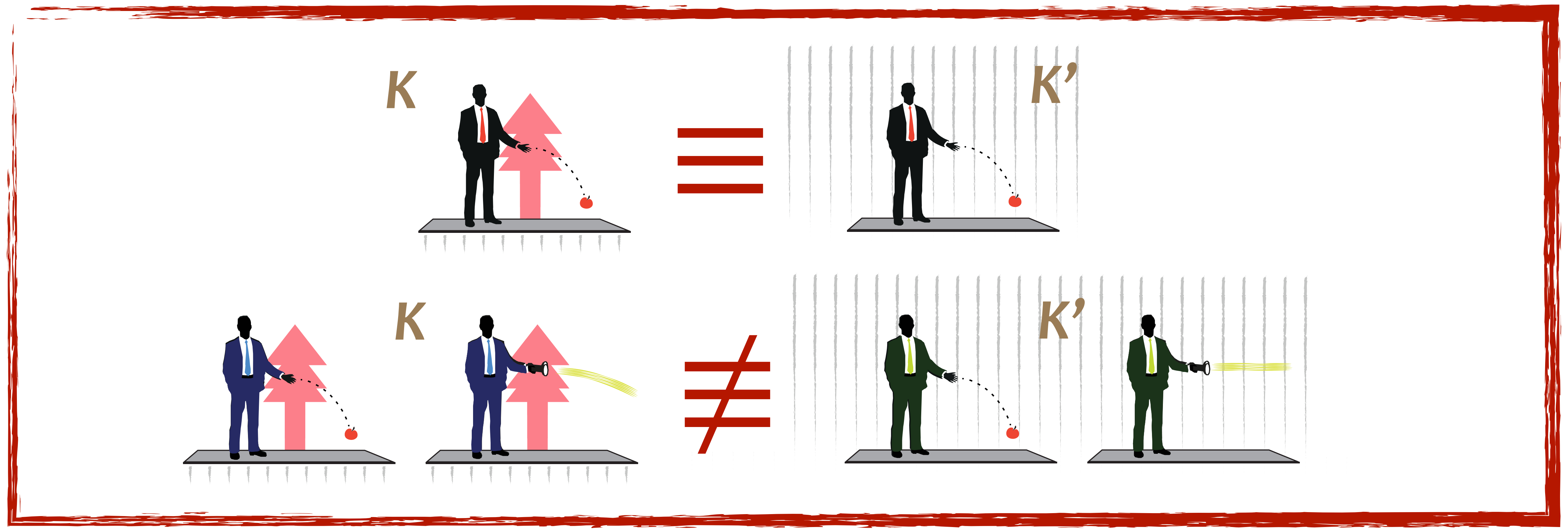
We can therefore do all calculations using **special relativity**!



PART 1 — THE ARGUMENT SO FAR…

1. THEOREM

Analysing **Newtonian physics**, and assuming that inertial and gravitational masses are equal, we have found two special observers that are equivalent w.r.t the laws of Newtonian physics, but *not* equivalent w.r.t other laws, e.g. those of **electrodynamics**.

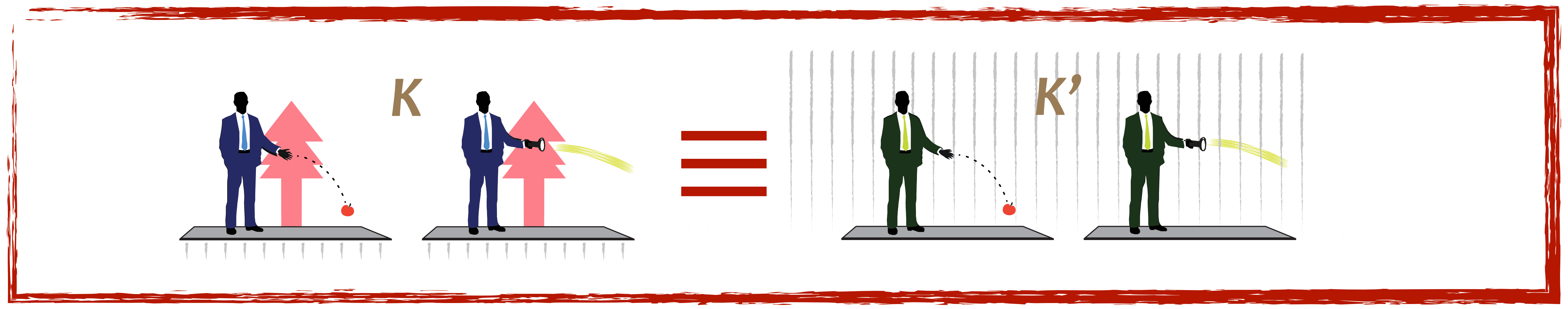


Newtonian physics

Classical physics

2. POSTULATE

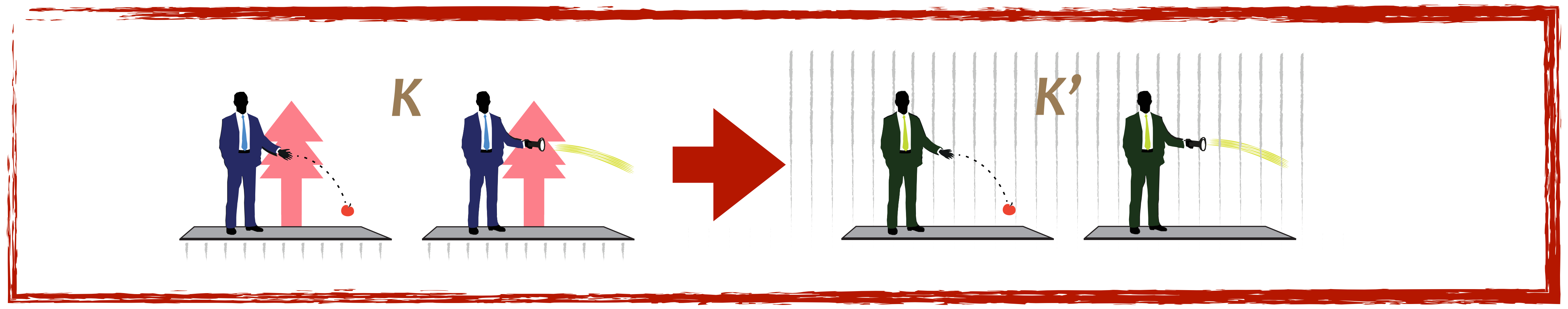
Äquivalenzprinzip. In **a relativistic theory of gravitation** the special observers ***K*** and ***K'*** must be equivalent with respect to **all laws of physics**.



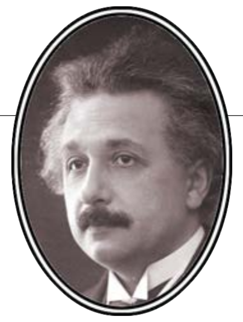
Unknown relativistic theory of gravity

3. HEURISTIC

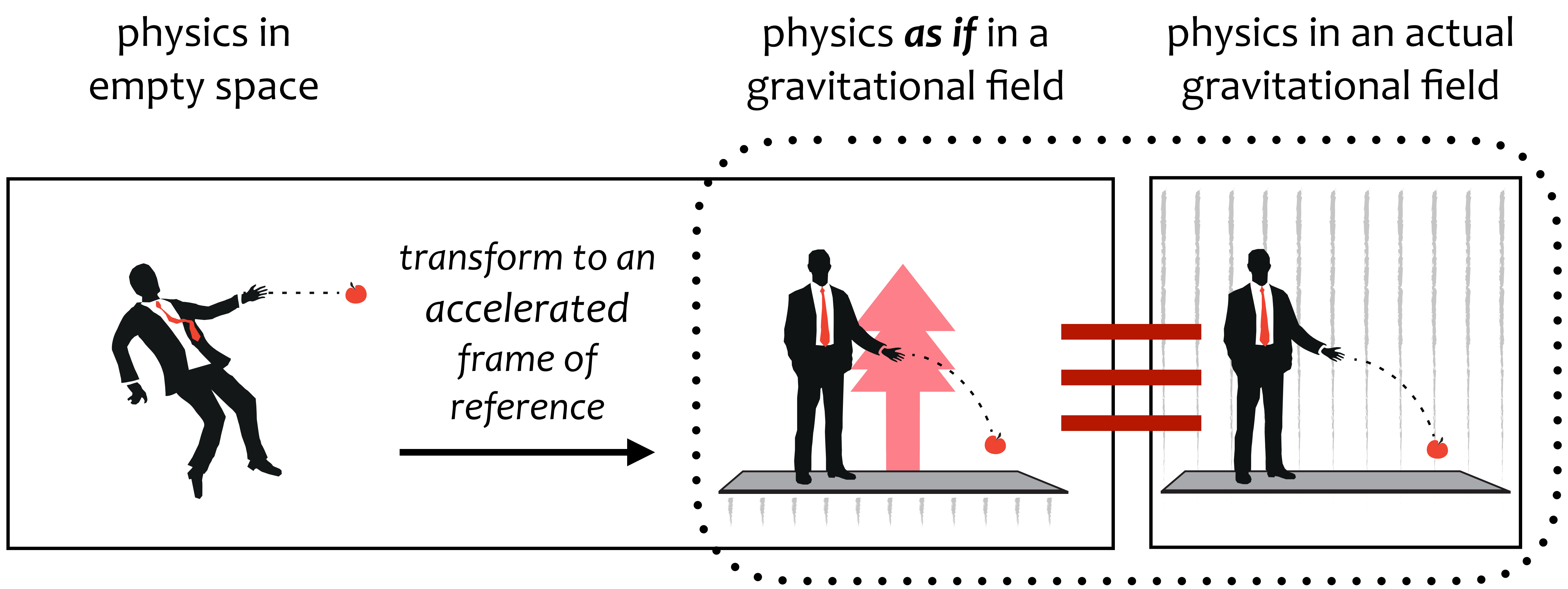
***Äquivalenzprinzip* Heuristic**. Analyse physical systems in **special relativity** w.r.t an accelerated observer. The results must also be valid for an observer in a gravitational field (in the unknown **relativistic theory of gravitation**).



Special relativity



Step 6 There is **a nice intuitive way** to talk about the heuristic procedure:

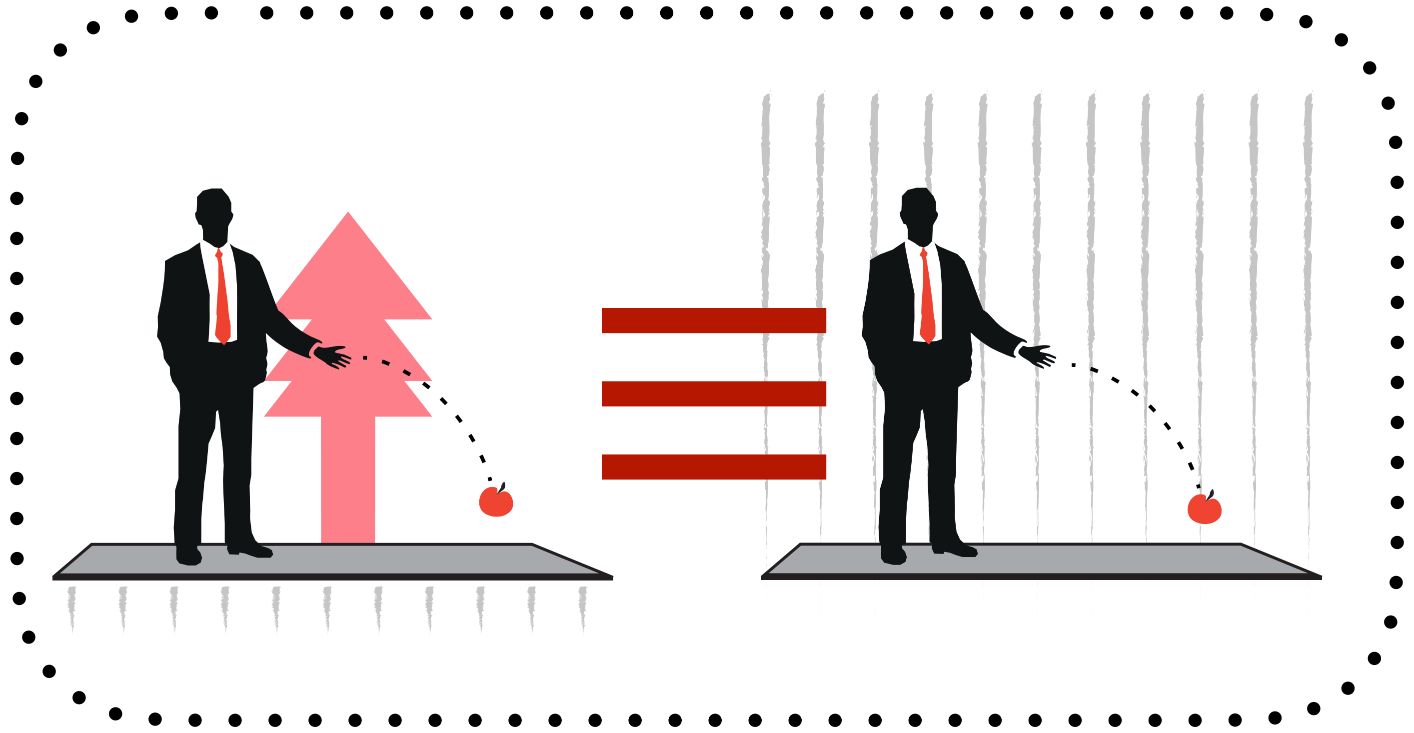


If you are an observer in free space **and start accelerating** upwards you become one of the observers that are equivalent (as far as the laws of Newtonian physics go) to an observer at rest in a gravitational field.

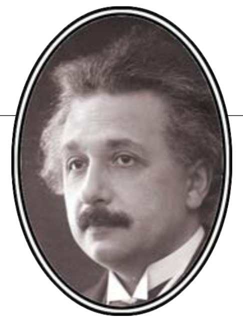
It is as if, **by going from rest to an accelerated frame of reference** (in empty space), **you have “created” a** (very special) **gravitational field.**

So let's just call these two equivalent observers

the "generative" observers



since they show us how one can "create" or "generate" a gravitational field by going to a state of acceleration.



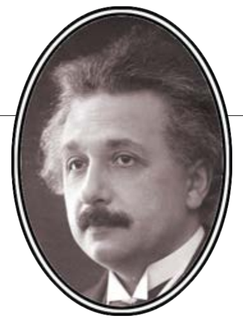
The first phase of Einstein’s search for GR (1907–1911)

These first clues to the new physical **effects** one could expect from a relativistic theory of gravity were obtained 1907–1911.

The second phase of Einstein’s search for GR (1912–1915)

Einstein’s **next step** would be to try to find the **causes** of the relativistic gravitational field (the *field equations*). He worked on this 1912–1915, with many false starts and problems.

This second phase **required new ideas,** beyond what the *Äquivalenzprinzip* could provide…



WHERE DID EINSTEIN GO FROM HERE?

The first phase of Einstein’s search for GR (1907–1911)

These first clues to the new physical **effects** one could expect from a relativistic theory of gravity were obtained 1907–1911.

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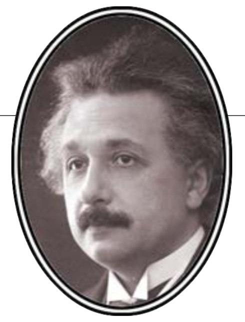
…but how Einstein found the final theory of general relativity is a different story!



WHERE DID EINSTEIN GO FROM HERE?

"… aber leichter ist ahnen als finden."

…but premonition is easier than discovery

Albert Einstein, letter to Rudolf Förster,
16 November 1917

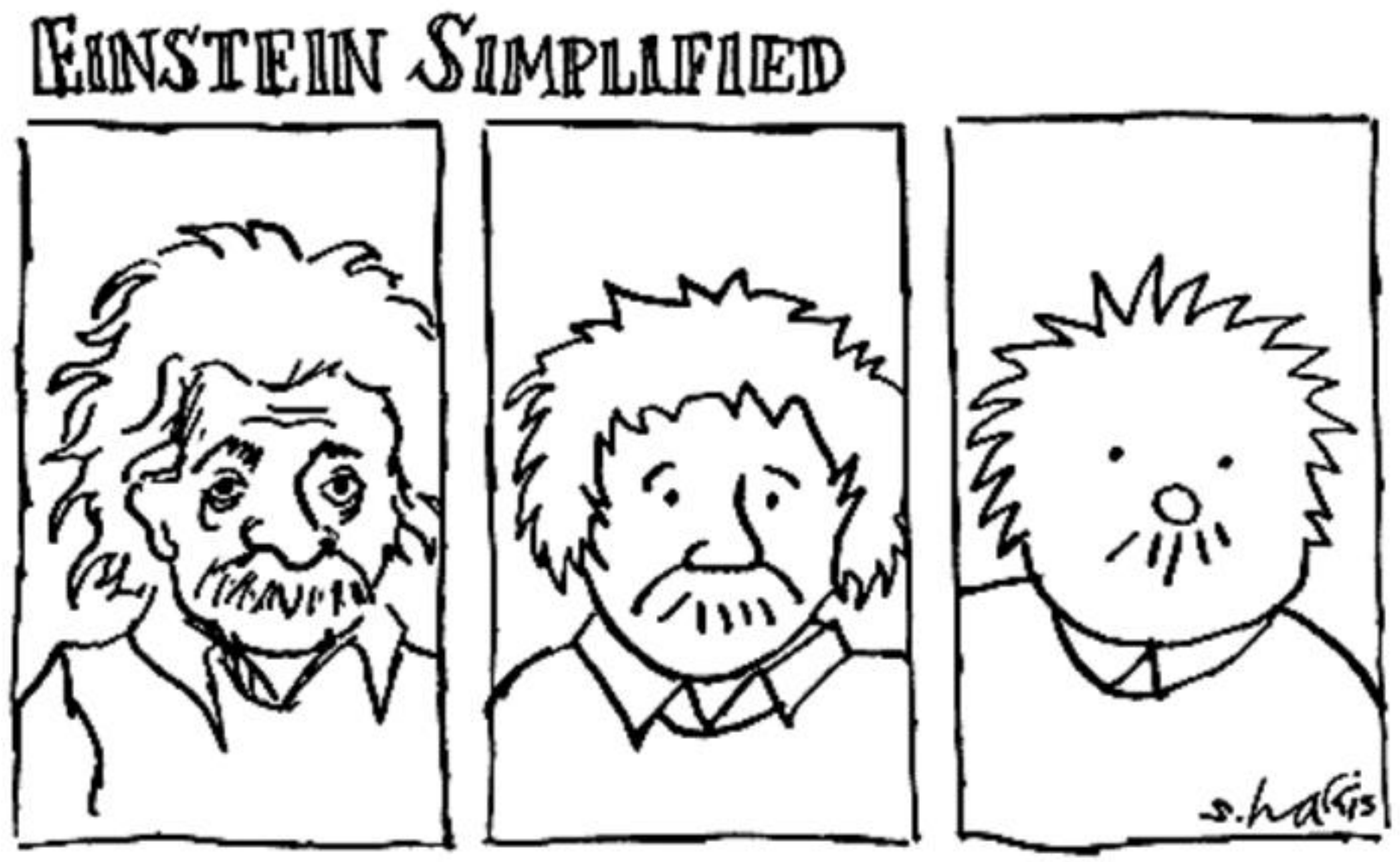
…but how Einstein found the final theory of general relativity is a different story!

I AM NOT FOLLOWING THE HISTORICAL DEVELOPMENT

I have told the story **as if** Einstein in 1907 was motivated mainly by **the search for a new relativistic theory of gravity**.

Actually, his main motivations lay elsewhere *.

But this would take too long for me to to explain…



Cartoon by **Sidney Harris**

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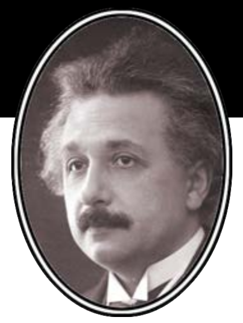
* *(extending the principle of relativity, and finding an explanation for the observed equality of inertial and gravitational mass)*

PART 2

THE MODERN EQUIVALENCE PRINCIPLES



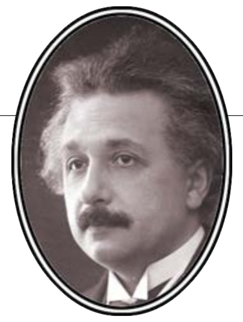
THE SECOND PAIR OF EQUIVALENT OBSERVERS

So how did **Einstein's *Äquivalenzprinzip*** argument develop into our modern **Weak and Strong Equivalence Principles**?

To understand this, we first need to explore an intermediate version of the equivalence principle:

the Infinitesimal Equivalence Principle

And just as in the case of the *Äquivalenzprinzip*, this principle emerges from **an analysis of a confused observer**…



THE SECOND PAIR OF EQUIVALENT OBSERVERS

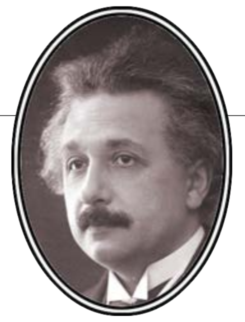
Step 1 In an otherwise empty region of space there is an observer performing **mechanical experiments:**

Observation

This time, when the observer throws a ball it **moves uniformly** along a **straight path.**

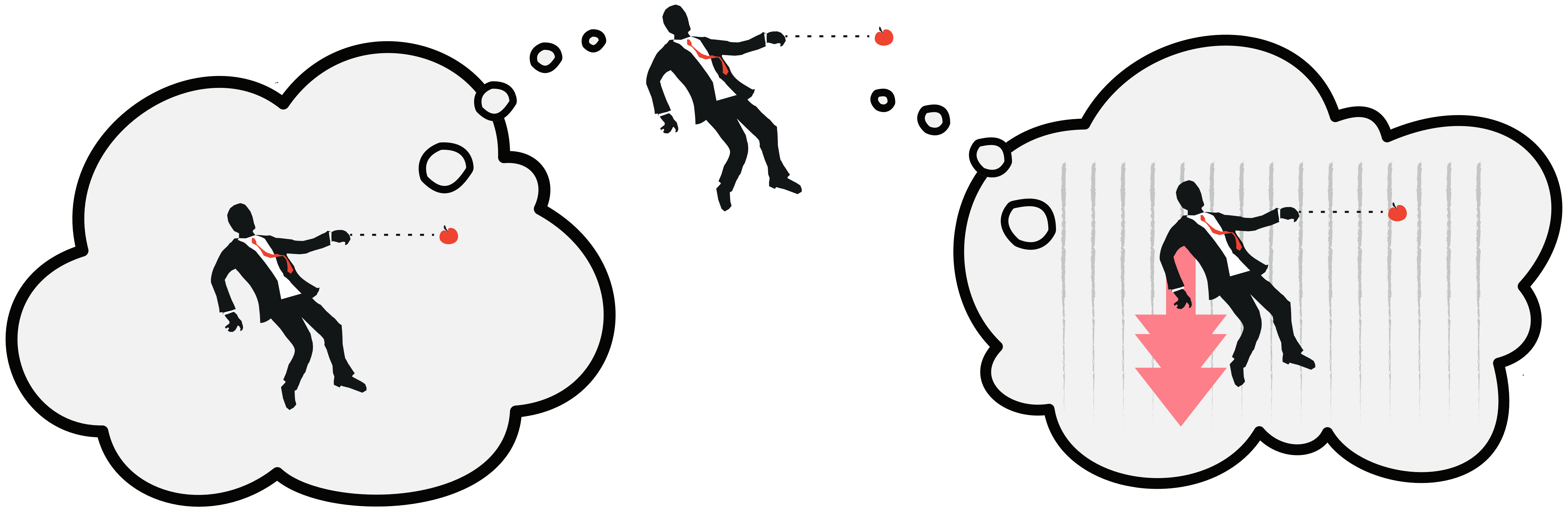
How would the observer explain* the observed behaviour of the thrown ball?

* *using only **Newtonian** physics!*



THE SECOND PAIR OF EQUIVALENT OBSERVERS

Again **Newtonian physics** provides us with two possible explanations, one in **empty space** and one in a **gravitational field**:



Explanation 1.

Equations of motion for body thrown by the **observer freely moving (or at rest) in a region without gravitational field** (in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2}=\frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2}=\frac{\mathrm{d}^2z^\mu}{\mathrm{d}t^2}=0$$

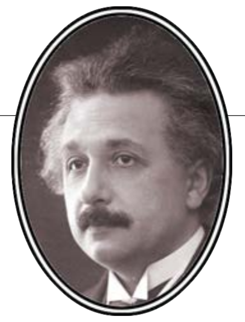
… **a straight line**

Explanation 2.

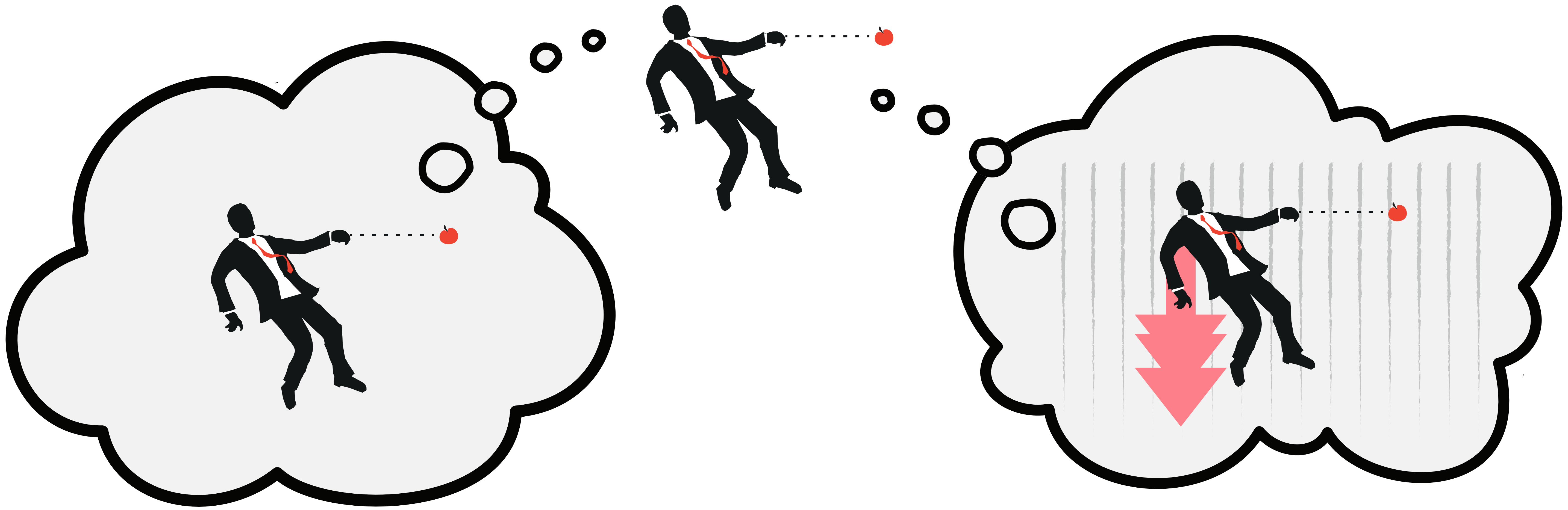
Equations of motion for body thrown by the **observer freely falling in a homogeneous gravitational field** (in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2}=\frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2}=0,\quad m_i\frac{\mathrm{d}^2z^\mu}{\mathrm{d}t^2}=0$$

… **a straight line**



THE SECOND PAIR OF EQUIVALENT OBSERVERS



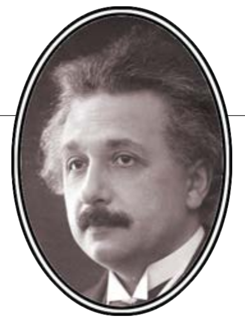
So **any mechanical process plays out exactly the same way**
for **an observer in free fall** as for an **observer at rest**!

THE SECOND PAIR OF EQUIVALENT OBSERVERS

This insight made a great impression on the young Einstein — it was nothing less than ***the happiest thought of his life***:

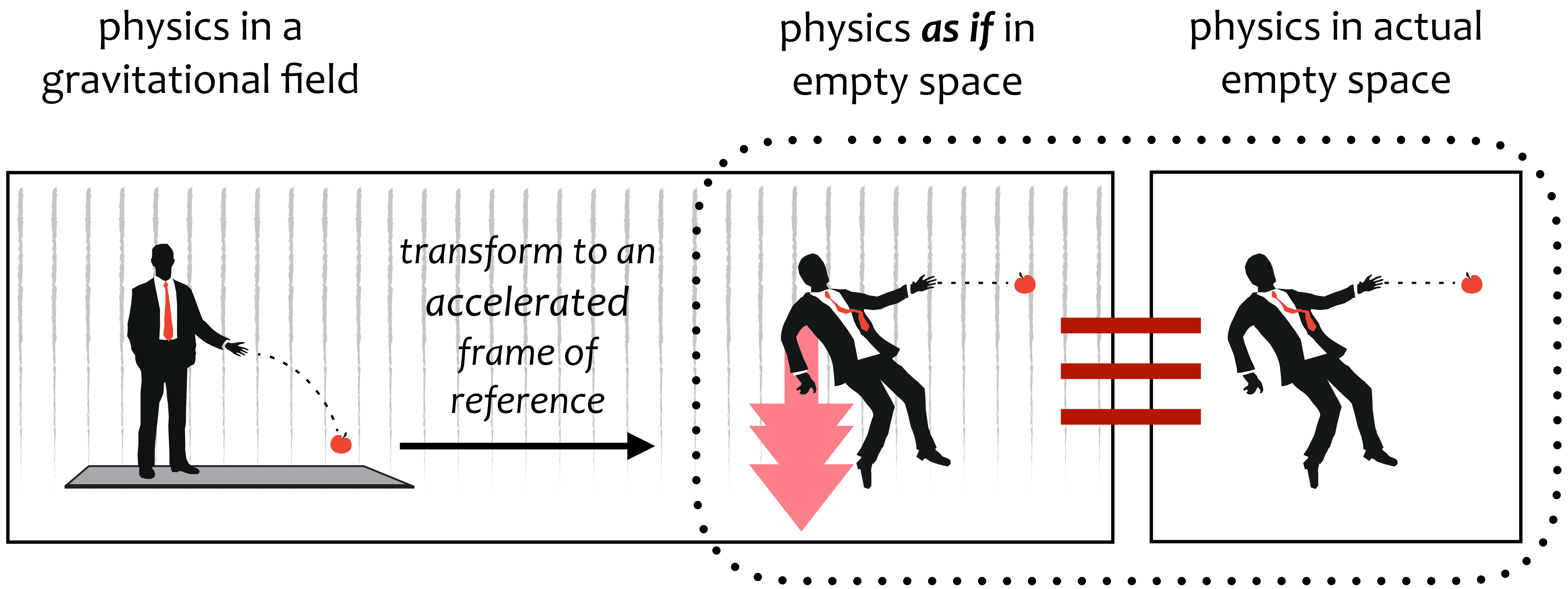
> At that moment [*in the Fall of 1907*] **I got the happiest thought of my life** in the following form:
>
> […] **for an observer in free-fall** from the roof of a house **there is during the fall** — at least in his immediate vicinity — **no gravitational field**.
>
> Namely, if the observer lets go of any bodies, they remain relative to him in a state of rest or uniform motion, independent of their special chemical or physical nature.

Albert Einstein, Unpublished manuscript, in all probability a draft for an article in *Nature* that was never published (c.1920)



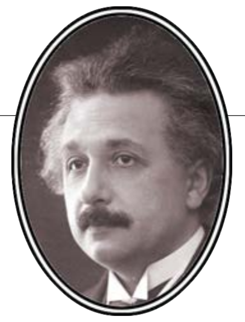
THE SECOND PAIR OF EQUIVALENT OBSERVERS

Step 2 The earlier naming convention “generative observer” inspires us to say:



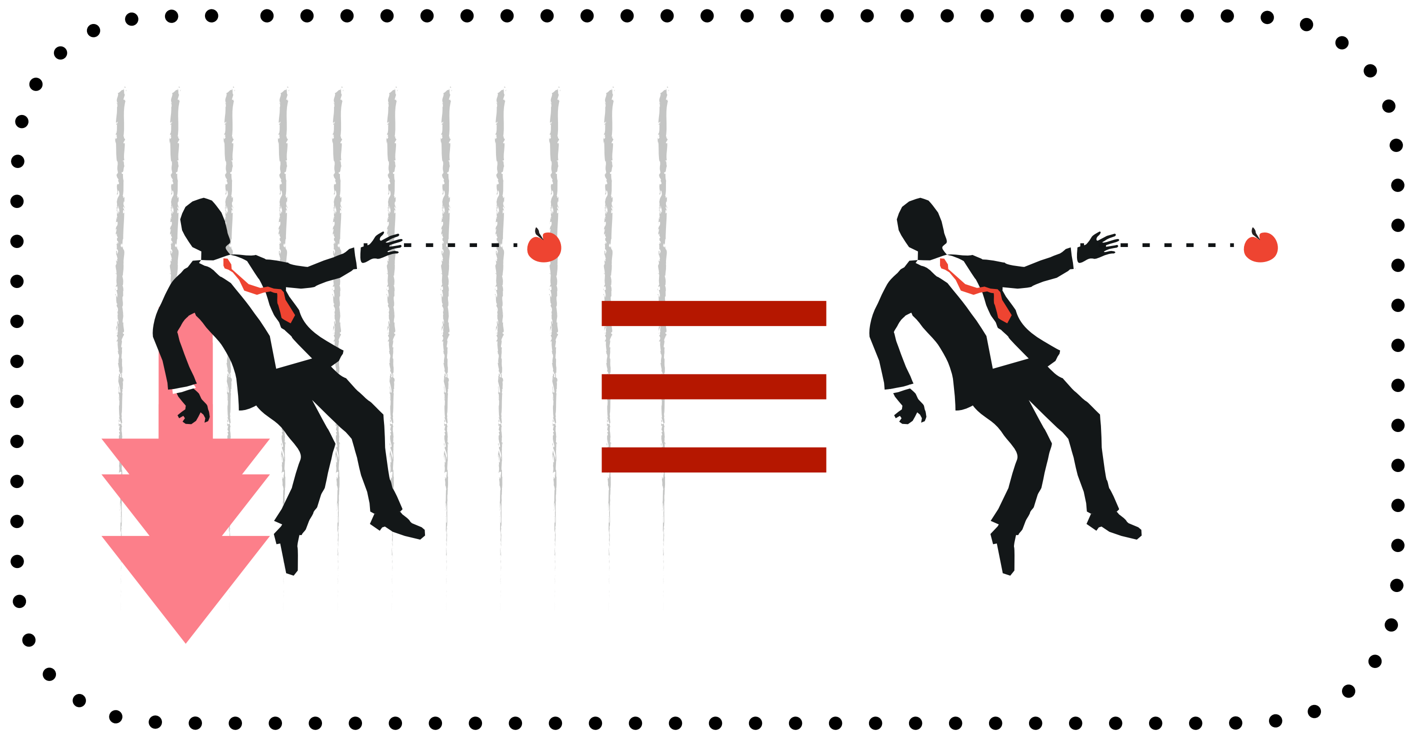
If you start out as an observer at rest in a gravitational field, you could let yourself fall freely, and the laws of physics play out just as for an equivalent free observer at rest.

It is as if, **by going from rest to an accelerated frame of reference** (in a gravitational field) **you have “eliminated” the gravitational field.**

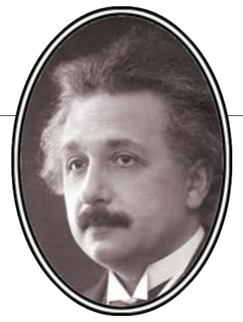


So let's call these two equivalent observers

the "eliminative" observers



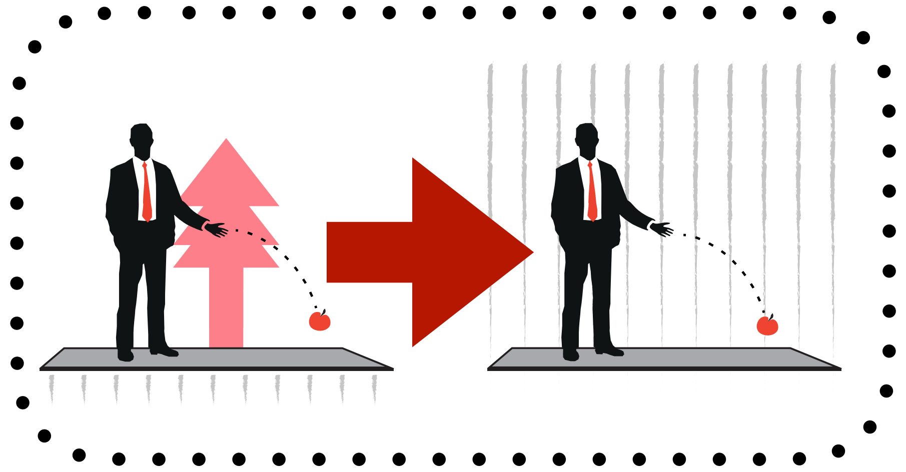
since they show us how one can "eliminate" the effects of a gravitational field by going to a state of acceleration.



Einstein used **the laws of Newtonian physics** to identify two pairs of special observers. The observers in each pair are equivalent w.r.t the laws of physics.

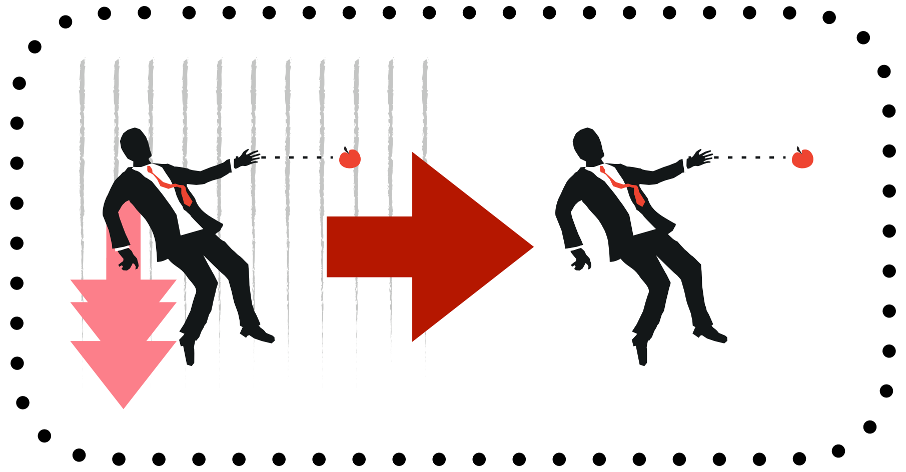
THEOREMS IN NEWTONIAN PHYSICS

The "**generative**" observers

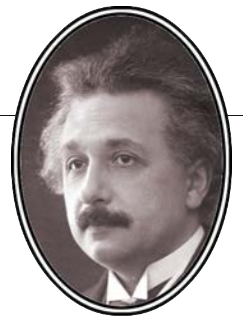


equivalent w.r.t. Newtonian physics

The "**eliminative**" observers



equivalent w.r.t. Newtonian physics



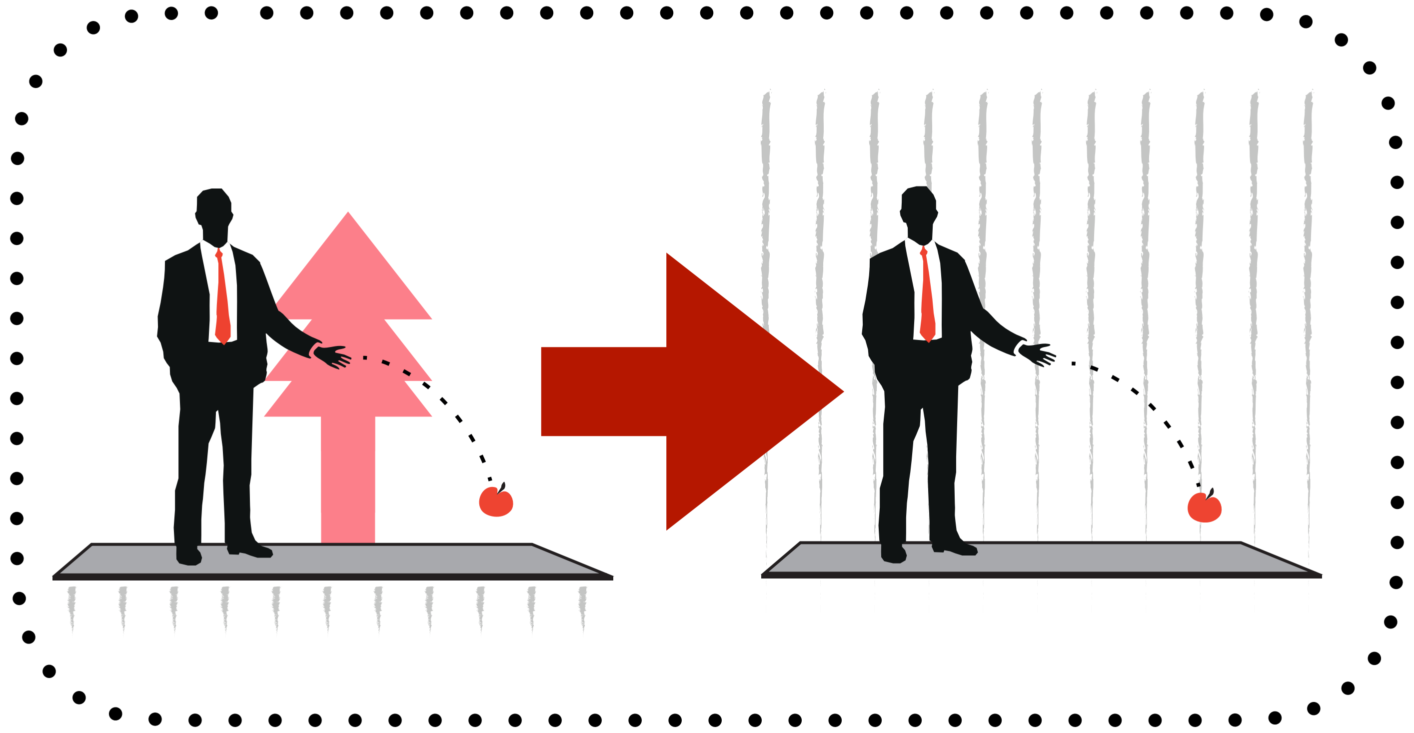
THE SECOND PAIR OF EQUIVALENT OBSERVERS

Step 3

Now —- remember how the ***Äquivalenzprinzip* heuristic** works:

We know how to calculate **kinematic** effects (in special relativity)

We do not know the properties of the unknown relativistic gravity theory



The original **"generative"** observers

Based on the **"generative"** observers:

1. We first study the (known) laws of physics for an **accelerated observer.**
2. We then translate the results to an **observer in an (unknown) gravity field.**

But — **can we use the "eliminative" observers in the same way,** to get a glimpse of what an (unknown) theory of relativistic gravity would look like?

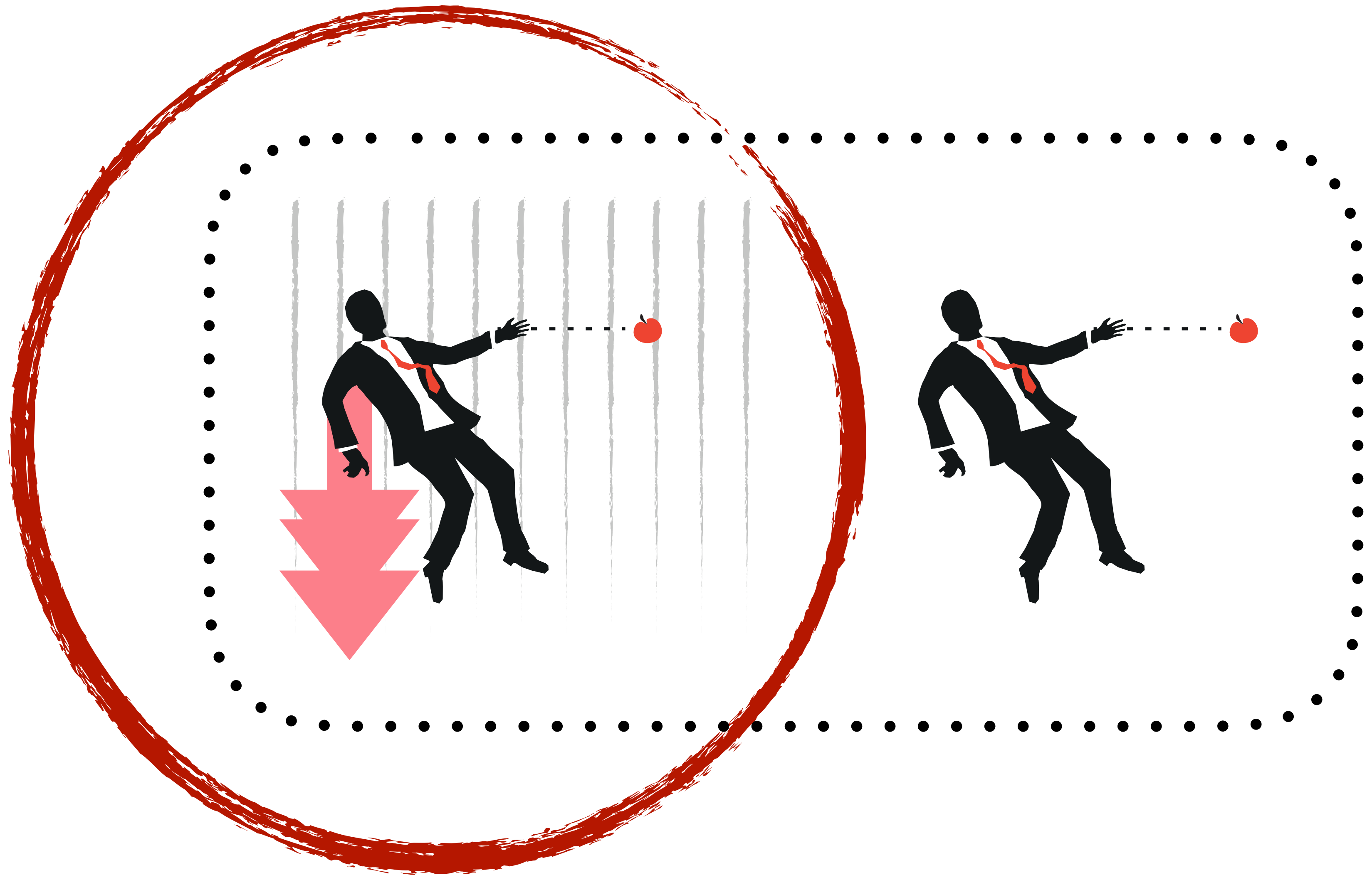
THE SECOND PAIR OF EQUIVALENT OBSERVERS

Well, not really…

The drawback is that with the **“eliminative”** observers we then would start in a frame of reference which ***mixes*** **the kinematic effects of acceleration** (which we know how to calculate) **with a gravitational field** (over which we have no control).

It is the ***clean*** **distinction between kinematics and dynamics** for the **“generative”** observer which *enables* the *Äquivalenzprinzip* heuristics.

