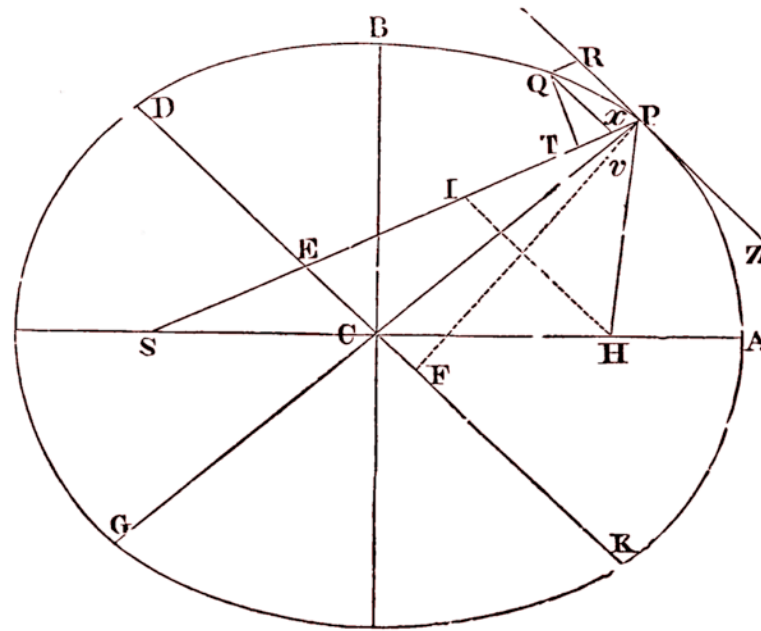


Newton's Critique of the Inverse Square Law of Gravity



Hans Mühlen
Nordita Day, 17 April 2015

Hans after office hours:

- **Gravitation** (any kind)
- **History of physics**
- **Foundational** questions

A case study —

The foundations of Newton's theory of Universal Gravity, as understood by himself and his contemporaries in the 17th century.

What did Newton think about the inverse square law?

A modern description of Newton's law of Universal Gravity:

- An **attractive** force acting between two massive bodies
- The force acts the same way for any massive bodies (**universal** gravity)
- The *strength* of the force is proportional to the **two masses**, and inversely proportional to the **square of the distance** between their centers of gravity
- The *direction* of the force is along the line joining their centers of gravity (**central force**)
- The force acts **instantaneously** and **at a distance**

$$F = G \frac{m_1 m_2}{r_{12}^2}$$

When you then look up
the original formulation
of the law it looks
slightly different.

...but this isn't really
very surprising
(this was 300 years ago,
long before TeX).

Proposition 7 from Book 3 of the
Philosophiae Naturalis Principia Mathematica,
first published in 1687

Prop. VII. Theor. VII.

Gravitatem in corpora universa fieri, eamque proportionalem esse quantitati materiae in singulis.

Planetam omnes in se mutuò graves esse jam ante probavimus, ut & gravitatem in unumquemque seorsim spectatum esse reciprocè ut quadratum distantiae locorum à centro Planetæ. Et inde consequens est, (per Prop. LXIX. Lib. I. & ejus Corollaria) gravitatem in omnes proportionalem esse materiae in iisdem.

Porro cum Planetæ cujuscvis *A* partes omnes graves sint in Planetam quemvis *B*, & gravitas partis cujusque sit ad gravitatem totius, ut materia partis ad materiam totius, & actioni omni reactio (per motus Legem tertiam) æqualis sit; Planeta *B* in partes omnes Planetæ *A* vicissim gravitabit, & erit gravitas sua in partem unamquamque ad gravitatem suam in totum, ut materia partis ad materiam totius. *Q. E. D.*

Corol. 1. Oritur igitur & componitur gravitas in Planetam totum ex gravitate in partes singulas. Cujus rei exempla habemus in attractionibus Magneticis & Electricis. Oritur enim attractio omnis in totum ex attractionibus in partes singulas. Res intelligetur in gravitate, concipiendo Planetas plures minores in unum Globum coire & Planetam majorem componere. Nam vis totius ex viribus partium componentium oriri debet. Siquis objiciat quod corpora omnia, quæ apud nos sunt, hac lege gravitare deberent in se mutuò, cum tamen ejusmodi gravitas neutiquam sentiat: Respondeo quod gravitas in hæc corpora, cum sit ad gravitatem in Terram totam ut sunt hæc corpora ad Terram totam, longe minor est quam quæ sentiri possit.

Corol. 2. Gravitatio in singulas corporis particulas æquales est reciprocè ut quadratum distantiae locorum à particulis. Patet per *Corol. 3.* Prop. LXXIV. Lib. I.

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PROPOSITION VII.

*That there is a power of gravity
tending to all bodies, proportional to
the several quantities of matter which
they contain.*

[...]

COR. 1. *Therefore the force of
gravity towards any whole planet arises
from, and is compounded of, the forces
of gravity towards all its parts. [...]*

COR. 2. *The force of gravity
towards the several equal particles of
any body is reciprocally as the square
of the distance of places from the
particles; [...]*

The real surprises come if you then continue to snoop around in other texts by Newton and his contemporaries.