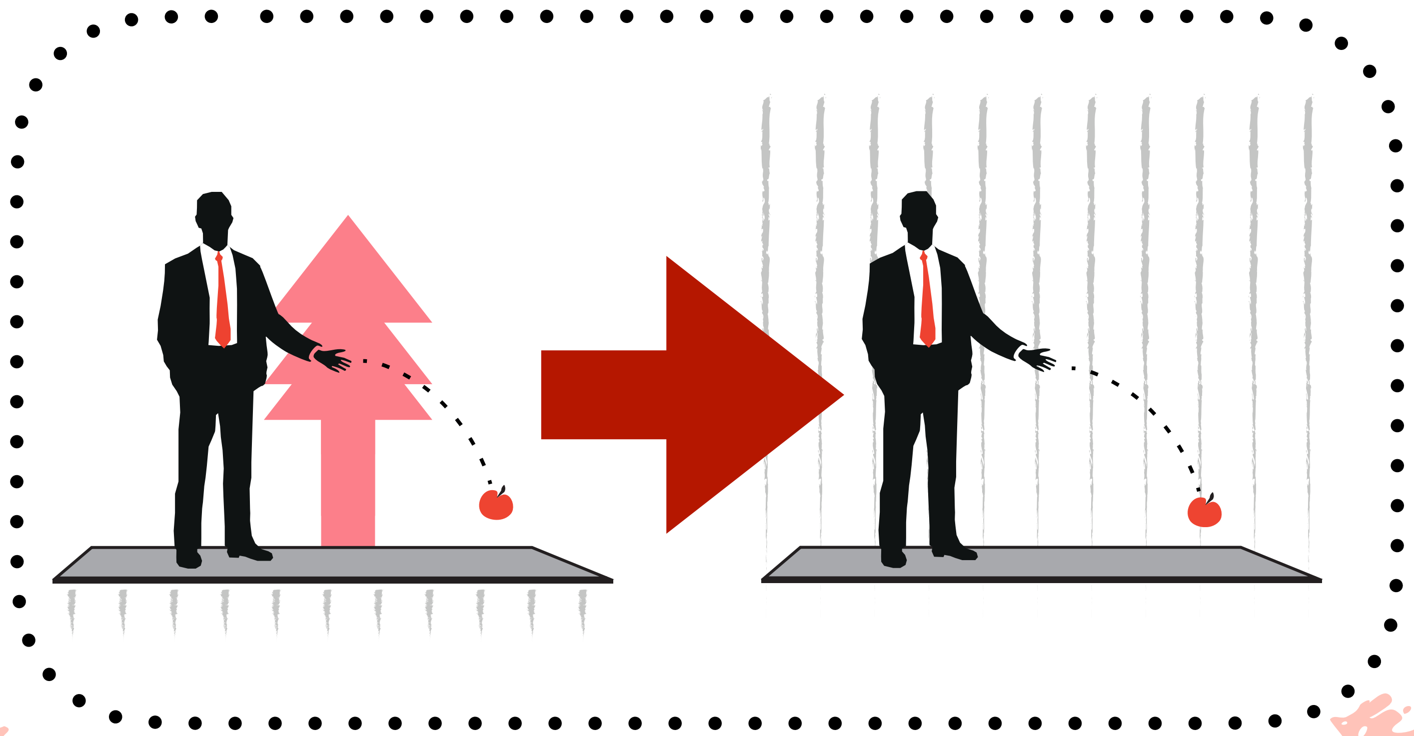
an observer both
(1) in accelerated motion
and
(2) in a gravitational field



The new
“**eliminative**”
observers

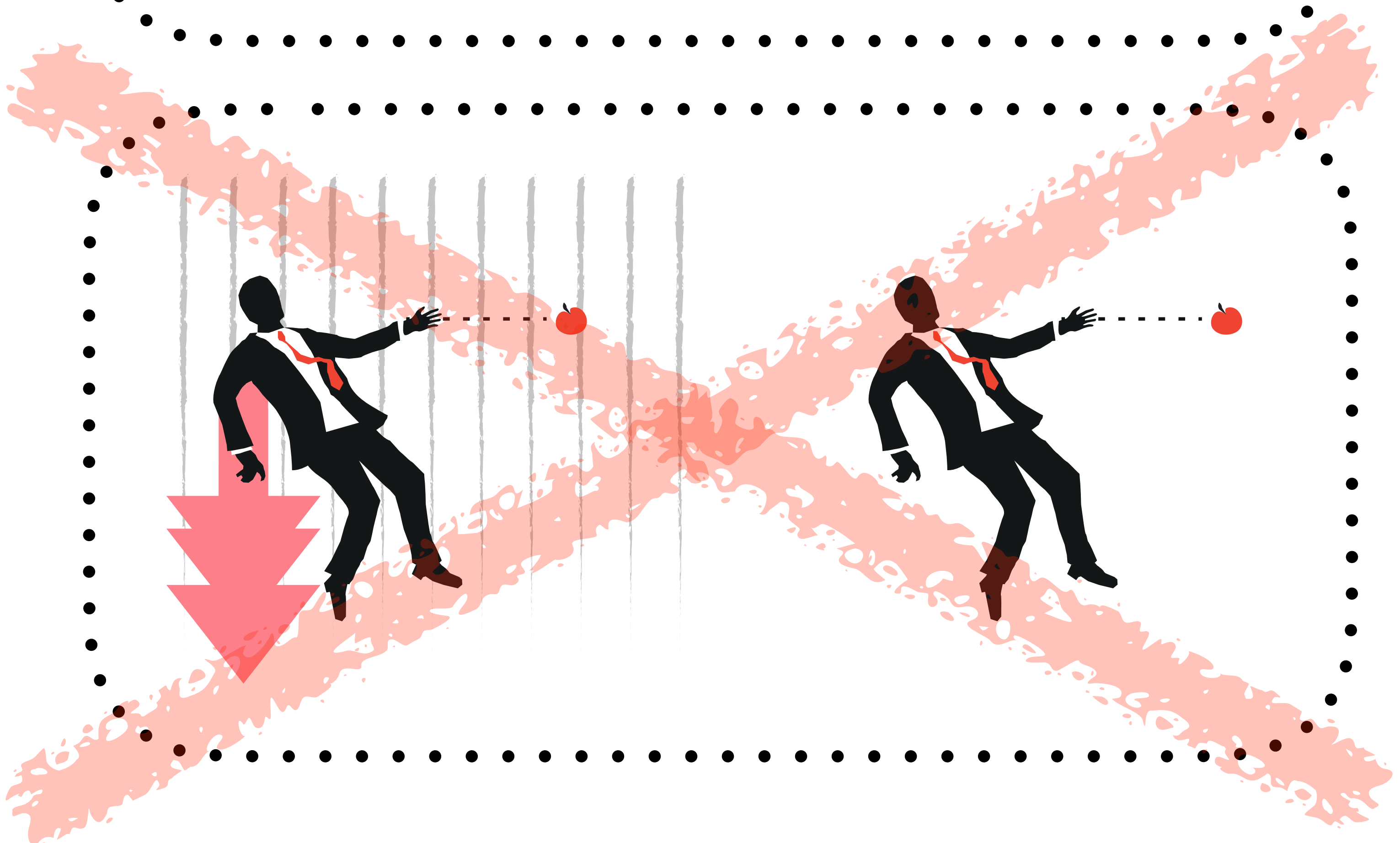
THE SECOND PAIR OF EQUIVALENT OBSERVERS

We can now understand why **Einstein consistently used the "generative" pair of equivalent observers** in his discussion and application of the *Äquivalenzprinzip*:



The original "**generative**" observers

true, but not useful



The new "**eliminative**" observers

SO WHAT KIND OF PRINCIPLE IS IT?

We have seen that Einstein based his 1907 ***Äquivalenzprinzip* heuristic** on an analysis of **two very special observers** (in *uniform acceleration* and in *a homogeneous gravitational field*).

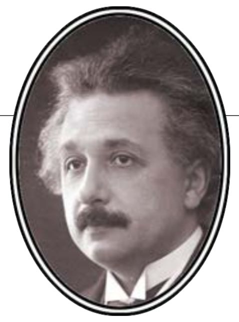
The observers were actually not very interesting in themselves!

Their only role was to reveal qualitative properties of the unknown relativistic gravitational field…

> If we accept this assumption, **we obtain a principle that possesses great heuristic significance […]**

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)

Guided by these insights, Einstein would then be able proceed **to find the actual theory** of the relativistic gravitational field (GR) **using additional methods** (1912–1915).



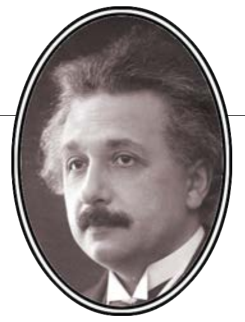
SO WHAT KIND OF PRINCIPLE IS IT?

We can conclude that:

Einstein’s 1907 *Äquivalenzprinzip* was a *heuristic* principle

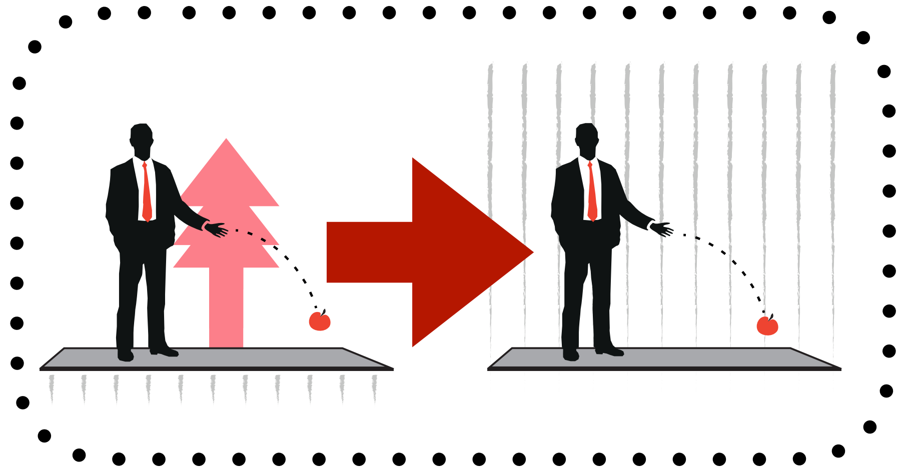
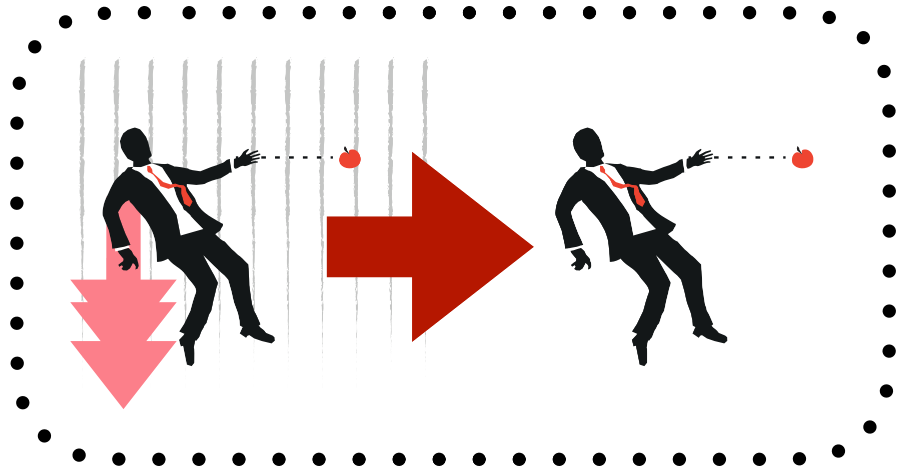
> **heu·ris·tic** *n.*
> *process or method enabling persons to discover or learn something*

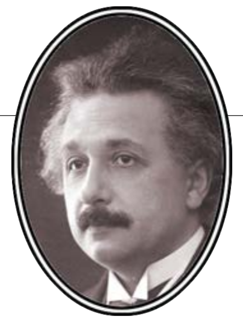
Note that *heuristics typically live dangerously*: they are essential to kickstart the discovery process, but **usually don’t make it into the final theory** (as theorems).



PART 2 — THE ARGUMENT SO FAR…

Einstein **generalised** the strictly Newtonoan result to obtain a forward-looking heuristic.

THEOREMS IN NEWTONIAN PHYSICS	**POSTULATED PRINCIPLES BEYOND NEWTONIAN PHYSICS**
The "**generative**" observers 	Einstein's ***Äquivalenzprinzip***
The "**eliminative**" observers 	*(not particularly useful heuristically)*

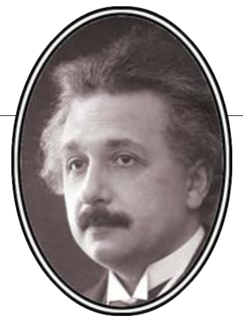


CAN WE BE SMARTER THAN EINSTEIN?

We have concluded that:

Einstein’s 1907 *Äquivalenzprinzip*
was a *heuristic* principle

But many of Einstein’s contemporaries completely missed this point…



CAN WE BE SMARTER THAN EINSTEIN?

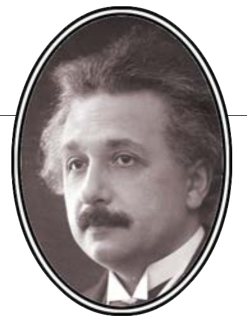
But many of Einstein's contemporaries completely missed this point.

Their concern seems to have been that **the *Äquivalenzprinzip* only talks about *a very special case*** of gravitational fields (those that can be simulated by acceleration).

Surely, anything called **a "principle" must have *general* validity**!

> Whatever the statement of the principle of equivalence (and there have been many) **its validity must extend beyond the framework of Newtonian gravitation or special relativity.**
>
> Otherwise, it would **hardly be justified to be called a principle.**

Fritz Rohrlich, "The Principle of Equivalence" (1963)



CAN WE BE SMARTER THAN EINSTEIN?

Thus started the transmogrification of Einstein's *Äquivalenzprinzip:*

from

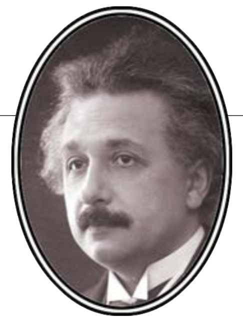
"heuristic principle" — *Einstein's Äquivalenzprinzip*

to

"principle of nature" — *modern equivalence principles*

with the understanding that a "principle of nature" is **a general fact that must be true in all theories of physics** (as theorems within each of these theories).

Like the first and second laws of thermodynamics, or the conservation of energy.



CAN WE BE SMARTER THAN EINSTEIN?

Einstein, however, did not change his mind on formulation of his "*Äquivalenzprinzip*" for the remainder of his life.

Here are quotes from the very first 1907 paper, and from one of the last texts he wrote about GR (he died in 1955).

> At our present state of experience we have thus no reason to assume that the systems Σ_1 and Σ_2 differ from each other in any respect, and in the discussion that follows, **we shall therefore assume the complete physical equivalence** of a gravitational field and a corresponding acceleration of the reference system.

Albert Einstein, "*On the relativity principle and the conclusions drawn from it*" (1907)

> An inertial space without gravitational field **is physically equivalent** to a uniformly accelerated space, in which there is a (homogeneous) gravitational field. (Equivalence hypothesis.)

Albert Einstein, letter to Jean Becquerel, 16 August 1951

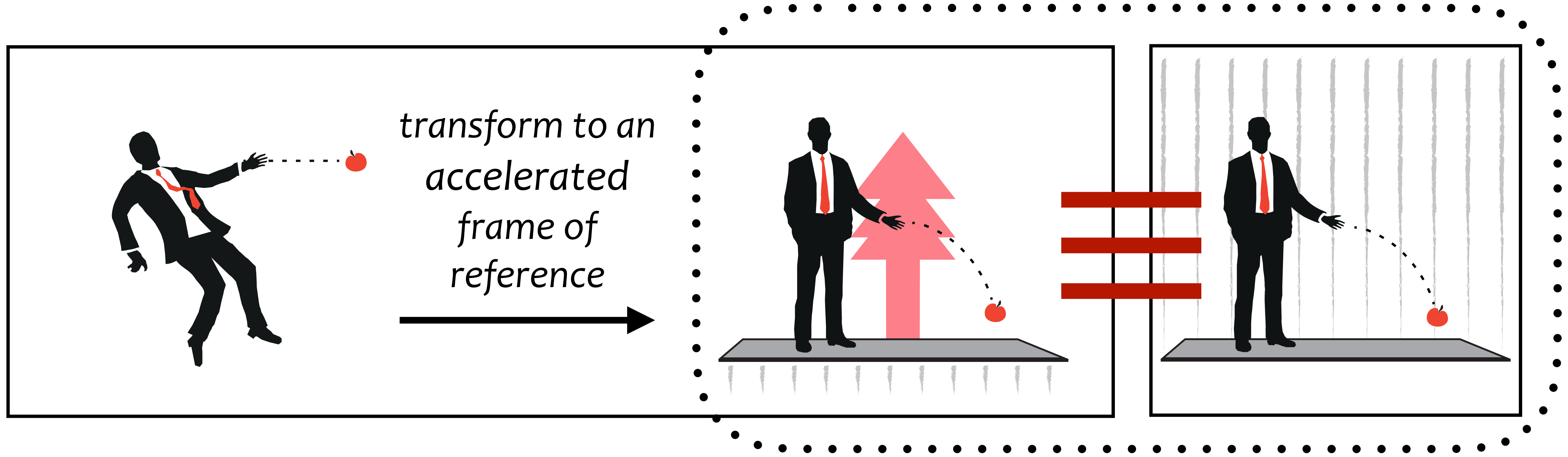
Let's have **a quick look at** **two proposed "generalisations"** **of Einstein's equivalence argument.**

One of the generalisations uses the "**generative**" equivalent observers, the other one starts with the "**eliminative**" equivalent observers.

And we will see how **Einstein criticised each of these two "generalisations"**.

Attempt 1

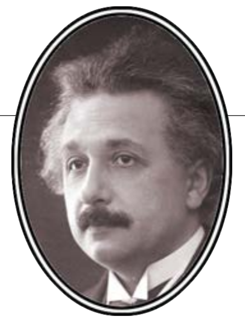
We know that, in Newtonian physics, by going from rest to a state of **very special acceleration** (uniform) we can "create" a **very special gravitational field** (homogeneous).



The equivalent **"generative"** observers **create** a *very special* gravitational field

*Proposed generalisation to a **generally valid** law*

But we should be able to **"create"**
any *arbitrary* gravitational field
by going to appropriate states
of acceleration!

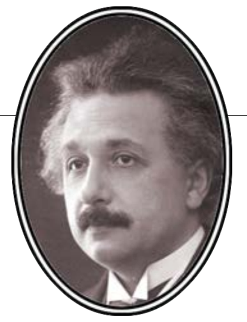


GENERALISATION 1: KINEMATIC GRAVITY

Einstein had to patiently, and again several times over the years, explain to his enthusiastic followers, that **such a "generalisation" of the equivalence argument is not possible.**

> The absurdity of such a hypothesis is plainly obvious. [...]
>
> [O]ne may never maintain that a gravitational field could be explained, so to speak, by pure kinematics; a "kinematic, nondynamic interpretation of gravitation" is not possible.
>
> By mere transformation from a Galilean [=*inertial*] system into another one by means of an acceleration transformation, **we do not learn about *arbitrary* gravitational fields** [*for example, that of a mass point at rest*] **but only some of a very special kind**.

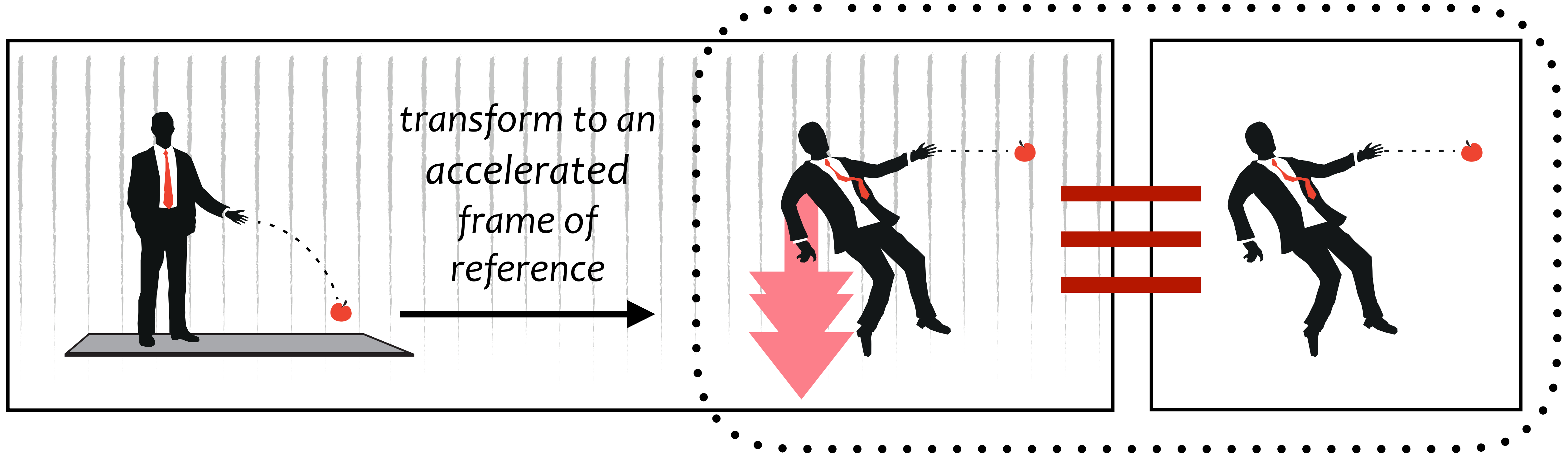
Albert Einstein, "On Friedrich Kottler's Paper: 'On Einstein's Equivalence Hypothesis and Gravitation' " (1916)



GENERALISATION 2: ELIMINATE GRAVITY

Attempt 2

We know that, in Newtonian physics, by going from rest in a **very special gravitational field** (homogeneous) to a state of **very special free fall** (uniform) we can "eliminate" the effects of gravity.



The equivalent "**eliminative**" observers **eliminate** a gravitational field

*Proposed generalisation to a **generally valid** law*

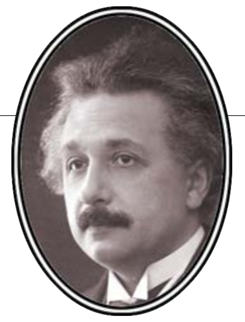
But we should be able to **"eliminate"** **any *arbitrary* gravitational field** by going to appropriate states of acceleration!

GENERALISATION 2: ELIMINATE GRAVITY

Again Einstein, still very patiently, had to remind his colleagues that **such a "generalisation" of the equivalence principle is not possible**:

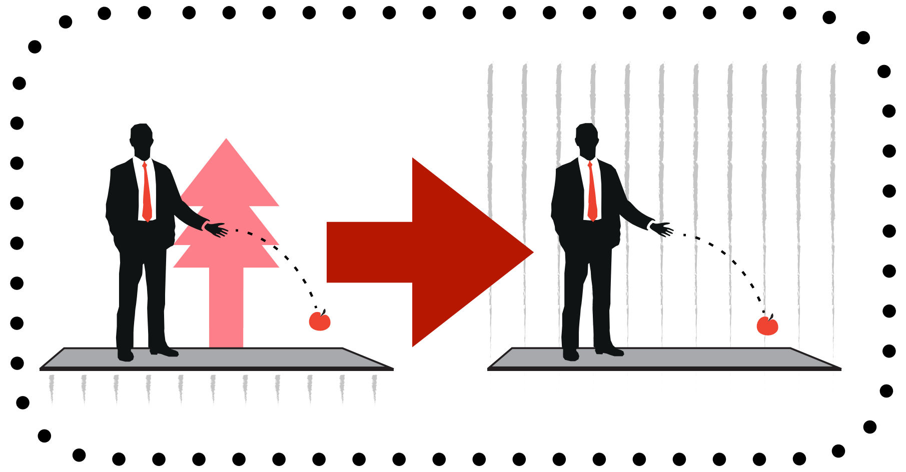
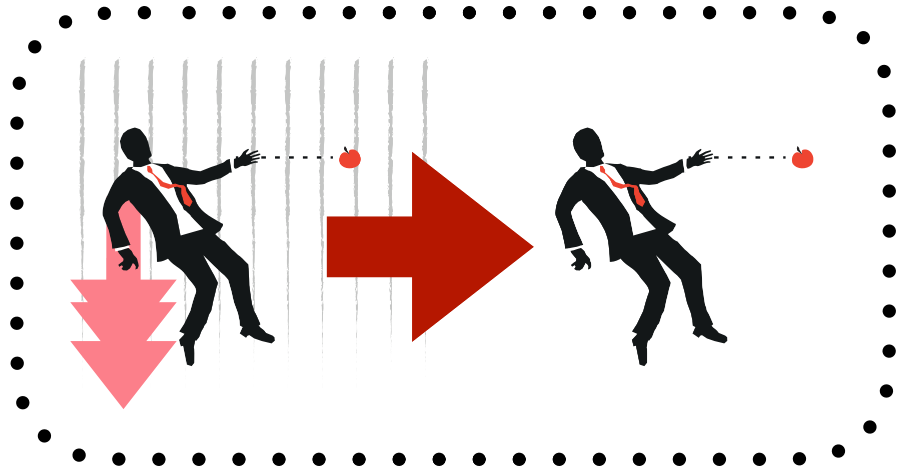
> **However, one must be careful not to assume that** […] **every gravitational field can be made to vanish,** *i.e.*, can be turned into a gravitation-free region, by means of a suitable choice of the coordinate system.
>
> For example, it is **impossible to make the gravitational field of the Earth vanish** by means of a suitable choice of the coordinate system. In fact, for a region of *finite extension* this is only possible with gravitational fields of a very special kind.

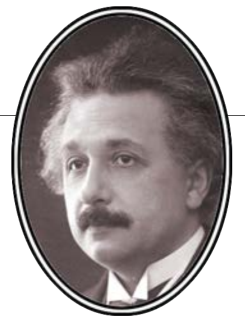
Albert Einstein, Theory of Relativity (1924)



PART 2 — THE ARGUMENT SO FAR…

Naive **generalisations** of the “generative” and “eliminative” strategies won’t work.

THEOREMS IN NEWTONIAN PHYSICS	POSTULATED PRINCIPLES BEYOND NEWTONIAN PHYSICS		
The “**generative**” observers 	Einstein’s ***Äquivalenzprinzip***	“Any gravitational field can be kinematically **generated**”	***Not valid*** (as a theorem)
The “**eliminative**” observers 		“Any gravitational field can be kinematically **eliminated**”	***Not valid*** (as a theorem)

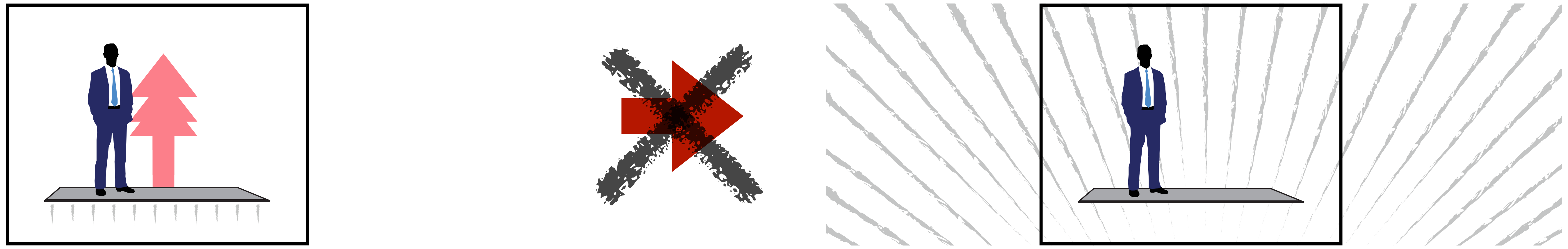


WHY THESE GENERALISATIONS WON'T WORK

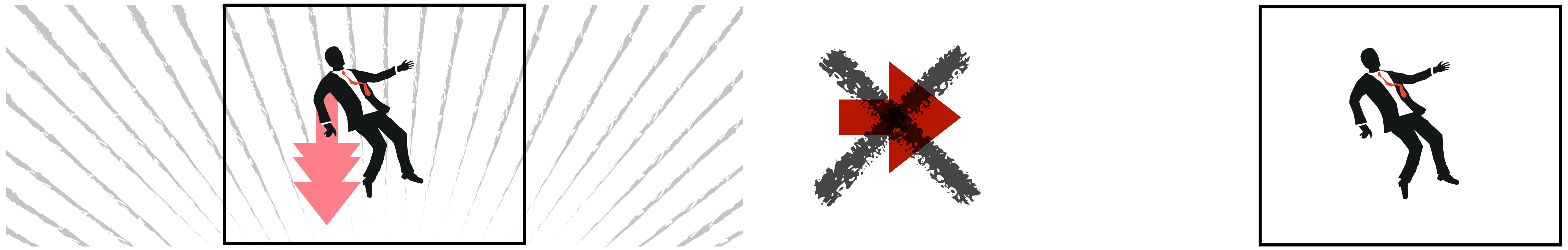
To summarise Einstein's objection to both generalisation proposals:

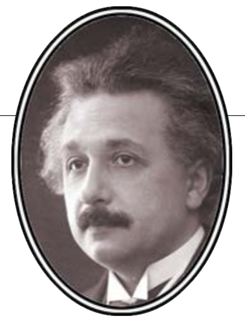
For an **observer in a rigid extended frame of reference…**

…there exists no state of acceleration that allows an observer in empty space to **generate** a spherically symmetric gravitational field…



…and there exists no state of acceleration that allows an observer in a spherically symmetric gravitational field to **eliminate** that field.

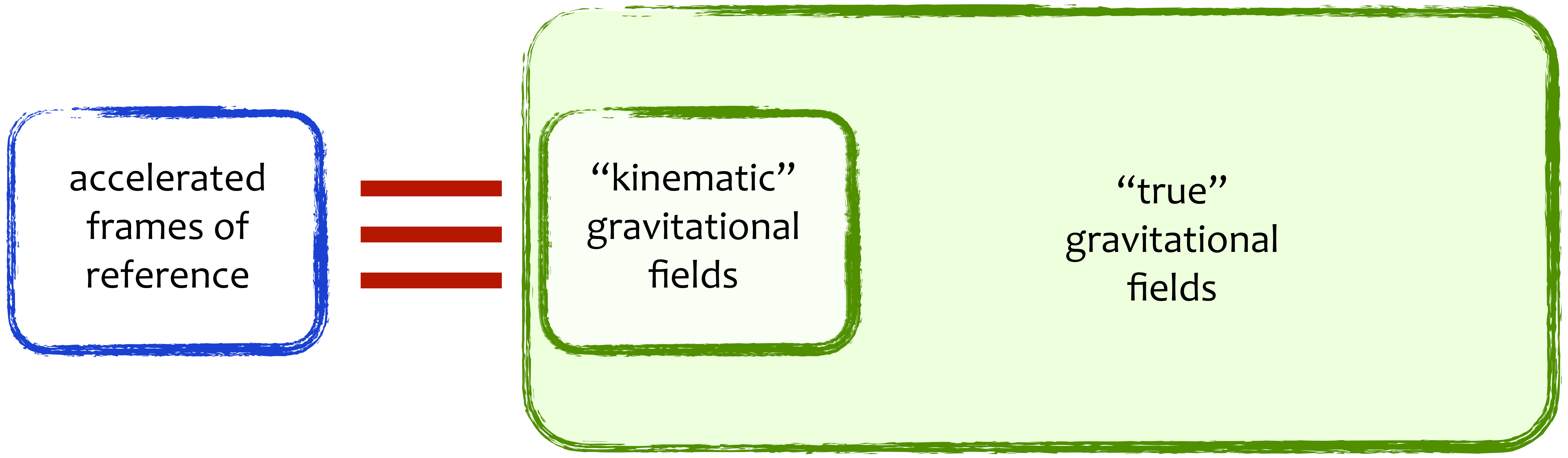




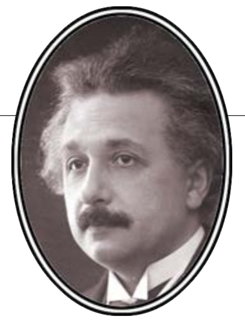
WHY THESE GENERALISATIONS WON’T WORK

The *Äquivalenzprinzip* only establishes a kind of equivalence between **all arbitrarily** accelerated frames of reference with **a very special subset** of all gravitational fields.

(In general relativity, all such “kinematical” gravitational fields are just ***flat Minkowski spacetime*** *seen from different observers.)*



gravitation and **acceleration**
are *not* equivalent



GENERALISATION 3: INFINITESIMAL REGIONS

But there is a loophole in Einstein's objection to the idea that **all** gravitational fields can be eliminated!

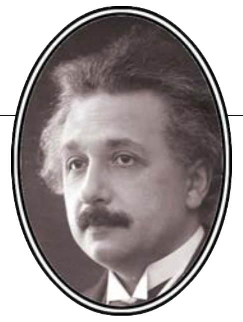
As he himself pointed out:

> **But for an infinitely small region** the coordinates can always be chosen such that no gravitational field will be present in it.

Albert Einstein, Theory of Relativity (1924)

True, **in a finite spacetime region** only **a very special kind of gravitational field** ("kinematical") can be "eliminated".

But in an infinitesimal spacetime region, you can "eliminate" **arbitrary gravitational fields**!



GENERALISATION 3: INFINITESIMAL REGIONS

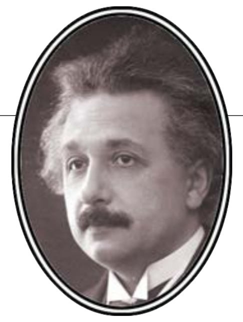
Attempt 3

Wolfgang Pauli seems to have been one of the first (1921) to formulate **a generalisation of the equivalence principle in terms of infinitesimal spacetime regions:**

Again this urge to **generalise** Einstein's argument

> Originally, the principle of equivalence had only been postulated for homogeneous gravitational fields. **For the general case, it can be formulated in the following way:**
>
> **For every infinitely small world region** […] there always exists a coordinate system K_o in which gravitation has no influence either on the motion of particles or any other physical processes.
>
> In short, **in an infinitely small world region every gravi-tational field can be transformed away.**

Wolfgang Pauli, "Relativitätstheorie" (1921)



GENERALISATION 3: INFINITESIMAL REGIONS

Proposed generalisation to a ***generally valid*** *law*

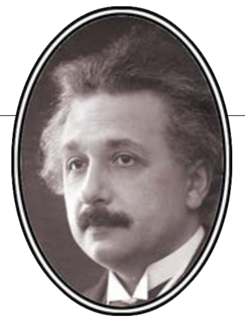
Infinitesimal Equivalence Principle

In an **infinitely small spacetime region** every gravitational field can be **transformed away.**

Intuitively, if you restrict your attention to a small enough (spatial) region *any* gravitational field will become (approximately) homogeneous.

You can now use the "eliminative" trick: by going to a freely falling frame of reference you get rid of the gravitational field.

But this of course **only works within that small region**…



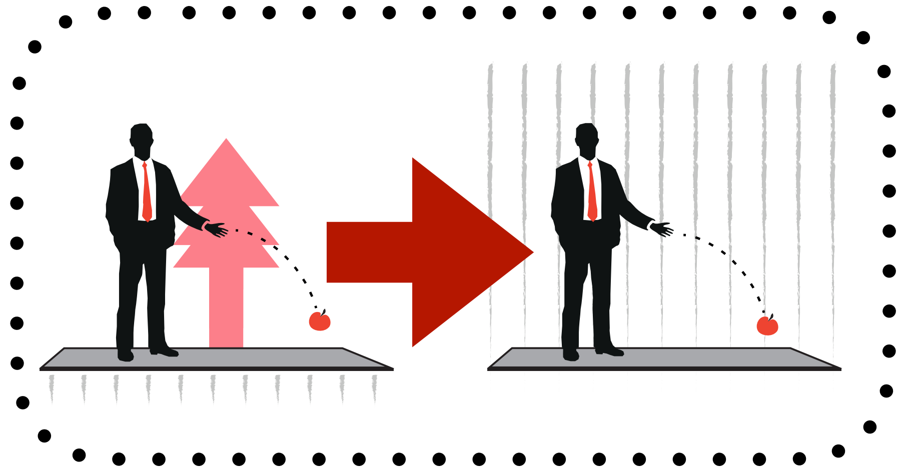
PART 2 — THE ARGUMENT SO FAR…

The “eliminative” generalisation is approximately valid only in the infinitesimal.

THEOREMS IN NEWTONIAN PHYSICS

POSTULATED PRINCIPLES BEYOND NEWTONIAN PHYSICS

The “**generative**” observers

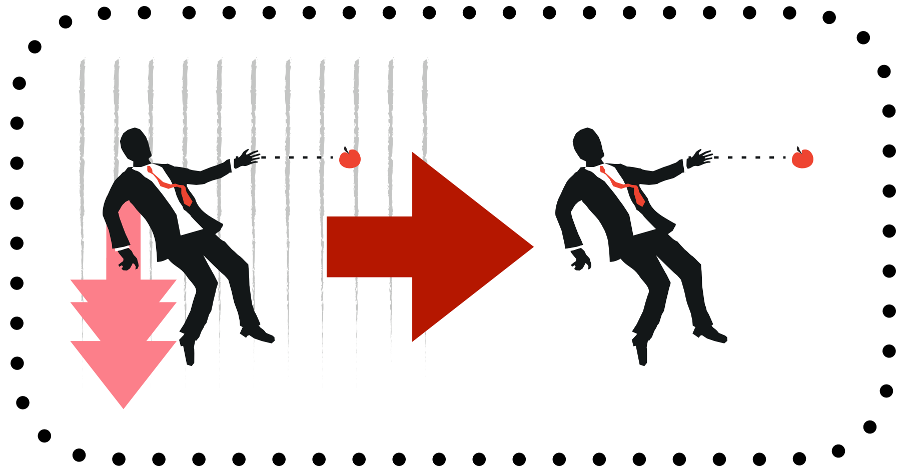


Einstein’s ***Äquivalenzprinzip***

“Any gravitational field can be kinematically **generated**”

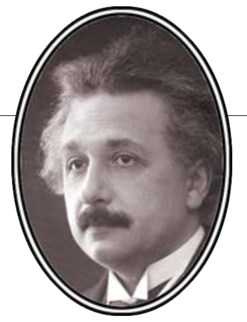
Not valid
(for extended regions)

The “**eliminative**” observers



Infinitesimal Equivalence Principle

Valid
(but only for infinitesimal regions)



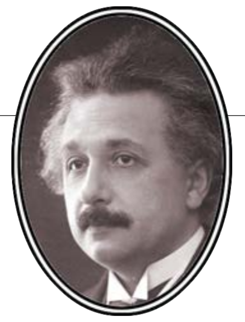
THE SR LIMIT IN INFINITESIMAL REGIONS

But you might wonder — **what is the point of eliminating gravitational fields** in an infinitesimal region?

Again Einstein comes to our help:

> **But for an infinitely small region the coordinates can always be chosen such that no gravitational field will be present in it.**
>
> **With respect to such an infinitely small region one may then assume that the special theory of relativity is valid.**
>
> That way the general theory of relativity is connected with the special theory of relativity, and the results of the latter can be utilized for the former.

Albert Einstein, Theory of Relativity (1924)



THE SR LIMIT IN INFINITESIMAL REGIONS

It seems we have stumbled across **the special-relativistic limit of general relativity**!

…except that the statement ***is not strictly speaking true***.

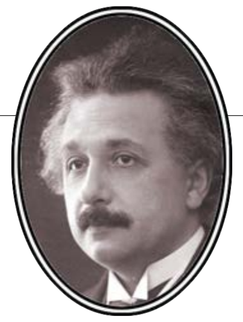
(sorry for all the technicalities — won't happen again!)

In general relativity, the curvature of the spacetime metric is expressed in the Riemann **tensor**.

And **you can't make a non-zero tensor vanish** just by transforming to a different frame of reference.

The Riemann tensor determines **tidal effects**, which would be detectable by a freely falling (infinitesimal) observer.

So what we actually have in an infinitesimal region around a spacetime point is a strange combination of special relativity (*defined* as having zero spacetime curvature) with a non-zero curvature tensor…



THE SR LIMIT IN INFINITESIMAL REGIONS

Incidentally, whenever Einstein discussed the special-relativistic limit of general relativity, he almost always referred to **finite regions**, which he called "**Galilean regions**".

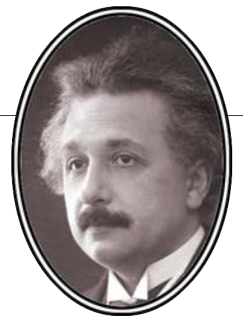
These would be regions **where the metric is approximately flat**. Here the laws of special relativity would be approximately true, and tidal effects negligible, since the curvature is almost zero.

Einstein's weak-field SR limit:

the weak-field approximation of GR **in an extended region**

The SR limit based on the infinitesimal equivalence principle:

SR laws approximately true only **at one spacetime point**



We have now come to the end of our story of the modern equivalence principles: **in the early 1960's, relativist Robert Dicke**

split the

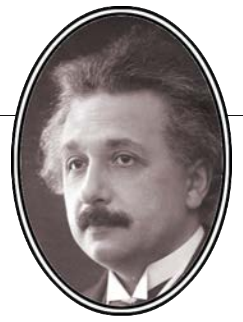
Infinitesimal Equivalence Principle

into two separate principles:

the **Weak Equivalence Principle**, and

the **Strong Equivalence Principle**

The "weak" principle was **that part which got empirical support from precision experiments determining the equality of inertial and gravitational mass,** while the "strong" principle was the rest.



Weak Equivalence Principle

This is Dicke's original statement of the **"Weak Equivalence Principle" (WEP):**

> The **weak principle of equivalence** states only that the local gravitational acceleration is substantially independent of the composition and structure of the matter being accelerated.

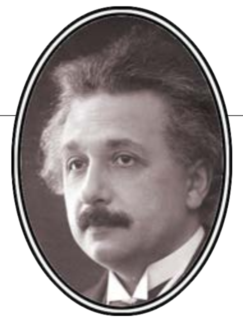
Robert Dicke, "Experimental Relativity" (1963)

The observation that all bodies fall with the same acceleration, regardless of their mass, is what is otherwise often called

the Universality of Free Fall (UFF),

usually credited to **Galileo**. But it remains a big mystery why the UFF should be confusingly renamed

the Weak Equivalence Principle



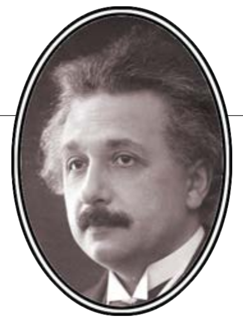
THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

If we fast forward to **general relativity**, the WEP is usually replaced by the statement that free particles **move along geodesics of the metric**, independently of their mass.

Originally this was known as the “Geodesic Postulate”. Later is was realised that it can actually be derived from the Einstein field equations (like with the **Geroch-Jang-Malament** or **Ehlers-Geroch theorems**).

But **these derivations only hold approximately**, and only for very special types of matter (non-rotating structureless test particles).

Therefore in GR the WEP is not a “**principle of nature**” (in the sense of being a strictly true theorem), but more like an **approximation scheme**.



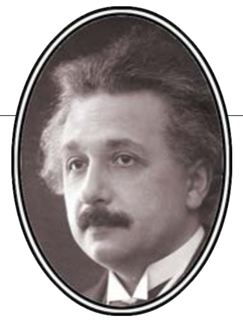
THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

Einstein never made the mistake of talking about the Universality of Free Fall as a “principle”, neither of “equivalence” nor of anything else:

In the Newtonian context of the *Äquivalenzprinzip*, referring back to Galilei, what Einstein actually often did talk about was…

> […] the old **experimental fact** that all bodies have the same acceleration in a gravitational field. This **law** […]

Albert Einstein, “Notes on the Origin of the General Theory of Relativity”, lecture manuscript (1933)



THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

Strong Equivalence Principle

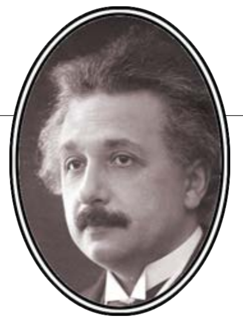
And this is Dicke's rendering of the **"Strong Equivalence Principle"** **(SEP)**:

> The **strong equivalence principle** might be defined as the assumption that in a freely falling, non-rotating, laboratory the local laws of physics take on some standard form, including a standard numerical content, independent of the position of the laboratory in space and time.

Robert Dicke, "Experimental Relativity" (1963)

This we immediately recognise as **the special-relativistic limit of general relativity** at a spacetime point, discussed earlier.

And we already know that Einstein didn't talk about the SR limit in connection with his *Äquivalenzprinzip*, and when he did, the limit was usually formulated in extended regions, not infinitesimal.



THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

We also saw earlier that the statement about the SR limit of GR in an infinitesimal region cannot be considered to be a “**principle of nature**” (in the sense of being a strictly true theorem), but that it is perfectly acceptable as an **approximation scheme.**

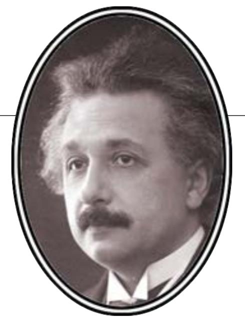
But it is of course trivially true that *any limit of any theory* must be formulated as an approximation scheme.

THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

Is there anything *wrong* with talking about the special-relativistic limit of general relativity? — **Of course not**!

The new theory of general relativity goes beyond the well-established earlier theories of Newtonian gravity and special-relativistic mechanics.

We should therefore **demand of the new theory that it recaptures the results of both old theories in well-defined limits**.



Is there anything *wrong* with talking about the special-relativistic limit of general relativity? — **Of course not!**

The new theory of general relativity goes beyond the well-established earlier theories of Newtonian gravity and special-relativistic mechanics.

We should therefore **demand of the new theory that it recaptures the results of both old theories in well-defined limits.**

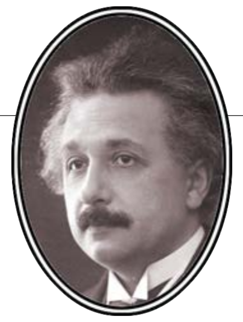
But while in modern texts the **Newtonian** limit of GR is called…

the Newtonian limit of general relativity

the **special-relativistic** limit of GR is called…

the strong equivalence principle

…No wonder people get confused!

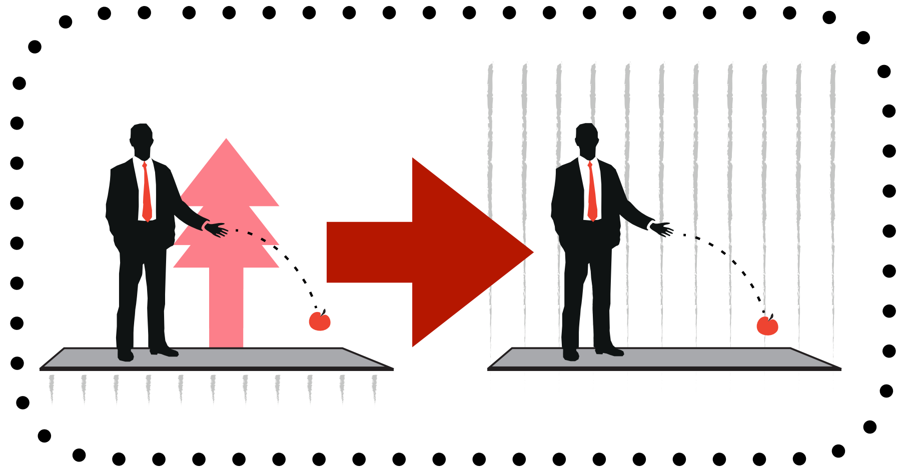


PART 2 — THE ARGUMENT SO FAR…

The Infinitesimal Equivalence Principle was split into “weak” and “strong” parts.

THEOREMS IN NEWTONIAN PHYSICS	POSTULATED PRINCIPLES BEYOND NEWTONIAN PHYSICS

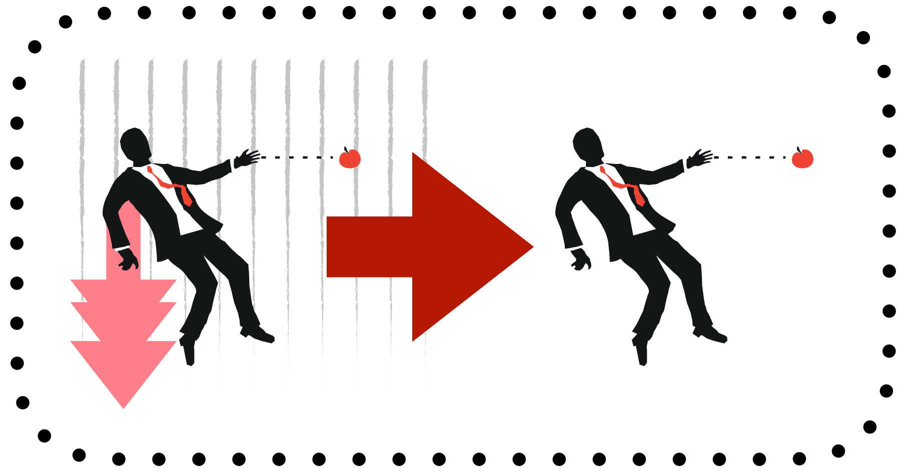
The “**generative**” observers



Einstein’s ***Äquivalenzprinzip***

“Any gravitational field can be kinematically **generated**”

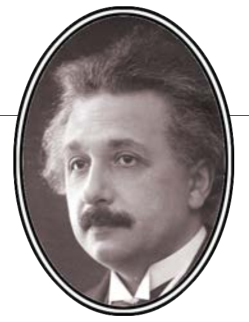
The “**eliminative**” observers



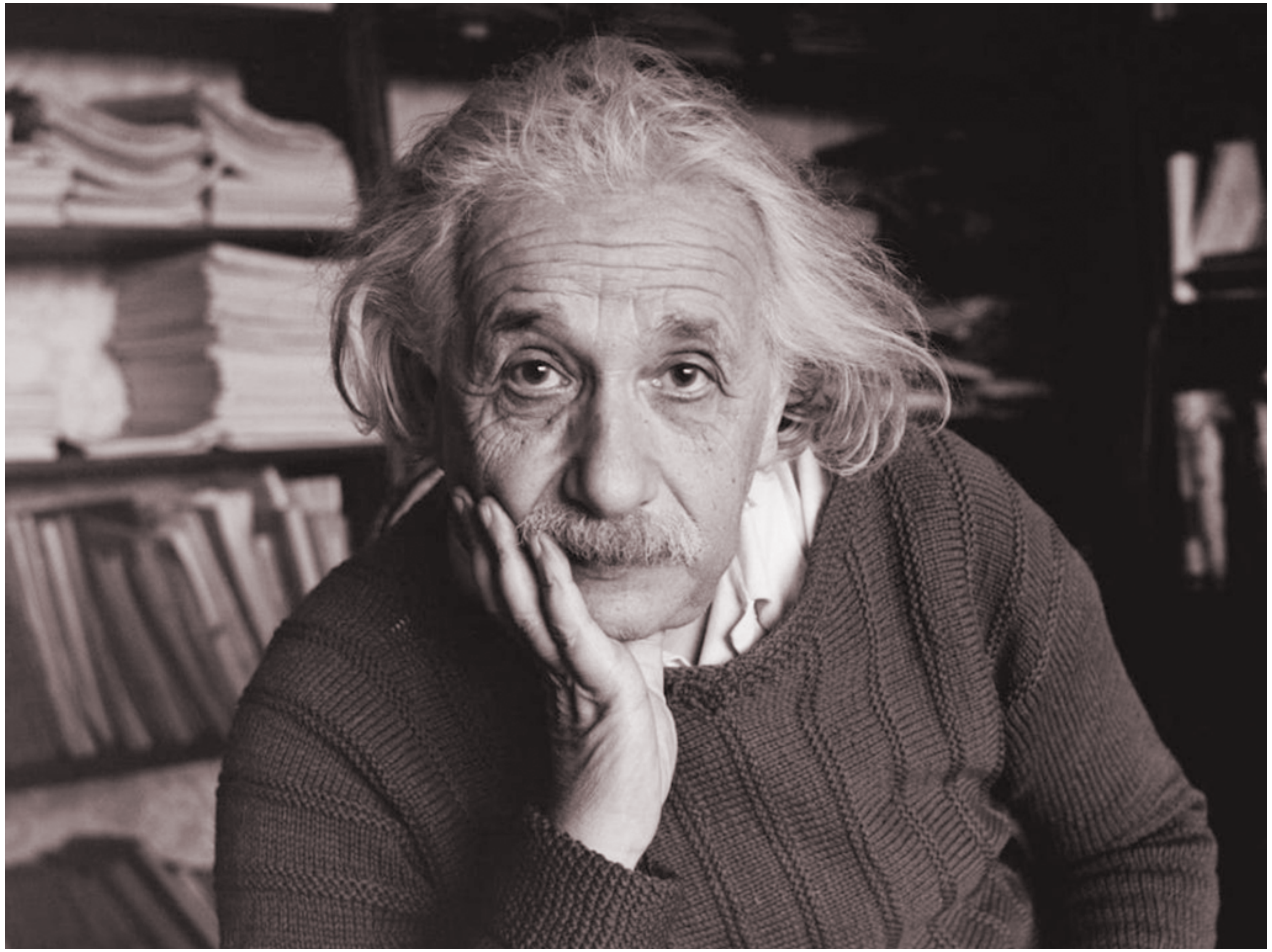
Infinitesimal Equivalence Principle

Strong Equivalence Principle

Weak Equivalence Principle



… SO, WHAT HAVE WE LEARNT?



CONCLUSION — PART 1

1. **Einstein's *Äquivalenzprinzip* was one step in a three-step argument,** starting with an analysis of Newtonian observers and culminating in a heuristic for discovering qualitative properties of the unknown theory of relativistic gravitation — but much more work was needed to finally arrive at GR!

1. THEOREM

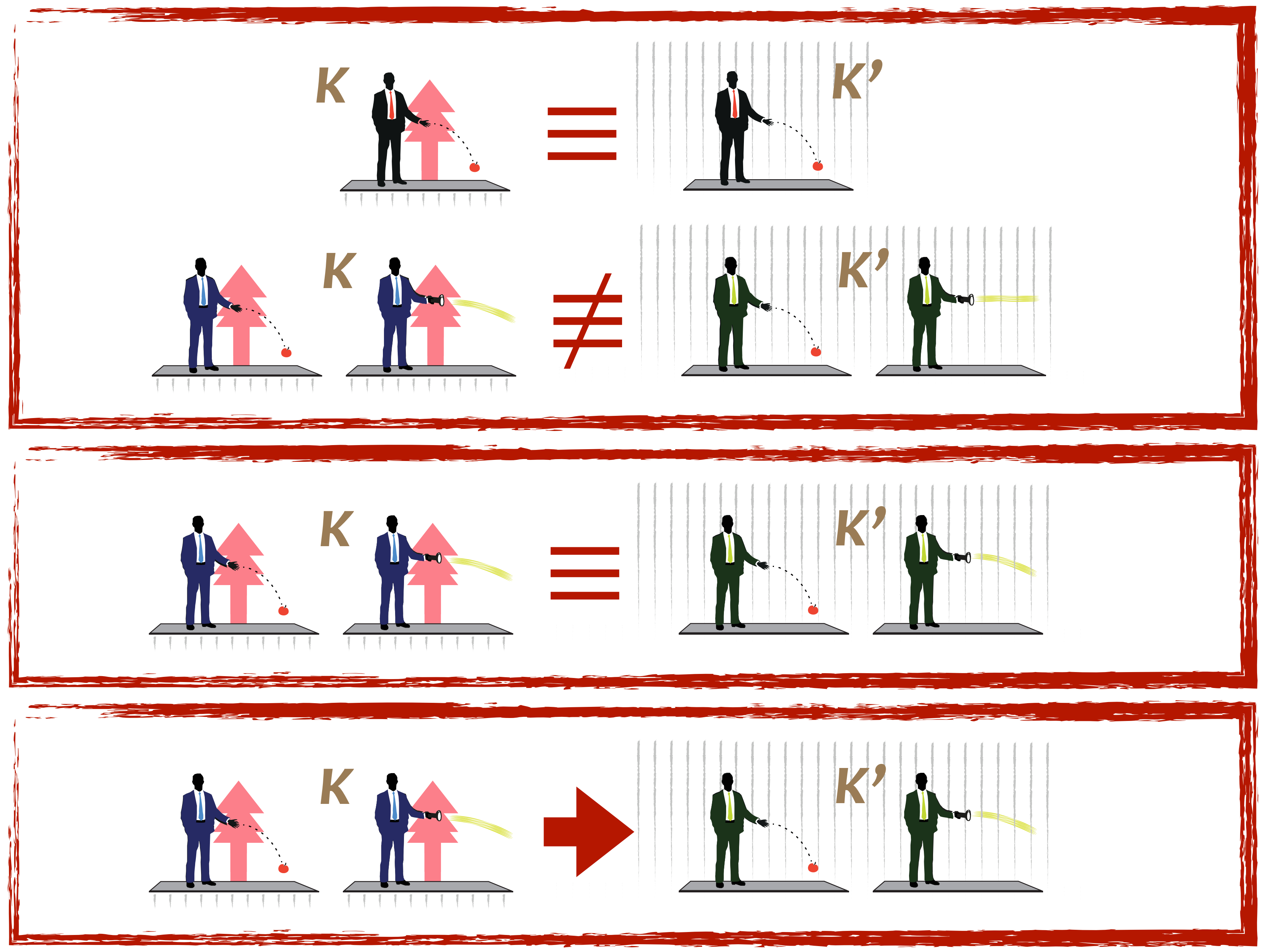
Analysing **Newtonian physics,** and assuming that inertial and gravitational masses are equal, we have found two special observers that are equivalent w.r.t the laws of Newtonian physics, but *not* equivalent w.r.t other laws, e.g. those of **electrodynamics**.

2. POSTULATE

Äquivalenzprinzip. In **a relativistic theory of gravitation** the special observers ***K*** and ***K'*** must be equivalent with respect to **all laws of physics.**

3. HEURISTIC

***Äquivalenzprinzip* Heuristic**. Analyse physical systems in **special relativity** w.r.t an accelerated observer. The results must also be valid for an observer in a gravitational field (in the unknown **relativistic theory of gravitation**).

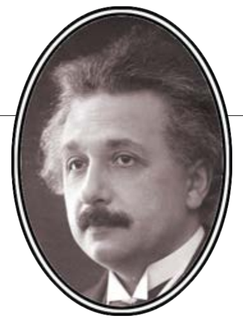


Newtonian physics

Classical physics

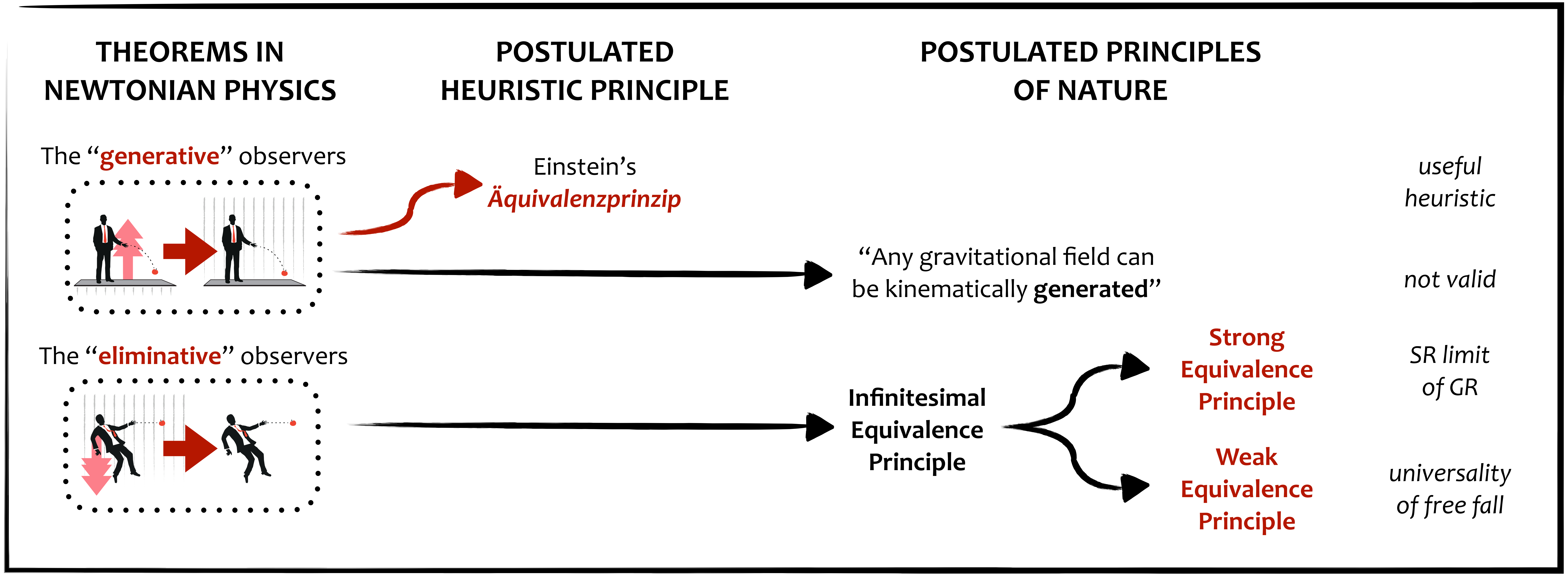
Unknown relativistic theory of gravity

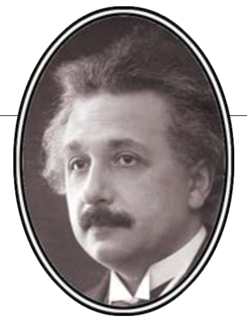
Special relativity



CONCLUSION — PART 2

2. What started as a naive generalisation of the “eliminative” strategy ended up as **“principles” of “equivalence” that neither are principles of nature nor have anything to do with “equivalence”** — an unfortunate naming convention that is a bottomless source of confusion in discussions of the foundations of GR.





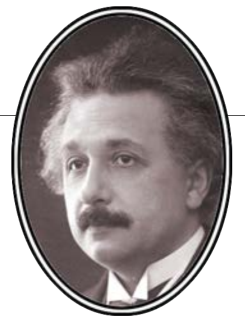
Take-Away Messages

PART 1

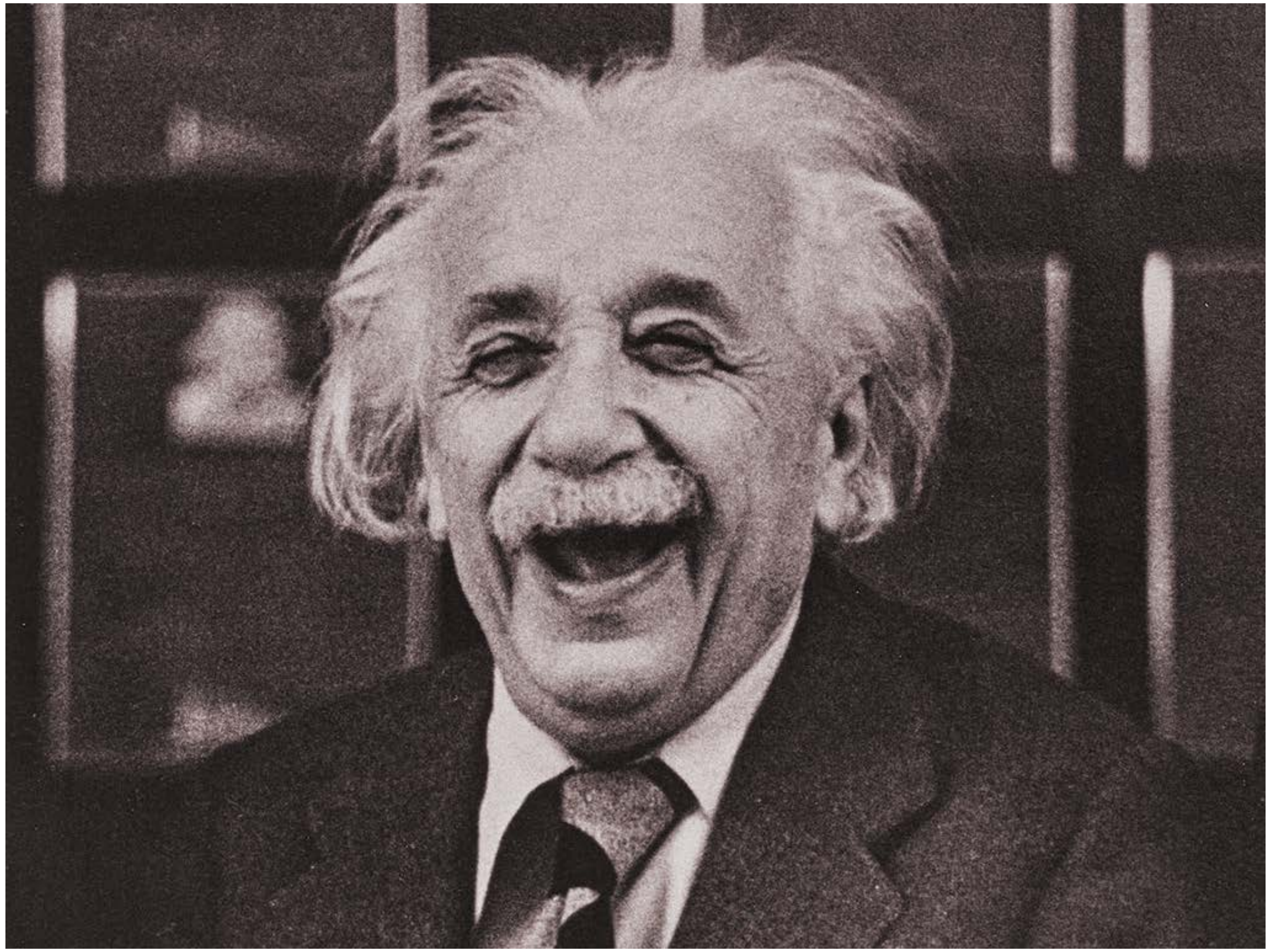
Einstein used features of *Newtonian* physics to get hints of a theory of *relativistic* gravity

PART 2

Don't use the term "*Equivalence Principle*" for anything that is neither.



Thank you for your attention!

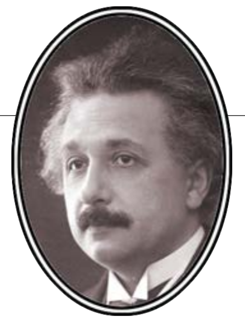


CONCLUSION

Unfortunately, Einstein's contemporaries seized upon one of Einstein's intermediate results, that in certain cases the gravitational fields […] have a relative existence, dependent on the choice of frame of reference. They sought to generalize this result from the simple cases in Minkowski spacetime which Einstein considered, to arbitrary gravitational fields. **It has rarely been acknowledged that Einstein never endorsed the principle which results, here called the "infinitesimal principle of equivalence."** […]

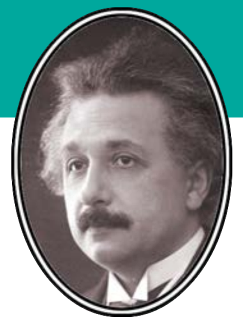
In recent decades there has been much criticism of "the" principle of equivalence. **But the principle under cogent attack has rarely been Einstein's version.**

John D. Norton, "What was Einstein's principle of equivalence?" (1985)



…WAIT, THERE IS MORE

FURTHER REFERENCES



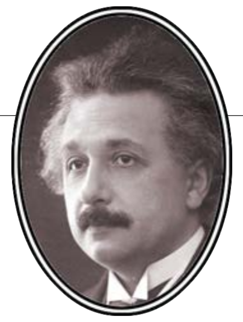
FURTHER REFERENCES

Tips for further reading…

The task of reconstructing Einstein's views is greatly simplified by the **easy access** we have today to everything that Einstein wrote:

All original texts by Einstein (currently **16 volumes,** up until ~1929) are available in German as well as in English translation at

einsteinpapers.press.princeton.edu



FURTHER REFERENCES

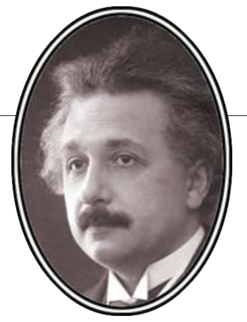
Tips for further reading…

Much of the relevant ***secondary literature*** has been collected in:

…

- the **Einstein Studies** anthology series from Springer (there are currently **16 titles** in the series)
  springer.com/series/4890

- the **The Genesis of General Relativity** anthology series (**4 volumes**) from the Max Planck Institute for the History of Science (MPIWG) in Berlin
  mpiwg-berlin.mpg.de



FURTHER REFERENCES

Tips for further reading…

The standard reference to the historical development and meaning of the *Äquivalenzprinzip* is

> *John D. Norton*
> "**What was Einstein's Principle of Equivalence?**"
> Studies in History and Philosophy of Science, **16** (1985) 203–246
> `doi.org/10.1016/0039-3681(85)90002-0`

More works by Norton on the history of relativity can be found at

`sites.pitt.edu/~jdnorton`

Tips for further reading…

Einstein's search for general relativity

Texts important for understanding Einstein's "Äquivalenzprinzip"

"On the relativity principle and the conclusions drawn from it" (Dec 1907)

"On the influence of gravitation on the propagation of light" (Sept 1911)

"On the Theory of the Static Gravitational Field" (May 1912)

"The speed of light and the statics of the gravitational field" (May 1912)

"Is There a Gravitational Effect Which Is Analogous to Electrodynamic Induction?" (July 1912)

"Relativity and gravitation. Reply to a comment by M. Abraham" (Aug 1912)

Zürich notebook (Summer 1912 – Spring 1913)

"Outline of a Generalized Theory of Relativity and of a Theory of Gravitation" (June 1913)

"On the Present State of the Problem of Gravitation" (Sept 1913)

"On the Foundations of the Generalized Theory of Relativity and the Theory of Gravitation" (Jan 1914)

"The Formal Foundation of the General Theory of Relativity" (Nov 1914)

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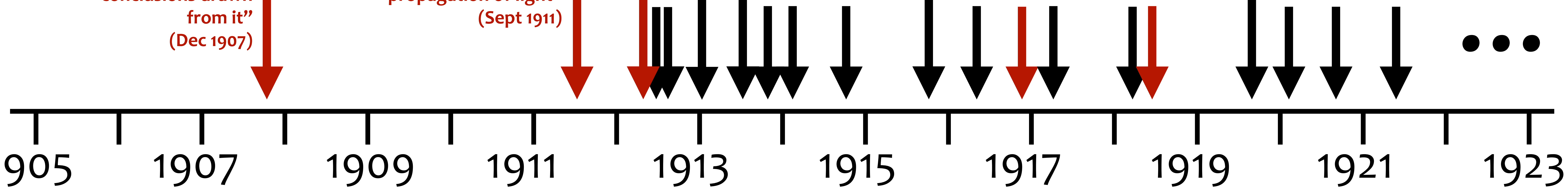

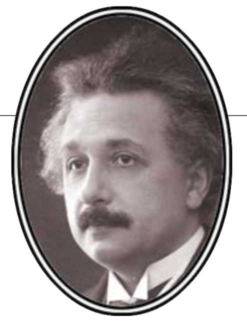

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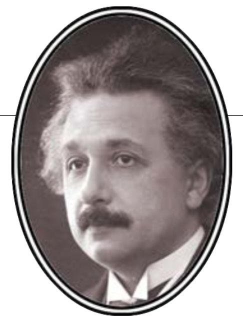


EINSTEIN QUOTES

EINSTEIN QUOTES

Einstein’s path to the theory of general relativity was a difficult one, both for himself and for those who tried to follow his various arguments along the line:

> My series of gravitation papers are a chain of wrong tracks, which nevertheless did gradually lead closer to the objectives. That is why now finally the basic formulas are good, but the derivations abominable.

Albert Einstein, letter to Hendrik A. Lorentz, 17 January 1916

A recollection from 1922 on how Einstein created the theory of general relativity:

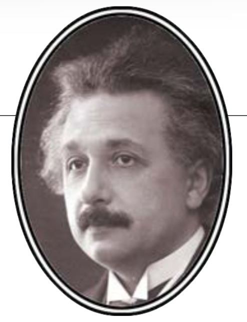
The first thought leading to the general theory of relativity occurred to me two years later, in 1907, and it did in a memorable setting.

I was already dissatisfied with the fact that the relativity of motion is restricted to motion with constant relative velocity and does not apply to arbitrary motion. I had always wondered privately whether this restriction could somehow be removed.

In 1907, while trying, at the request of Mr. Stark, to summarize the results of the special theory of relativity for the *Jahrbuch der Radioaktivität und Elektronik* of which he was the editor, I realized that, while all other laws of nature could be discussed in terms of the special theory of relativity, the theory could not be applied to the law of universal gravitation. I felt a strong desire to somehow find out the reason behind this. But this goal was not easy to reach. What seemed to me most unsatisfactory about the special theory of relativity was that, although the theory beautifully gave the relationship between inertia and energy, the relationship between inertia and weight, *i.e.*, the energy of the gravitational field, was left completely unclear. I felt that the explanation could probably not be found at all in the special theory of relativity.

continued...

Albert Einstein, "How I Created the Theory of Relativity " (1922)



EINSTEIN QUOTES

A recollection from 1921 on how Einstein created the theory of general relativity:

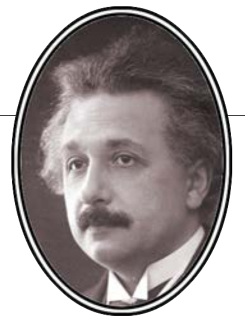
…continued

I was sitting in a chair in the Patent Office in Bern when all of a sudden I was struck by a thought: "If a person falls freely, he will certainly not feel his own weight."

I was startled. This simple thought made a really deep impression on me. My excitement motivated me to develop a new theory of gravitation. My next thought was: "When a person falls, he is accelerating. His observations are nothing but observations in an accelerated system." Thus, I decided to generalize the theory of relativity from systems moving with constant velocity to accelerated systems. I expected that this generalization would also allow me to solve the problem of gravitation. This is because the fact that a falling person does not feel his own weight can be interpreted as due to a new additional gravitational field compensating the gravitational field of the Earth, in other words, because an accelerated system gives a new gravitational field.

I could not immediately solve the problem completely on the basis of this insight. It would take me eight more years to find the correct relationship. In the meantime, however, I did come to recognize part of the general basis of the solution.

Albert Einstein, "How I Created the Theory of Relativity ",
Jun Ishiwara's Notes of Einstein's Lecture at Kyoto University (1922)



EINSTEIN QUOTES

The first presentation of the *Äquivalenzprinzip* argument in Einstein's 1907 paper.

§17. Accelerated reference system and gravitational field

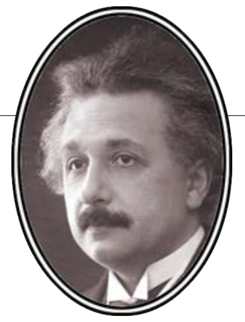
So far we have applied the principle of relativity, *i.e.*, the assumption that the physical laws are independent of the state of motion of the reference system, only to non-accelerated reference systems. Is it conceivable that the principle of relativity also applies to systems that are accelerated relative to each other?

While this is not the place for a detailed discussion of this question, it will occur to anybody who has been following the applications of the principle of relativity. Therefore I will not refrain from taking a stand on this question here.

We consider two systems Σ_1 and Σ_2 in motion. Let Σ_1 be accelerated in the direction of its X-axis, and let G be the (temporally constant) magnitude of that acceleration. Σ_2 shall be at rest, but it shall be located in a homogeneous gravitational field that imparts to all objects an acceleration $-\gamma$ in the direction of the X-axis.

continued…

Albert Einstein, "*On the relativity principle and the conclusions drawn from it*" (1907)



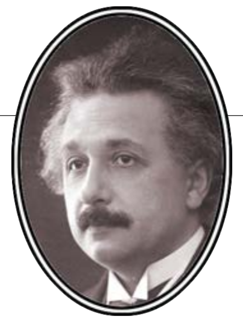
EINSTEIN QUOTES

The first presentation of the *Äquivalenzprinzip* argument in Einstein’s 1907 paper.

…continued

> As far as we know, the physical laws with respect to Σ_1 do not differ from those with respect to Σ_2; this is based on the fact that all bodies are equally accelerated in the gravitational field. At our present state of experience we have thus no reason to assume that the systems Σ_1 and Σ_2 differ from each other in any respect, and in the discussion that follows, we shall therefore assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system.
>
> This assumption extends the principle of relativity to the uniformly accelerated translational motion of the reference system. The heuristic value of this assumption rests on the fact that it permits the replacement of a homogeneous gravitational field by a uniformly accelerated reference system, the latter case being to some extent accessible to theoretical treatment.

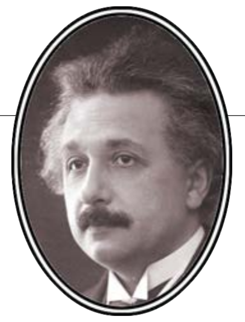
Albert Einstein, “*On the relativity principle and the conclusions drawn from it*” (1907)



Einstein's second paper from 1911 in his quest for a relativistic theory of gravitation again explores the kinematical consequences of the *Äquivalenz-prinzip*. Here is the statement of the principle (still without the name):

> As long as we confine ourselves to purely mechanical processes within the range of validity of Newton's mechanics, we can be sure of the equivalence of the systems *K* [*at rest in a homogeneous gravitational field*] and *K'* [*in uniform acceleration in empty space*]. However, for our conception [*of equivalence*] to acquire deeper significance, the systems *K* and *K'* must be equivalent with respect to all physical processes, i.e., the natural laws with respect to *K* must coincide completely with those with respect to *K'*. If we accept this assumption, we obtain a principle that possesses great heuristic significance…

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



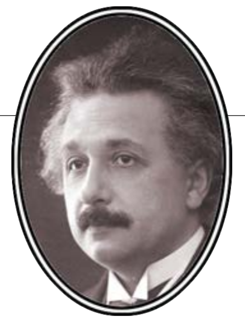
EINSTEIN QUOTES

Here is the full quote from the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

> **§1. A Hypothesis Concerning the Physical Nature of the Gravitational Field**
>
> In a homogeneous gravitational field (acceleration due to gravity γ) let there be a coordinate system at rest *K*, which is oriented in such a way that the lines of force of the gravitational field run in the direction of the negative *z*-axis. In a space free of gravitational fields, let there be another coordinate system *K'* that moves with a uniform acceleration (acceleration γ) in the direction of its positive *z*-axis. So as not to complicate the analysis unnecessarily, we will disregard the theory of relativity for the time being, and consider, instead, the two systems according to conventional kinematics, and the motions occurring in them according to customary mechanics.

continued…

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



Here is the full quote form the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

… continued

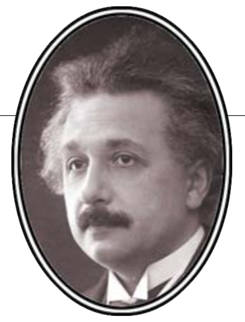
Material points not subjected to actions of other material points move relative to *K* as well as relative to *K’* according to the equations

$$\frac{d^2 x_\nu}{dt^2} = 0, \quad \frac{d^2 y_\nu}{dt^2} = 0, \quad \frac{d^2 z_\nu}{dt^2} = -\gamma$$

For the accelerated system *K’*, this follows directly from Galileo's principle, but for the system *K* at rest in a homogeneous gravitational field, this follows from the experience that all bodies undergo the same, constant, acceleration in such a field. This experience of the identical falling of all bodies in the gravitational field is one of the most universal experiences that the observation of nature has yielded to us; nevertheless, this law has not been granted a place in the foundation of our physical edifice.

continued…

Albert Einstein, “On the Influence of Gravitation on the Propagation of Light” (1911)



EINSTEIN QUOTES

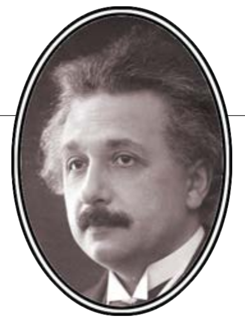
Here is the full quote form the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

… continued

> But we arrive at a very satisfactory interpretation of the empirical law if we assume that the systems K and *K'* are, physically, perfectly equivalent, *i.e.*, if we assume that the system *K* could likewise be conceived as occurring in a space free of a gravitational field; but in that case we must consider K as uniformly accelerated. Given this conception, one can no more speak of the absolute acceleration of the reference system than one can speak of a system's absolute velocity in the ordinary theory of relativity. [Footnote: Of course, one cannot replace an arbitrary gravitational field by a state of motion of the system without a gravitational field, just as one cannot transform to rest all the points of an arbitrarily moving medium by means of a relativistic transformation.] With this conception, the equal falling of all bodies in a gravitational field is self-eveident.

continued…

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



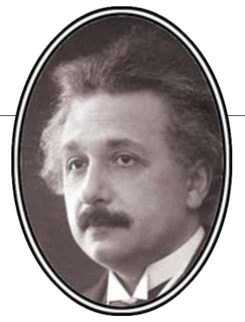
EINSTEIN QUOTES

Here is the full quote form the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

… continued

As long as we confine ourselves to purely mechanical processes within the range of validity of Newton's mechanics, we can be sure of the equivalence of the systems *K* and *K'*. However, for our conception to acquire deeper significance, the systems *K* and *K'* must be equivalent with respect to all physical processes, *i.e.*, the natural laws with respect to *K* must coincide completely with those with respect to *K'*. If we accept this assumption, we obtain a principle that possesses great heuristic significance, provided that it is really correct. For through a theoretical analysis of processes taking place relative to a uniformly accelerating reference system, we obtain information about the course of processes taking place in a homogeneous gravitational field. [*Footnote*: It will be shown in a subsequent paper that the gravitational field considered here is homogeneous only to first approximation.] In what follows, I shall first show that from the point of view of the ordinary theory of relativity our hypothesis has considerable probability.

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



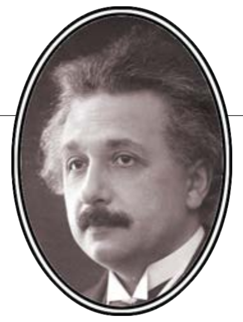
Einstein’s 1916 reply to Friedrich Kottler (who proposed a “kinematic, nondynamic interpretation of gravitation”) is a concise summary of the *Äquivalenzprinzip* argument.

Kottler claims I had abandoned in my later papers the “principle of equivalence” which I did introduce in order to unify the concepts of “inertial mass” and “gravitational mass.” This opinion must be based upon the fact that we both do not denote the same thing as “the principle of equivalence”; because in my opinion my theory rests exclusively upon this principle. Therefore I repeat the following:

1. The Limiting Case of the Special Theory of Relativity. Let a finite space-time-like domain be without a gravitational field; i.e., let it be possible to introduce a system of reference *K* (“Galilean system”) relative to which the following is true for the domain. As is usually presupposed in the special theory of relativity, let the coordinates be directly measurable in known manner by means of a unit measuring stick, and the times by a unit clock. Relative to this system an isolated material point shall move uniformly in a straight line, just as it was postulated by Galileo.

continued...

Albert Einstein, “On Friedrich Kottler's Paper: ‘On Einstein's Equivalence Hypothesis and Gravitation’ ” (1916)



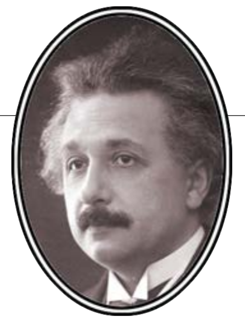
Einstein's 1916 reply to Friedrich Kottler (who proposed a "kinematic, nondynamic interpretation of gravitation") is a concise summary of the *Äquivalenzprinzip* argument.

… continued

2. The Principle of Equivalence. Starting from the limiting case of the special theory of relativity, one may ask if in the domain under consideration an observer, who is uniformly accelerated relative to *K*, must necessarily judge his state as accelerated, or whether he has an option left — according to the (approximately) known laws of nature — to interpret his state as "at rest" Or, to phrase it more precisely: Do the laws of nature, known to us in some approximation, allow us to consider a reference system *K'* as being at rest if it is in uniform acceleration with respect to *K*? Or, somewhat more generally: Can the principle of relativity be extended such as to encompass reference systems that are in (uniform) accelerated motion relative to one another? The answer is: insofar as we really know the laws of nature, nothing prevents us from considering a system *K'* as at rest, provided we assume a gravitational field (homogeneous in first approximation) relative to *K'*. Because in a homogeneous gravitational field, as with regard to our system *K'*, all bodies fall with the same acceleration independent of their physical nature. I call "principle of equivalence" the assumption that *K'* can be treated with all rigor as being at rest, such that no law of nature fails to be satisfied relative to *K'*.

continued…

Albert Einstein, "On Friedrich Kottler's Paper: 'On Einstein's Equivalence Hypothesis and Gravitation' " (1916)



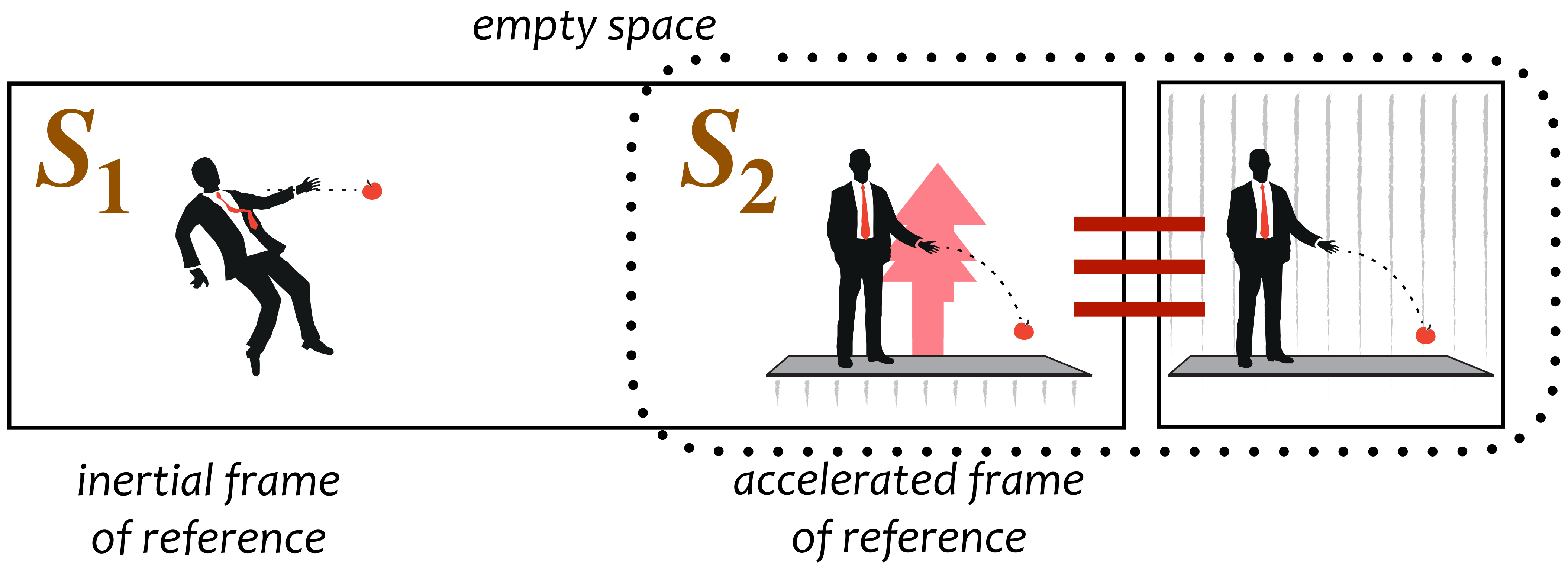
Einstein's 1916 reply to Friedrich Kottler (who proposed a "kinematic, nondynamic interpretation of gravitation") is a concise summary of the *Äquivalenzprinzip* argument.

…continued

3. The Gravitational Field Not Only Kinematically Caused. The previous consideration can also be inverted. Let the system *K'* with the gravitational field, which we considered above, be the original system. One can then introduce a new system *K* that is accelerated with respect to *K'* such that (isolated) masses move uniformly in straight lines (within the domain of consideration). But one must *not* go beyond this and say: If *K'* is a reference system with an *arbitrary* gravitational field, then one can always find a system *K* relative to which isolated masses move uniformly in straight lines, i.e., relative to which no gravitational field exists. The absurdity of such a hypothesis is plainly obvious. If the gravitational field relative to *K'* is, for example, that of a mass point at rest, then not even the most refined trick of transformation can transform the field away in the entire neighborhood of the mass point. Therefore, one may never maintain that a gravitational field could be explained, so to speak, by pure kinematics; a "kinematic, nondynamic interpretation of gravitation" is not possible. By mere transformation from a Galilean system into another one by means of an acceleration transformation, we do not learn about *arbitrary* gravitational fields but only some of a very special kind; but these too must — of course — obey the same laws as all other fields of gravitation. This is again just another formulation of the principle of equivalence (specialized in its application to gravitation).

Albert Einstein, "On Friedrich Kottler's Paper: 'On Einstein's Equivalence Hypothesis and Gravitation' " (1916)

In one of his last texts on GR from 1952, Einstein again presents the basic idea of the *Äquivalenzprinzip* in a way which can be illustrated like this:

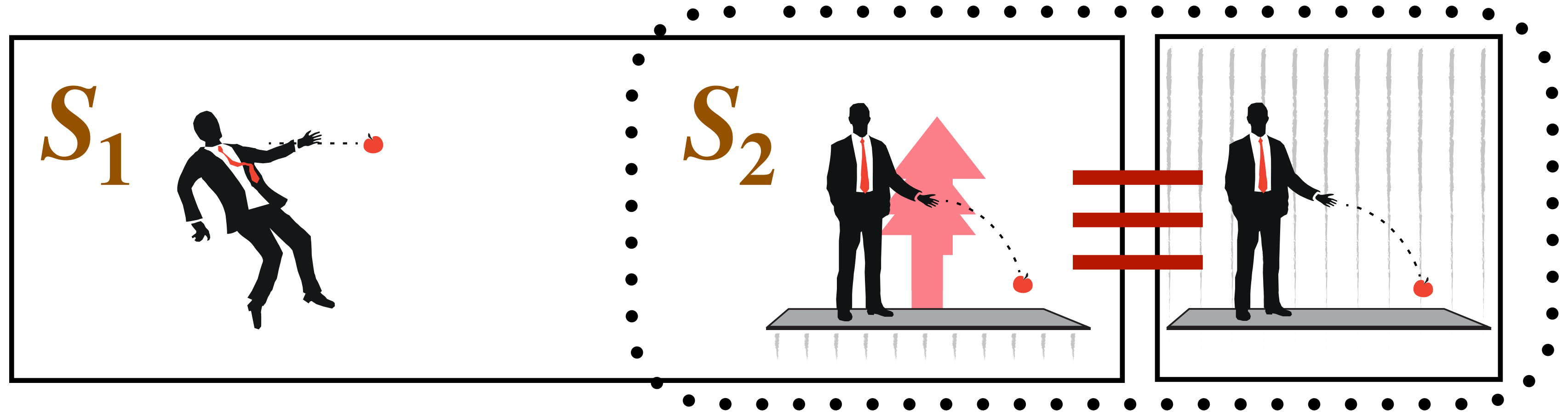


> This theory arose primarily from the endeavour to understand the equality of inertial and gravitational mass. We start out from an inertial system S_1, whose space is, from the physical point of view, empty. In other words, there exists in the part of space contemplated neither matter (in the usual sense) nor a field (in the sense of the special theory of relativity).

continued…

Albert Einstein, "Relativity and the Problem of Space" (1952)

In one of his last texts on GR from 1952, Einstein again presents the basic idea of the *Äquivalenzprinzip* in a way which can be illustrated like this:



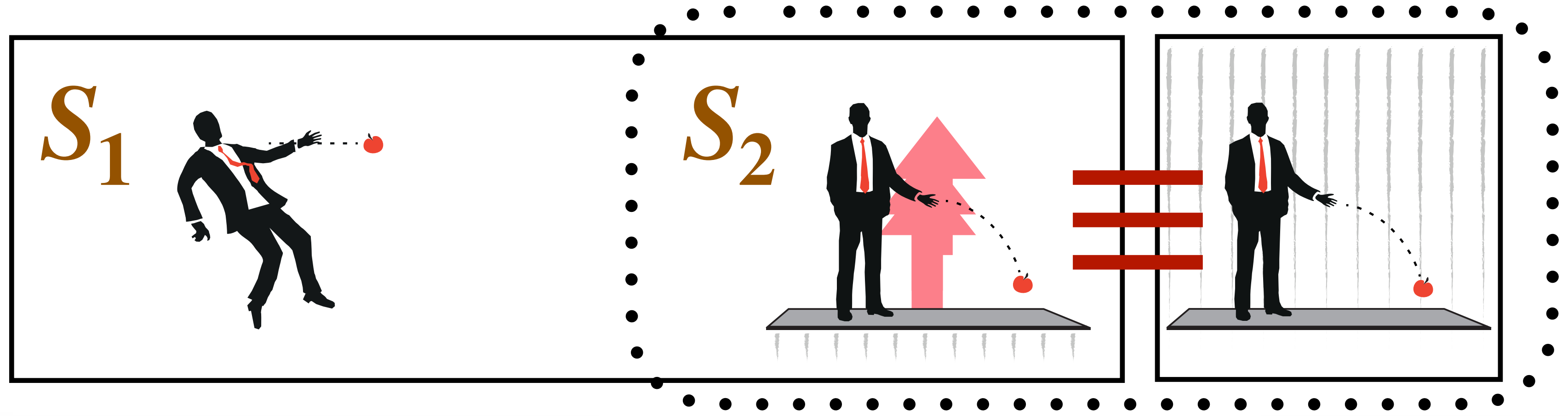
…continued

> With reference to S_1 let there be a second system of reference S_2 in uniform acceleration. Then S_2 is thus not an inertial system. With respect to S_2 every test mass would move with an acceleration, which is independent of its physical and chemical nature. Relative to S_2, therefore, there exists a state which, at least to a first approximation, cannot be distinguished from a gravitational field. The following concept is thus compatible with the observable facts: S_2 is also equivalent to an "inertial system"; but with respect to S_2 a (homogeneous) gravitational field is present (about the origin of which one does not worry in this connection).

continued…

Albert Einstein, "Relativity and the Problem of Space" (1952)

In one of his last texts on GR from 1952, Einstein again presents the basic idea of the *Äquivalenzprinzip* in a way which can be illustrated like this:



…continued

Thus when the gravitational field is included in the framework of the consideration, the inertial system loses its objective significance, assuming that this "principle of equivalence" can be extended to any relative motion whatsoever of the systems of reference. If it is possible to base a consistent theory on these fundamental ideas, it will satisfy of itself the fact of the equality of inertial and gravitational mass, which is strongly confirmed empirically.

Albert Einstein, "Relativity and the Problem of Space" (1952)

EINSTEIN QUOTES

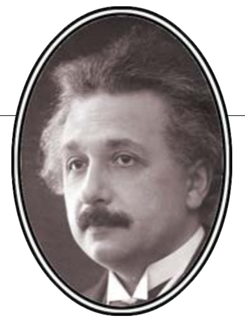
A selection of further statements of the *Äquivalenzprinzip:*

> In a homogeneous gravitational field all motions take place in the same way as in the absence of a gravitational field in relation to a uniformly accelerated coordinate system. If this principle held good for any events whatever (the "principle of equivalence") […] we were to reach a natural theory of the gravitational fields.

Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)

> An inertial space without gravitational field is physically equivalent to a uniformly accelerated space, in which there is a (homogeneous) gravitational field. (Equivalence hypothesis.)