For instance **this comment on *action-at-a-distance*** by Newton, from a letter he wrote to Rev. Richard Bentley five years after the publication of the *Principia*:

*[...] That gravity should be innate, inherent
& essential to matter so that one body may
act upon another at a distance through a
vacuum without the mediation of anything
else [...] is to me so great an absurdity that
I believe no man who has in philosophical
matters a competent faculty of thinking can
ever fall into it.*

Isaac Newton to Richard Bentley, 25 February 1693

Now, what is this?

It *appears* that **Newton is criticizing**
a central aspect of his own theory:
that universal gravity acts at a
distance.

[...] *That gravity should be innate, inherent
& essential to matter so **that one body
may act upon another at a
distance** through a vacuum without the
mediation of anything else [...] **is to me
so great an absurdity** that I believe
no man who has in philosophical matters a
competent faculty of thinking can ever fall
into it.*

But surely, Newton's gravitational force *must* be understood as *acting at a distance*.

The expression for the force contains only quantities referring to the properties of the two gravitating bodies (their mass) and their relative distance.

There is no place for gravity being “*mediated by anything else*” here.

This is a common problem you encounter when you start acquainting yourself with ideas and theories of the past.

Actual historical texts don't fit easily with ***our preconceived ideas*** of how things are (based on several hundred years worth of benefit of hindsight).

Is this what is going on here?

Do we understand Newtonian gravity better *now* than *Newton* did *then*?

Is Newton just confused?

No, probably not. And I'll tell you why i think so.

We have made one of the most common mistakes of historical tourists:

Just reading one snippet of historical text, taken out of context, without any other understanding of the subject matter than what modern knowledge would suggest.

What we need to do is first to **read as much as possible about the period in question**, and try to understand the people and ideas — as well as one can — as they were understood back then.

Without cheating by looking into their “future”, or our “present”.

Exploring the history of science — beyond the usual “just-so stories” — is an interesting exercise in **being forced to forget everything you know in order to learn something new.**



So, let's pretend we have spent a couple of years reading all there is to know about the science of the 17th century.

Now we go back to the Newton quote, and see if we can understand it better.

Some of the text I omitted earlier will give us some valuable clues...

*'Tis inconceivable that inanimate brute matter should (without the mediation of something else which is not material) operate upon & affect other matter, without mutual contact; as it must do if **gravitation in the sense of Epicurus** be essential & inherent in it.*

[...] That gravity should be innate, inherent & essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of anything else [...] is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it.

There is a reference here to *the natural philosophy of the Epicureans*.

Without going into details: might it be that **Newton's critique is not directed at his own theory, but rather against some other older theories of gravity?**

There are keywords in the text that support this reading of the quote:

'Tis inconceivable that inanimate brute matter should (without the mediation of something else which is not material) operate upon & affect other matter, without mutual contact; as it must do if gravitation in the sense of Epicurus be essential & inherent in it. [...] That gravity should be innate, inherent & essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of anything else [...] is to me so great an absurdity [...]

Other texts by Newton confirm that he distinguished between “**inanimate brute matter**” and “**immaterial substances**” (like God, angels or the human mind).

Ordinary matter is *passive* and therefore *cannot act directly* on other matter, not without external intervention.

Let’s check the quote again, with this philosophical distinction in mind.

*'Tis inconceivable that inanimate brute matter should
(without the mediation of something else
which is not material) operate upon & affect
other matter, without mutual contact; as it must
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may act upon another at a distance
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of anything else [...] is to me so great an absurdity*

So Newton didn't criticize his own theory.

He was just furious about the accusations of some of his contemporaries (especially Huyghens and Leibniz) who had unjustly accused Newton of holding absurd Epicurean ideas about *active matter* and *inherent gravity*.

(Nishant, is it too late to change
the title of my talk?)

~~Newton's Critique of the Inverse Square Law of Gravity~~

Newton's Critique of the Critique of the Inverse Square Law of Gravity

Take-away message:

Don't try to interpret a (historical) text without a thorough understanding of the context.

Well, this is rather obvious really.

But it is so *easy* to make this mistake when you are in unfamiliar historical environments.

Let's go back to my original question:

What did Newton think about the inverse square law?

Let me reinstate a few more sentences I left out earlier in the quote.

[...] that one body may act upon another at a distance through a vacuum without the mediation of anything else by & through which their action and force may be conveyed from one to another is to me so great an absurdity [...] Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent be material or immaterial is a question I have left to the consideration of my readers.

So — what kind of “material or immaterial agents” *causing* gravitational interaction did Newton have in mind?

Actually, it is clear from other texts that **Newton didn't pretend to know the cause of gravity.**

This is a typical quote, taken from the “*General Scholium*”, a discussion section at the end of the *Principia*:

*Hitherto we have explain'd the phaenomena of the heavens and of our sea, by the power of Gravity [...] but **I have not been able to discover the cause of those properties of gravity from phaenomena**, and I frame no hypothesis. For whatever is not deduc'd from the phaenomena, is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy.*

Principia, 3rd edⁿ, 1729, General Scholium

This famous quote expresses
Newton's **empiricist** credo:

*Knowledge can only be obtained from
an **analysis of the phenomena**
(experiment, observation); through
induction we arrive at **general laws**;
from these laws predictions can then be
obtained through **deduction**;
hypotheses (a priori speculation) must
be banned from science.*

Nothing new here from the point of view of modern science, perhaps.