Albert Einstein, letter to Jean Becquerel, 16 August 1951



Einstein's "happiest thought" (once again):

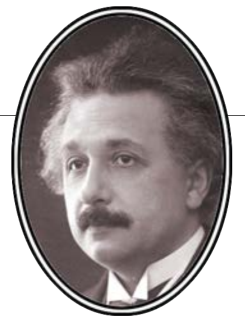
In 1907 [...] I realized that, while all other laws of nature could be discussed in terms of the special theory of relativity, the theory could not be applied to the law of universal gravitation. [...]

I was sitting in a chair in the Patent Office in Bern when all of a sudden I was struck by a thought: "If a person falls freely, he will certainly not feel his own weight."

I was startled. This simple thought made a really deep impression on me. My excitement motivated me to develop a new theory of gravitation. My next thought was: "When a person falls, he is accelerating. His observations are nothing but observations in an accelerated system." Thus, I decided to generalize the theory of relativity from systems moving with constant velocity to accelerated systems. I expected that this generalization would also allow me to solve the problem of gravitation. [...]

I could not immediately solve the problem completely on the basis of this insight. It would take me eight more years to find the correct relationship.

Albert Einstein, "How I Created the Theory of Relativity ",
Jun Ishiwara's Notes of Einstein's Lecture at Kyoto University (1922)

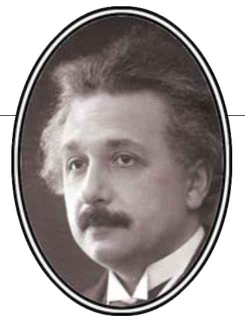


EINSTEIN QUOTES

Another recollection of Einstein's "happiest thought":

> At that moment I got the happiest thought of my life in the following form: [...] for an observer in free-fall from the roof of a house there is during the fall — at least in his immediate vicinity — no gravitational field. Namely, if the observer lets go of any bodies, they remain relative to him in a state of rest or uniform motion, independent of their special chemical or physical nature. The observer, therefore, is justified in interpreting his state as being "at rest."

Albert Einstein, Unpublished manuscript, in all probability a draft for an article in *Nature* that was never published (c.1920)



EINSTEIN QUOTES

Einstein on the special-relativistic limit of general relativity:

> This led me to the view that the customary theory of relativity provides only an approximation to reality; it should apply only in the limit case where differences in the gravitational potential in the space-time region under consideration are not too great.

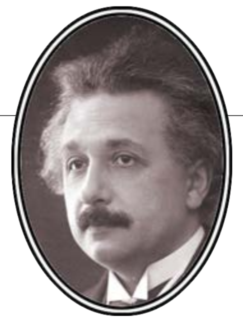
Albert Einstein, Marcel Grossmann, "Outline of a Generalized Theory of Relativity and of a Theory of Gravitation" (1913)

EINSTEIN QUOTES

Arbitrary gravitational fields cannot be generated by acceleration.

> The equivalence principle does not assert that every gravitational field (e.g. the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field.

Albert Einstein, Letter to A. Rehtz, 12 July 1953

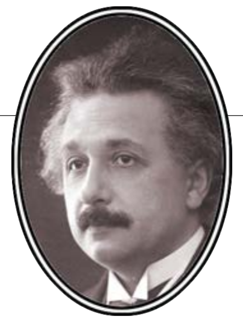


EINSTEIN QUOTES

On the impossibility to transform away arbitrary gravitational fields in finite regions, and on the recovery of special relativity in infinitesimal regions.

> However, one must be careful not to assume that, conversely, every gravitational field can be made to vanish, *i.e.*, can be turned into a gravitation-free region, by means of a suitable choice of the coordinate system. For example, it is impossible to make the gravitational field of the Earth vanish by means of a suitable choice of the coordinate system. In fact, for a region of *finite extension* this is only possible with gravitational fields of a very special kind. But for an infinitely small region the coordinates can always be chosen such that no gravitational field will be present in it. With respect to such an infinitely small region one may then assume that the special theory of relativity is valid. That way the general theory of relativity is connected with the special theory of relativity, and the results of the latter can be utilized for the former.

Albert Einstein, Theory of Relativity (1924)



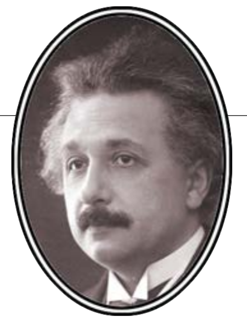
EINSTEIN QUOTES

Einstein often found it necessary to reply to critics of GR and overenthusiastic followers alike, who hadn’t quite understood the theory they were criticising or promoting. Here is a quote from a reply to a paper by Ernst Reichenbächer, relevant to the observation that arbitrary gravitational fields cannot be transformed away in extended spacetime regions:

> I now turn to the objections against the relativistic theory of the gravitational field. Here, Herr Reichenbächer first of all forgets the decisive argument, namely, that the numerical equality of inertial and gravitational mass must be traced to an equality of essence. [Footnote: Instead of “Since gravitation... shows its effect in acceleration,” Herr Reichenbächer should have said, “Since the gravitational acceleration is independent of the material and the state of the body influenced by the gravitational force.” The latter property only, and alone, distinguishes the gravitational field from the other fields of force.] It is well known that the principle of equivalence accomplishes just that. He (like Herr Kottler) raises the objection against the principle of equivalence that gravitational fields for finite space-time domains in general cannot be transformed away. He fails to see that this is of no importance whatsoever.

continued...

Albert Einstein, “Response to Ernst Reichenbächer, ‘To What Extent Can Modern Gravitational Theory Be Established Without Relativity?’ " (1920)



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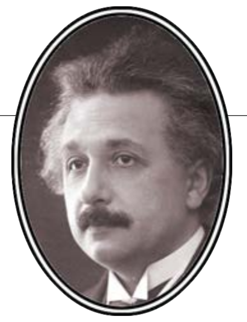
…continued

> What is important is only that one is justified at any instant and at will (depending upon the choice of a system of reference) to explain the mechanical behavior of a material point either by gravitation or by inertia. More is not needed; to achieve the essential equivalence of inertia and gravitation it is not necessary that the mechanical behavior of two or more masses must be explainable as a mere effect of inertia by the same choice of coordinates. After all, nobody denies, for example, that the theory of special relativity does justice to the nature of uniform motion, even though it cannot transform all acceleration-free bodies together to a state of rest by one and the same choice of coordinates.

Albert Einstein, “Response to Ernst Reichenbächer, ‘To What Extent Can Modern Gravitational Theory Be Established Without Relativity?’ " (1920)



EINSTEIN QUOTES

The “Einstein Box” [*in German: “Kasten”*]…

Two physicists, A and B, awake from a narcotic sleep and notice that they are in a closed box that has nontransparent walls and is equipped with all their instruments. They have no idea where the box is situated and whether it moves...

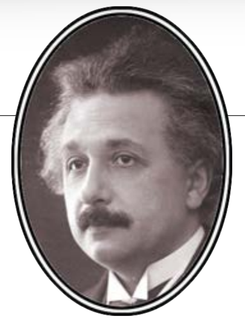
Albert Einstein, “On the Present State of the Problem of Gravitation” (1913)

An observer enclosed in a box can in no way decide whether the box is at rest in a static gravitational field, or whether it is in accelerated motion, maintained by forces acting on the box, in a space that is free of gravitational fields.

A. Einstein, M. Grossmann, “Outline of a Generalized Theory of Relativity and of a Theory of Gravitation” (1913)

Let us imagine a spacious chest resembling a room with an observer inside who is equipped with apparatus […] To the middle of the lid of the chest is fixed externally a hook with rope attached, and now a ‘being’ (what kind of a being is immaterial to us) begins pulling at this with a constant force...

Albert Einstein, “Relativity. The Special and the General Theory. A Popular Exposition” (1917)



EINSTEIN QUOTES

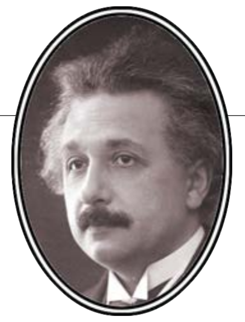
…proportionality of inertial and gravitational mass…

The principle of the equality of inertial and gravitational mass could now be formulated quite clearly as follows: In a homogeneous gravitational field all motions take place in the same way as in the absence of a gravitational field in relation to a uniformly accelerated coordinate system…

Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)

…If this principle held good for any events whatever (the "principle of equivalence"), this was an indication that the principle of relativity needed to be extended to coordinate systems in non-uniform motion with respect to each other, if we were to reach a natural theory of the gravitational fields.

Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)



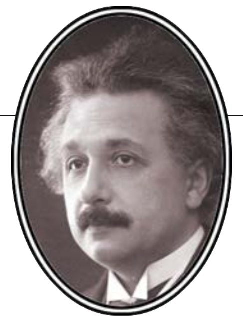
On of the main motivations for Einstein to look for a relativistic theory of gravity was to explain **the observed equality of inertial and gravitational mass,** unexplained by Newtonian theory.

> The theory of general relativity owes its origin primarily to the experimental fact of the numerical equality of inertial and gravitational mass of a body, a fundamental fact for which classical mechanics has given no interpretation.

Albert Einstein, King's College Lecture (1921)

> This experience of the identical falling of all bodies in the gravitational field is one of the most universal experiences that the observation of nature has yielded to us; nevertheless, this law has not been granted a place in the foundation of our physical edifice.

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)

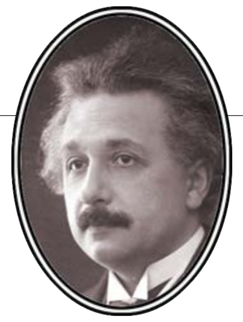


EINSTEIN QUOTES

…proportionality of inertial and gravitational mass…

> The theory expounded in what follows derives from the conviction that the proportionality between the inertial and the gravitational mass of bodies is an exactly valid law of nature that must already find expression in the very foundation of theoretical physics.

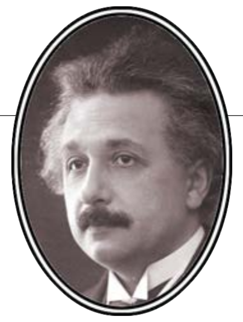
Albert Einstein, Marcel Grossmann, "Outline of a Generalized Theory of Relativity and of a Theory of Gravitation" (1913)



prop of mi mg

> [...] the old experimental fact that all bodies have the same acceleration in a gravitational field. This law [...]

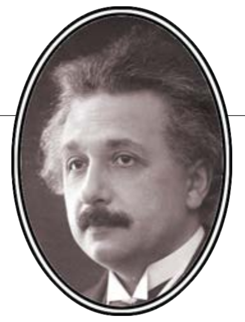
Albert Einstein, "Notes on the Origin of the General Theory of Relativity", lecture manuscript (1933)



…proportionality of inertial and gravitational mass…

[…] This did not fit in with the old experimental fact that all bodies have the same acceleration in a gravitational field. This law, which may also be formulated as the law of the equality of inertial and gravitational mass, was now brought home to me in all its significance. I was in the highest degree amazed at its existence and guessed that in it must lie the key to a deeper understanding of inertia and gravitation. […] I now abandoned as inadequate the attempt to treat the problem of [relativistic] gravitation within the framework of the special theory of relativity. It clearly failed to do justice to the most fundamental property of gravitation.

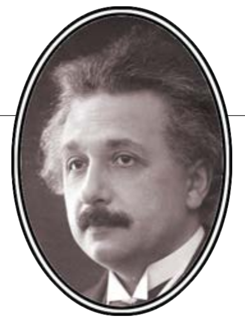
Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)



no infintesimal geodesics

The derivation of the law of motion of a point [*in the manuscript of your paper*] assumes that, seen from the local coordinate system, the point moves in a straight line. Nothing can be derived from this, however. The local coordinate system is generally of importance only at the infinitesimal level, and at the infinitesimal level every uninterrupted line is straight. [...] That the point's world line is a geodetic line otherwise as well (if others do not exert gravitational influence) is a hypothesis, albeit one that suggests itself.

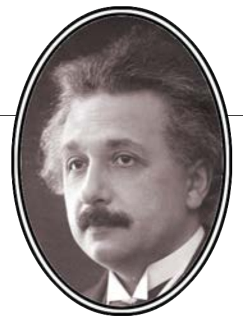
Albert Einstein, Letter to Moritz Schlick, 21 March 1917



no space without grav.field

> There is no space without gravitational or inertial field. What one calls empty space in the sense of classical or Maxwell's theory, is a gravitational field of a special kind, that is one in which the gravitational potentials are constant with an appropriate choice of coordinates.

Albert Einstein, Letter to H. Titze, 16 January 1954



ÄP

> The "acceleration field" is a special case of the gravitational field.

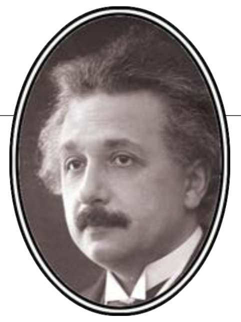
Albert Einstein, “The speed of light and the statics of the gravitational field” (1912)

EINSTEIN QUOTES

ÄP

> … the theory rests on the conviction that acceleration of a reference system is equivalent to a gravitational field.

Albert Einstein, Letter to Hendrik A. Lorenz, 14 August 1913

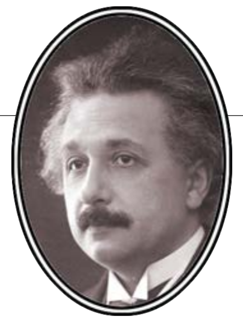


EINSTEIN QUOTES

wesensgleich

Principle of Equivalence: Inertia and gravitation are phenomena identical in nature [“*wesensgleich*”]. From this and from the special theory of relativity it follows necessarily that the symmetric “fundamental tensor” ($g_{\mu\nu}$) determines the metric properties of space, the inertial behaviour of bodies in this space, as well as the gravitational effects. We shall call the state of space which is described by this fundamental tensor the “G-field”.

Albert Einstein, “On the Foundations of the General Theory of Relativity” (1918)



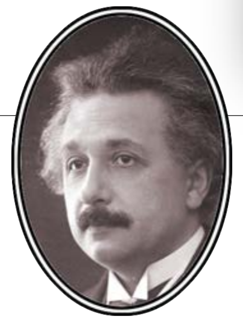
wesensgleich

> ...the equivalence of the inertial and gravitational mass is to be traced to **the essential identity [*Wesensgleichheit*]** of these two elementary qualities of matter and energy...

Albert Einstein, "Relativity and gravitation. Reply to a comment by M. Abraham" (1912)

> [...] the gravitational and inertial masses of a body are numerically equal to each other. **This numerical equality suggests identity in character [*Wesensgleichheit*].**
> Can gravitation and inertia be identical? This question leads directly to the General Theory of Relativity.

Albert Einstein, "A brief outline of the development of the theory of relativity"

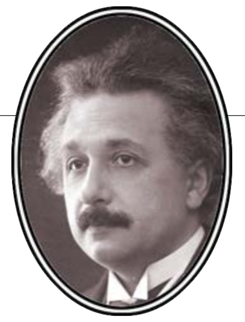


EINSTEIN QUOTES

wesensgleich

...the equivalence of the inertial and gravitational mass is to be traced to **the essential identity [*Wesensgleichheit*]** of these two elementary qualities of matter and energy...

Albert Einstein, "Relativity and gravitation. Reply to a comment by M. Abraham" (1912)



wesensgleichheit

It must be a task of the immediate future to create a relativity-theoretical scheme in which the equivalence of inertial and gravitational mass finds expression. I sought to make a first, quite modest contribution to the attainment of this goal in my papers on the static gravitational field. There I started from the most obvious conception that the equivalence of the inertial and gravitational mass is to be traced to the essential identity of these two elementary qualities of matter and energy by conceiving the static gravitational field as physically identical with an acceleration of the reference system.

Albert Einstein, "Relativity and gravitation. Reply to a comment by M. Abraham" (1912)

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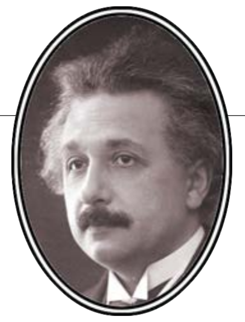
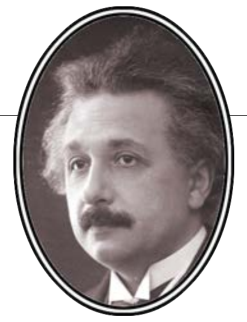


wesensgleichheit

Furthermore, it was soon found impossible to link up the science of gravitation with the special theory of relativity in a natural manner. In this connection I was struck by the fact that the force of gravitation possesses a fundamental property, which distinguishes if from electro-magnetic forces. All bodies fall in a gravitational field with the same acceleration, or — what is only another formulation of the same fact — the gravitational and inertial masses of a body are numerically equal to each other. This numerical equality suggests identity in character. Can gravitation and inertia be identical? This question leads directly to the General Theory of Relativity.

Albert Einstein, "A brief outline of the development of the theory of relativity" (*Nature*, 13 February 1921)



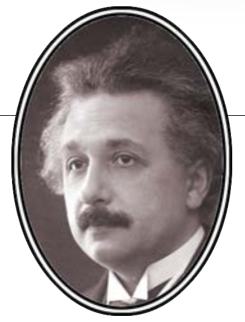
EINSTEIN QUOTES

Ontological unification

If we speak about a unified theory we have two possible points of view [...]:

(1) That the field appear as a unified covariant entity. As an example I cite the unification of the electric and magnetic fields by the special theory of relativity. The unification here consists in this that the entire field considered is described as a skew-symmetric tensor. The basic group of Lorentz transformations does not enable us to split this field independently of the system of coordinates, into an electric and a magnetic one. [...]

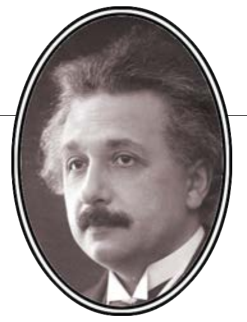
Albert Einstein, "A generalization of the relativistic theory of gravitation" (1945)



Ontological unification

According to the theory of special relativity, **the four-dimensional continuum**, formed by the union of space and time, retains **that kind of absolute character** that in previous theories space and time held separately (Minkowski).

Albert Einstein, Typescript of a lecture held at King's College London on 13 June 1921



EINSTEIN QUOTES

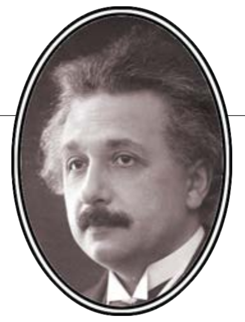
Ontological unification

> According to [*the equations for the Lorentz transformation of E and B*], electric and magnetic field strengths do not have an existence per se, since it may depend on the choice of the coordinate system whether an electric or magnetic field strength is or is not present at a location.

Albert Einstein, "On the relativity principle and the conclusions drawn from it" (1907)

> [...] only the electric and magnetic fields combined, aside from the state of motion of the observer or coordinate system, could be granted a kind of objective reality.

Albert Einstein, "Fundamental Ideas and Methods of the Theory of Relativity, Presented in Its Development", Manuscript for an article in Nature (c.1920)



EINSTEIN QUOTES

XXX

The following idea on Faraday's magneto-electric induction – so far not mentioned – played a leading role for me when I established the
relativity.

According to Faraday, during the movement of a magnet relative to an electric circuit, an electric current is induced in the latter. It is irrel
magnet or the conductor is moved; what counts only is the relative motion. But according to the Maxwell-Lorentz theory, the theoretica
phenomenon is very different for the two cases:

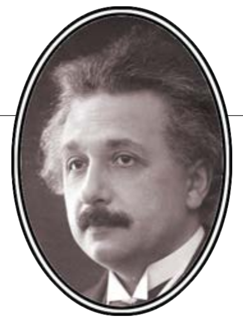
If the magnet is moved, there exists in space a magnetic field variable with time, which, according to Maxwell, forms closed lines of an el
physically real electric field; this electric field then puts the movable electric masses in the conductor into motion. However, no electric fi
at rest and the circuit is moved; instead, the current in the conductor is created because the electricities moving with it due to the (mech
movement relative to the magnetic field suffer an electromotive force, which Lorentz introduced hypothetically.

The idea that these two cases should essentially be different was unbearable to me. According to my conviction, the difference betwee
the choice of the point of view, but not in a real difference in the reality of nature. **As seen from the magnet, there was certainly no elec
from the circuit there certainly was an electric field. Therefore, the existence of the electric field was a relative one, depending on the
coordinate system used; and only the electric and magnetic fields combined, aside from the state of motion of the observer or coordin
granted a kind of objective reality.** This phenomenon of magneto-electric induction forced me to postulate the principle of (special) rela

(1920 Nature manuscript)

(See also 1914 overview of Entwurf)

Albert Einstein, "Fundamental Ideas and Methods of the Theory of Relativity, Presented in Its Development", Manuscript for an article in Nature (c.1920)



EINSTEIN QUOTES

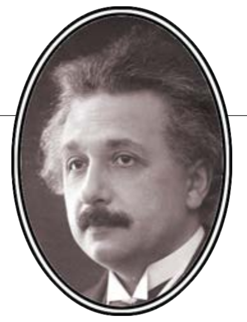
XXX

Following the spirit of the equivalence principle, we will have to assume that the motion of a material point, subject only to inertia **and gravity**, is described by the equation

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0$$

Indeed, this equation becomes that of a straight line if the components of [*the* $\Gamma^\mu{}_{\alpha\beta}$] vanish.

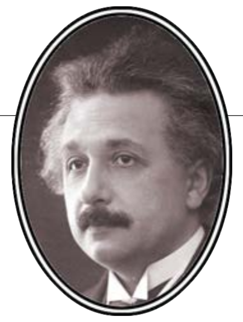
Albert Einstein, "Four lectures on the theory of relativity" (Princeton lectures, 1921)



the naming of parts

> The naming of the parts, which I have introduced, is in principle meaningless and only meant to **appeal to our physical habit of thinking** [...] and **in order to maintain the continuity of ideas**.

Albert Einstein, “On Friedrich Kottler's Paper: ‘On Einstein's Equivalence Hypothesis and Gravitation’ ” (1916)



EINSTEIN QUOTES

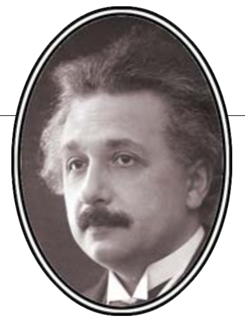
gr not geometrising gravity

I cannot admit that the assertion that the theory of relativity traces physics back to geometry has a clear meaning.

[…] this is by no means a justification for denoting as "geometry" every area of study in which this formal structure [*the metric*] plays a role, not even if for the sake of illustration one makes use of notions which one knows from geometry.

Using a similar reasoning Maxwell and Hertz could have denoted the electromagnetic equations of the vacuum as "geometrical" because the geometrical concept of a vector occurs in these equations.

Albert Einstein, Review of Emile Meyerson's book "*La deduction relativiste*" (1927)



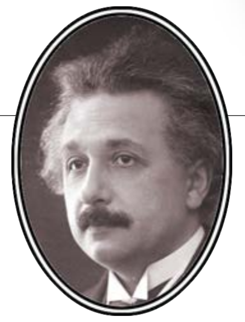
EINSTEIN QUOTES

Riem=0, not vanishing of grav field

It is true that in that case the [*components of the curvature tensor*] R_{iklm} vanish, so that one could say: “There is no gravitational field present”. However, what characterizes the existence of a gravitational field from the empirical standpoint is the non-vanishing of the [*components of the affine connection*] Γ^{l}_{ik}, not the non-vanishing of the R_{iklm}.

If one does not think intuitively [*anschaulich*] in such a way, one cannot grasp why something like a curvature should have any thing to do with gravitation. **In any case, no reasonable person would have hit upon such a thing**. The key for the understanding of the equality of inertial and gravitational mass is missing.

Albert Einstein, Letter to Max v. Laue, 12 September 1950

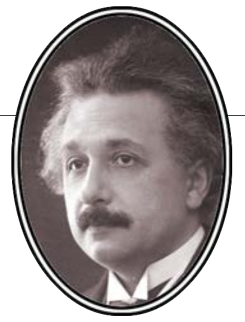


EINSTEIN QUOTES

On attempts to bootstrap the full non-linear theory of general relativity from linearised GR:

> Contemporary physicists do not see that **it is hopeless to take a theory that is based on an independent rigid space** (Lorentz-invariance) **and later hope to make it general relativistic** (in some natural way).

Albert Einstein, Letter to Georg Jaffe, 19 January 1954

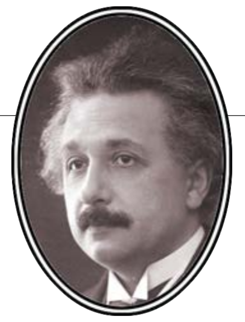


removing privileged motion

You consider the transition to special relativity as **the most essential thought of relativity,** not the transition to general relativity.

I consider the reverse to be correct. **I see the most essential thing in the overcoming of the inertial system,** a thing that acts upon all processes but undergoes no reaction.

Albert Einstein, Letter to Georg Jaffe (1954)



EINSTEIN QUOTES

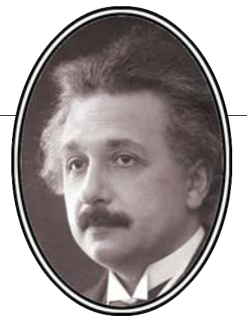
removing privileged motion

In classical mechanics, and no less in the special theory of relativity, there is **an inherent epistemological defect …**

Albert Einstein, "The Foundation of the General Theory of Relativity" (1916)

… no person whose mode of thought is logical can rest satisfied with this state of affairs. He asks: 'How does it come that certain reference-bodies … are given priority over other reference-bodies … ? **What is the reason for this Preference**?'

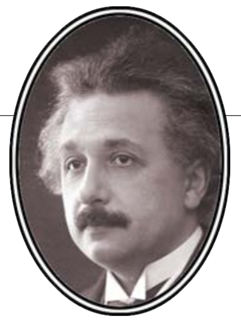
Albert Einstein, "Relativity. The Special and the General Theory. A Popular Exposition" (1917)



generalised action-reaction

It is contrary to the scientific mode of understanding to postulate a thing (the space-time continuum) **that acts, but which cannot be acted upon.**

Albert Einstein, "Four lectures on the theory of relativity" (Princeton lectures, 1921)

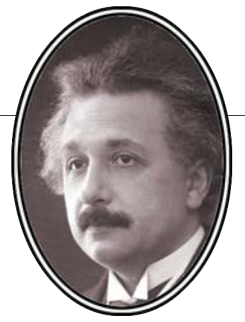


EINSTEIN QUOTES

UFF

> If the observer lets go of any bodies, they remain relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature...

Albert Einstein, xxx (1954)



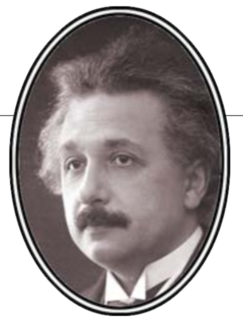
EINSTEIN QUOTES

Footnote: For a brief period in 1912, Einstein … worried that ÄP only valid in inf. regions… since the field equations for his theory of the static gravitational field did not the principle of equality of action and reaction (kolla)

In paper 1912d:

> Thus, it seems that the only way to avoid a contradiction with the reaction principle is to replace equations (3) and (3a) with other equations homogeneous in c for which the reaction principle is satisfied when the force postulate (4) is applied. I hesitate to take this step because by doing so I am leaving the territory of the unconditional equivalence principle. It seems that the latter can be maintained for inifinitely small fields only. Our derivations of the equations of motion of the material point and of the electromagnetic equations do not thereby become illusory, because they apply equations (2) only to infinitesimally small spaces.

Albert Einstein, "On the Theory of the Static Gravitational Field" (1912)

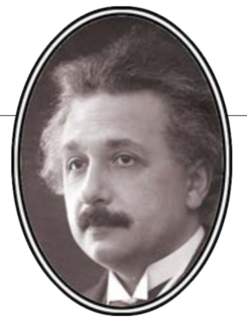


Footnote: For a brief period in 1912, Einstein … worried that ÄP only valid in inf. regions… since the field equations for his theory of the static gravitational field did not the principle of equality of action and reaction (kolla)

In paper 1912d, and here in a letter to Ehrenfest:

I still do not understand the theorem on gravitational fields that you are telling me about. You must explain it to me more clearly. To make this easier for you, I am sending you my papers on gravitation, the latest of which you do not have. According to it, it appears that the equivalence principle can be valid only for infinitely small fields, and that, therefore, Born's accelerated finite system cannot be considered a static gravitational field, i.e., cannot be generated by masses at rest. A rotating ring does not generate a static field in this sense, even though it is a temporally invariant field. The reversibility of light paths will not hold in such a field. My case corresponds to the electrostatic field in the theory of electricity, whereas the more general static case would also include the analogue of the static magnetic field. I haven't got as far as that yet. The equations I found apply only to the static case of masses at rest. Born's field of finite extension does not fall into this category. It has not yet become clear to me why the equivalence principle fails for finite fields (Born).

Albert Einstein, Letter to Paul Ehrenfest, June 1912

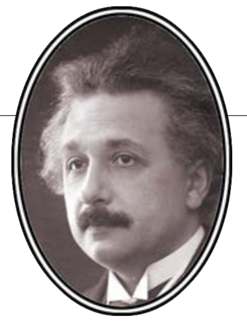


EINSTEIN QUOTES

For a brief period in 1912, Einstein … worried that ÄP only valid in inf. regions…

It must be a task of the immediate future to create a relativity-theoretical scheme in which the equivalence of inertial and gravitational mass finds expression. I sought to make a first, quite modest contribution to the attainment of this goal in my papers on the static gravitational field. There I started from the most obvious conception that the equivalence of the inertial and gravitational mass is to be traced to the essential identity of these two elementary qualities of matter and energy by conceiving the static gravitational field as physically identical with an acceleration of the reference system. I have to admit that I was able to carry through this conception in a consistent way only for inifinitely small spaces, and that I cannot give any satisfactory reason for that fact. But I do not see this as any reason to reject the equivalence principle for the infinitely small as well; no one can deny that this principle is a natural extrapolation of one of the most general empirical laws of physics.

Albert Einstein, "Relativity and gravitation. Reply to a comment by M. Abraham" (1912)



EINSTEIN QUOTES

The problem that suggested that ÄP could only be valid in inf.regions was a defect of the preliminary theory of the static gravitational field. With the development of the full theory th problem went away and trust in the ÄP could be restored.

This has not prevented some commentators to interpret Einstein's statements from 1912 as a precursor to the infinitesimal EP (or SEP).

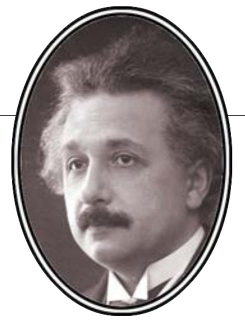
Here is a quote to that effect from Einstein biographer Abraham Pais:

It had been a grave decision to make this last modification of the c-field equation, Einstein wrote, "since [as a result] I depart from the foundation of the unconditional equivalence principle." [...] "It seems that [the equivalence principle] holds only for infinitely small fields...".

This is the dawn of the correct formulation of equivalence as a principle that holds only locally.

Abraham Pais, "Subtle is the Lord. The Science and the Life of Albert Einstein" (198

A beautiful example of the art of cherry-picking historical quotes.

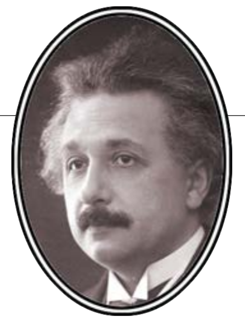


EINSTEIN QUOTES

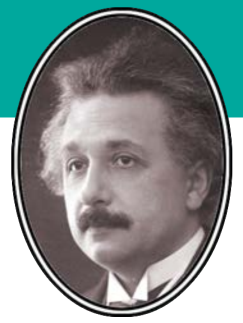
"… aber leichter ist ahnen als finden."

Albert Einstein, letter to Rudolf Förster,
16 November 1917

"… but premonition is easier than discovery."



BRIEF SUMMARY OF GENERAL RELATIVITY

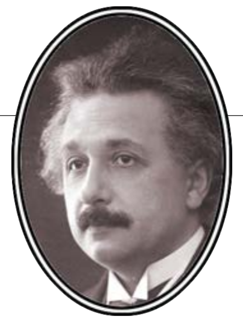


SUMMARY OF GENERAL RELATIVITY

If you are curious about General Relativity, here is
a short but comprehensive summary of the theory:

“General Relativity —
A reconciliation between ***gravitational structure*** *and* ***chronogeometry***
which excludes any ***prior geometry.”***

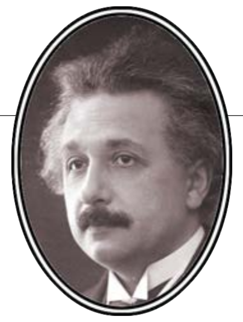
Jürgen Renn, Tilman Sauer, “Heuristics and Mathematical Representation in Einstein’s Search for a Gravitational Field Equation” (1999)



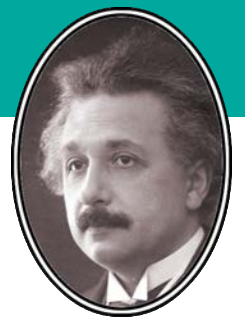
For completeness, here is the original quote for the snappy characterisation of general relativity:

> In spite of the mathematical and conceptual difficulties that he encountered, [Einstein's] heuristic principles guided **a reconciliation between gravito-inertial structure and chronogeometry which excluded any prior geometry**, thus effectively determining the characteristic features of general relativity, even as we understand it today.

Jürgen Renn, Tilman Sauer, "Heuristics and Mathematical Representation in Einstein's Search for a Gravitational Field Equation" (1999)



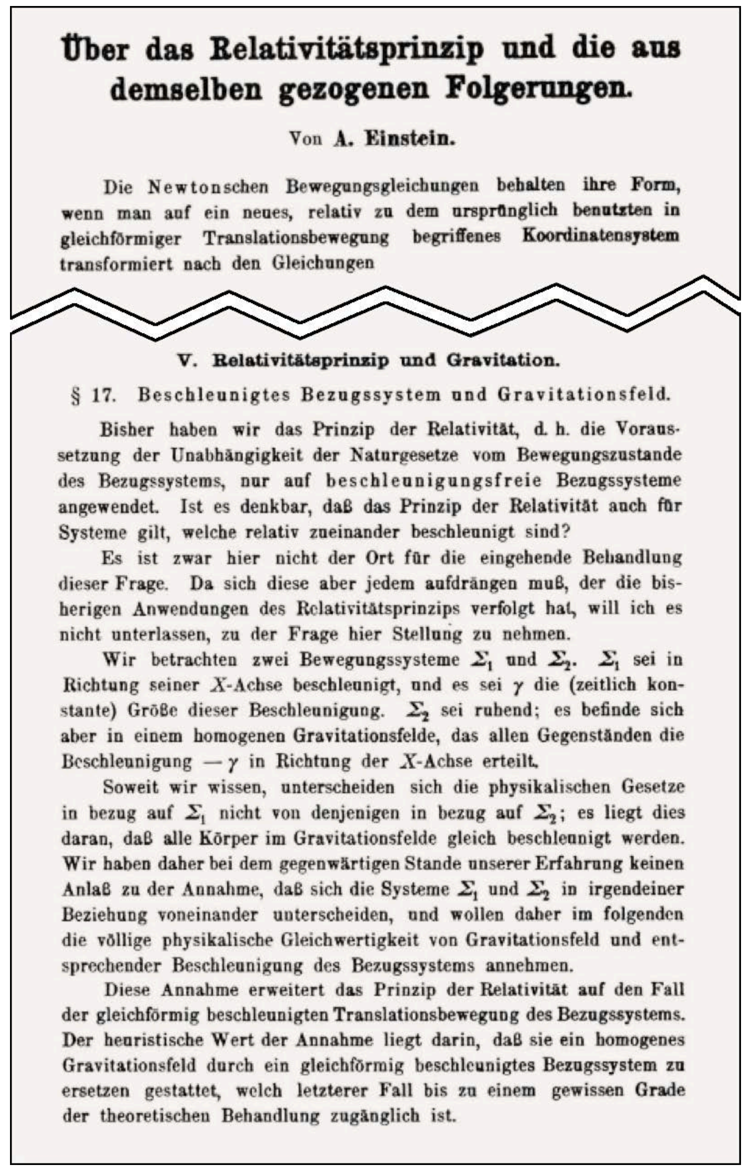
A TIMELINE FOR EINSTEIN’S SEARCH FOR GENERAL RELATIVITY



EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

It took Einstein eight long years to find the theory of general relativity:

Submitted
4 December
1907



Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen.

Von **A. Einstein.**

Die Newtonschen Bewegungsgleichungen behalten ihre Form, wenn man auf ein neues, relativ zu dem ursprünglich benutzten in gleichförmiger Translationsbewegung begriffenes Koordinatensystem transformiert nach den Gleichungen

V. Relativitätsprinzip und Gravitation.

§ 17. Beschleunigtes Bezugssystem und Gravitationsfeld.

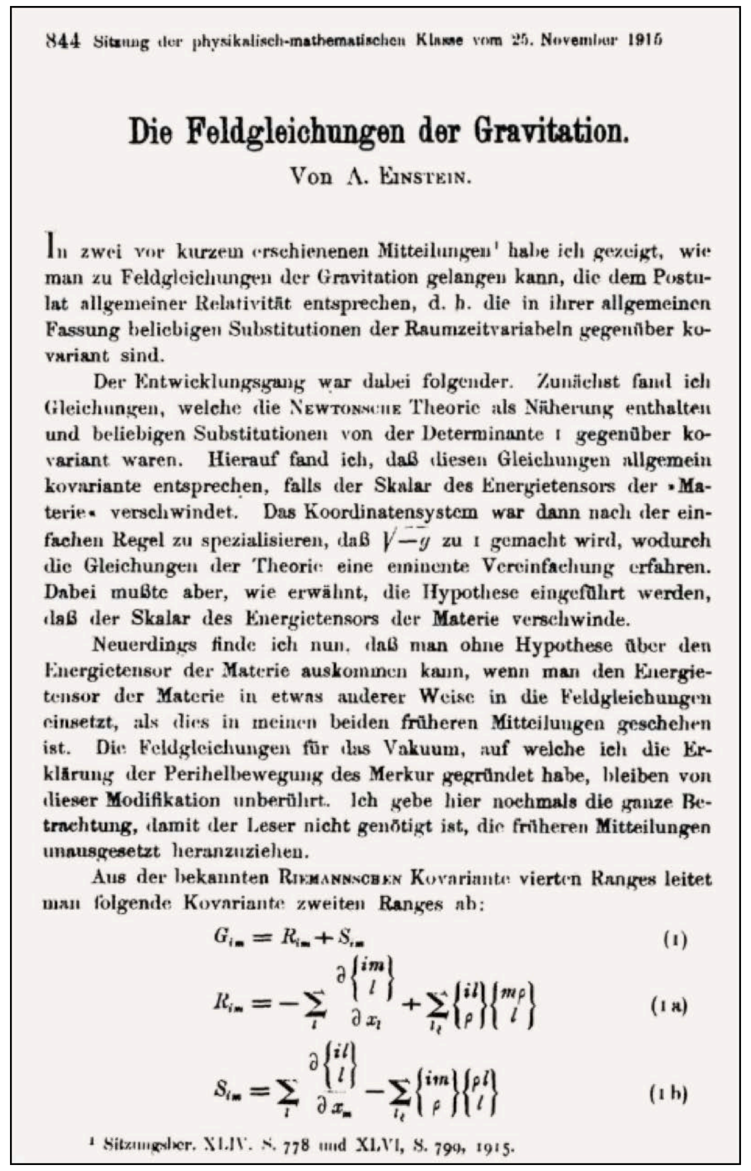
Bisher haben wir das Prinzip der Relativität, d. h. die Voraussetzung der Unabhängigkeit der Naturgesetze vom Bewegungszustande des Bezugssystems, nur auf beschleunigungsfreie Bezugssysteme angewendet. Ist es denkbar, daß das Prinzip der Relativität auch für Systeme gilt, welche relativ zueinander beschleunigt sind?

Es ist zwar hier nicht der Ort für die eingehende Behandlung dieser Frage. Da sich diese aber jedem aufdrängen muß, der die bisherigen Anwendungen des Relativitätsprinzips verfolgt hat, will ich es nicht unterlassen, zu der Frage hier Stellung zu nehmen.

Wir betrachten zwei Bewegungssysteme Σ_1 und Σ_2. Σ_1 sei in Richtung seiner X-Achse beschleunigt, und es sei γ die (zeitlich konstante) Größe dieser Beschleunigung. Σ_2 sei ruhend; es befinde sich aber in einem homogenen Gravitationsfelde, das allen Gegenständen die Beschleunigung $-\gamma$ in Richtung der X-Achse erteilt.

Soweit wir wissen, unterscheiden sich die physikalischen Gesetze in bezug auf Σ_1 nicht von denjenigen in bezug auf Σ_2; es liegt dies daran, daß alle Körper im Gravitationsfelde gleich beschleunigt werden. Wir haben daher bei dem gegenwärtigen Stande unserer Erfahrung keinen Anlaß zu der Annahme, daß sich die Systeme Σ_1 und Σ_2 in irgendeiner Beziehung voneinander unterscheiden, und wollen daher im folgenden die völlige physikalische Gleichwertigkeit von Gravitationsfeld und entsprechender Beschleunigung des Bezugssystems annehmen.

Diese Annahme erweitert das Prinzip der Relativität auf den Fall der gleichförmig beschleunigten Translationsbewegung des Bezugssystems. Der heuristische Wert der Annahme liegt darin, daß sie ein homogenes Gravitationsfeld durch ein gleichförmig beschleunigtes Bezugssystem zu ersetzen gestattet, welch letzterer Fall bis zu einem gewissen Grade der theoretischen Behandlung zugänglich ist.



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \tag{1}$$

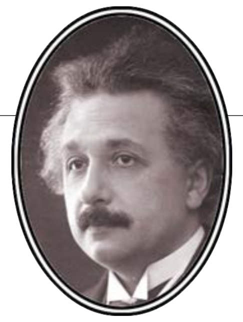
$$R_{im} = -\sum_l \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \tag{1a}$$

$$S_{im} = \sum_l \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \tag{1b}$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915.

Submitted
25 November
1915

1905 **1907** 1909 1911 1913 **1915** 1917 1919 1921 1923



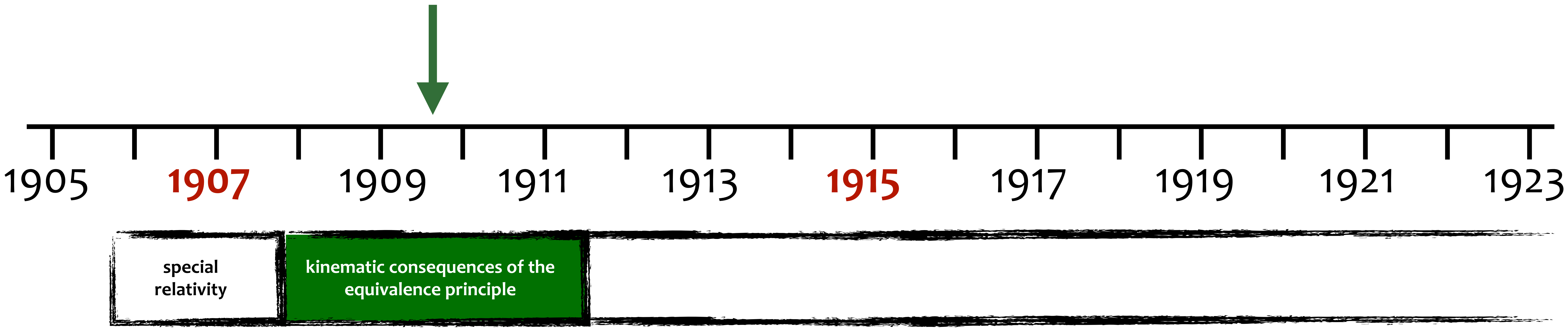
Einstein's Critique of the Equivalence Principle

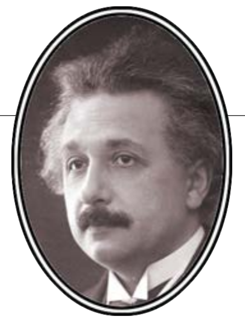
NORDITA

EINSTEIN’S SEARCH FOR GENERAL RELATIVITY

In 1907 Einstein introduced the idea that he later (in 1912) would call the “*Equivalence Principle*”.

Exploring the kinematical consequences of this *heuristic principle* he obtained a rough idea of the properties one should expect in a relativistic theory of gravity — such as light bending and gravitational redshift.

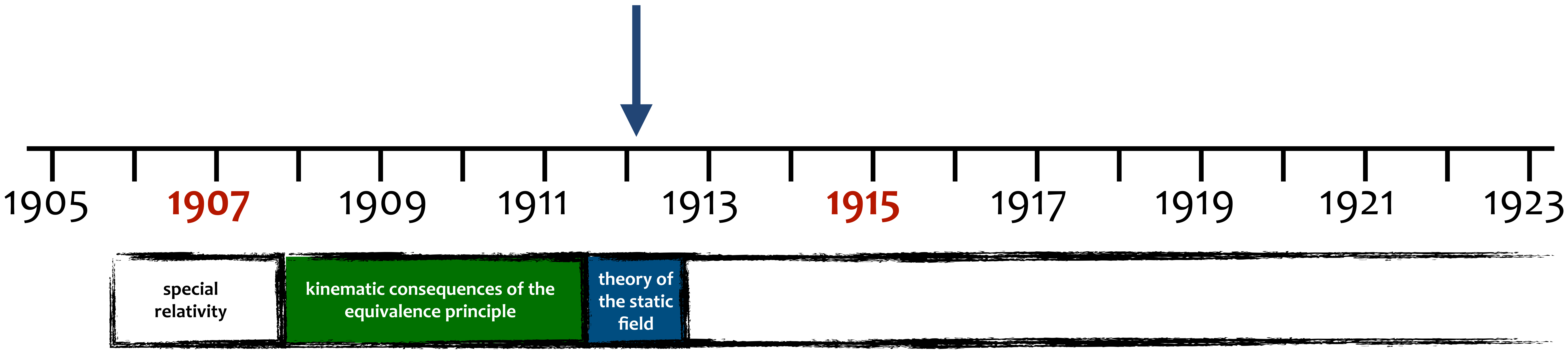


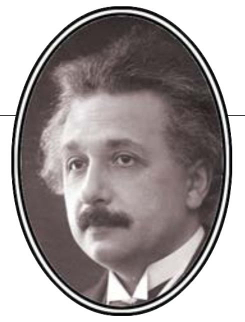


EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

When Einstein got a professorship in physics in Prag 1911–1912, he started to work on the dynamics of a simpler case, static gravitational fields.

In this theory the constant speed of light c of SR became a field in 3-space $c(x, y, z)$ representing the *scalar* gravitational potential.



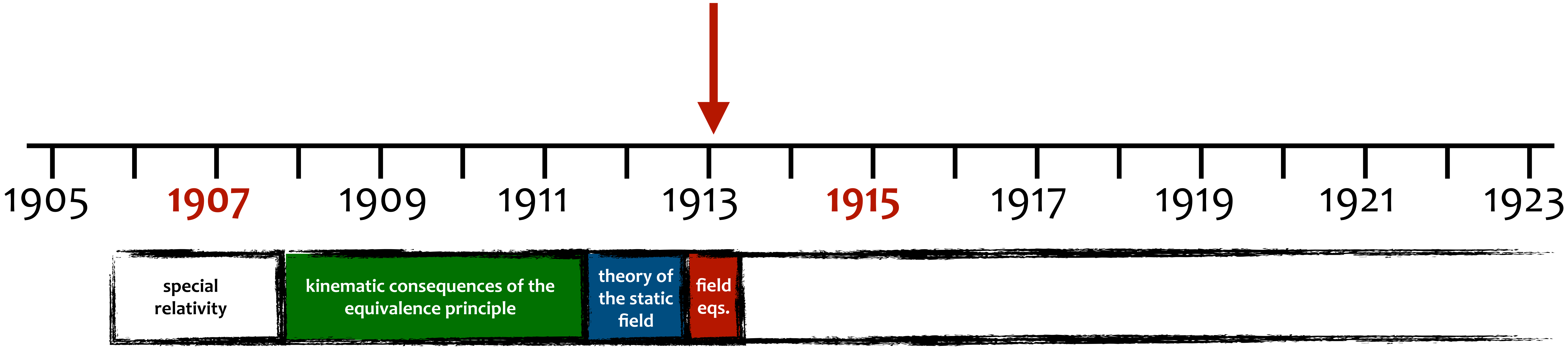


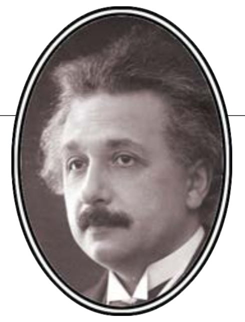
EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

By the time he came back to Zürich in August 1912, Einstein had understood that the full theory of relativistic gravity must be a theory of the spacetime metric, governed by generally covariant field equations sourced by the energy-momentum content of spacetime.

He asked his friend Marcel Grossmann to help him find the appropriate mathematics for such a theory: tensor calculus in a Riemannian geometry.

We can follow Einstein's struggle to find the correct field equations in the "Zürich notebook".



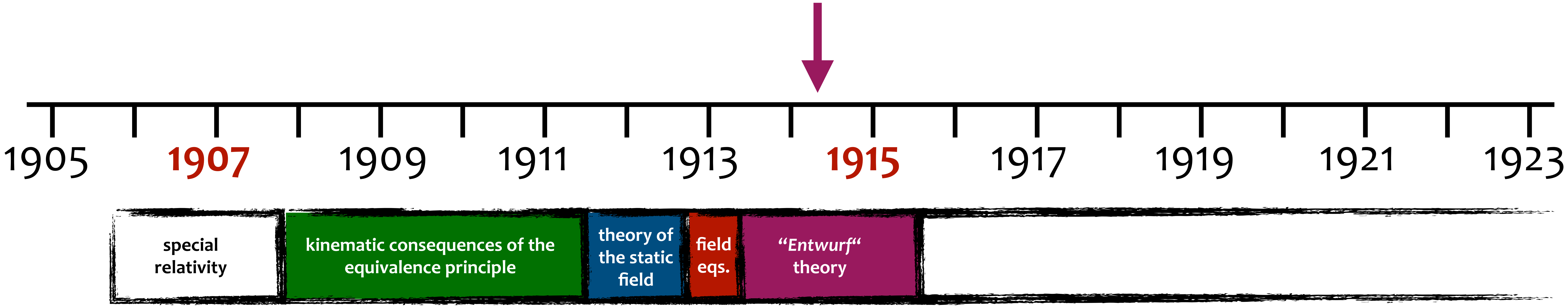


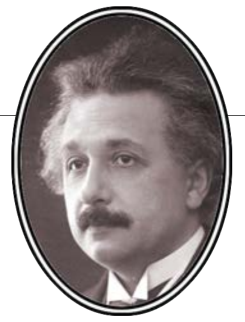
EINSTEIN’S SEARCH FOR GENERAL RELATIVITY

But Einstein and Grossmann failed to find field equations that would satisfy all requirements that Einstein had set up.

They finally settled on field equations that were *not* generally covariant: the Einstein-Grossmann (or “*Entwurf*”) theory.

Einstein came up with clever arguments why the equations could not be generally covariant, the most famous being the “*hole argument*”.

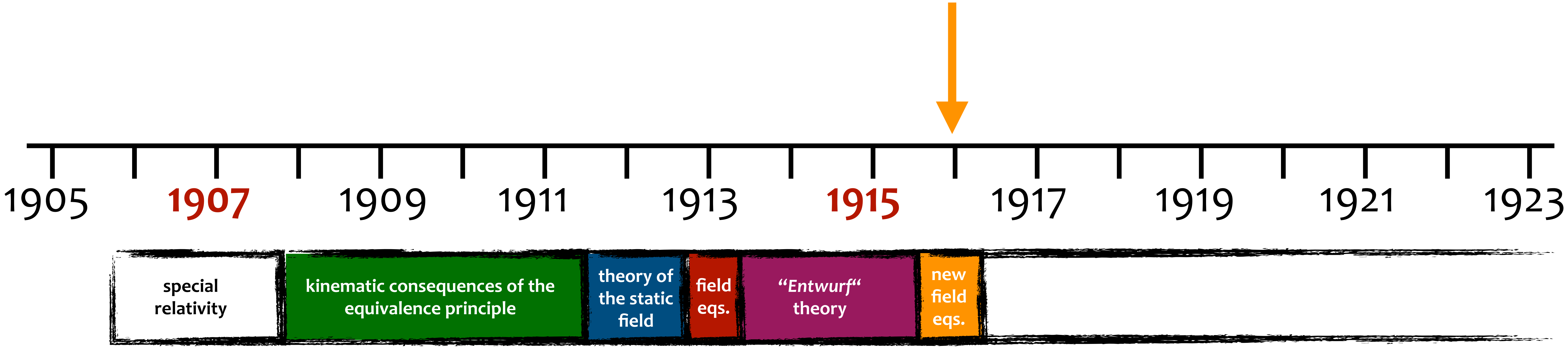


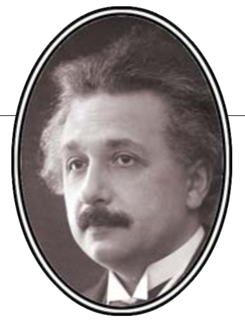


EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

By November 1915 Einstein had concluded that the *Entwurf* theory could not be correct, and that his arguments against general covariance were wrong.

After some failed attempts he finally found acceptable generally covariant field equations — completing the theory of General Relativity.

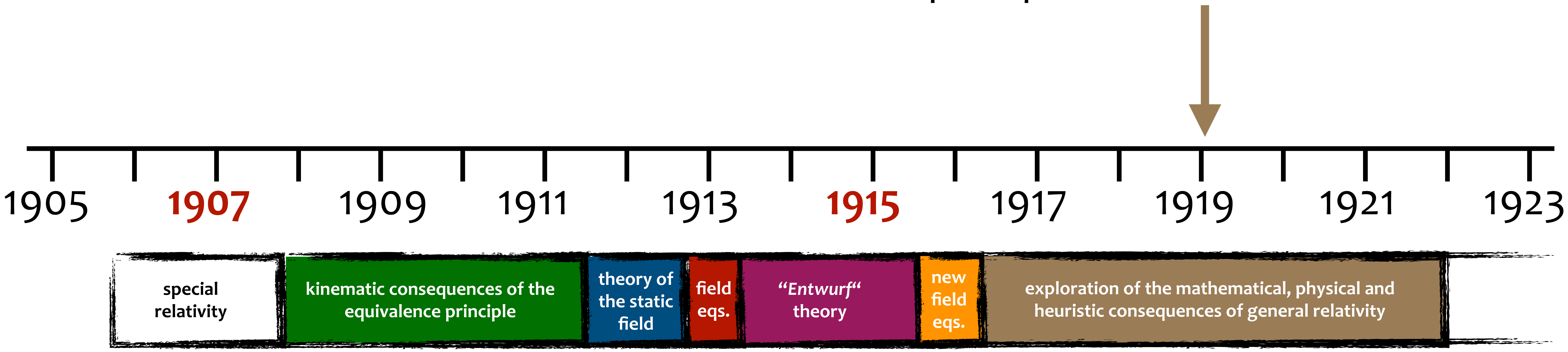


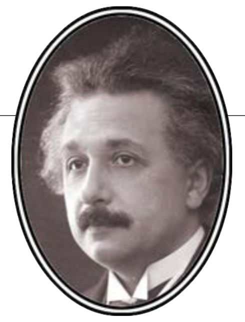


EINSTEIN’S SEARCH FOR GENERAL RELATIVITY

The new theory fulfilled all physical requirements, but did not agree with some of the intuitive heuristics that had guided Einstein during his search — in particular “Mach’s Principle”.

Einstein spent the next couple of years exploring the properties of the new theory (cosmological constant, gravitational waves), and very reluctantly accepted that he had to give up some of his heuristic principles.



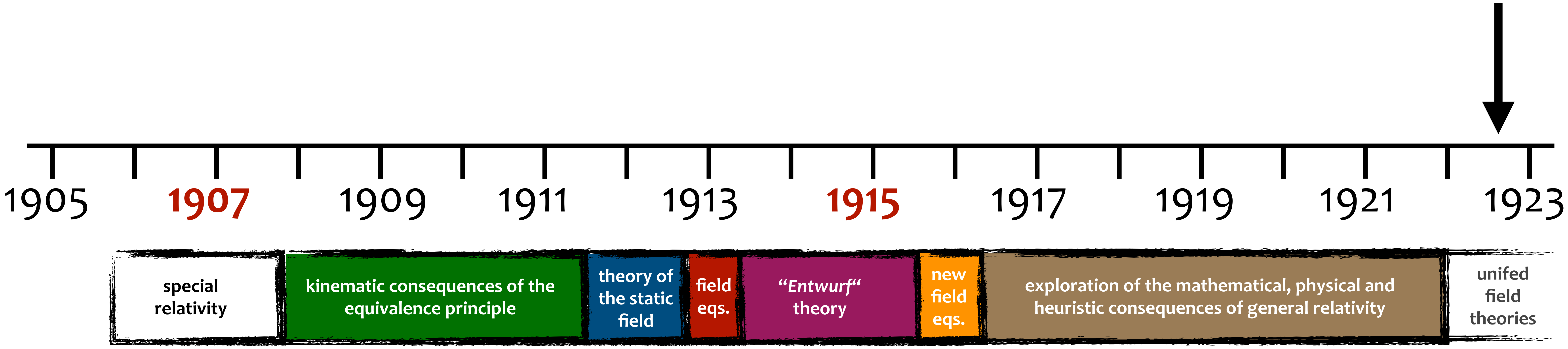


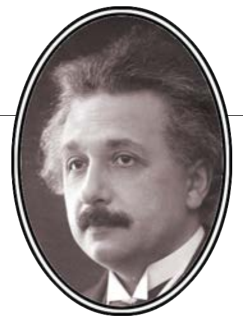
NORDITA

EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

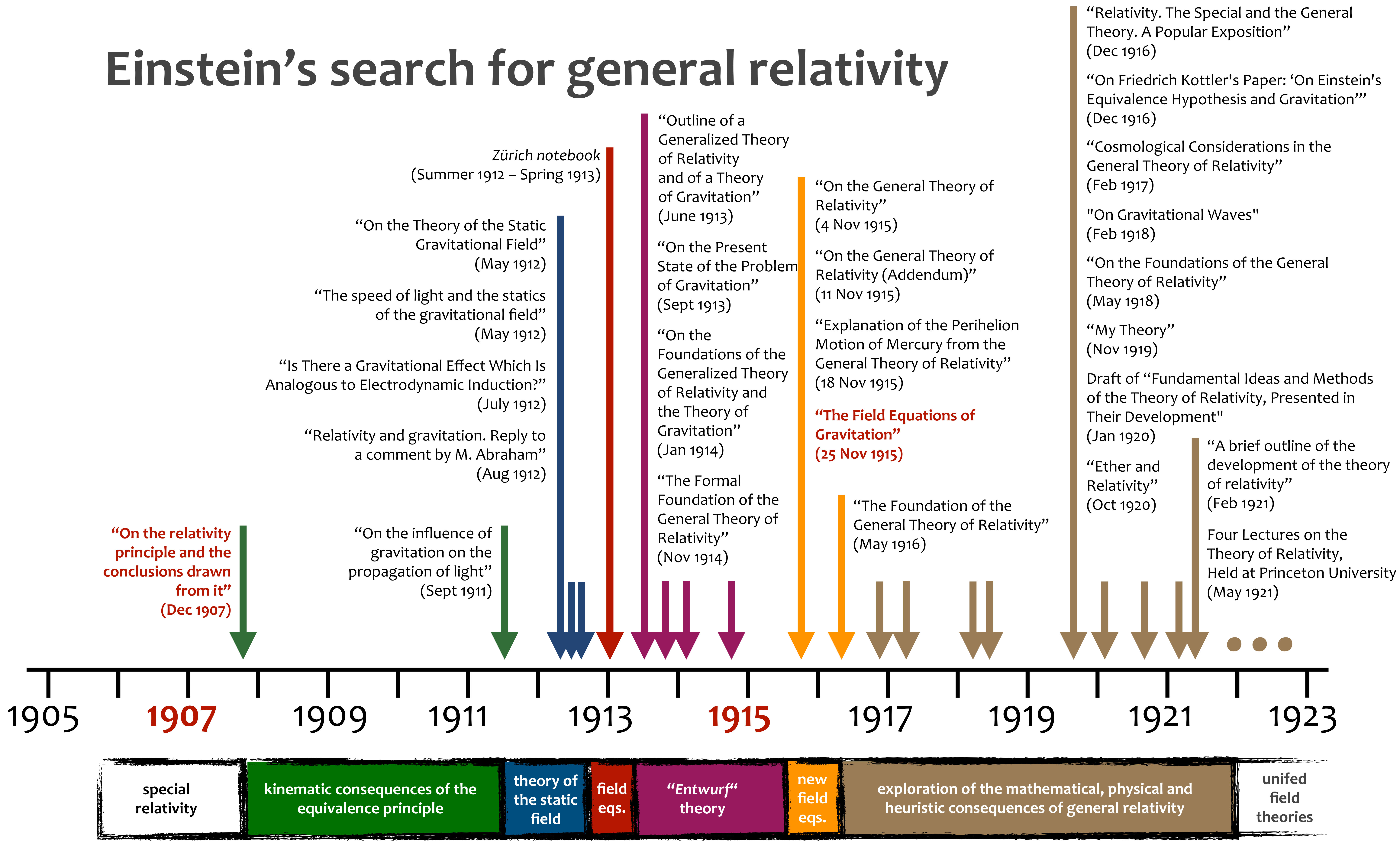
From the 1920s until his death in 1955 Einstein turned to what he saw as the next step — finding a "*unified field theory*" that would unite gravitation and electromagnetism, and explain the structure of matter.

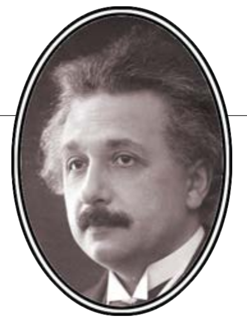
He would not succeed.





EINSTEIN’S SEARCH FOR GENERAL RELATIVITY





Einstein’s Critique of the Equivalence Principle

NORDITA

Einstein's search for general relativity

Texts important for understanding Einstein's "*Äquivalenzprinzip*"

"On the relativity principle and the conclusions drawn from it" (Dec 1907)

"On the influence of gravitation on the propagation of light" (Sept 1911)

"On the Theory of the Static Gravitational Field" (May 1912)

"The speed of light and the statics of the gravitational field" (May 1912)

"Is There a Gravitational Effect Which Is Analogous to Electrodynamic Induction?" (July 1912)

"Relativity and gravitation. Reply to a comment by M. Abraham" (Aug 1912)

Zürich notebook (Summer 1912 – Spring 1913)

"Outline of a Generalized Theory of Relativity and of a Theory of Gravitation" (June 1913)

"On the Present State of the Problem of Gravitation" (Sept 1913)

"On the Foundations of the Generalized Theory of Relativity and the Theory of Gravitation" (Jan 1914)

"The Formal Foundation of the General Theory of Relativity" (Nov 1914)

"On the General Theory of Relativity" (4 Nov 1915)

"On the General Theory of Relativity (Addendum)" (11 Nov 1915)

"Explanation of the Perihelion Motion of Mercury from the General Theory of Relativity" (18 Nov 1915)

"The Field Equations of Gravitation" (25 Nov 1915)

"The Foundation of the General Theory of Relativity" (May 1916)

"Relativity. The Special and the General Theory. A Popular Exposition" (Dec 1916)

"On Friedrich Kottler's Paper: 'On Einstein's Equivalence Hypothesis and Gravitation'" (Dec 1916)

"Cosmological Considerations in the General Theory of Relativity" (Feb 1917)

"On Gravitational Waves" (Feb 1918)

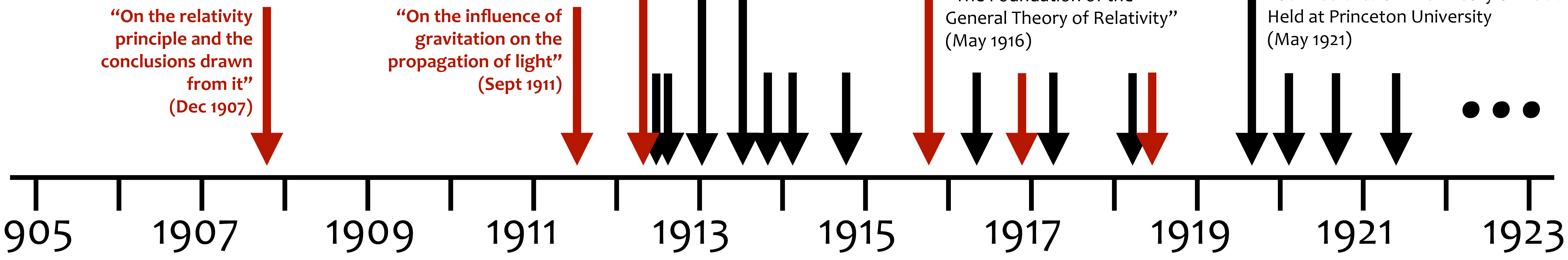
"On the Foundations of the General Theory of Relativity" (May 1918)

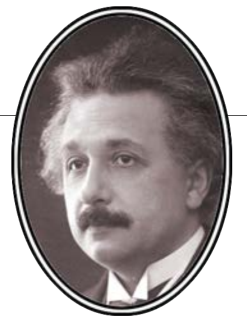
"My Theory" (Nov 1919)

Draft of "Fundamental Ideas and Methods of the Theory of Relativity, Presented in Their Development" (Jan 1920)

"Ether and Relativity" (Oct 1920)

Four Lectures on the Theory of Relativity, Held at Princeton University (May 1921)

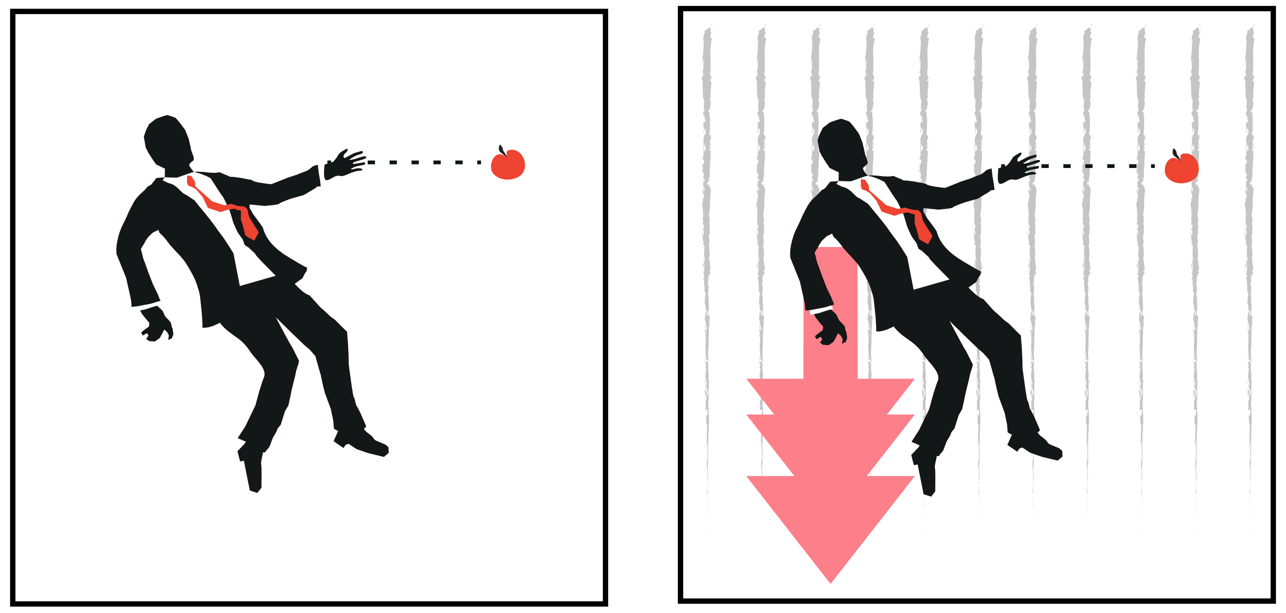
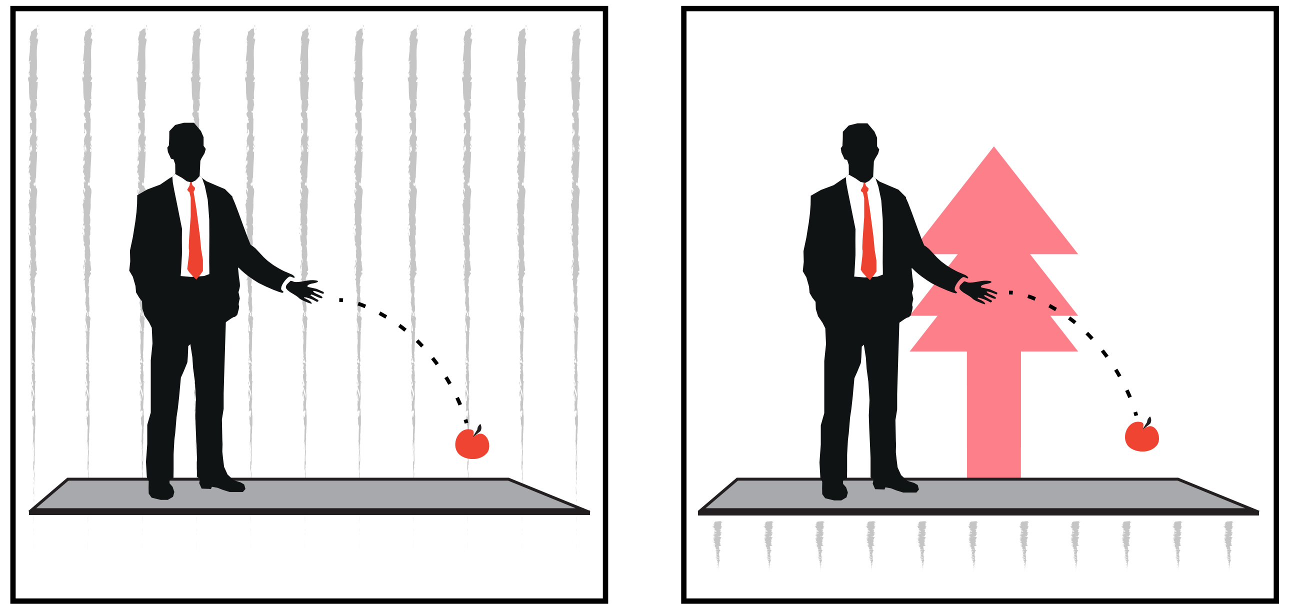




A QUIZ FOR THE GR EXPERTS

A quiz for the experts:

1) In NM, which of these four observers are **inertial observers**?

		"happiest thought" observers		"Einstein elevator" observers	
					
Newtonian mechanics	NM	?	?	?	?

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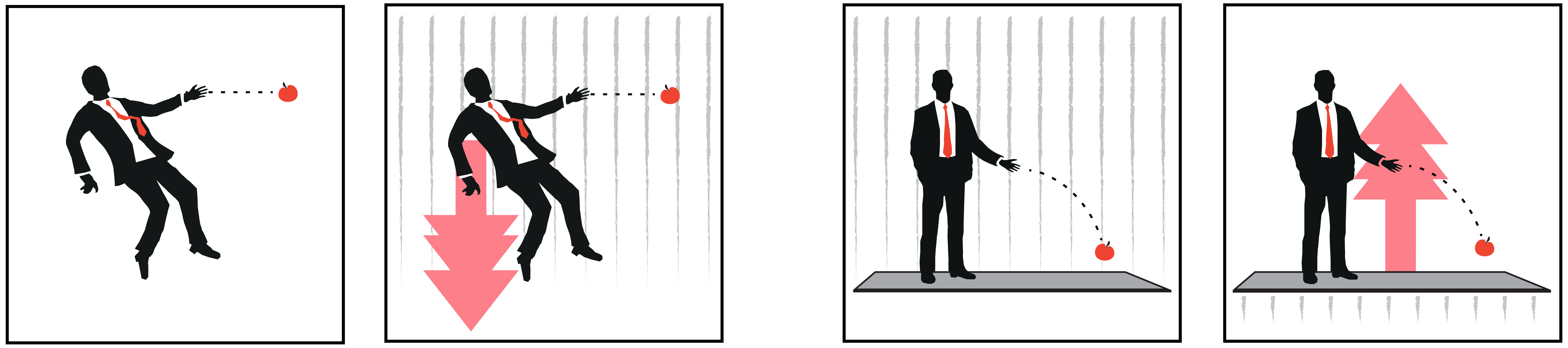
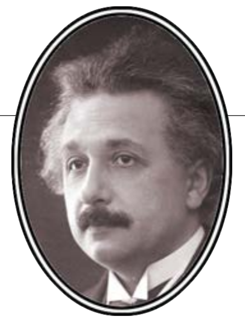
"happiest thought" observers — "Einstein elevator" observers



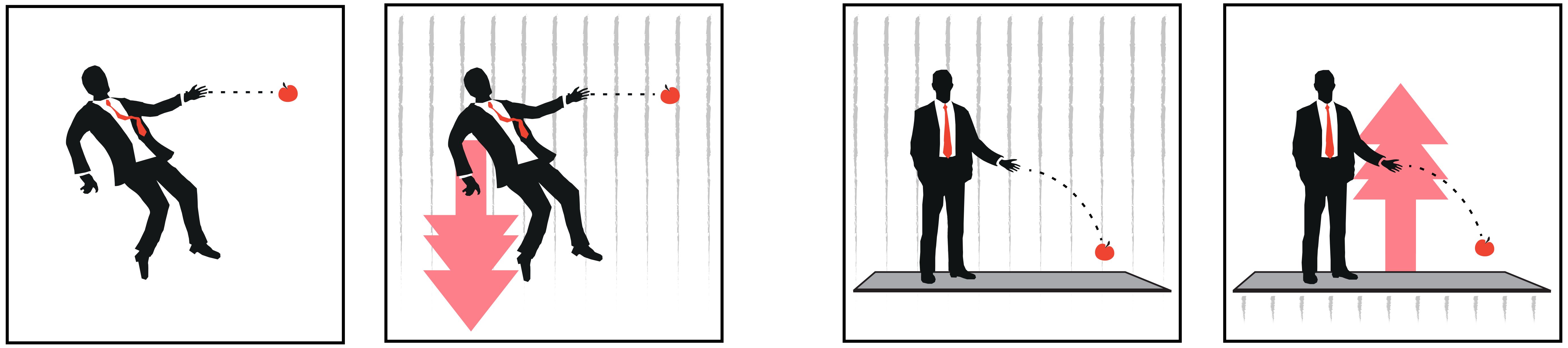
Newtonian mechanics	NM	**Inertial** observer	**Non-inertial** observer	**Inertial** observer	**Non-inertial** observer



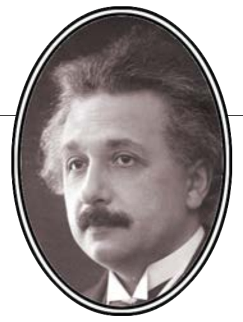
***A quiz for the experts*:**

1) In NM, which of these four observers are **inertial observers**?
2) In GR, which of the four observers are **moving on a geodesic**?

"happiest thought" observers — "Einstein elevator" observers



Newtonian mechanics	NM	Inertial observer	Non-inertial observer	Inertial observer	Non-inertial observer
General relativity	GR	?	?	?	?



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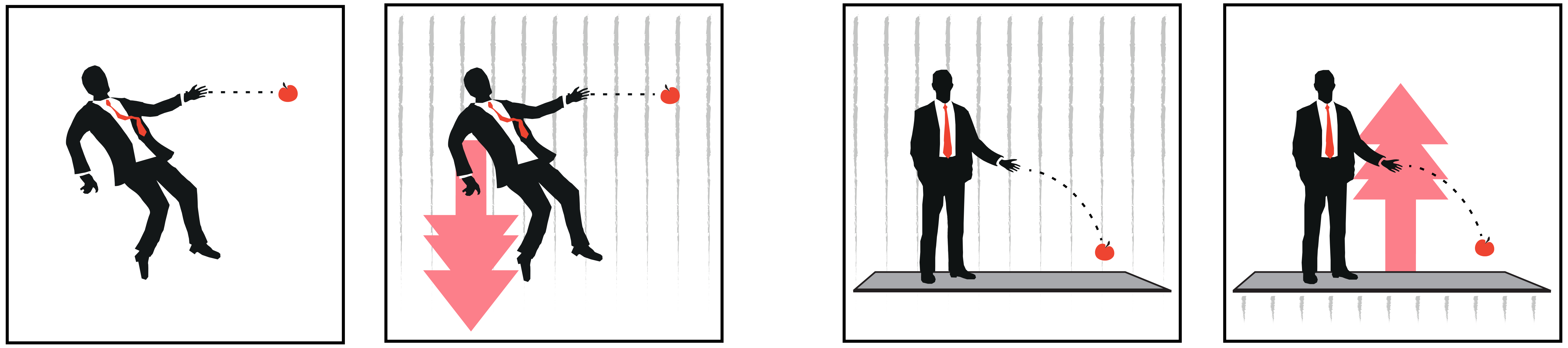
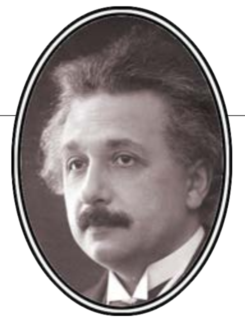
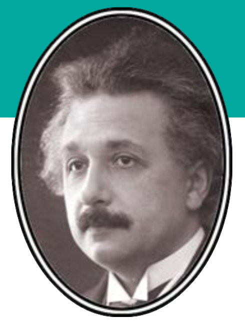
"happiest thought" observers | "Einstein elevator" observers



Newtonian mechanics	NM	**Inertial** observer	**Non-inertial** observer	**Inertial** observer	**Non-inertial** observer
General relativity	GR	Observers **moving on geodesics** of the space-time metric		Observers **in non-geodesic motion**	

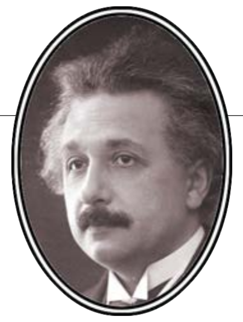


OPEN PROBLEMS
IN SPECIAL RELATIVITY



OPEN PROBLEMS IN SPECIAL RELATIVITY

Einstein was particularly concerned about four closely related problems left unresolved in special relativity:



OPEN PROBLEMS IN SPECIAL RELATIVITY

1. Why are there *privileged motions* in NM and SR (inertial frames of reference) ?
2. What is the origin of inertia (inertial mass)?
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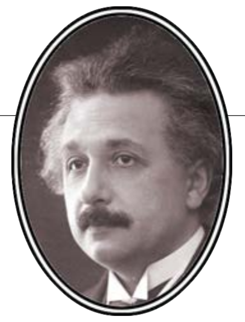
4. How can Newton's law of gravity be extended to SR?

In 1907 Einstein was particularly concerned about **four closely related problems** left unresolved in special relativity



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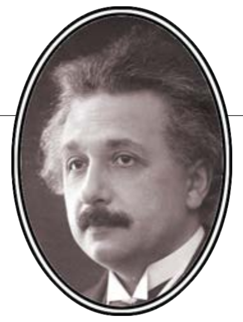
Special relativity had inherited the first three problems from Newtonian physics, without being able to resolve them.



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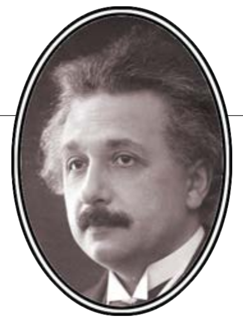

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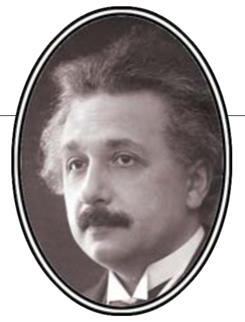
The first unresolved question is actually the one which was **Einstein's main motivation** for pushing beyond SR in 1907 (*but that is a different talk…*).



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For the first **problem** Einstein refers to the writings of Ernst Mach, who in the 1880s had criticised the foundations of Newtonian mechanics.

Einstein would later sometimes refer to this problem as “**Mach’s Paradox**”.

OPEN PROBLEMS IN SPECIAL RELATIVITY

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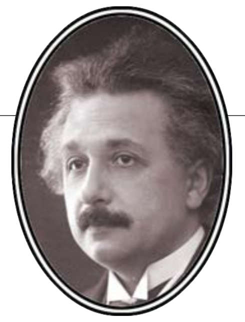
→ *Äquivalenzprinzip*

Principle of General Relativity

Principle of General Covariance

It is actually this first unresolved question which was **Einstein's main motivation** for pushing beyond SR in 1907.

It was to solve it that he introduced the ***Äquivalenzprinzip*** as a heuristic tool, and it would also eventually give the new theory its name, **general relativity**.

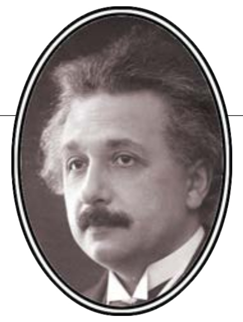


OPEN PROBLEMS IN SPECIAL RELATIVITY

1. **Why are there *privileged motions* in NM and SR (inertial frames of reference) ?**
2. **What is the origin of inertia (inertial mass)?** → *Mach's Principle*
3. **Why is the *inertial* mass of a body numerically equal to its *gravitational* mass, $m_i = m_g$?**
4. **How can Newton's law of gravity be extended to SR?**

For the **second problem** Einstein was also influenced by Ernst Mach.

But important as it was to Einstein's search for a theory of relativistic gravity, I will have to skip discussing his use of "**Mach's Principle**".



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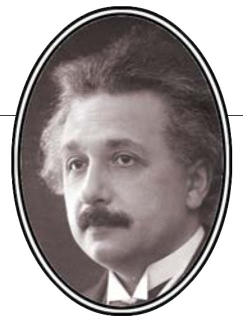
4. **How can Newton's law of gravity be extended to SR?**

Instead I will give **a condensed version of Einstein's arguments,** centering around the **third** of the unresolved questions.

The end result is going to be the same, **culminating in Einstein's most succinct formulation of the *Äquivalenzprinzip* from 1918.**



OPEN PROBLEMS IN SPECIAL RELATIVITY

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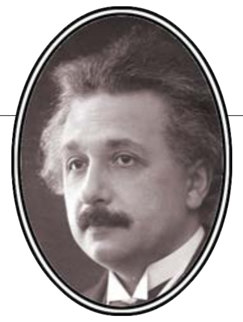
4. **How can Newton's law of gravity be extended to SR?** → *Äquivalenzprinzip heuristic*

… the **fourth problem** Einstein xxxx forced to consider it while writing the 1907 review article

also tried to find a rel grav theory based on SR, but never published since it didn't work as expected.



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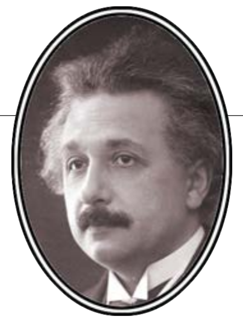
… but in the interest of time **I will have to skip the description of this line of inquiry**, even though it is historically important.



WHAT'S THE POINT?

Einstein was particularly concerned about these **four closely related problems left unresolved in special relativity.**

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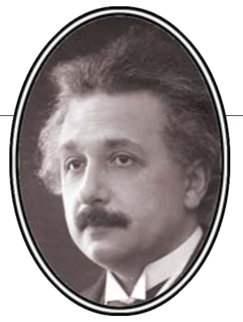
To understand why Einstein introduced the *Äquivalenzprinzip* it is sufficient to keep **the last motivating question** in mind.



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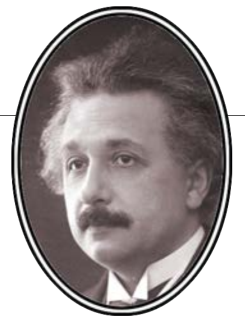
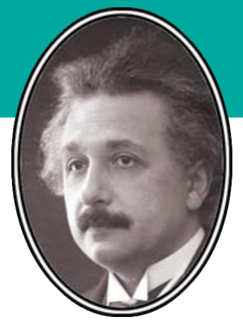
But to understand why Einstein introduced the *Äquivalenzprinzip* it is sufficient to keep **the last two motivating questions** in mind.



EINSTEIN’S ORIGINAL MOTIVATION FOR THE *ÄQUIVALENZPRINZIP*



EINSTEIN'S ORIGINAL MOTIVATION FOR THE *ÄQUIVALENZPRINZIP*

SOM TILLÄGG TILL "EINSTEIN'S CRITIQUE"

Jag har beskrivit HUR Einstein resonerade sig fram till ÄP, men inte nämnt något om VARFÖR han ville postulera just denna generalisering av hissargumentet. (När han väl hade ÄP var det naturligt att gå vidare till ÄP-heuristiken för att hitta en relativistis gravitationsteori, men det var alltså inte hans ursprungliga motivation för ÄP.)

Två initiala motiveringar, båda inspirerade av Mach: (1) eliminera priviligierade referenssystem genom en utvidgning av relativitetsprincipen till alla rörelser, (2) förklara m_i=m_g genom wesensgleichheit.

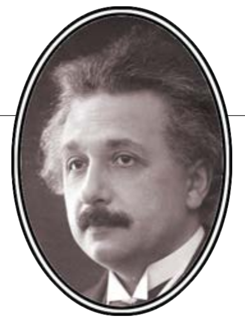
I bägge fall lovar ekvivalensen för de generativa observatörerna i Newtonsk teori en väg i rätt riktning: (1) genom att introducera gravitationsfältet i diskussionen kan en likformigt accelererad observatör se sig som i vila, men i ett gravitationsfält, (2) en sammansmältning av tröghet och gravitation skulle förklara masslikheten.

Och i bägge fall sätter icke-ekvivalensen mellan de två observatörerna map icke-mekaniska lagar stopp för båda heuristikerna. Därav behovet av att postulera ekvivalens map alla naturlagar (dvs ÄP).

Efter detta postulat delar sig vidareutvecklingen av de två ursprungliga motiveringarna.

(1) Einstein trodde sig ha eliminerat priviligierade observatörer genom allmänkovariansen hos GR. Men det funkar inte. I själva verket har GR fortfarande priviligierade rörelser: geodetisk rörelse.

(2) [Kolla] ÄP borde vara tillräckligt för att säkerställa wesensgleichheit i en icke-GR-kontext. Generaliseringen till relativsitsk gravitation (pre-GR) är naturlig. Därefter följer sedan motivationen till att gå från UFM+Euklidisk geometri till UFF+dynamisk geometri, dvs NC resp GR.



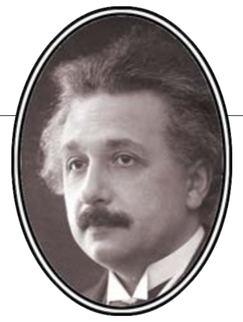
HOW TO ELIMINATE PRIVILEGED MOTIONS

Why are there *privileged* motions (inertial frames of reference) in Newtonian mechanics and special relativity?

E inspired by Mach, but has a different approach.

Mach: cannot explain inertial frames by referring to Newton's absolute space, not accessible to us; could explain them as a kind of dependence on all masses in the universe.

Rather than *explaining* the privileged inertial frames, Einstein decides to try to *eliminate* them instead: if all motions are equally privileged, the problem disappears.



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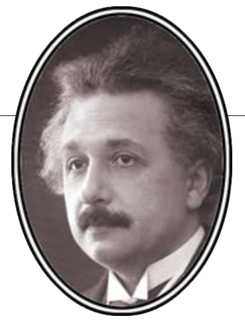
This problem was of such great concern to Einstein that — much later — he would consider its resolution **the most important success of general relativity**:

> You consider the transition to special relativity as **the most essential thought of relativity,** not the transition to general relativity.
>
> I consider the reverse to be correct. **I see the most essential thing in the overcoming of the inertial system,** a thing that acts upon all processes but undergoes no reaction.

Albert Einstein, Letter to Georg Jaffe (1954)



HOW TO ELIMINATE PRIVILEGED MOTIONS

Why are there *privileged* motions (inertial frames of reference) in Newtonian mechanics and special relativity?

[why is this a problem?]

> It is contrary to the scientific mode of understanding to postulate a thing (the space-time continuum) **that acts, but which cannot be acted upon.**

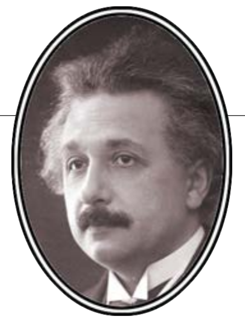
Albert Einstein, “Four lectures on the theory of relativity” (Princeton lectures, 1921)

[action-reaction in E’s arguments only after abandoning Mach’s principle]

HOW TO ELIMINATE PRIVILEGED MOTIONS

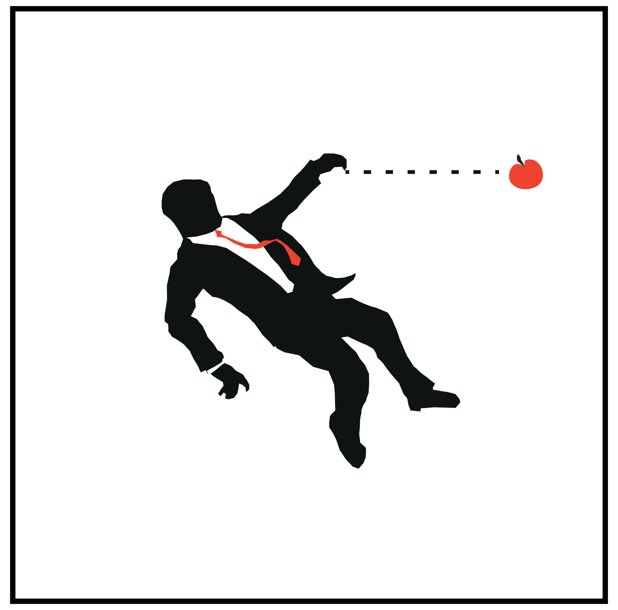
In order to understand Einstein's arguments, and how these led him to the first formulation of the *Äquivalenzprinzip*, we need to **understand the meaning of several key words** that Einstein used:

- simpler (or standard) form of the laws of physics
- observer at rest
- observer in privileged motion
- equivalent observers
- relativity principle



HOW TO ELIMINATE PRIVILEGED MOTIONS

Let's first look at an **observer at rest** (in a theory which has a notion of absolute rest, like Newtonian mechanics or ether-based electro-magnetism).



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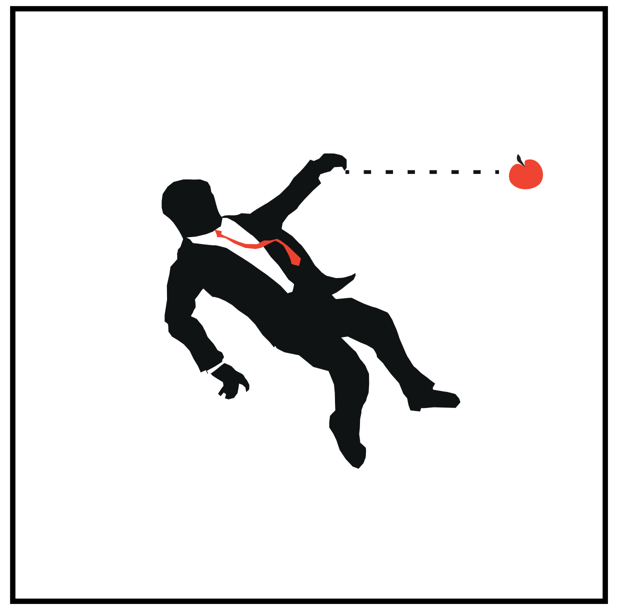
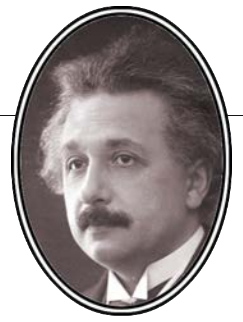


Observer at rest

We note that **the laws of physics take on an especially simple form** for this observer (all expressions depending on velocities or accelerations vanish)



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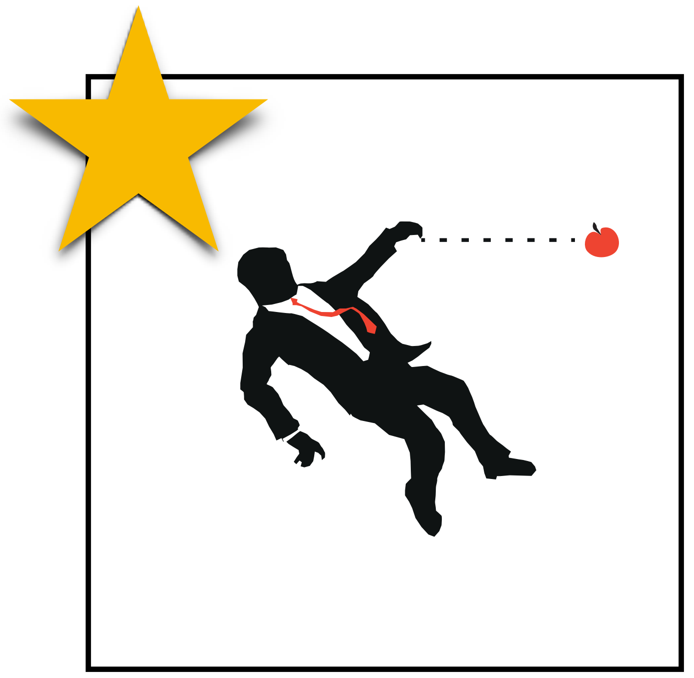

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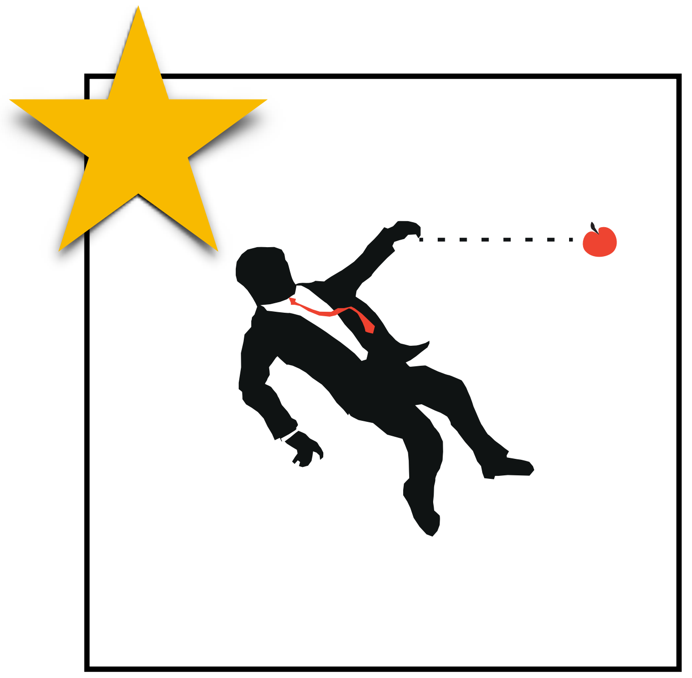
In this sense, the observer at rest holds a **privileged** position among all possible observers.