But in Newton's time this empiricist philosophy was highly controversial.

Newton is here again distancing himself from contemporary philosophies of nature.

This time the target is the **rationalist** philosophy of **René Descartes**, which was dominating scientific discussions in the 17th century.

The *Principia* can be seen a frontal assault on the dominant Cartesian physics.

But it would take almost a century until Newtonian physics and an empiricist view of scientific method would have replaced Cartesianism rationalism.

In his investigation of the phenomena, Newton has not yet been able to find a mechanism that would explain, to his satisfaction, the gravitational force.

Rather than “feigning hypotheses”, Newton chooses to leave the theory in its current form, where **the law of Universal Gravity is purely phenomenological.**

So the law only describes the **how** of gravity, not the **why**.

To a modern reader, this sounds very much like **positivism**.

Was Newton an early positivist?

No: he expected there to be a causal explanation of the gravitational force. He just hadn't been able to find one.

Incidentally, this is very similar to the present status of our currently best theory of gravity, ***General Relativity***.

It is today often seen as an “effective theory”, awaiting its ultimate causal explanation by a quantum micro-theory.

(The only difference is, perhaps, that today no-one has any qualms about “feigning hypotheses”.)

So, the official version of Newton's **theory** was a purely phenomenological account of gravity.

But in private, Newton tried hard to find an underlying cause of gravity for the remainder of his life.

In printed texts he only dared to hint at various possible models he had come up with, using **very speculative language**.

Here is one example of a mechanism that Newton suggested might *explain* the observed phenomenon of gravity.

Newton assumes **there might exist an “aether”**, a medium consisting of **very small particles**, which pervades both matter and void throughout the universe.

The aether is then used to explain both gravity, electricity, magnetism, and more.

The aether is assumed to get more dense further away from massive bodies.

So there is a pressure gradient in the aether surrounding bodies.

What we perceive as gravity is just the action of this pressure gradient, pushing smaller bodies towards larger ones.

QUERY 21. Is not this [Aethereal] Medium much rarer within the dense Bodies of the Sun, Stars, Planets and Comets, than in the empty celestial Spaces between them? And in passing from them to great distances, doth it not grow denser and denser perpetually, and thereby cause the gravity of those great Bodies towards one another, and of their parts towards the Bodies; every Body endeavouring to go from the denser parts of the Medium towards the rarer?

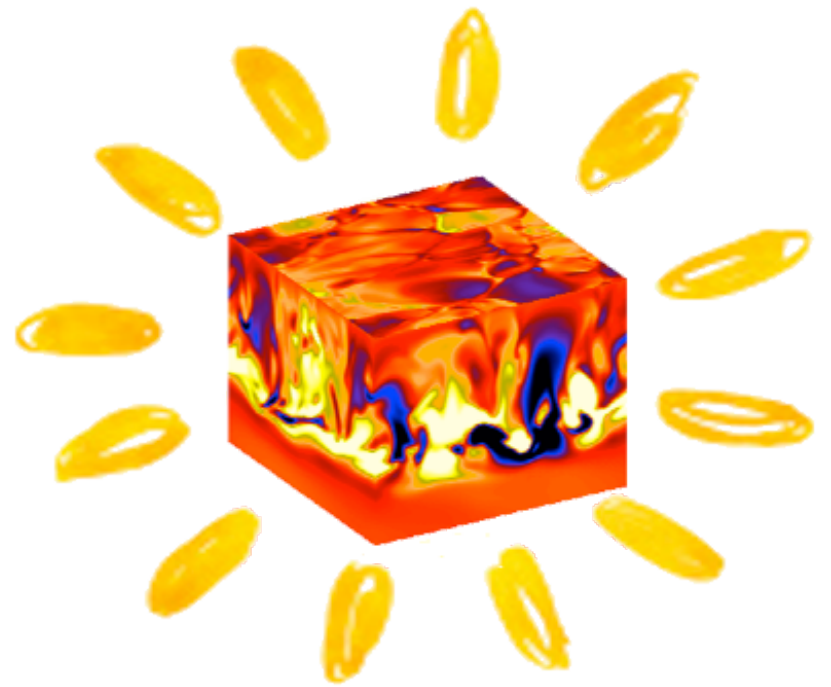
Opticks, 2nd English edⁿ, 1717, Queries

Again I can't help thinking about similarities with modern ideas about gravity.

Every time I read about the Newtonian mechanical gravitational aether models (or the competing Cartesian aether vortices), I get reminded of current models involving (quantum) mechanical particles — the **gravitons** — that supposedly explain the ultimate cause of gravity.

CONCLUSION:

With a historical perspective on the progress of science, you realize that...



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With a historical perspective on the progress of science, you realize that

nothing is new under the sun.