HOW TO ELIMINATE PRIVILEGED MOTIONS

In theories where the laws do not depend on absolute or relative velocity, the **laws of physics take on a simpler form** also for observers in **uniform motion** (inertial observers).



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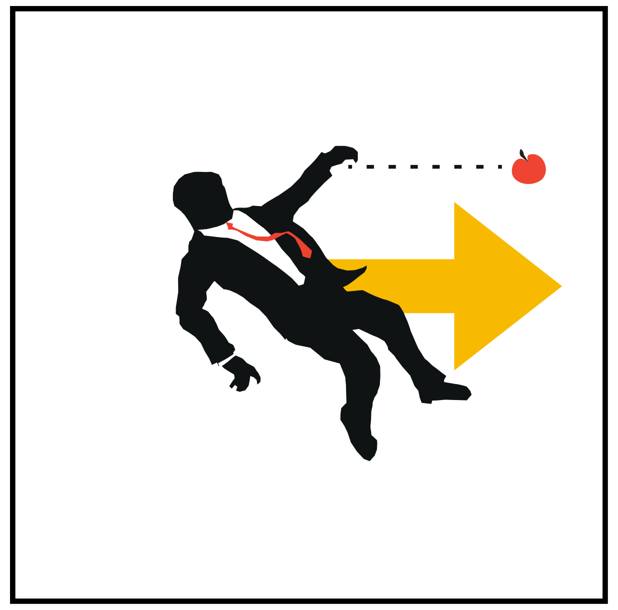
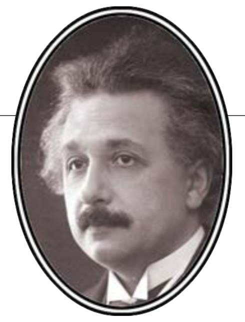


Uniformly moving observer

[kolla med Es “standard form” för lagar]



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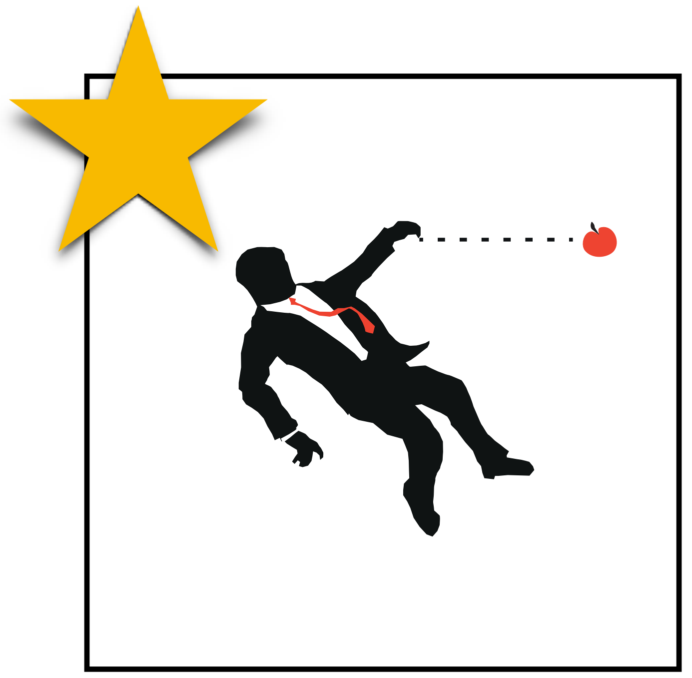

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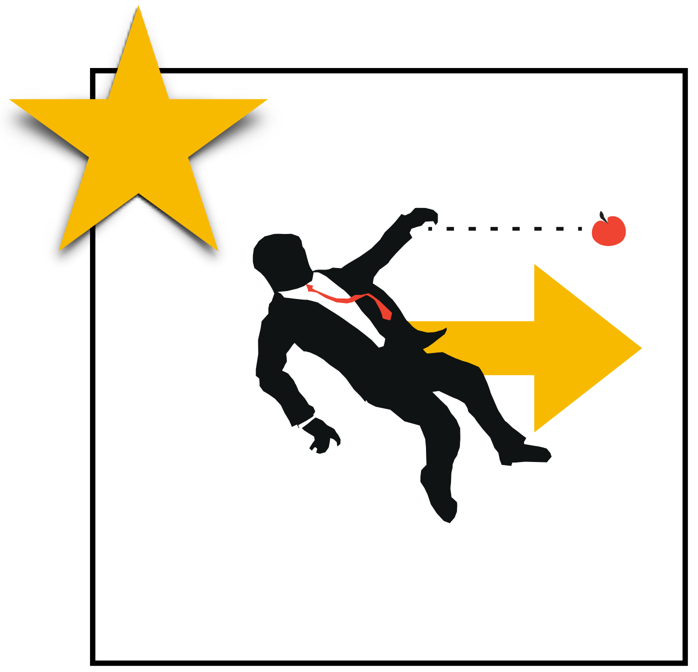
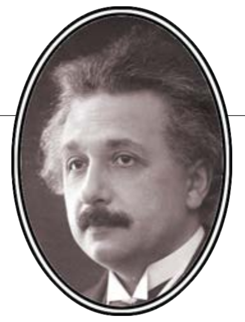


Uniformly moving observer

Observers in uniform motion are therefore equally **privileged** as observers at rest.



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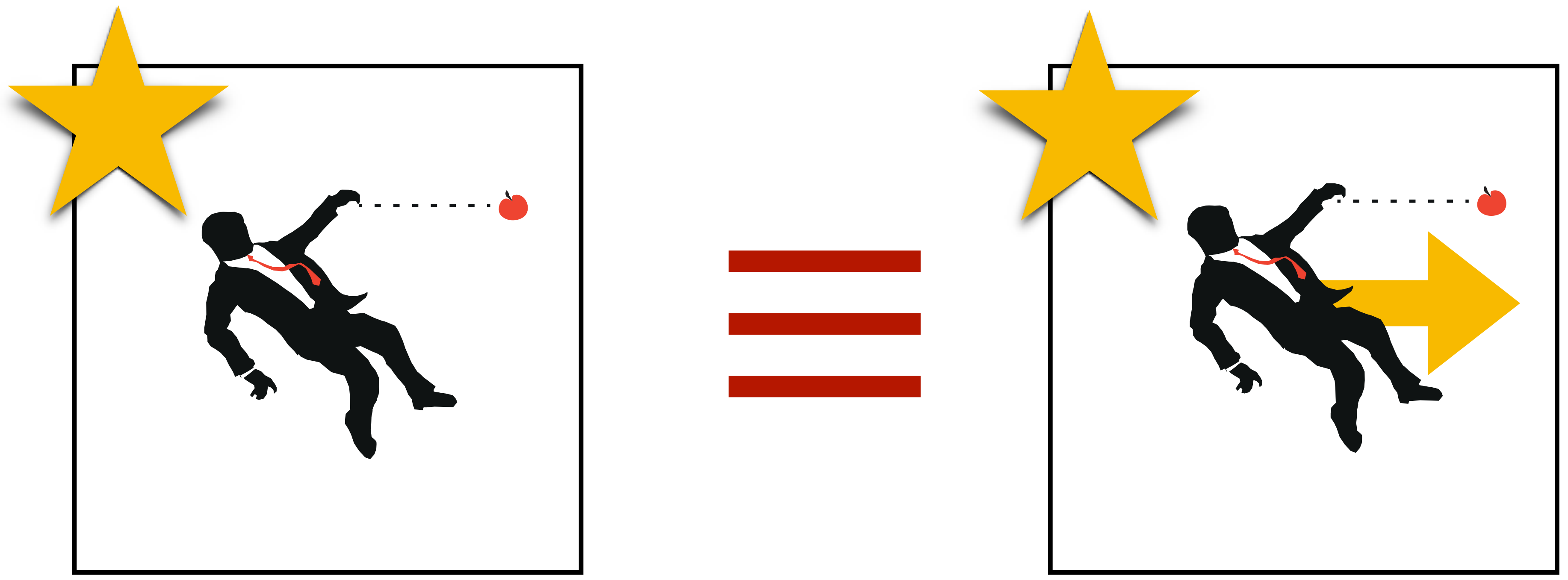
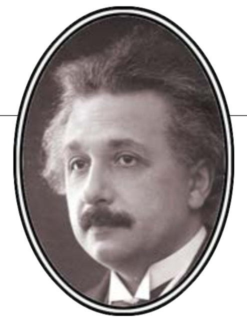


Observer at rest

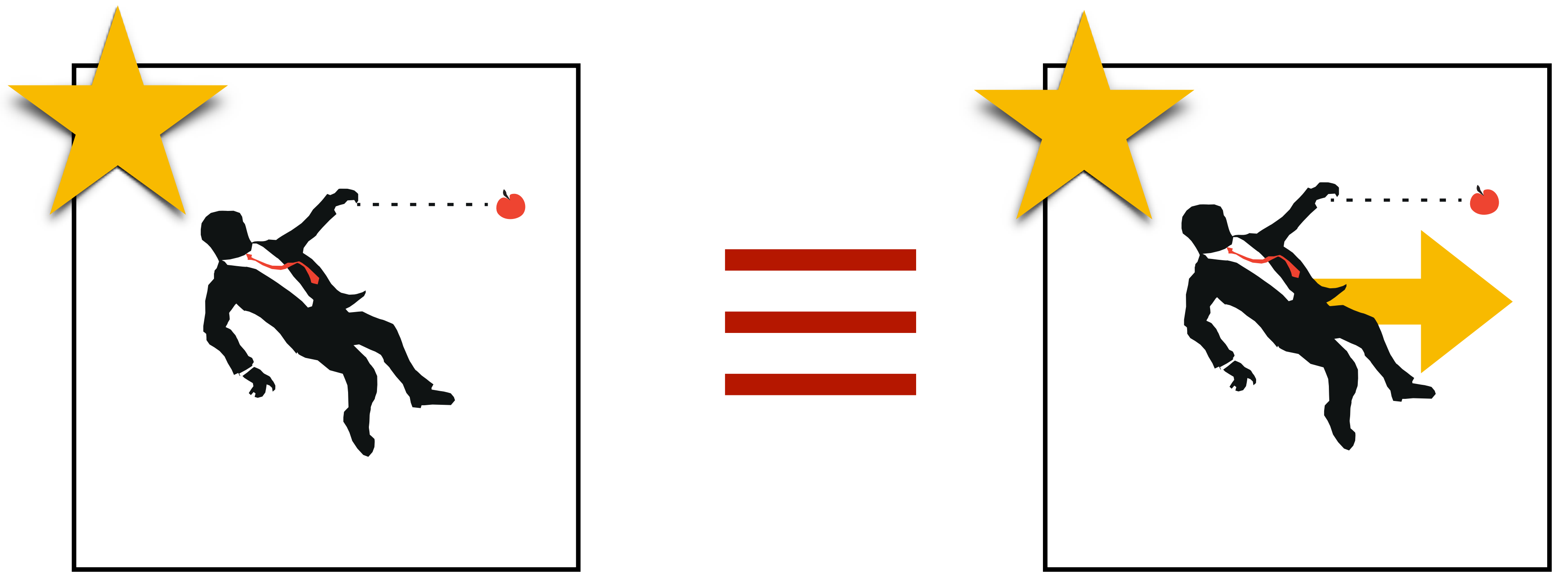
Uniformly moving observer

Observers in uniform motion are therefore equally **privileged** as observers at rest.

We say that all such observers are **equivalent** (with respect to the form of the laws of physics). They are all equally justified to claim to be “at rest”.



A theory with such a **class of privileged (equivalent) observers** is said to satisfy a **relativity principle**.

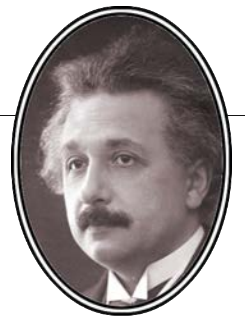


Observer at rest

Uniformly moving observer

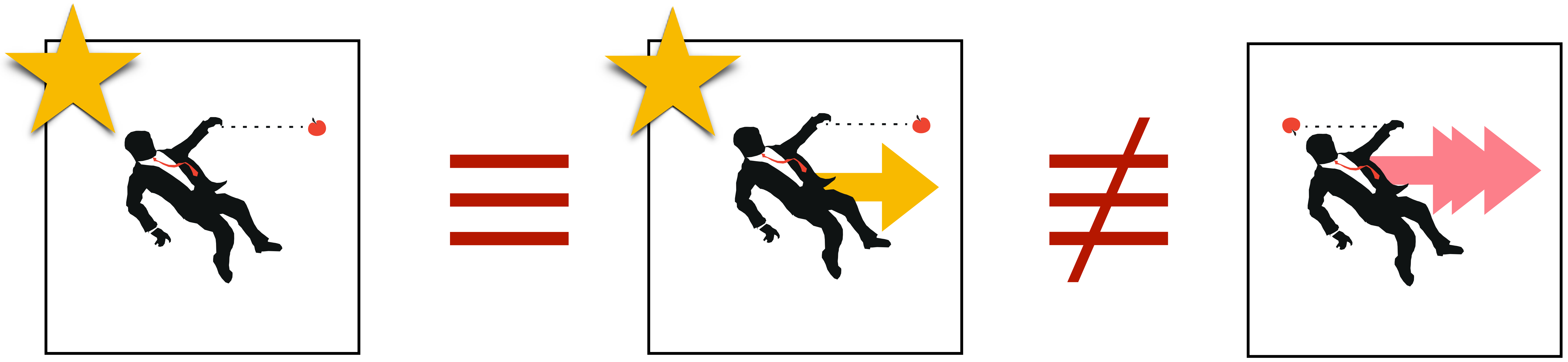
Examples are Newtonian mechanics and special relativity (but not ether-based electrodynamics).

The statement that in SR **all inertial observers are equivalent** is simply a statement of the **principle of special relativity.**



HOW TO ELIMINATE PRIVILEGED MOTIONS

For all other observers (*i.e.* accelerated observers) the **laws of physics will take on a more complicated form** (*e.g.* involving inertial force terms).



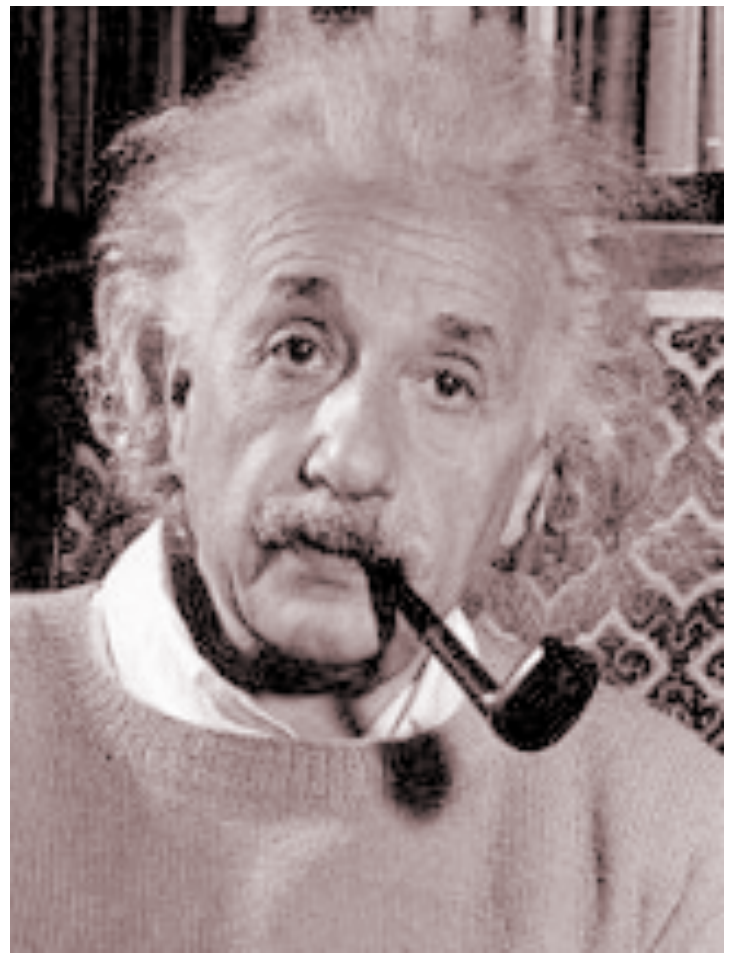
Observer at rest

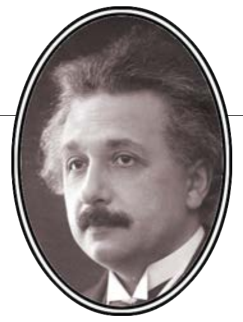
Uniformly moving observer

Accelerated observer

The accelerated observers are obviously neither **privileged** nor **equivalent** to observers at rest.

But Einstein was not happy about this state of affairs…



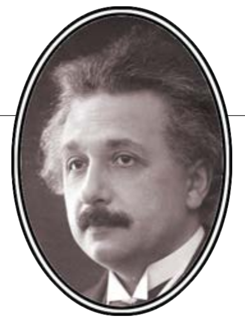


HOW TO ELIMINATE PRIVILEGED MOTIONS

First we need to clarify some important expressions:

An observer is said to be a **PRIVILEGED** observer if **the laws of physics reduce to a simpler form** in their frame of reference, compared to the form of laws in arbitrary frames of reference.

> *Examples*: an observer **AT REST** in absolute space in NM, or at rest with respect to the luminiferous ether in pre-relativistic EM.



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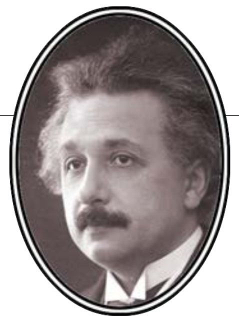
Two observers are **EQUIVALENT** if **the laws of physics take on the same form in both frames of reference.**

> *Example*: the class of inertial observers in NM or SR.



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First we need to clarify some important expressions:

An observer is said to be a **PRIVILEGED** observer if **the laws of physics reduce to a simpler form** in their frame of reference, compared to the form of laws in arbitrary frames of reference.

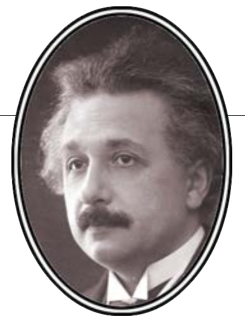
> *Examples*: an observer **AT REST** in absolute space in NM, or at rest with respect to the luminiferous ether in pre-relativistic EM.

Two observers are **EQUIVALENT** if **the laws of physics take on the same form in both frames of reference.**

> *Example*: the class of inertial observers in NM or SR.

A theory satisfies a **RELATIVITY PRINCIPLE** if it allows for **a whole class of equivalent privileged observers**, not just the single observer "at rest".

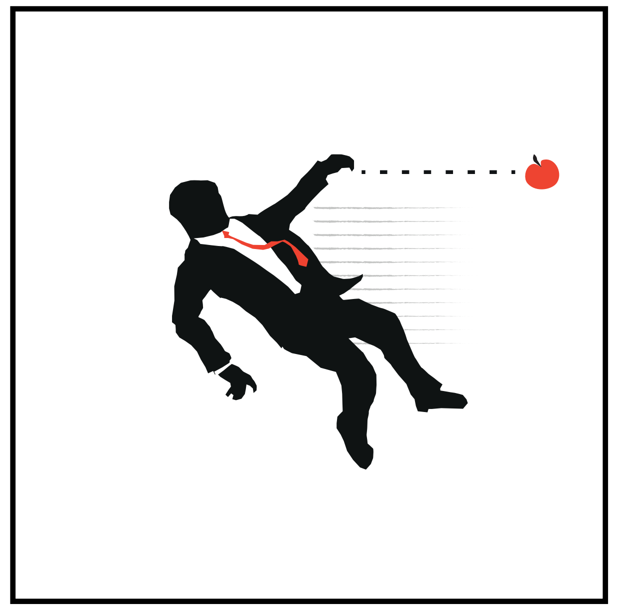
> *Examples*: Galilean-invariant NM, Lorentz-invariant SR



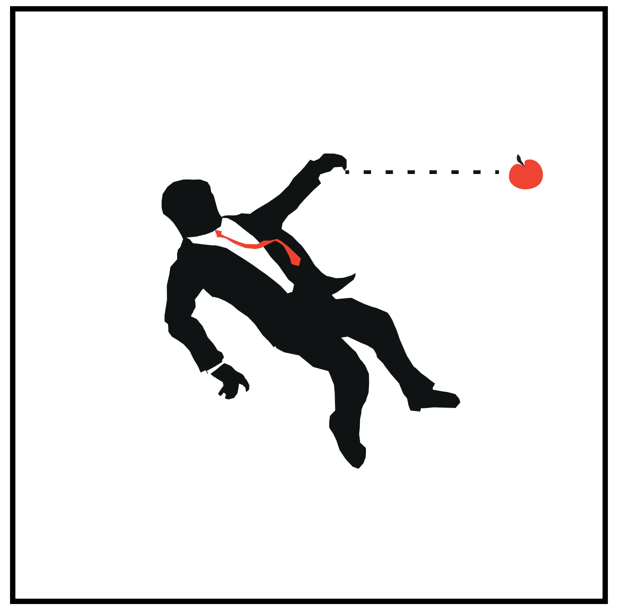
HOW TO ELIMINATE PRIVILEGED MOTIONS

Why are there *privileged* motions?

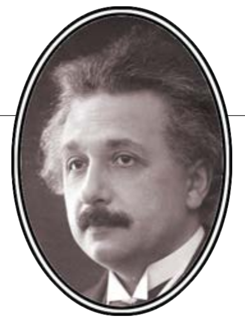
- All **privileged** (inertial) observers can **claim to be "at rest"** (there is no *absolute rest* in theories obeying the Principle of Relativity)



Moving observer



Observer at rest



HOW TO ELIMINATE PRIVILEGED MOTIONS

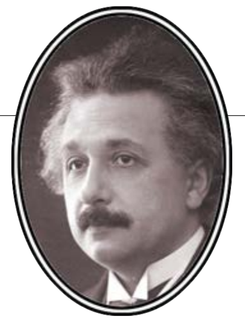
Why are there *privileged* motions (inertial frames of reference) in Newtonian mechanics and special relativity?

> In classical mechanics, and no less in the special theory of relativity, there is **an inherent epistemological defect** …

Albert Einstein, "The Foundation of the General Theory of Relativity" (1916)

> … no person whose mode of thought is logical can rest satisfied with this state of affairs. He asks: 'How does it come that certain reference-bodies … are given priority over other reference-bodies …? **What is the reason for this Preference**?'

Albert Einstein, "Relativity. The Special and the General Theory. A Popular Exposition" (1917)



But then!

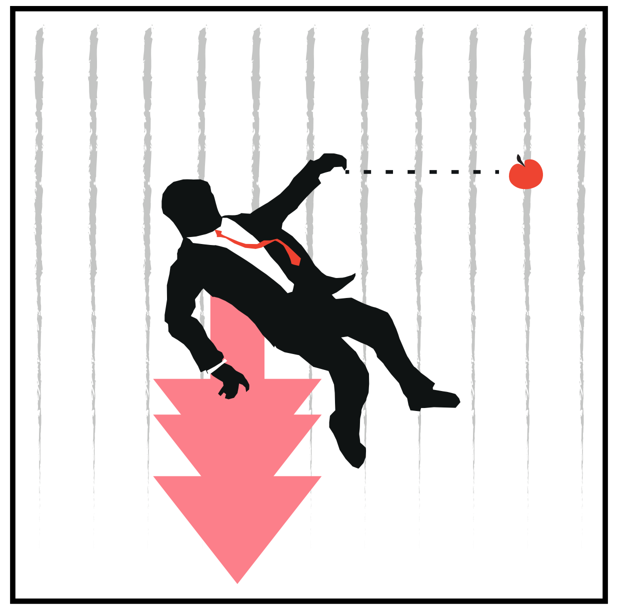
Einstein had the happiest thought of his life…

HOW TO ELIMINATE PRIVILEGED MOTIONS

At that moment **I got the happiest thought of my life** in the following form:

[…] **for an observer in free-fall** from the roof of a house **there is during the fall** — at least in his immediate vicinity — **no gravitational field.**

Albert Einstein, Unpublished manuscript, in all probability a draft for an article in *Nature* that was never published (c.1920)



An observer falling freely in a uniform gravitational field doesn't see any gravitational effects

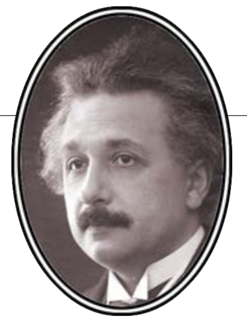
HOW TO ELIMINATE PRIVILEGED MOTIONS

At that moment **I got the happiest thought of my life** in the following form:

[…] **for an observer in free-fall** from the roof of a house **there is during the fall** — at least in his immediate vicinity — **no gravitational field**.

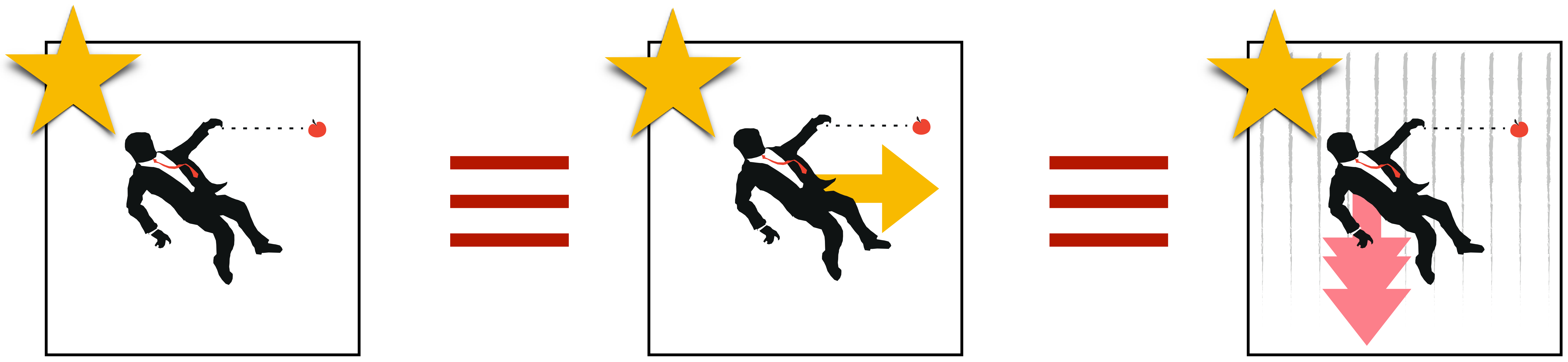
Namely, if the observer lets go of any bodies, they remain relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature. **The observer, therefore, is justified in interpreting his state as being "at rest."**

Albert Einstein, Unpublished manuscript, in all probability a draft for an article in *Nature* that was never published (c.1920)



HOW TO ELIMINATE PRIVILEGED MOTIONS

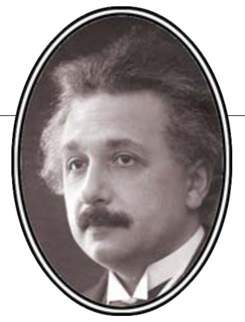
This allowed Einstein to **extend the class of privileged observers** (w. r. t. the laws of mechanics) to include also freely falling observers:



Observer at rest

Uniformly moving observer

Observer falling freely in a uniform gravitational field



HOW TO ELIMINATE PRIVILEGED MOTIONS

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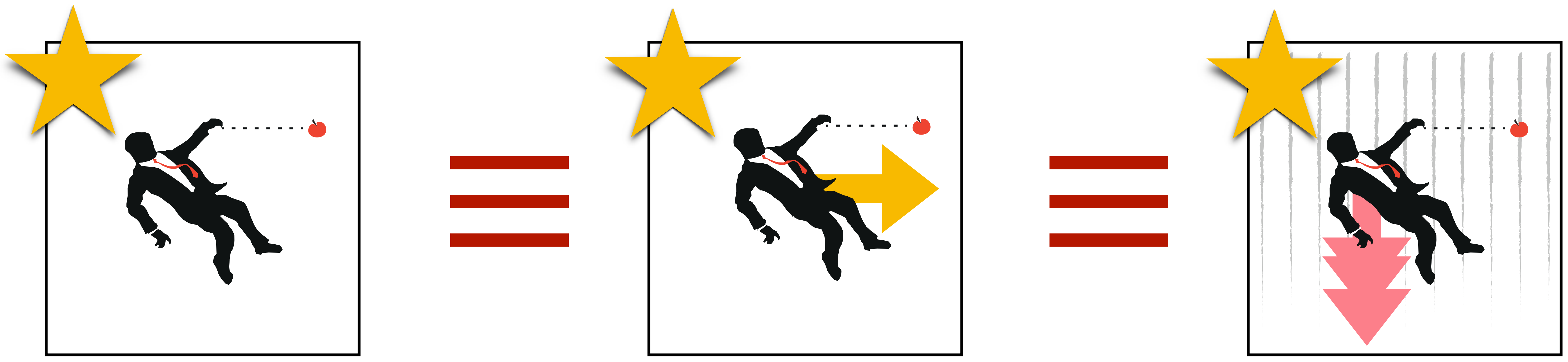


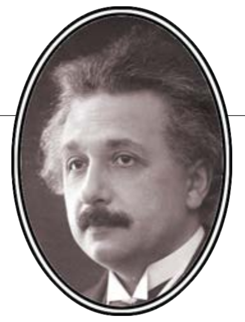
Observer at rest ≡ Uniformly moving observer ≡ Observer falling freely in a uniform gravitational field

This is still far from Einstein's lofty goal to make *all* observers equally privileged (*i.e.*, to find a theory satisfying a Principle of ***General*** Relativity)…

HOW TO ELIMINATE PRIVILEGED MOTIONS

…but even this very simplistic “happiest thought” argument allowed Einstein to **draw one heuristically very important conclusion**:

> To find a theory satisfying a “Principle of General Relativity” **one needs to take gravitation into consideration.**



HOW TO ELIMINATE PRIVILEGED MOTIONS

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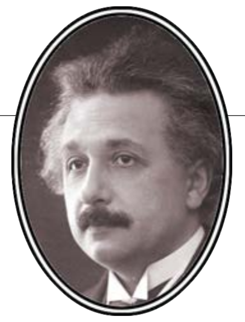
> To find a theory satisfying a “Principle of General Relativity” **one needs to take gravitation into consideration.**

All he need to do now was to find **a theory of relativistic gravitation,** and his goal to find a “**theory of general relativity**” would be fulfilled!



HOW TO ELIMINATE PRIVILEGED MOTIONS

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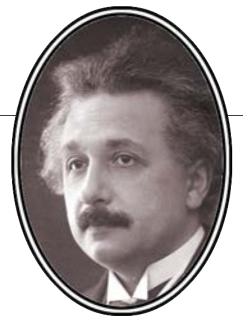
> To find a theory satisfying a "Principle of General Relativity" **one needs to take gravitation into consideration.**

All he need to do now was to find **a theory of relativistic gravitation,** and his goal to find a "**theory of general relativity**" would be fulfilled!

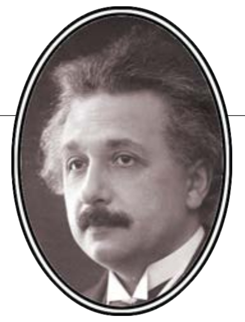
> ***"… Aber leichter ist ahnen als finden."***
>
> **Albert Einstein**, letter to Rudolf Förster, 16 November 1917



Einstein continues his analysis of the falling man, and here **it is important to note his choice of words:**

> *If the observer lets go of any bodies, they remain relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature...*

Albert Einstein, xxx (1954)



HOW TO ELIMINATE PRIVILEGED MOTIONS

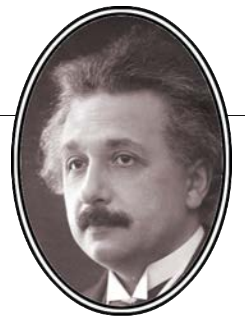
Why are there *privileged* motions in NM and SR (inertial frames of reference) ?

Einstein was convinced that the best way to eliminate the privileged status of inertial frames of reference was by extending the Relativity Principle to all observers.

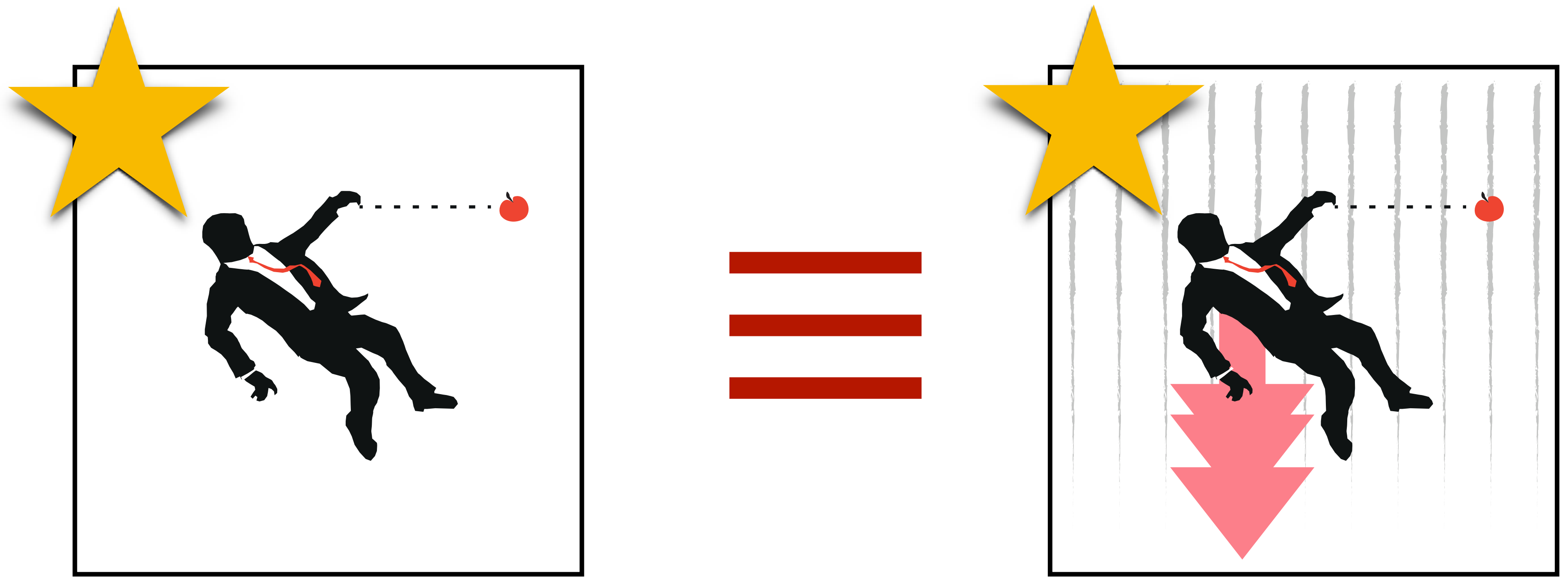
(It would turn out that this ambitious goal would not be satisfied in the final theory — "General Relativity" — but would anyhow give the theory its name.)

I will come back to how Einstein thought he could achieve this using his *Äquivalenzprinzip*.

But first…



HOW TO ELIMINATE PRIVILEGED MOTIONS



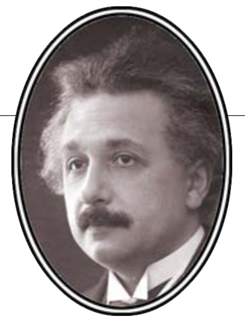
Observer at rest

Observer falling freely in a uniform gravitational field

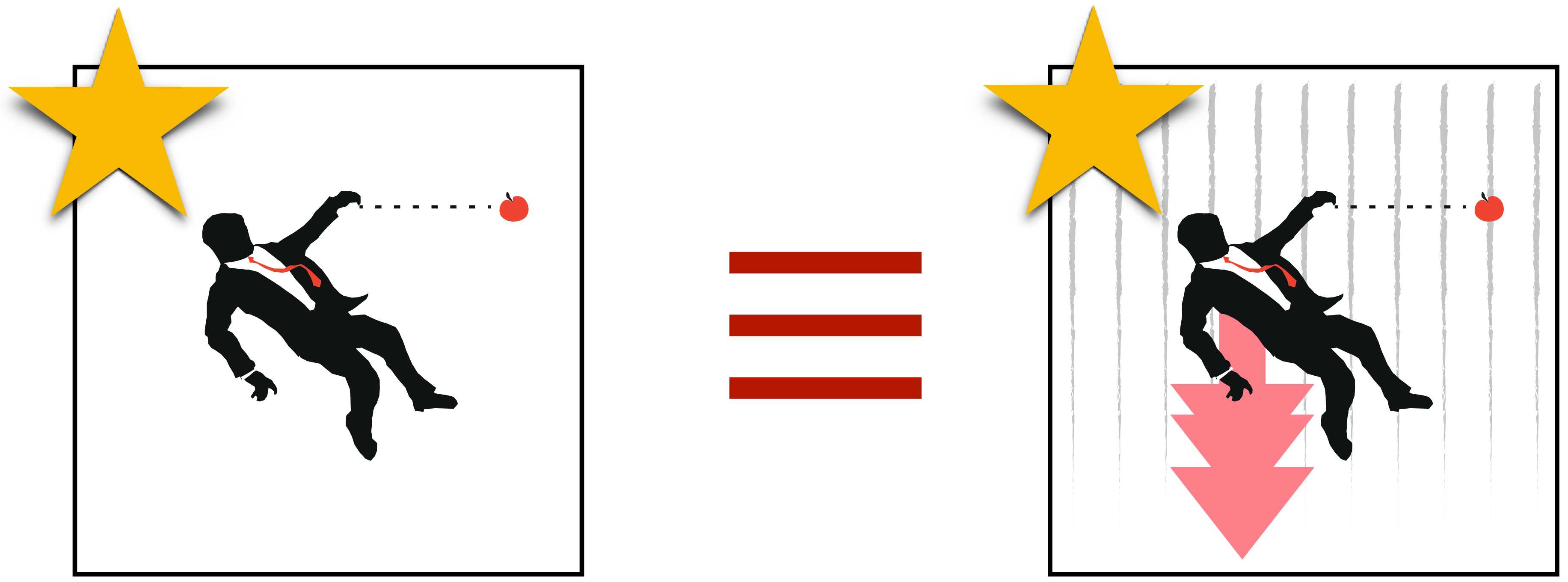
The equivalent "**happiest thought**" observers

The first time Einstein made use of his "happiest thought" in print was in a paper in 1907.

It was in the form of the "**Einstein elevator argument**", well known form general relativity textbooks.



HOW TO ELIMINATE PRIVILEGED MOTIONS



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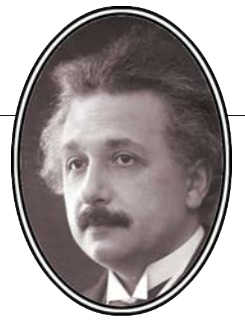
The equivalent “**happiest thought**” observers

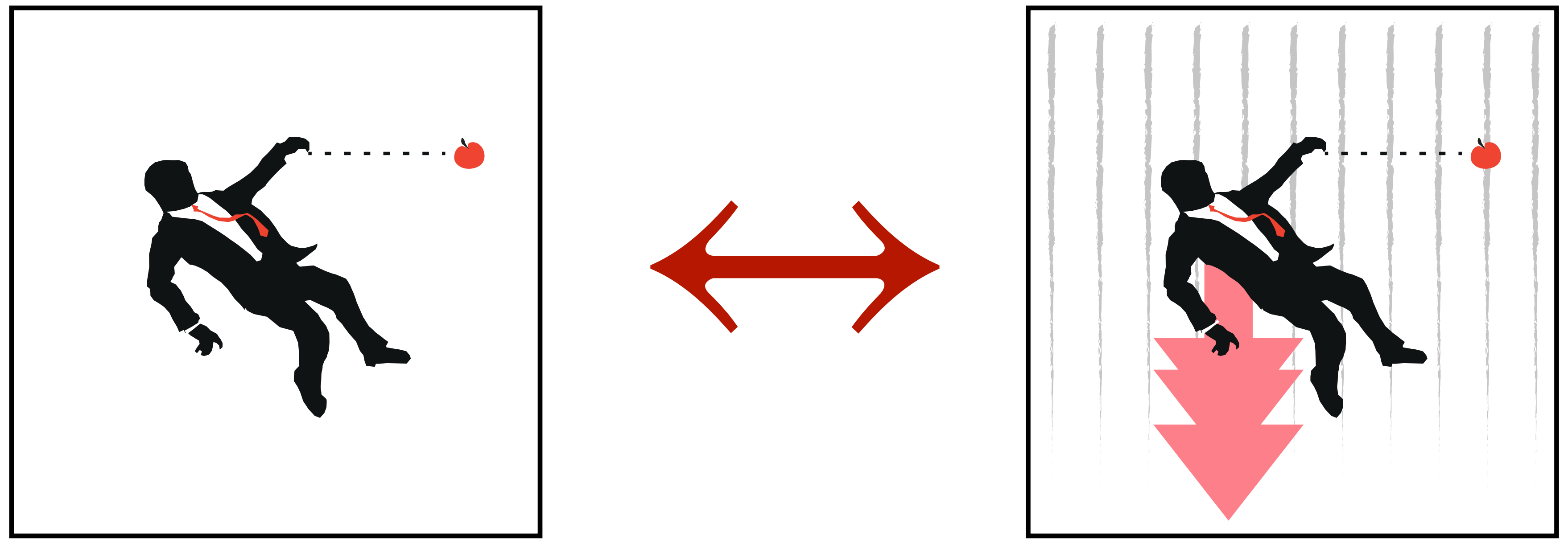
The first time Einstein made use of his “happiest thought” in print was in a paper in 1907.

It was in the form of the “**Einstein elevator argument**”, well known form general relativity textbooks.

This argument is formulated using **a different pair of equivalent observers** than the pair we know from the “happiest thought”.



HOW TO ELIMINATE PRIVILEGED MOTIONS

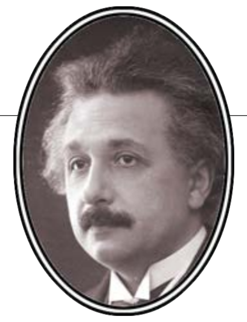


The equivalent
“**happiest thought**”
observers

So far we have compared
one observer **at rest**
(*i.e.* an **inertial observer**) …

… with another observer who is
both **accelerating** (free fall)
and **is in a gravitational field.**

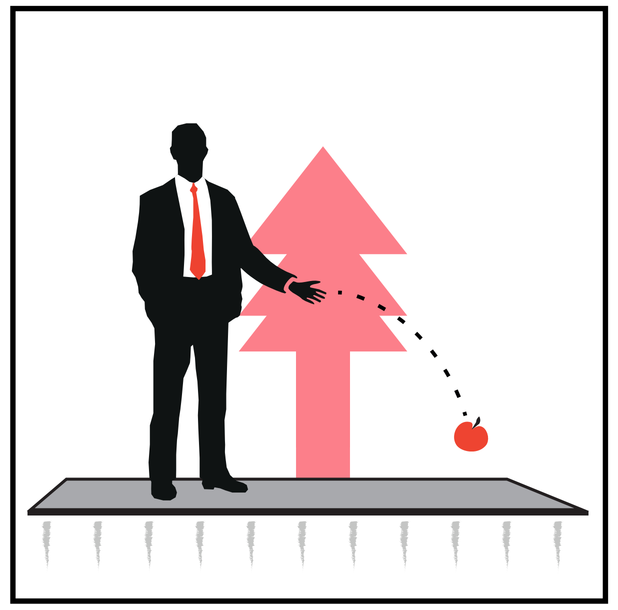
But this is a bit messy, so…



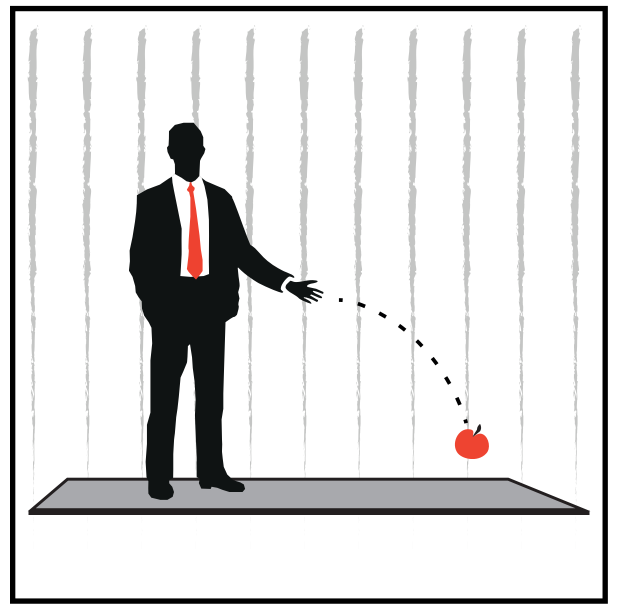
Things become clearer if we make a clean split *between the* **kinematic** *and the* **dynamic** *phenomena.* So from now on we will compare:

one observer who is **accelerating**
in gravity-free space…

…with another observer who is
at rest in a gravitational field.



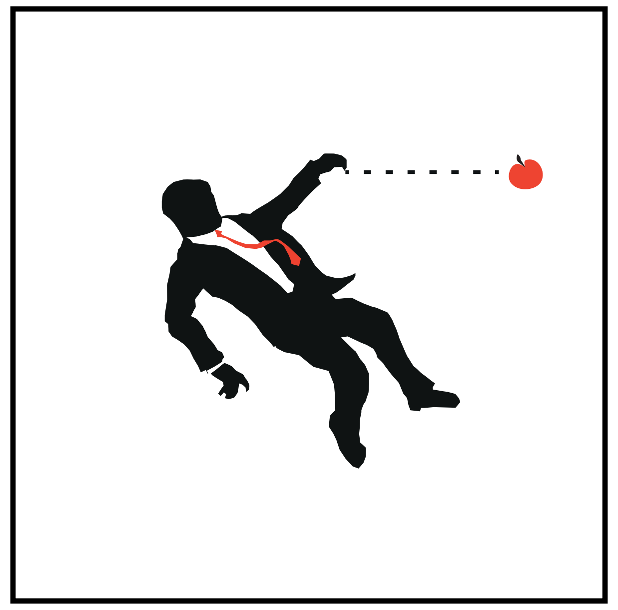
The observed curved path is a **kinematic effect** (acceleration).



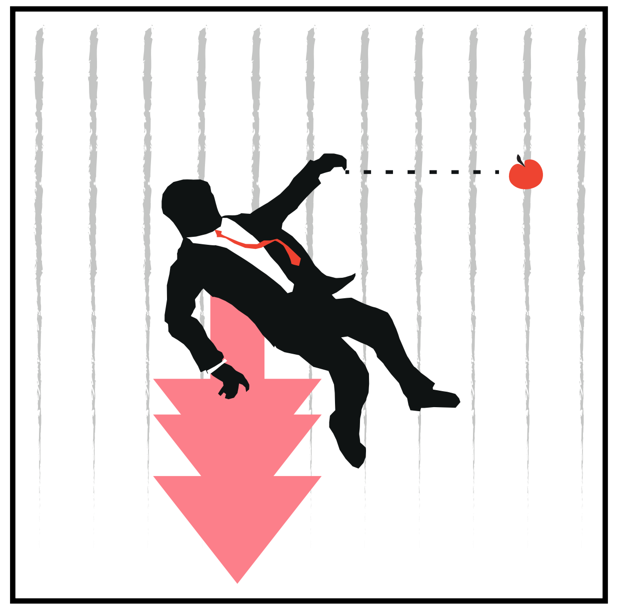
The observed curved path is a **dynamical effect** (gravitational force).

The equivalent "**Einstein elevator**" observers

HOW TO ELIMINATE PRIVILEGED MOTIONS

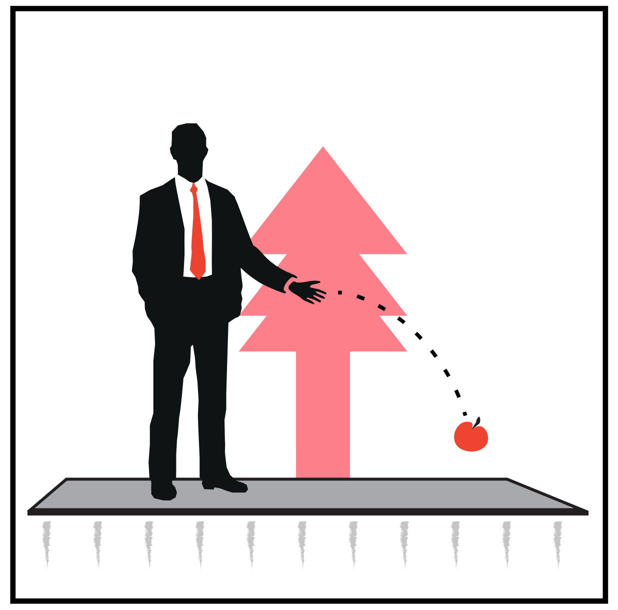


Observer at rest

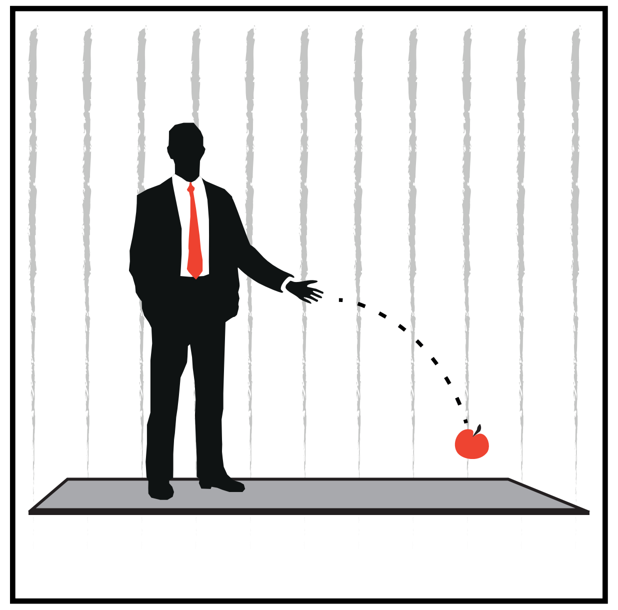


Observer falling freely in a uniform gravitational field

The equivalent "**happiest thought**" observers

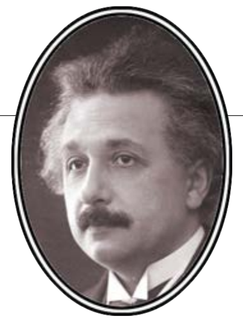


Uniformly accelerated observer in a region with no gravity field

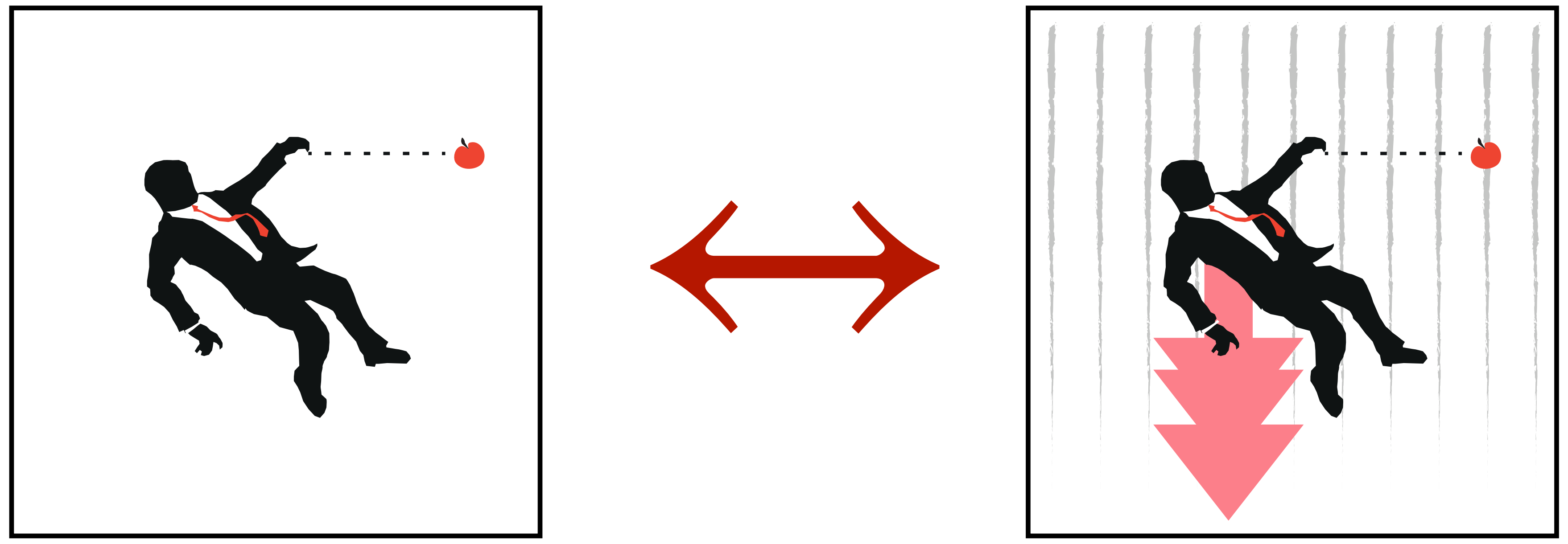


Observer at rest in a uniform gravitational field

The equivalent "**Einstein elevator**" observers



HOW TO ELIMINATE PRIVILEGED MOTIONS



Observer at rest

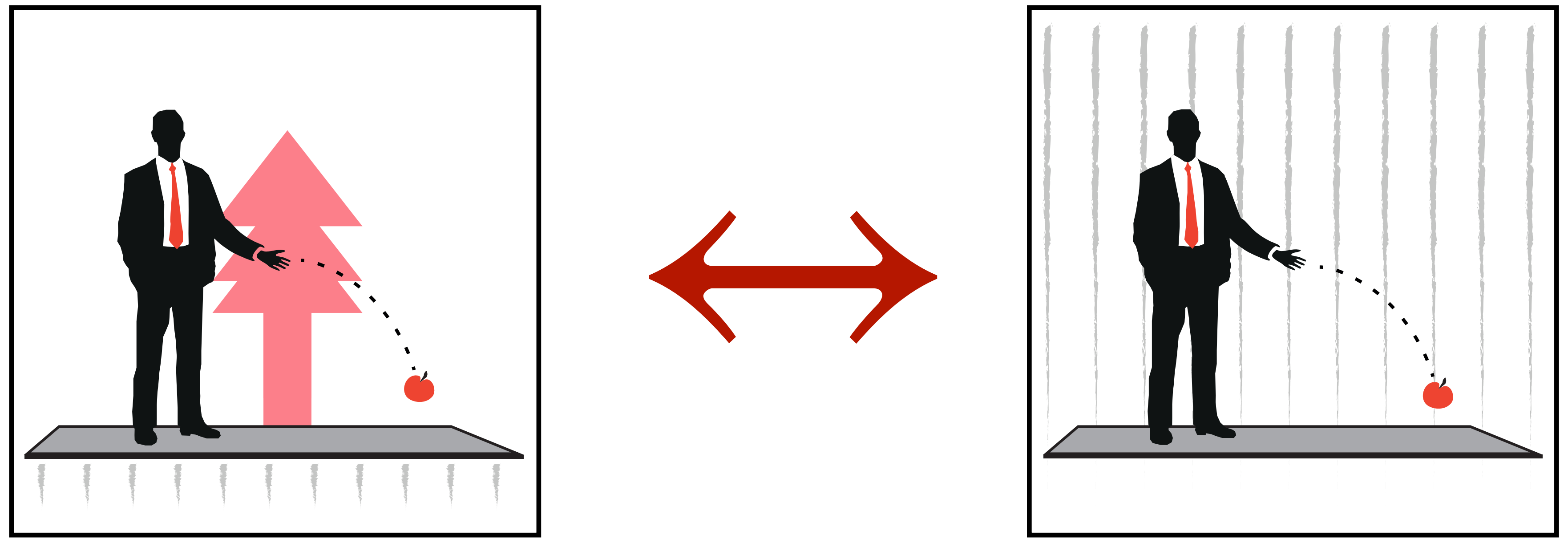
Observer falling freely in a uniform gravitational field

“happiest thought” observers

Equations of motion for body dropped by the observer:

$$\frac{d^2x^\mu}{dt^2} = \frac{d^2y^\mu}{dt^2} = 0, \quad \boldsymbol{\frac{d^2z^\mu}{dt^2} = 0}$$

… **a straight line**



Uniformly accelerated observer in a region with no gravity field

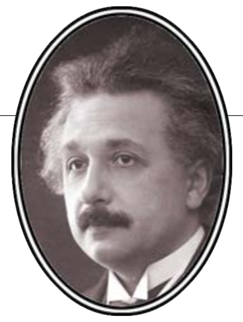
Observer at rest in a uniform gravitational field

“Einstein elevator” observers

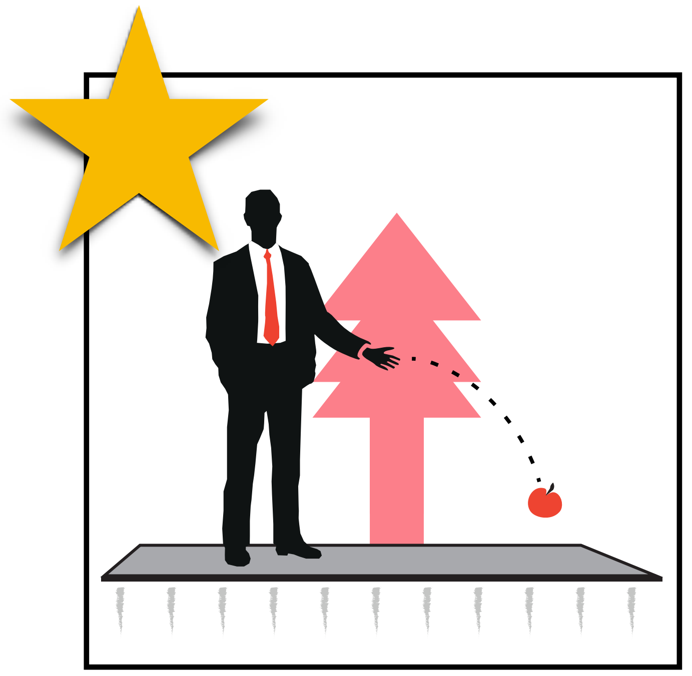
Equations of motion for body dropped by the observer:

$$\frac{d^2x^\mu}{dt^2} = \frac{d^2y^\mu}{dt^2} = 0, \quad \boldsymbol{\frac{d^2z^\mu}{dt^2} = -a}$$

… **a parabola**

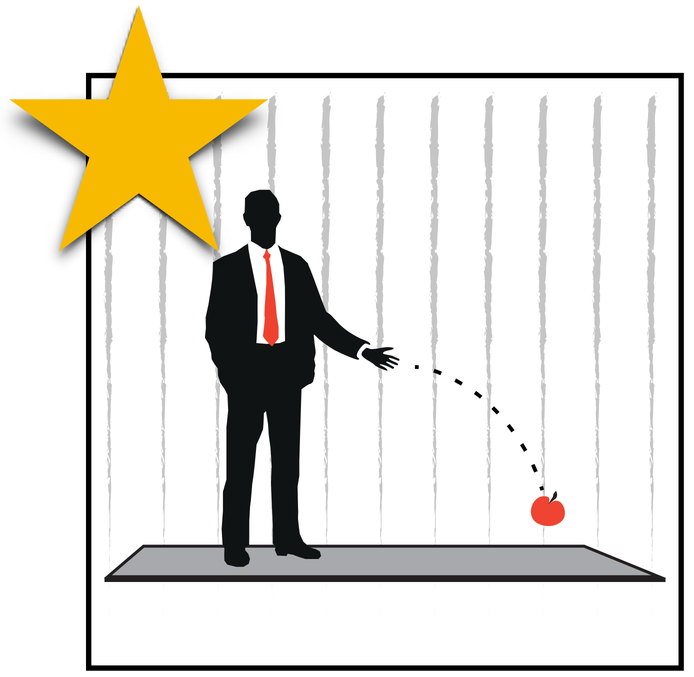


HOW TO ELIMINATE PRIVILEGED MOTIONS



Uniformly accelerated observer in a region with no gravity field

$\equiv$

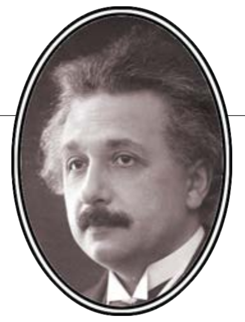


Observer at rest in a uniform gravitational field

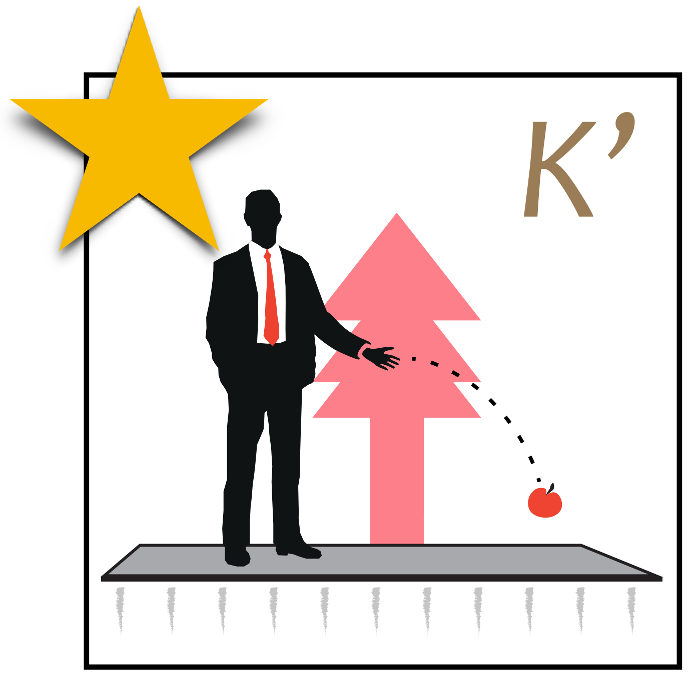
The equivalent "**Einstein elevator**" observers

The "elevator" observer in the gravitational field [*right*] is **privileged** simply because they are **at rest** (inertial observer).

The uniformly accelerated observer [*left*] can claim to be **at rest** (and hence, to be **privileged**) since, thanks to $m_i = m_g$, there is no experiment which can prevent them from claiming to be at rest *in a homogeneous gravitational field*.

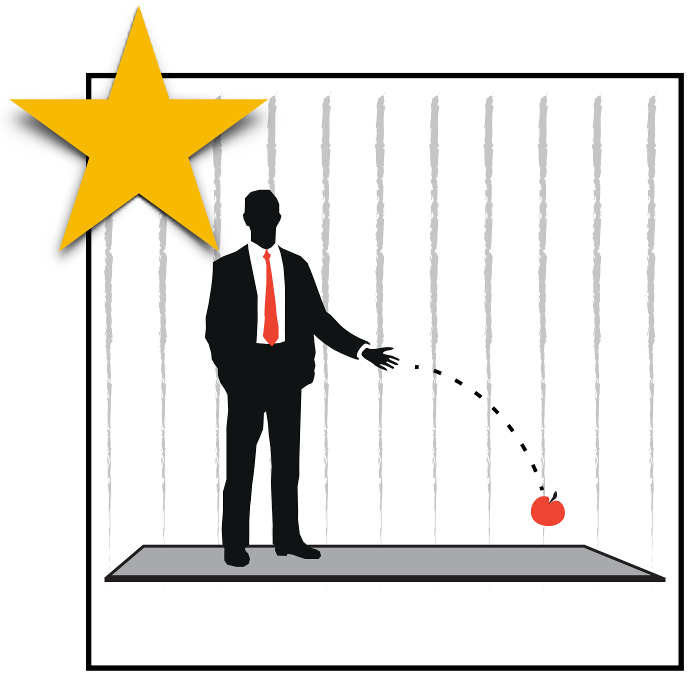


HOW TO ELIMINATE PRIVILEGED MOTIONS



Uniformly accelerated observer in a region with no gravity field

$\equiv$

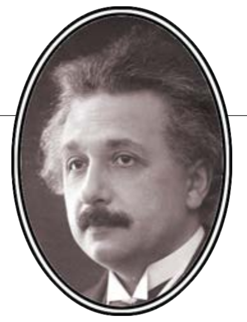


Observer at rest in a uniform gravitational field

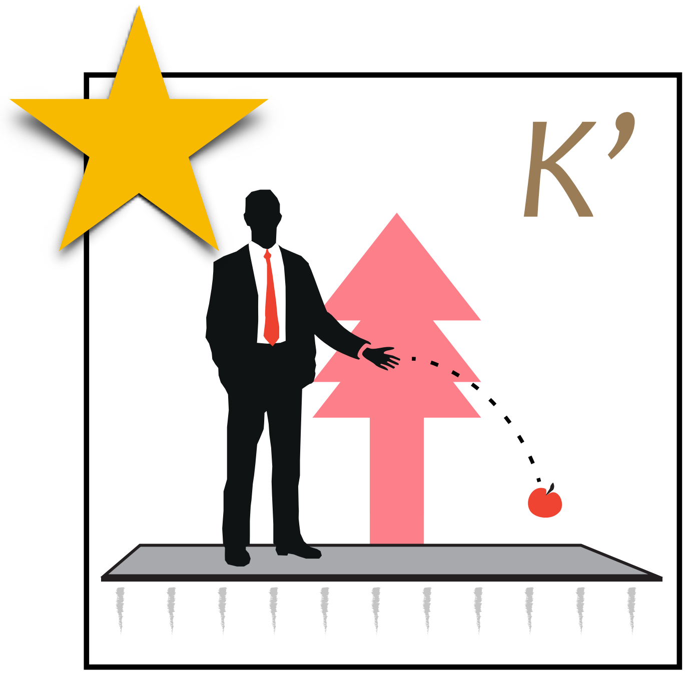
The equivalent “**Einstein elevator**” observers

…[*in a region without gravitational field,*] insofar as we really know the laws of nature, **nothing prevents us from considering a system K'** [*uniformly accelerated relative to an inertial observer*] **as at rest, provided we assume a gravitational field** (homogeneous in first approximation) **relative to K'**.

Albert Einstein, “On Friedrich Kottler's Paper: ‘On Einstein's Equivalence Hypothesis and Gravitation’ ” (1916)

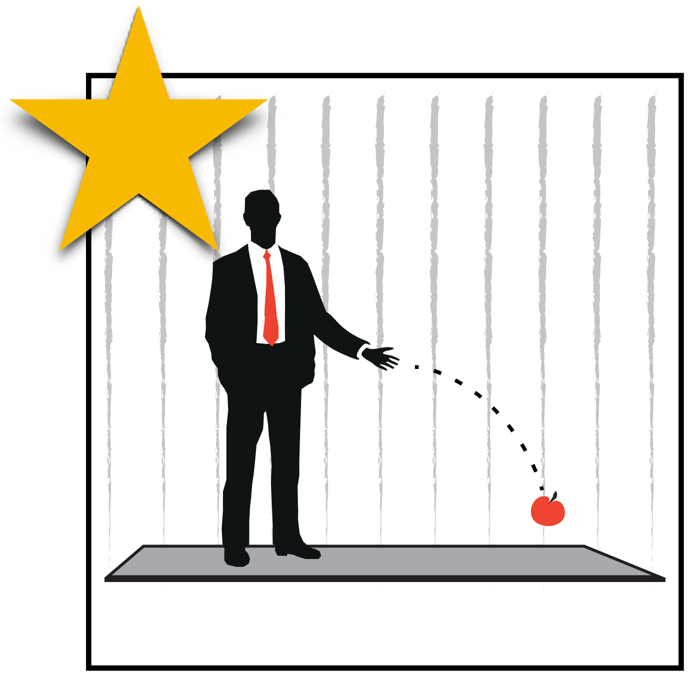


HOW TO ELIMINATE PRIVILEGED MOTIONS



Uniformly accelerated observer in a region with no gravity field

$\equiv$

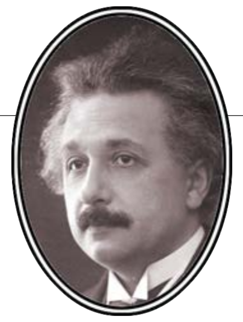


Observer at rest in a uniform gravitational field

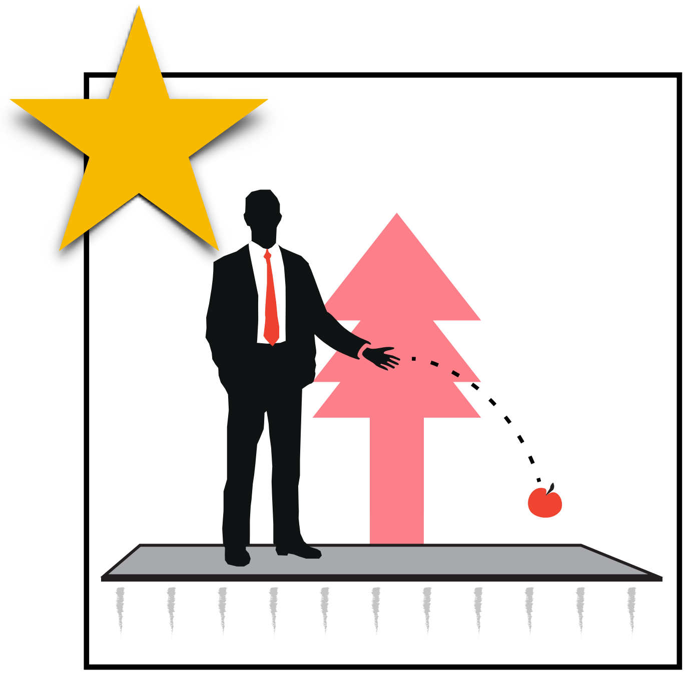
The equivalent "**Einstein elevator**" observers

[In his writings] Einstein described this observation as **a concrete [physical?] way of *expressing*** $m_i = m_g$, the equality of inertial and gravitational mass [or of UFF].

Citat? Se t.ex Autobiographisches

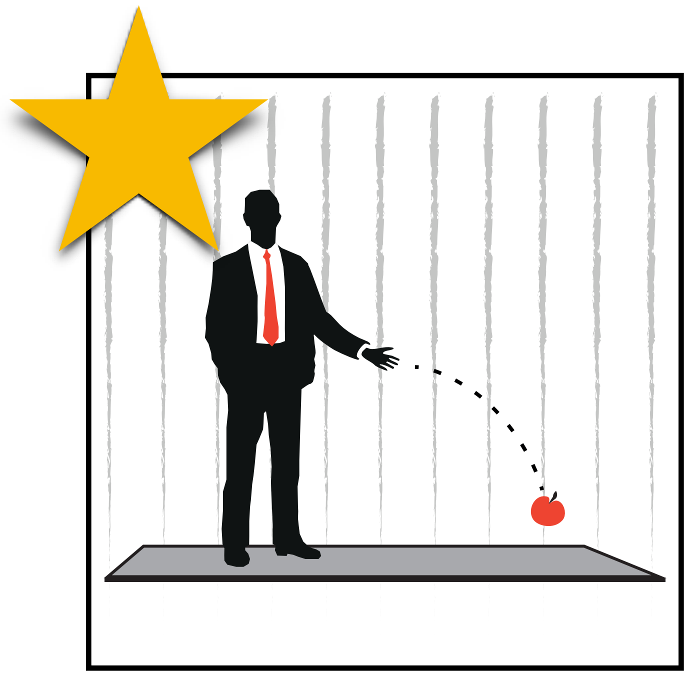


HOW TO ELIMINATE PRIVILEGED MOTIONS



Uniformly accelerated observer in a region with no gravity field

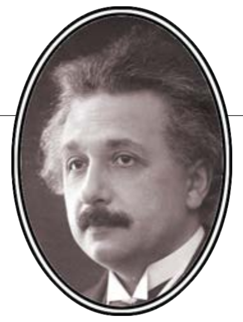
$\equiv$



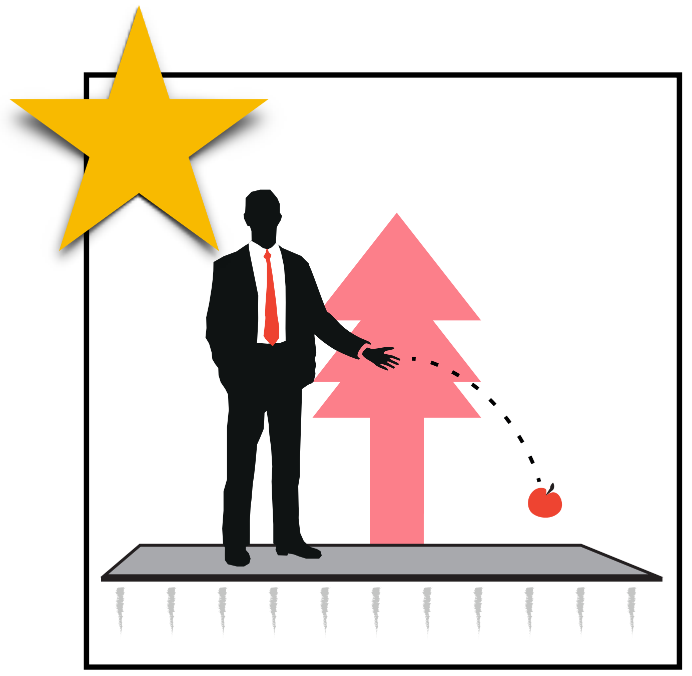
Observer at rest in a uniform gravitational field

The equivalent "**Einstein elevator**" observers

But **there is a problem with the privilege claim** of the accelerated observer…

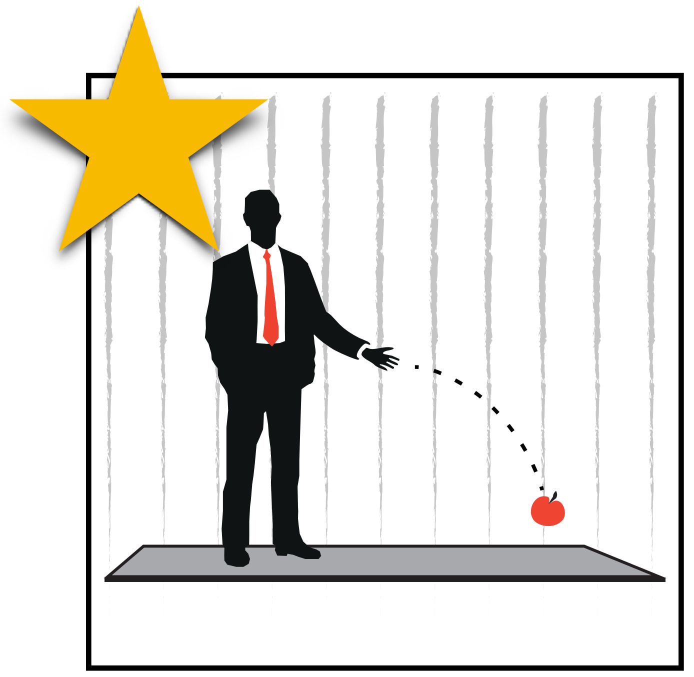


HOW TO ELIMINATE PRIVILEGED MOTIONS



Uniformly accelerated observer in a region with no gravity field

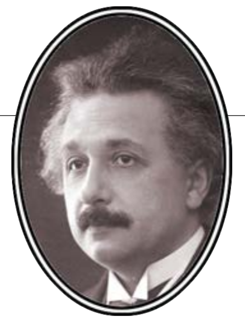
$\equiv$



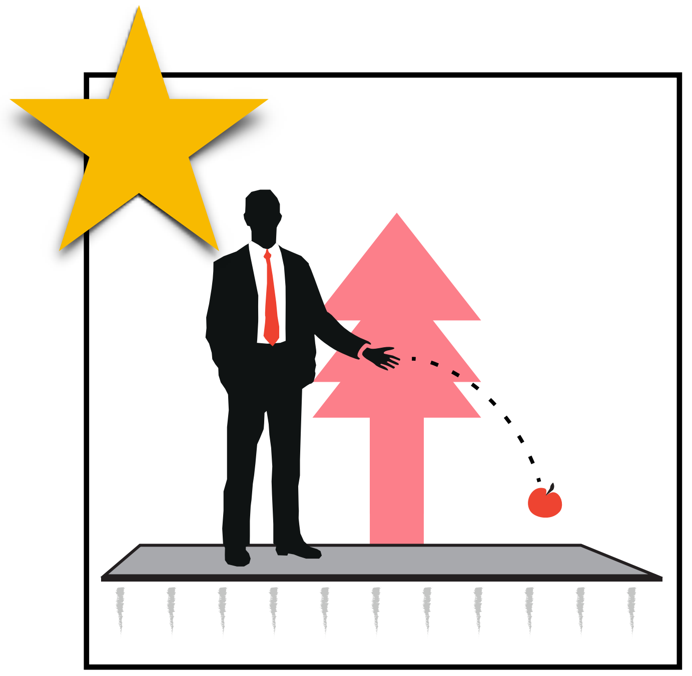
Observer at rest in a uniform gravitational field

The two observers are indeed **equivalent** with respect to the laws of **mechanics**…

(*thanks to* $m_i = m_g$)

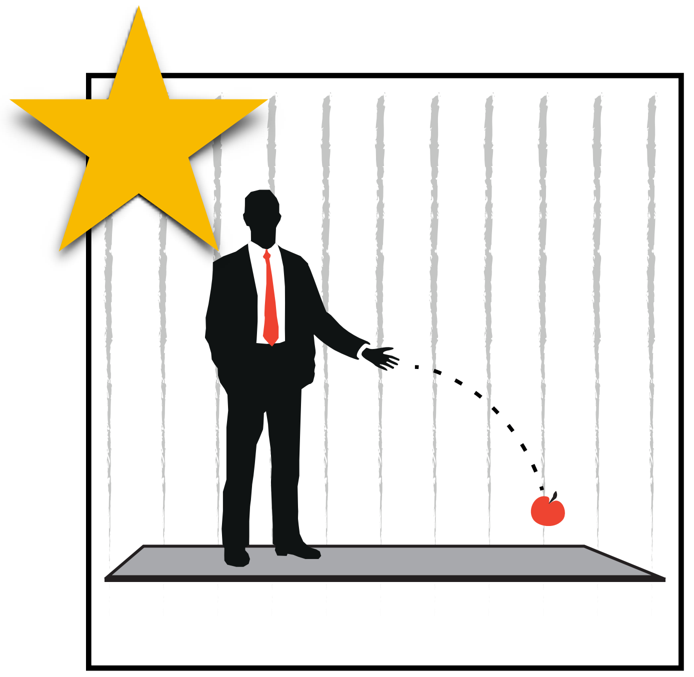


HOW TO ELIMINATE PRIVILEGED MOTIONS



Uniformly accelerated observer in a region with no gravity field

$$\equiv$$



Observer at rest in a uniform gravitational field

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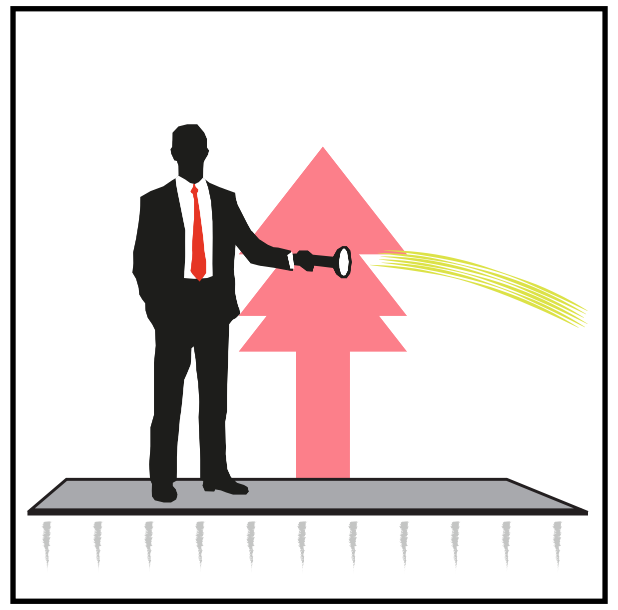
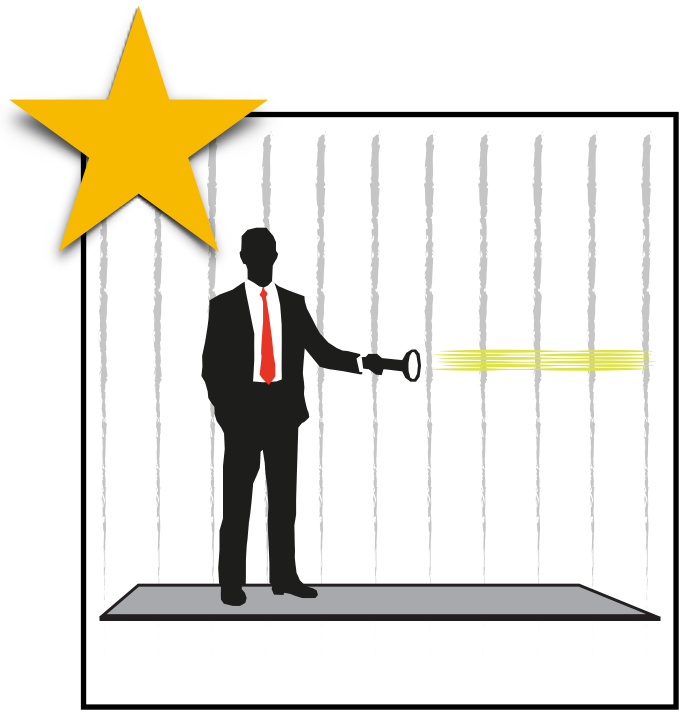


Uniformly accelerated observer in a region with no gravity field

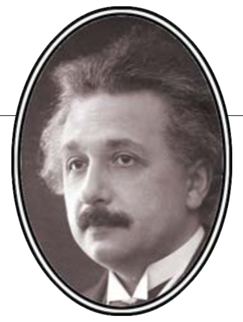
$$\not\equiv$$



Observer at rest in a uniform gravitational field

… but they are **not equivalent** with respect to the laws of **electrodynamics**!

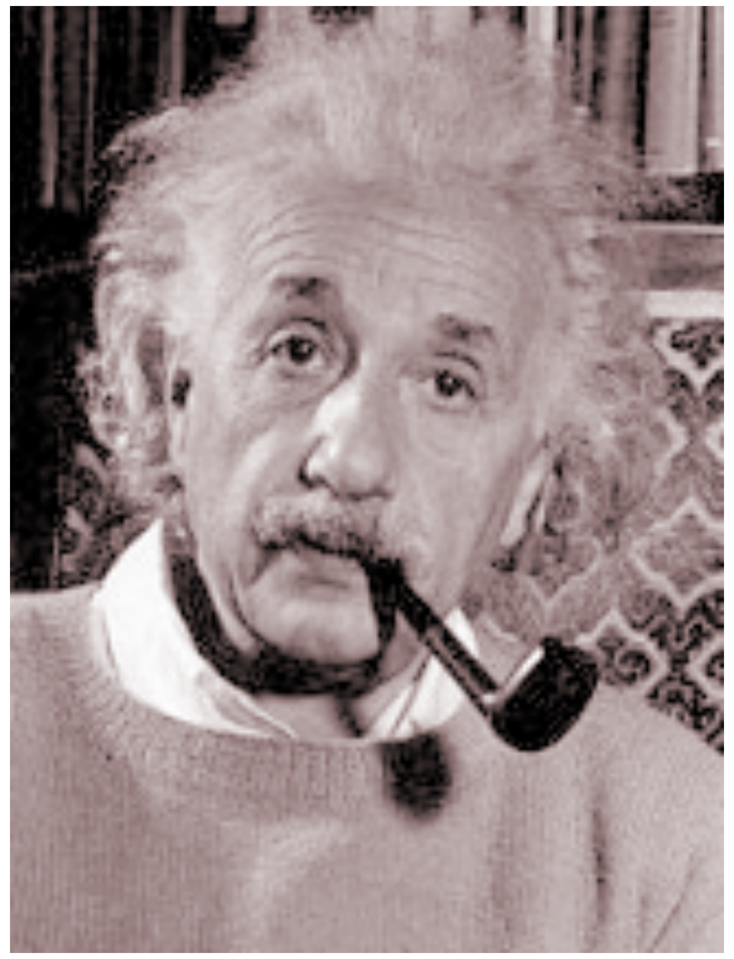
(*Newtonian gravity doesn't bend light*)



… here we go again.

This **non-equivalence seems to spoil Einstein's hope** for an extension of the Relativity Principle

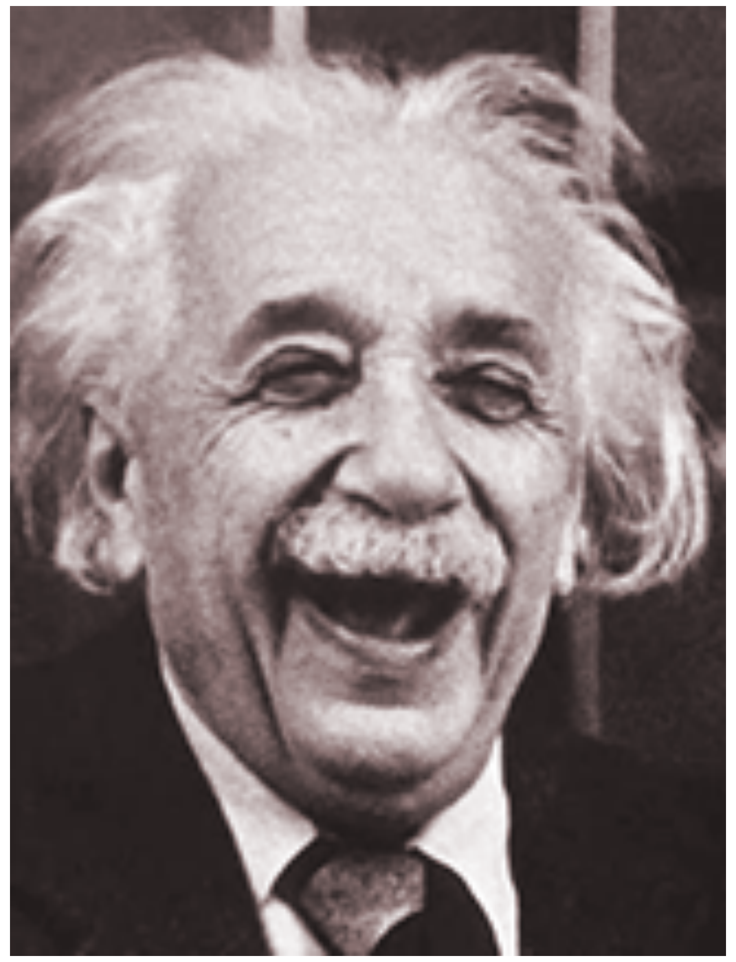
(since the "happiest thought" only works in Newtonian physics).



HOW TO ELIMINATE PRIVILEGED MOTIONS

But Einstein would not
give up that easily!

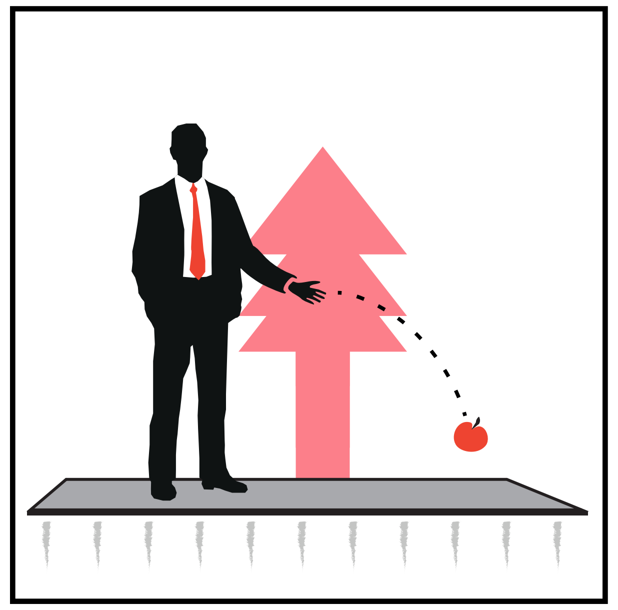
…he just **postulated that** (the still
to be discovered new theory of
gravitation should be such that)
**the two observers are equivalent
with respect to *all* laws of physics.**



HOW TO ELIMINATE PRIVILEGED MOTIONS

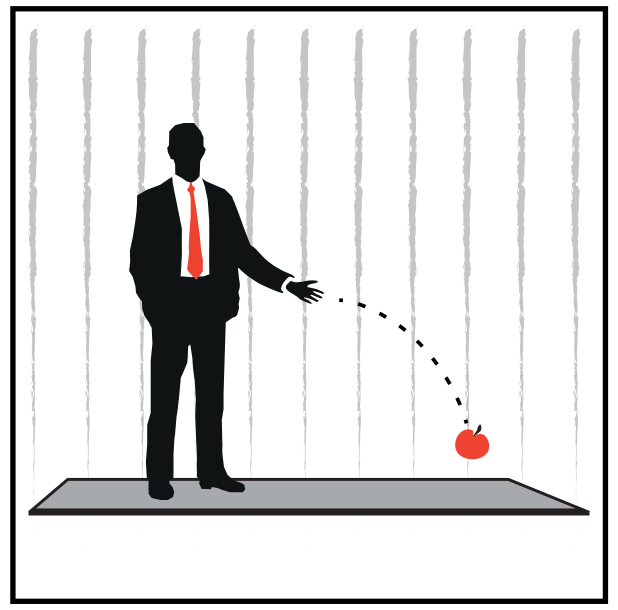
> The principle of the equality of inertial and gravitational mass could now be formulated quite clearly as follows: **In a homogeneous gravitational field all motions take place in the same way as in the absence of a gravitational field in relation to a uniformly accelerated coordinate system…**

Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)

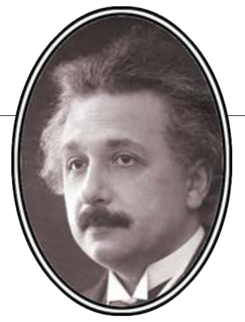


Uniformly accelerated
observer in a region
with no gravity field

$\equiv$



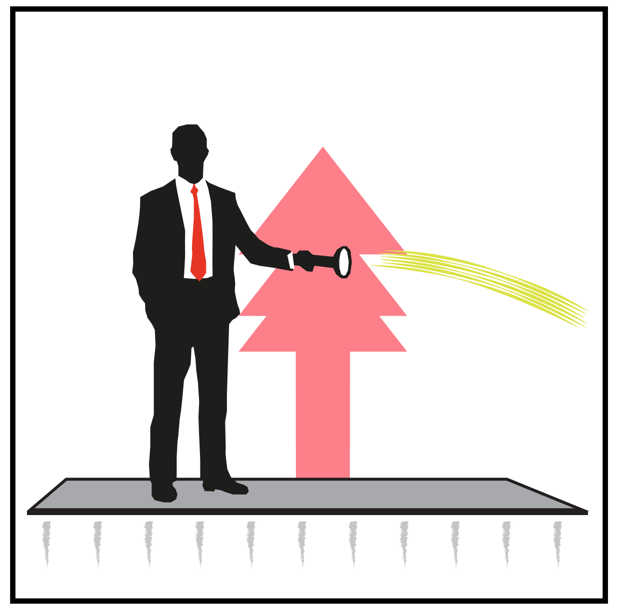
Observer at rest
in a uniform
gravitational field



HOW TO ELIMINATE PRIVILEGED MOTIONS

…If this principle held good for any events whatever (the "principle of equivalence"), this was an indication that **the principle of relativity needed to be extended** to coordinate systems in non-uniform motion with respect to each other, if we were to **reach a natural theory of the gravitational fields.**

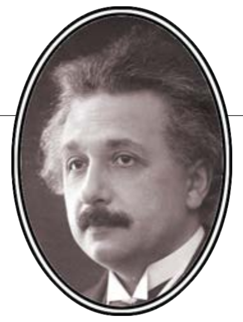
Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)



Uniformly accelerated observer in a region with no gravity field

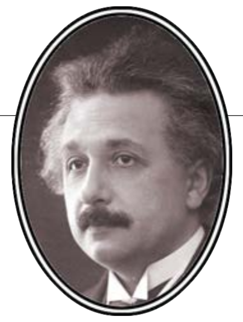
≡

Observer at rest in a uniform gravitational field



Another Einstein quote clarifies the logic of the equivalence argument even further:

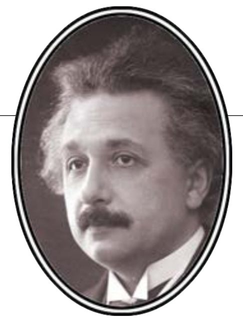
- Newtonian theory ensures the equivalence of the systems **only for mechanical processes**
- Only if by hypothesis we **extend equivalence to all physical processes** can we use equivalence as an argument to eliminate privileged motions, or to explain the observed numerical equivalence of the inertial and gravitational masses.



Another Einstein quote clarifies the logic of the "equivalence hypothesis" argument even further:

> **As long as we confine ourselves to purely mechanical processes** within the range of validity of Newton's mechanics, **we can be sure of the equivalence of the systems *K* and *K'*.**
>
> However, for our conception [*of equivalence*] to acquire deeper significance, **the systems *K* and *K'* must be equivalent with respect to all physical processes,** i.e., the natural laws with respect to *K* must coincide completely with those with respect to *K'*.
>
> If we accept this assumption, **we obtain a principle that possesses great heuristic significance…**

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



HOW TO ELIMINATE PRIVILEGED MOTIONS

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* *Methodological footnote:*

We can describe Einstein's "**equivalence hypothesis**" from 1907 like this:

The validity in Newtonian mechanics of *the universality of free fall* is a source of inspiration — but **its full heuristic potential** for finding a relativistic theory of gravitation **can only be cached out if we extend its validity beyond Newtonian mechanics to all physical phenomena.**

This is a little reminiscent of Einstein's treatment in 1905 of the original Galilean **relativity principle:**

The relativity principle is strictly valid only in Newtonian mechanics (and not in ether-based electrodynamics) — but **its full heuristic potential** for resolving conceptual problems in classical mechanics **can only be cached out if we extend its validity beyond Newtonian mechanics to all physical phenomena.**

We can now get an overview of the long-term project [ifr diagram med "ontological unification"]...

RP: all inertial observers equivalent wrt mechanics

SR (1905)

RP: all inertial observers equivalent wrt mechanics and electrodynamics

ÄP (1907)

RP: all inertial observers and observers in uniform acceleration equivalent wrt mechanics, electrodynamics and uniform gravitational fields

GR (1915)

RP: all observers equivalent wrt all physical phenomena

(*) As always "equivalence of observers" means that the laws of physics will take on exactly the same form for them all.

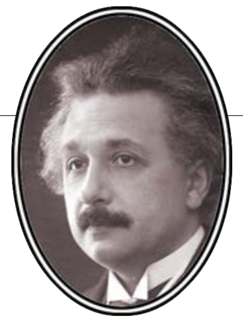
THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

We have seen that the "*Äquivalenzprinzip* Heuristic" was originally formulated for **uniform linearly accelerated motion.**

From it Einstein could deduce **a number of properties that the new relativistic theory of gravity ought to have:**

- *rate of clocks depends on gravitational potential*
  — gravitational redshift
- *light rays bend in a gravitational field*
  — electromagnetism couples to gravity
  — no global Lorentz invariance (need to break out of SR)
- *energy has weight (not just inertia as in SR)*

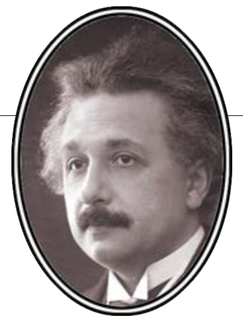


THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

Later Einstein applied the "*Äquivalenzprinzip* Heuristic" also to **uniform rotational motion.**

From this follow **a few more tentative properties of relativistic gravitational fields:**

- *3-geometry need not be Euclidean*
  — space-time geometry not flat
- *may not be possible to be define global coordinate time*
- *gravitational induction*



THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

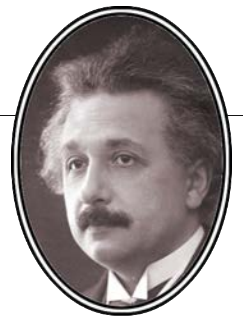
TO AVOID ANY MISUNDERSTANDINGS…

…it is important to remember that

all the results obtained from the
"*Äquivalenzprinzip* Heuristic"
must be seen as *purely qualitative*,

They are just ***hints*** at what we ***might expect*** form the still elusive complete theory of relativistic gravity.

After all: these results **have only been obtained for a *very special* gravitational field,** the perfectly homogeneous one.



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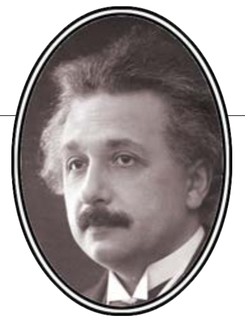
After all: these results **have only been obtained for a *very special* gravitational field,** the perfectly homogeneous one.

An example: the value obtained this way for **the bending of light** is **a factor 1/2 off** from the correct GR prediction.

…But hey! That’s not bad for a rough heuristic!



DICKE'S SPLIT OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE INTO WEP AND SEP

THE MODERN EQUIVALENCE PRINCIPLES

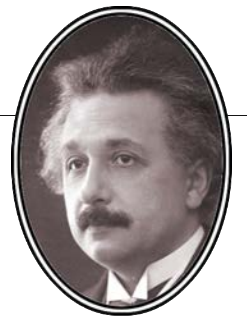
The distinction between and naming convention for the two versions of the Equivalence Principle was **introduced in the early 1960's by Robert Dicke**.

THE MODERN EQUIVALENCE PRINCIPLES

The distinction between and naming convention for the two versions of the Equivalence Principle was **introduced in the early 1960's by Robert Dicke**.

Dicke's group had just improved on earlier experiments by Roland Eötvös, and had **determined the numerical equality of inertial and gravitational mass** to a precision of $3 \cdot 10^{-11}$.

Dicke wanted to distinguish clearly between **that part of the Equivalence Principle which gained empirical support from the Eötvös-type experiments** (the "weak" version) and that part which was unaffected by the experimental results (the "strong" version).



RECONSIDERING THE SEP

eak Equivalence Principle valid in NM? — ***no***

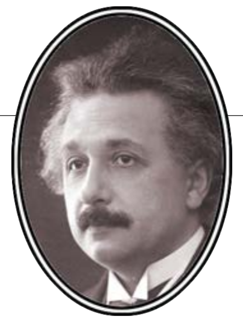
eak Equivalence Principle valid in GR? — ***no***

eak Equivalence Principle valid in QM? — ***no***

Conclusion:

In GR the proposition **SEP must be formulated in terms of an approximation scheme.**

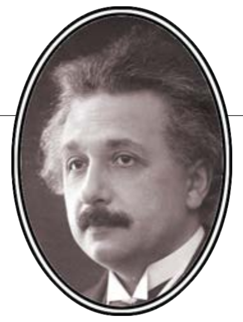
But what justification is there for calling an approximation scheme a "principle"?



XXX

"In order to understand the limited conclusions to be drawn from the Eötvös experiment, we must first consider the significance of the equivalence principle for relativity. In this connection it may be convenient to make a distinction between the strong equivalence principle upon which Einstein's general relativity is based and the weak equivalence principle supported by Eötvös experiment."

Robert Dicke, "Experimental Relativity" (1963)



MODERN EQUIVALENCE PRINCIPLES — WEP

Dicke's **original statement** of the Weak Equivalence Principle:

> The **weak principle of equivalence** states only that the local gravitational acceleration is substantially independent of the composition and structure of the matter being accelerated.

Robert Dicke, "Experimental Relativity" (1963)

The **standard statement** of the WEP is due to Clifford Will:

> **Weak equivalence principle (WEP):**
>
> The property of a body called "mass" is proportional to the "weight". Or, the trajectory of a freely falling "test" body is independent of its internal structure and composition.

Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)

MODERN EQUIVALENCE PRINCIPLES — WEP

Notice that Will's statement "*the trajectory of a freely falling 'test' body is independent of its internal structure and composition*" is just **our old friend the Universality of Free Fall (UFF).**

> **Universality of Free Fall (UFF):**
>
> Two bodies dropped in a gravitational field fall with the same acceleration (simplest application of WEP).

Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)

[In their 1973 textbook "Gravitation", Misner, Thorne and Wheeler preferred to **talk about *UFF* rather than use Dicke's label *WEP* "to avoid confusion"**, but unfortunately this good advice has since been ignored.]

De facto standard definition by Will

Nowadays, **the standard formulation of the two modern equivalence principles** comes from Clifford Will:

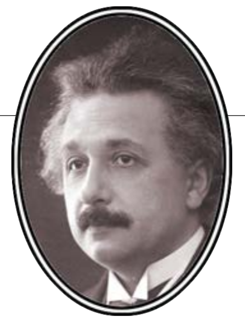
Weak equivalence principle (WEP):

The property of a body called "mass" is proportional to the "weight". Or, the trajectory of a freely falling "test" body (one not acted upon by such forces as electromagnetism and too small to be affected by tidal gravitational forces) is independent of its internal structure and composition.

Universality of Free Fall (UFF):

Two bodies dropped in a gravitational field fall with the same acceleration (simplest application of WEP).

Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)

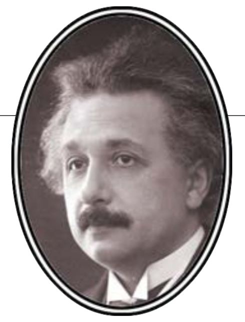


MODERN EQUIVALENCE PRINCIPLES — WEP

Weinberg's formulation of the Weak Equivalence Principle:

> **"Weak" Principle of Equivalence:**
> At every space-time point in an arbitrary gravitational field it is possible to choose a "locally inertial coordinate system" such that, within a sufficiently small region of the point in question, the laws of motion of freely falling particles take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation.

Steven Weinberg, "Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity" (1972)

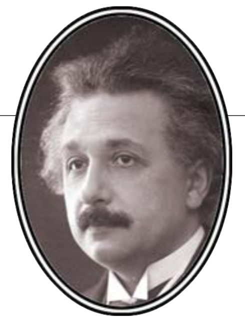


MODERN EQUIVALENCE PRINCIPLES — WEP

Hartle's formulation in his 2003 textbook is not framed in terms of motion of test bodies, but is a statement about properties of (a special case of) gravitational fields:

> **Einstein's equivalence principle**
> is the idea that *there is no experiment that can distinguish a uniform acceleration from a uniform gravitational field*. The two are fully "equivalent".

James Hartle, "An Introduction to Einstein's General Relativity" (2003)



Alternative naming convention by MTW

Universality of Free Fall (UFF):

Two bodies dropped in a gravitational field fall with the same acceleration (simplest application of WEP).

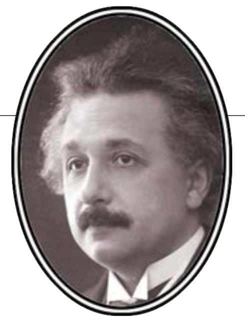
Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)

Principle of the uniqueness of free fall:
The world line of a freely falling test body is independent of its composition or structure.

Charles Misner, Kip Thorne, John A. Wheeler, "Gravitation" (1973)

One fundamental building block common to Einstein's theory of gravity and to almost all other modern theories is the principle of "uniqueness of free fall". R. H. Dicke calls this principle "The weak equivalence principle." We prefer to avoid confusion with the [strong] equivalence principle.

Charles Misner, Kip Thorne, John A. Wheeler, "Gravitation" (1973)



MODERN EQUIVALENCE PRINCIPLES — WEP

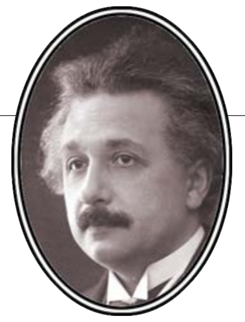
→ ***Is the Weak Equivalence Principle valid in GR?***

We have seen that the universality of free fall **WEP/UFF holds in Newtonian physics**, but in general relativity the situation is more complicated.

In GR the claim is that free particles move along geodesics of the metric, and that this is not a postulate but can be derived from the Einstein field equations (like with the Geroch-Jang or Ehlers-Geroch theorems).

But **these derivations only hold approximately**, and only for very special types of matter (non-rotating structureless test particles).

So, as a general statement, **WEP/UFF is not valid in GR**.



MODERN EQUIVALENCE PRINCIPLES — WEP

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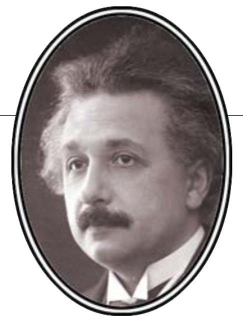
In addition, **WEP/UFF is not valid in QM**: fluctuations around the Ehrenfest-theorem path of a quantum particle are mass-dependent $\sim \hbar / m_i$, so UFF is only approximately valid (in the limit of heavy quantum particles).



→ ***Is the Weak Equivalence Principle valid in GR?***

Conclusion:

In GR the proposition **WEP/UFF must be formulated in terms of an approximation scheme.**

But what justification is there for calling an approximation scheme a "principle"?

How Einstein understood it

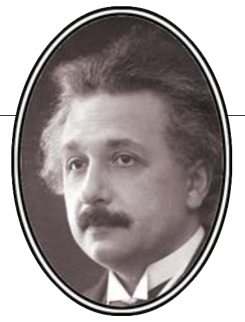
Einstein, however, never talked about the Universality of Free Fall as a "principle", neither of "equivalence" or of anything else:

Referring back to Galilei

> [...] the old **experimental fact** that all bodies have the same acceleration in a gravitational field. This **law** [...]

Albert Einstein, "Notes on the Origin of the General Theory of Relativity", lecture manuscript (1933)

Let the WEP go back to what it was before Dicke's intervention: an experimental fact (relevant in the context of Newtonian theory) that we often call UFF.



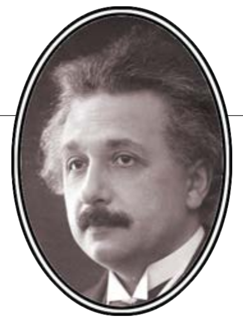
MODERN EQUIVALENCE PRINCIPLES — SEP

This is how Robert Dicke characterized the idea that he called the "Strong Equivalence Principle", and which he mistakenly thought originated with Einstein:

> The **strong equivalence principle** might be defined as the assumption that in a freely falling, non-rotating, laboratory the local laws of physics take on some standard form, including a standard numerical content, independent of the position of the laboratory in space and time.

Robert Dicke, "Experimental Relativity" (1963)

The "**standard form**" of the local laws of physics is commonly assumed to be the form they have according to **special relativity**.



MODERN EQUIVALENCE PRINCIPLES — SEP

Nowadays, **the standard formulation of the strong equivalence principle** comes from Clifford Will. It is a complicated way of saying that laws describing local non-gravitational physics must be Poincaré-invariant (special relativity again!):

Einstein equivalence principle (EEP):

- The Weak Equivalence Principle (**WEP**) is valid…
- …and in addition *the outcome of any local non-gravitational experiment* is independent of the velocity of the freely-falling reference frame in which it is performed (**Local Lorentz Invariance, LLI**)
- …and is independent of where and when in the universe it is performed (**Local Position Invariance LPI**).

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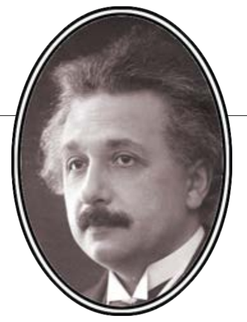

MODERN EQUIVALENCE PRINCIPLES — SEP

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“Medium-Strong” Principle of Equivalence: …

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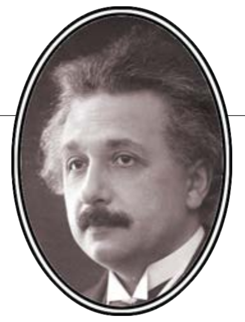
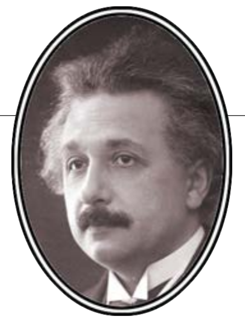


→ ***Is the Strong Equivalence Principle valid in GR?***

The common idea to all variations of the “SEP” is that **we can recover the laws of gravitation-free special relativity** if we restrict our attention to special frames of reference **in infinitesimal regions of space-time**.

But xxx [Ohanian quote?]…

Therefore, strictly speaking, **SR is *not* recovered in infinitesimal regions** around a space-time point in curved space-time models in GR, since the curvature (tidal effects) is a tensor and therefore does not vanish at a point.



MODERN EQUIVALENCE PRINCIPLES — SEP

→ ***Is the Strong Equivalence Principle valid in GR?***

This can be avoided by **an epsilon-delta-type formulation of the SEP**, restricting its validity to small enough space-time regions for tidal effects not to make any detectable difference:

> **$\forall\delta\,\exists\varepsilon$ Equivalence Principle (DEEP):**
>
> In any theory of physics which satisfactorily includes gravitation, for any order of approximation $\delta > 0$, we can always find a small neighbourhood of size $\varepsilon > 0$ around any point and a chart thereon in which the laws of motion of matter (but not gravitation) approximate their standard special relativistic form to order δ.

Michel Ghins, Tim Budden, "The Principle of equivalence" (2001)

MODERN EQUIVALENCE PRINCIPLES — SEP

→ ***Is the Strong Equivalence Principle valid in GR?***

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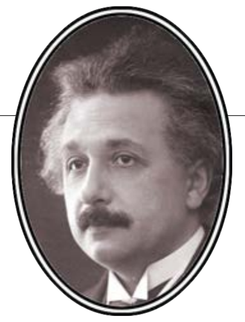

MODERN EQUIVALENCE PRINCIPLES — SEP

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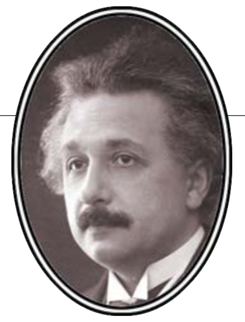
> **But what justification is there for calling an approximation scheme a "principle"?**



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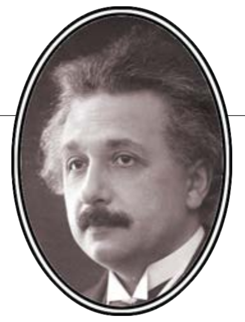

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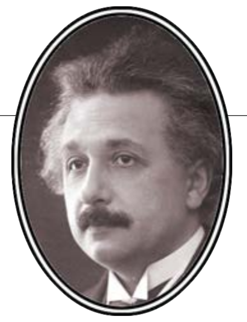

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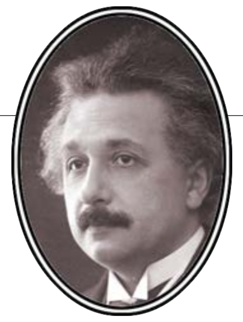

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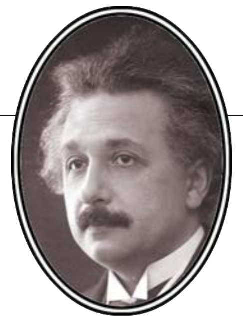
The common idea to all variations of the “SEP” is that **we can recover the laws of gravitation-free special relativity** if we restrict our attention to special frames of reference **in infinitesimal regions of space-time**.

But xxx [Ohanian quote?]...

Therefore, strictly speaking, **SR is *not* recovered in infinitesimal regions** around a space-time point in curved space-time models in GR, since the curvature (tidal effects) is a tensor and therefore does not vanish at a point.



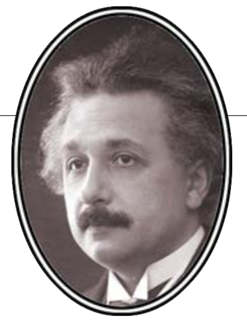
MODERN EQUIVALENCE PRINCIPLES — SEP

→ ***Is the Strong Equivalence Principle valid in GR?***

This can be avoided by **an epsilon-delta-type formulation of the SEP**, restricting its validity to small enough space-time regions for tidal effects not to make any detectable difference:

> **$\forall\delta\,\exists\varepsilon$ Equivalence Principle (DEEP):**
>
> In any theory of physics which satisfactorily includes gravitation, for any order of approximation $\delta > 0$, we can always find a small neighbourhood of size $\varepsilon > 0$ around any point and a chart thereon in which the laws of motion of matter (but not gravitation) approximate their standard special relativistic form to order δ.

Michel Ghins, Tim Budden, "The Principle of equivalence" (2001)



→ ***Is the Strong Equivalence Principle valid in GR?***

Xxx

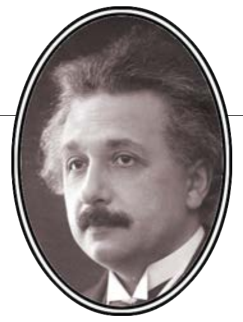
MODERN EQUIVALENCE PRINCIPLES — SEP

→ ***Is the Strong Equivalence Principle valid in GR?***

Conclusion:

In GR the proposition **SEP must be formulated in terms of an approximation scheme.**

But what justification is there for calling an approximation scheme a “principle”?



How Einstein understood it

But this is of course basically a statement of how to find the **special relativity limit of general relativity.**

Einstein, however, never talked about the special relativity limit as a "principle", either of "equivalence" or anything else.

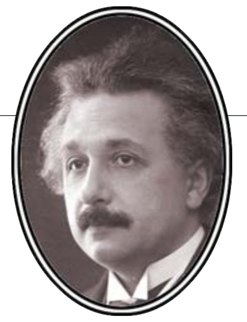
SEP = SR LIMIT

E. Citat om sr limit.

Albert Einstein, xxx (19xx)

This led me to the view that the customary theory of relativity provides only an approximation to reality; it s apply only in the limit case where differences in the gravitational potential in the space-time region under consideration are not too great.

Albert Einstein, Marcel Grossmann, "Outline of a
Theory of Relativity and of a Theory of Gravitation"



MODERN EQUIVALENCE PRINCIPLES — SEP

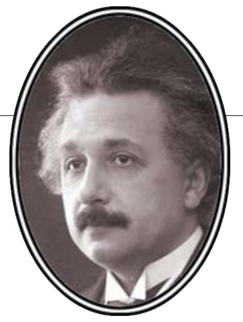
How Einstein understood it

SEP = SR LIMIT

However, **one must be careful not to assume that,** conversely, **every gravitational field can be made to vanish,** i.e., can be turned into a gravitation-free region, by means of a suitable choice of the coordinate system. For example, it is impossible to make the gravitational field of the Earth vanish by means of a suitable choice of the coordinate system.

In fact, for a region of *finite extension* this is only possible with gravitational fields of a very special kind. But for an infinitely small region the coordinates can always be chosen such that no gravitational field will be present in it. **With respect to such an infinitely small region one may then assume that the special theory of relativity is valid.** That way the general theory of relativity is connected with the special theory of relativity, and the results of the latter can be utilized for the former.

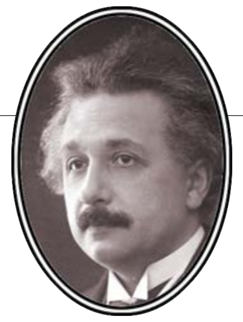
Albert Einstein, Theory of Relativity (1924)



MODERN EQUIVALENCE PRINCIPLES — SEP

In his book "Principles of Relativity Physics" from 1967, J. L. Anderson discusses the same ideas as those which Dicke at the same time calls SEP, but calls them a "**principle of minimal coupling**". …

> "But, unlike Dicke and other authors, **we do not hold that such a principle is essential to general relativity;** it is only a useful principle of selection that may or may not be valid". (In "Covariance, Invariance, and Equivalence" 1970)



MODERN EQUIVALENCE PRINCIPLES — SEP

Alternative naming convention by MTW

In the spirit of Einstein we can reinterpret SEP as a heuristic for GR<->SR

Einstein's equivalence principle:

In any and every local Lorentz frame, anywhere and anytime

physics

ns.

Comma-goes-to-semicolon rule:

The laws of physics, written in component form, change on passage from flat spacetime to curved spacetime by a mere displacement of all commas by semicolons (nothing but a rephrased version of the equivalence principle).

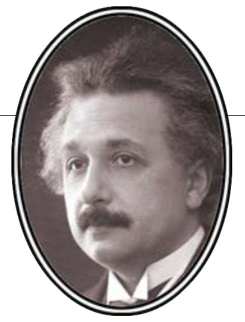
Charles Misner, Kip Thorne, John A. Wheeler, "Gravitation" (1973)

Minimal coupling

Comma… rule is the heuristic content of SEP

MTW citat om att SEP är relaterat till (,->;)

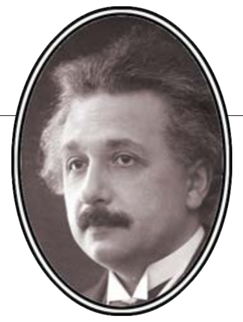
Charles Misner, Kip Thorne, John A. Wheeler, "Gravitation" (1973)



We can **summarise the physical content** of the two modern equivalence principles as follows:

$$\text{WEP} \quad \equiv \quad \text{UFF}$$

$$\text{SEP} \quad \equiv \quad \begin{cases} \text{GR} \rightarrow \text{SR limit} \\ \text{SR} \rightarrow \text{GR bootstrap } (,\!\rightarrow;) \end{cases}$$



TAKEAWAY

The physical content of the two modern Equivalence principles is:

XXX

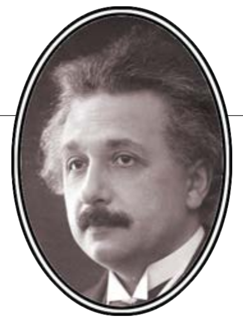
XXX

THE MODERN EQUIVALENCE PRINCIPLES

Notice how the WEP and SEP are usually formulated using concepts that are only well defined in Newtonian mechanics and gravity:

… citat…

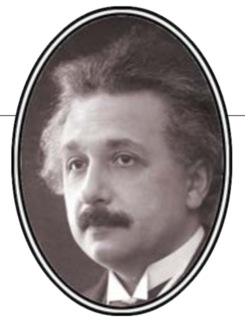
And we have seen how, in Einstein's view, the notions of "inertia" and "gravitational field" cannot be defined independently of each other, so therefore it makes no sense — within GR — to make any statements about relations between them (relations of "equivalence" or of anything else).



THE MODERN EQUIVALENCE PRINCIPLES

WEP, SEP and ÄP **have in common the intuitive idea** that "gravitational and inertial effects are indistinguishable, at least locally" (Einstein's "happiest thought").

But they differ both in the way they try to make this intuitive idea more precise, and, more importantly, in their respective heuristic value and the role they play in the finished theory of general relativity.



THE MODERN EQUIVALENCE PRINCIPLES

Trautmann's "weak EP" (not to be confused with Dicke's WEP), suggesting that what is essential to general relativity is that the requirement that all inertial effects can be duplicated by gravitational effects. (OBS, alltså bara att tröghet och "kinematiska grav.fält" är likvärda)

"**To the extent that we can neglect their action back on the sources of the gravitational field present in a given space-time region, measurements made on any physical system will serve to determine the same affinity in this region**. By affinity, we mean here that quantity which must be used to convert ordinary derivatives into covariant derivatives in the covariant form of the dynamical laws of of the system. This weak form of the principle allows the possibility of systems that are not coupled minimally to the gravitational field while, at the same time, it retains the essential content of equivalence, namely that inertial effects are indistinguishable from gravitational effects." (Citat från JL Anderson "Covariance…" 1970). Som kontrast mot Dickes tidigare def av SEP som "reduction to laws of SR locally"

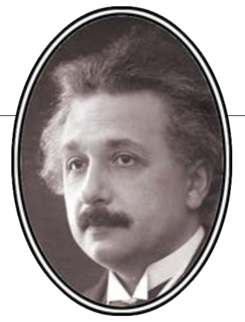
THE MODERN EQUIVALENCE PRINCIPLES

In making this statement I follow one of the most popular traditions in the literature on general relativity: **to claim that everyone else has misunderstood either Einstein and/or GR, and that only my interpretation makes sense.**

Here is a typical quote:

> … his [*Einstein's*] formulation is deeply flawed when taken literally… **the strict adoption of his principle has led to a pointless literature of apparent paradoxes, debates and conundrum…** Einstein's original formulation [*of the principle of equivalence*] is not an equivalence but a tautology.

Eric Poisson, Clifford M. Will, "Gravity. Newtonian, Post-Newtonian, Relativistic" (2014)



CONCLUSION — MODERN EQUIVALENCE PRINCIPLES

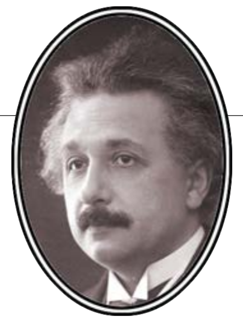
The only rational conclusion is to **stop thinking of the Strong Equivalence Principle as a principle of nature,** i.e. insisting on that it should be valid in GR (and all other gravitational theories).

… because ***the SEP is not valid in GR***. Get over it.

Instead we should learn to treat the SEP as an **approximation scheme** for obtaining the special relativistic limit of GR.

This is just the same way as we have always treated the Newtonian limit. Why threat the SR limit differently?

And we can always continue to use the "comma-goes-to-semicolon" or "minimal coupling" version of the SEP as a **heuristic** to guess the form of laws in GR from laws in SR.



THE MIDWIFE EUOLOGY

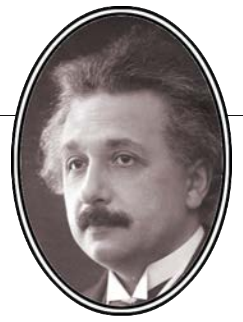
THE MIDWIFE EULOGY

The perhaps most quoted **critique of the Equivalence Principle** comes from American relativist J. L. Synge.

In the introduction to his influential textbook on relativity from 1960 he concludes that what his relativist colleagues call “the principle of equivalence” is either **trivial** or **wrong**.

And as a statement about the modern formulations of the principle I suppose he is right in being critical.

I like to think of Synge’s elegant dismissal of (what people then believed to be) Einstein’s central heuristic as “The Midwife Eulogy”:



THE MIDWIFE EULOGY

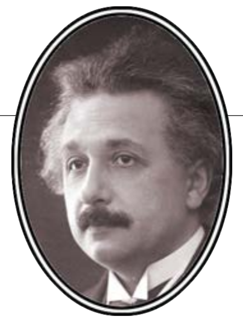
I have never been able to understand this Principle [*of Equivalence*]. Does it mean that the signature of the space-time metric is +2 (or -2 if you prefer the other convention)? **If so, it is important, but hardly a Principle…**

... Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? **If so, it is false.** In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property …

… The Principle of Equivalence performed the essential office of midwife at the birth of general relativity ...

I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced.

J. L. Synge, "Relativity: The General Theory" (1960)

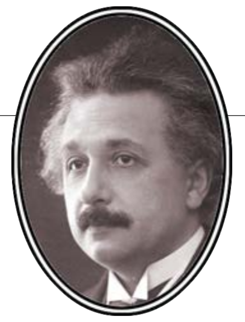


INFINITESIMAL REGIONS IN CHICAGO

The philosopher of science **Moritz Schlick** wrote one of the first philosophical assessments of general relativity already in 1917, published in the semi-popular journal *Die Naturwissenschaften*.

In that paper Schlick tried to **justify the geodesic principle** (that freely falling test bodies follow geodesics of the curved space-time metric) **from the observation that the laws of special relativity are valid at a space-time point.**

The argument goes roughly as follows: *In SR the path of a freely moving body is a straight line; in an infinitesimal space-time region one can find a coordinate system such that any geodesic is a straight line; therefore it is justified to assume that freely moving bodies move along geodesics in extended space-time regions.*



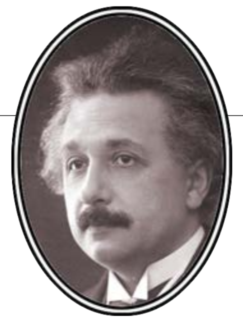
INFINITESIMAL REGIONS IN CHICAGO

Einstein very much appreciated Schlick's paper, but had to correct him on this point:

> The derivation of the law of motion of a point [*in the manuscript of your paper*] assumes that, seen from the local coordinate system, the point moves in a straight line. Nothing can be derived from this, however. The local coordinate system is generally of importance only at the infinitesimal level, and **at the infinitesimal level every uninterrupted line is straight**. [...] That the point's world line is a geodetic line otherwise as well (if others do not exert gravitational influence) is a hypothesis, albeit one that suggests itself.

Albert Einstein, Letter to Moritz Schlick, 21 March 1917

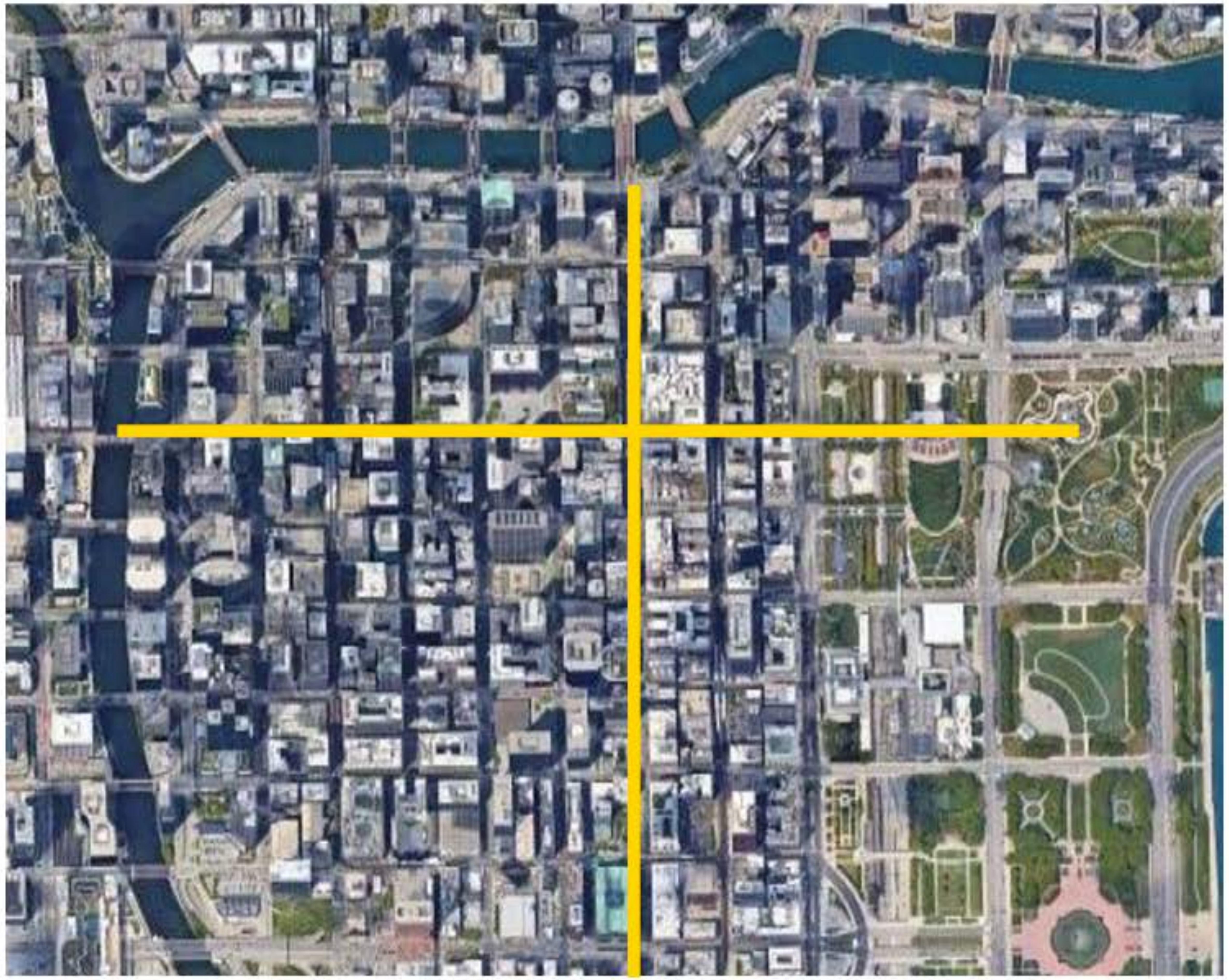
The relativist **Roberto Torretti** gives a nice illustration of the phenomenon that Einstein mentions:



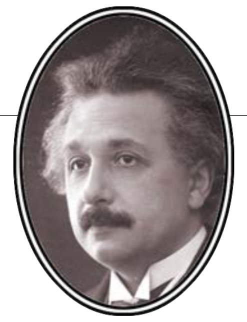
INFINITESIMAL REGIONS IN CHICAGO

Downtown Chicago the streets follow a rectangular grid.

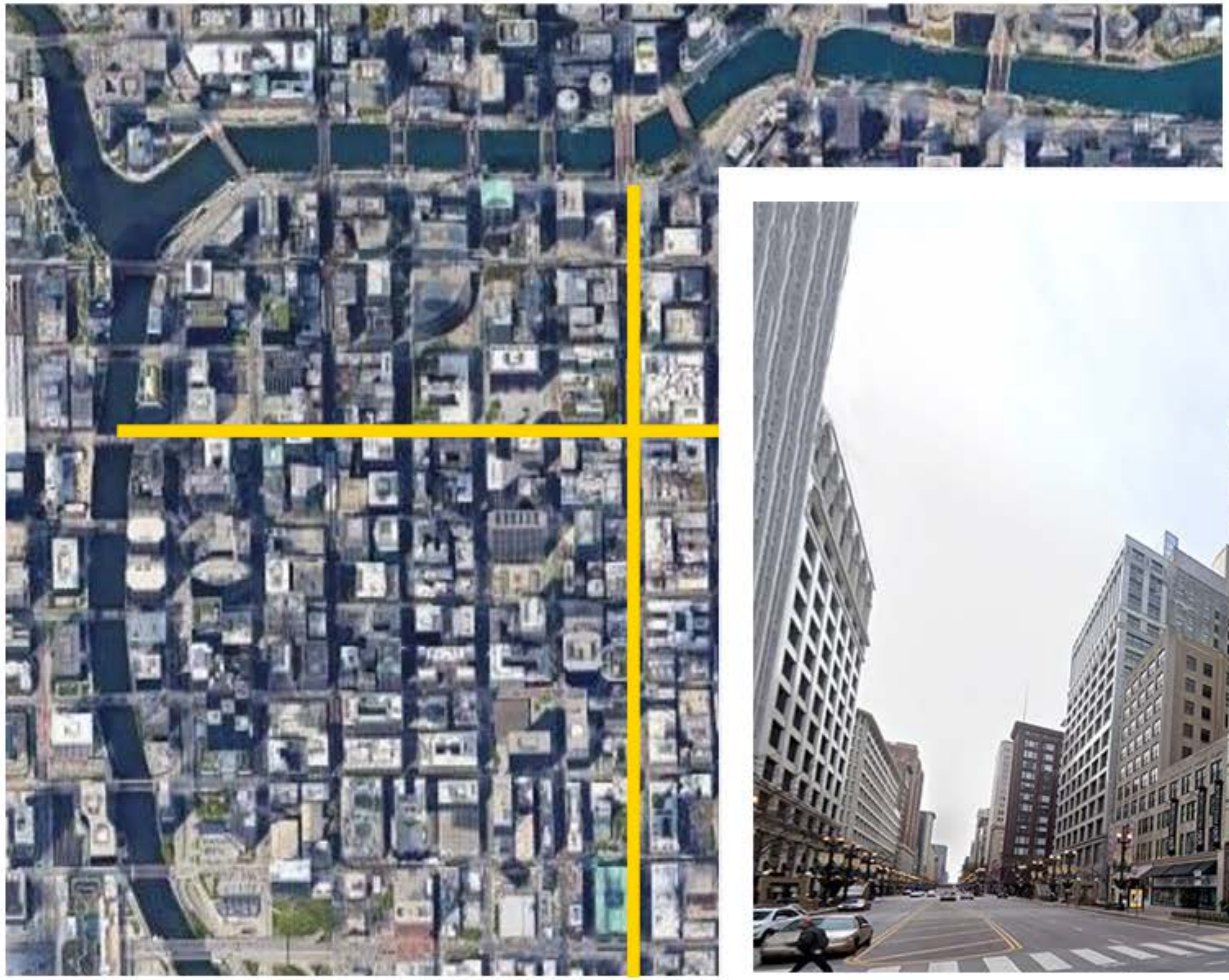
INFINITESIMAL REGIONS IN CHICAGO



North-South **meridians** (part of great circles) are geodesics on the sphere, East-West **parallels** are not.

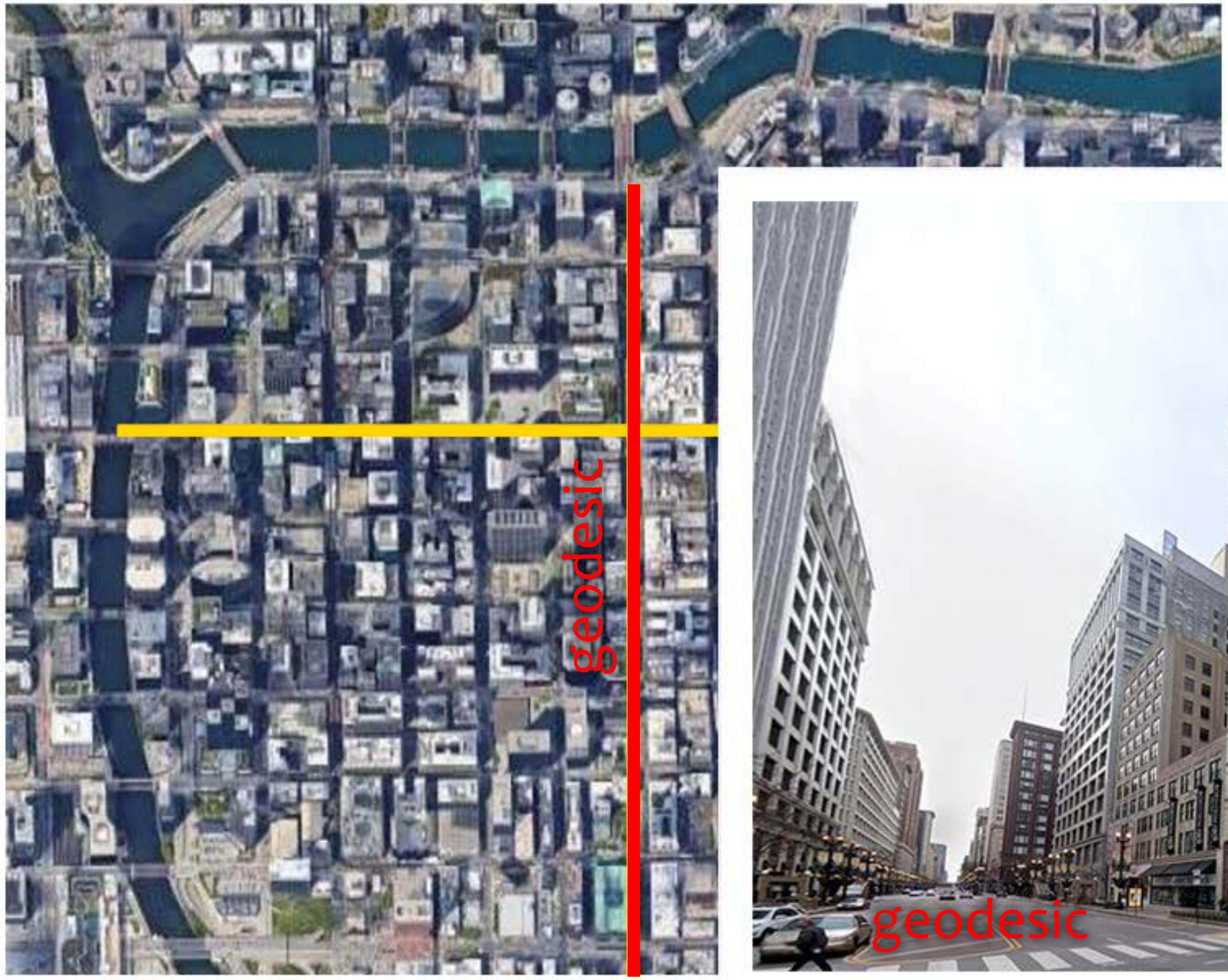


INFINITESIMAL REGIONS IN CHICAGO

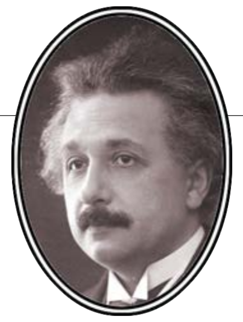


Can you tell which of the Chicago streets in the photo runs along a geodesic?

INFINITESIMAL REGIONS IN CHICAGO



Conclusion: **in infinitesimal regions, geodesics cannot be distinguished from non-geodesics.**

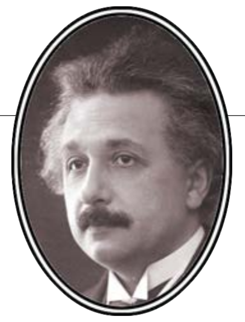


INFINITESIMAL REGIONS IN CHICAGO

To suggest as some have done that the worldline of a freely falling particle must be a geodesic because, according to the EP, each line segment of it is mapped onto a straight in R^4 by a suitable local Lorentz chart, is like arguing that the parallels of latitude on the surface of the Earth are no less geodesic than the meridians, because they agree, for instance, with the avenues that cross Chicago from East to West, which are no less straight than those leading from North to South.

In a Riemannian manifold *every* curve is "straight in the infinitesimal".

Roberto Torretti, "Relativity and Geometry" (1984)



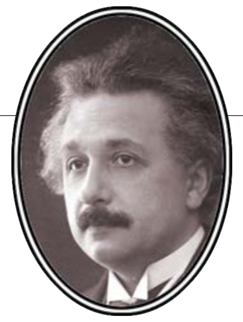
RECONSIDERING THE WEAK EQUIVALENCE PRINCIPLE

RECONSIDERING THE WEP

The **standard statement** of the WEP is due to Clifford Will:

> **Weak equivalence principle (WEP):**
>
> The property of a body called “mass” is proportional to the “weight”. Or, the trajectory of a freely falling “test” body is independent of its internal structure and composition.

Clifford M. Will, “The Confrontation between General Relativity and Experiment” (2014)



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The property of a body called "mass" is proportional to the "weight". Or, the trajectory of a freely falling "test" body is independent of its internal structure and composition.

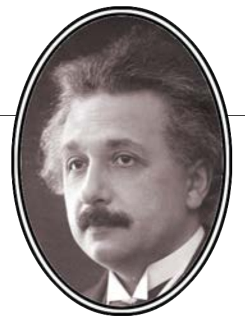
Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)

Notice that the last part of Will's definition is what is otherwise often called **the Universality of Free Fall (UFF).**

Universality of Free Fall (UFF):

Two bodies dropped in a gravitational field fall with the same acceleration (simplest application of WEP).

Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)

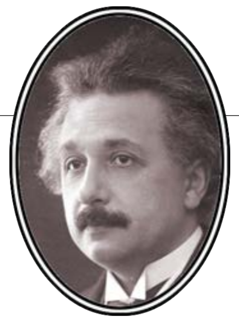


RECONSIDERING THE WEP

→ *Is the Weak Equivalence Principle valid in NM?*

In Newtonian physics, the equation of motion for a freely falling body is $m_i \ddot{x} = -m_g g$, so the acceleration will be independent of the mass only if $m_i = m_g$. Experimentally this is true to high precision, but Newtonian theory cannot explain why.

So, as a general statement, **WEP/UFF remains unexplained in NM**.



RECONSIDERING THE WEP

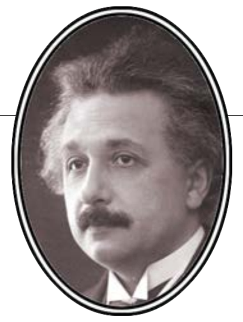
→ ***Is the Weak Equivalence Principle valid in NM?*** ***— no***

→ ***Is the Weak Equivalence Principle valid in GR?***

In GR the claim is that **free particles move along geodesics of the metric**, independently of their mass. This is not a postulate but can actually be derived from the Einstein field equations (like with the **Geroch-Jang-Malament** or **Ehlers-Geroch theorems**).

But **these derivations only hold approximately**, and only for very special types of matter (non-rotating structureless test particles).

So, as a general statement, **WEP/UFF is not strictly valid in GR**.



RECONSIDERING THE WEP

→ ***Is the Weak Equivalence Principle valid in NM?*** — ***no***

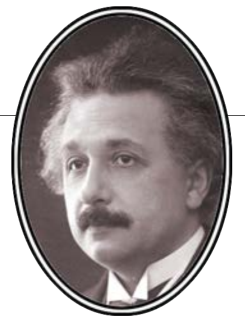
→ ***Is the Weak Equivalence Principle valid in GR?*** — ***no***

→ ***Is the Weak Equivalence Principle valid in QM?***

The Schrödinger equation for a freely falling quantum particle is $-\hbar^2 \nabla^2 \Psi / 2 m_i + m_g \, \varphi \, \Psi = i\hbar \, \dot{\Psi}$, but unlike in the Newtonian case the masses don't drop out if $m_i = m_g$. However, they do drop out in the Ehrenfest-theorem equation of motion.

But fluctuations around the Ehrenfest-theorem path of a quantum particle are mass-dependent $\sim \hbar / m_i$, so UFF is only approximately valid (and only in the limit of heavy quantum particles).

So, as a general statement, **WEP/UFF is not strictly valid in QM**.



RECONSIDERING THE WEP

→ ***Is the Weak Equivalence Principle valid in NM?*** — ***no***

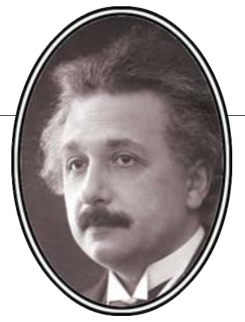
→ ***Is the Weak Equivalence Principle valid in GR?*** — ***no***

→ ***Is the Weak Equivalence Principle valid in QM?*** — ***no***

Conclusion:

In GR the proposition **WEP/UFF must be formulated in terms of an approximation scheme.**

But what justification is there for calling **an approximation scheme** a “principle”?



RECONSIDERING THE WEP

Einstein never made the mistake of talking about the Universality of Free Fall as a "principle", neither of "equivalence" or of anything else:

In the Newtonian context of the *Äquivalenzprinzip*, referring back to Galilei, what Einstein actually did talk about was…

> […] the old **experimental fact** that all bodies have the same acceleration in a gravitational field. This **law** […]

Albert Einstein, "Notes on the Origin of the General Theory of Relativity", lecture manuscript (1933)