This was... ...but the...

News and Views

...118 years ago... ...that I will discuss might be new to some of you

Einstein's Critique of the Equivalence Principle

How does Einstein's Original 1907 Equivalence Principle Differ from the Modern Equivalence Principles?

Hans Mühlen

2025

This talk is based on a true story

Some of the arguments have been changed for dramatic purposes

BEFORE I START…

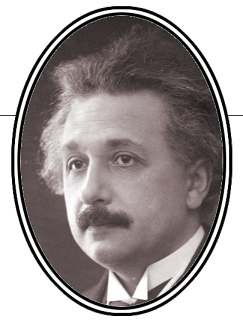
Today I want to talk about
the early development phase of
the theory of general relativity

…but first I need to ask…

QUESTION TO THE AUDIENCE

Who of you have *any* experience with general relativity (GR)?

(Taken a course in GR,
taught a course in GR,
used GR in your research…)

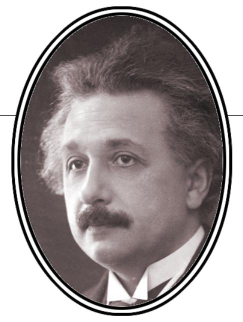


Don’t worry —

The only theoretical tools I will allow myself to use in this talk are the same that were available to the young Einstein **in 1907**:

- **Newtonian mechanics** (**NM**),
- the new mechanics of **special relativity** (**SR**),
- **Newton’s theory of gravity** (**NG**), and
- the **Maxwell-Lorentz theory of electromagnetism** (**EM**).

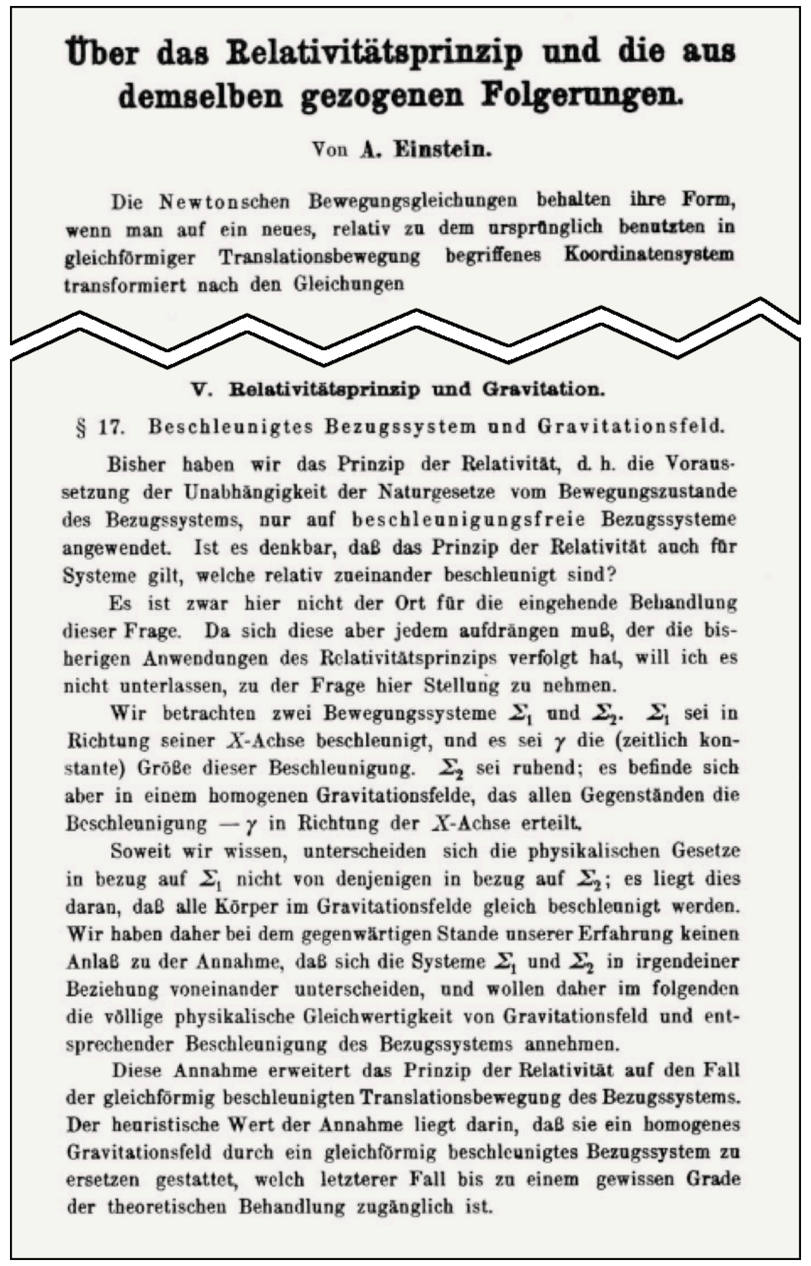
[*It’s very tempting to mix in arguments from the future* ***theory of general relativity*** (**GR**), *but we must try to resist that temptation.*]



It took Einstein eight long years to find the theory of general relativity:

It's a fascinating but complex story…

Submitted
4 December
1907



Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen.

Von **A. Einstein.**

Die Newtonschen Bewegungsgleichungen behalten ihre Form, wenn man auf ein neues, relativ zu dem ursprünglich benutzten in gleichförmiger Translationsbewegung begriffenes Koordinatensystem transformiert nach den Gleichungen

V. Relativitätsprinzip und Gravitation.

§ 17. Beschleunigtes Bezugssystem und Gravitationsfeld.

Bisher haben wir das Prinzip der Relativität, d. h. die Voraussetzung der Unabhängigkeit der Naturgesetze vom Bewegungszustande des Bezugssystems, nur auf beschleunigungsfreie Bezugssysteme angewendet. Ist es denkbar, daß das Prinzip der Relativität auch für Systeme gilt, welche relativ zueinander beschleunigt sind?

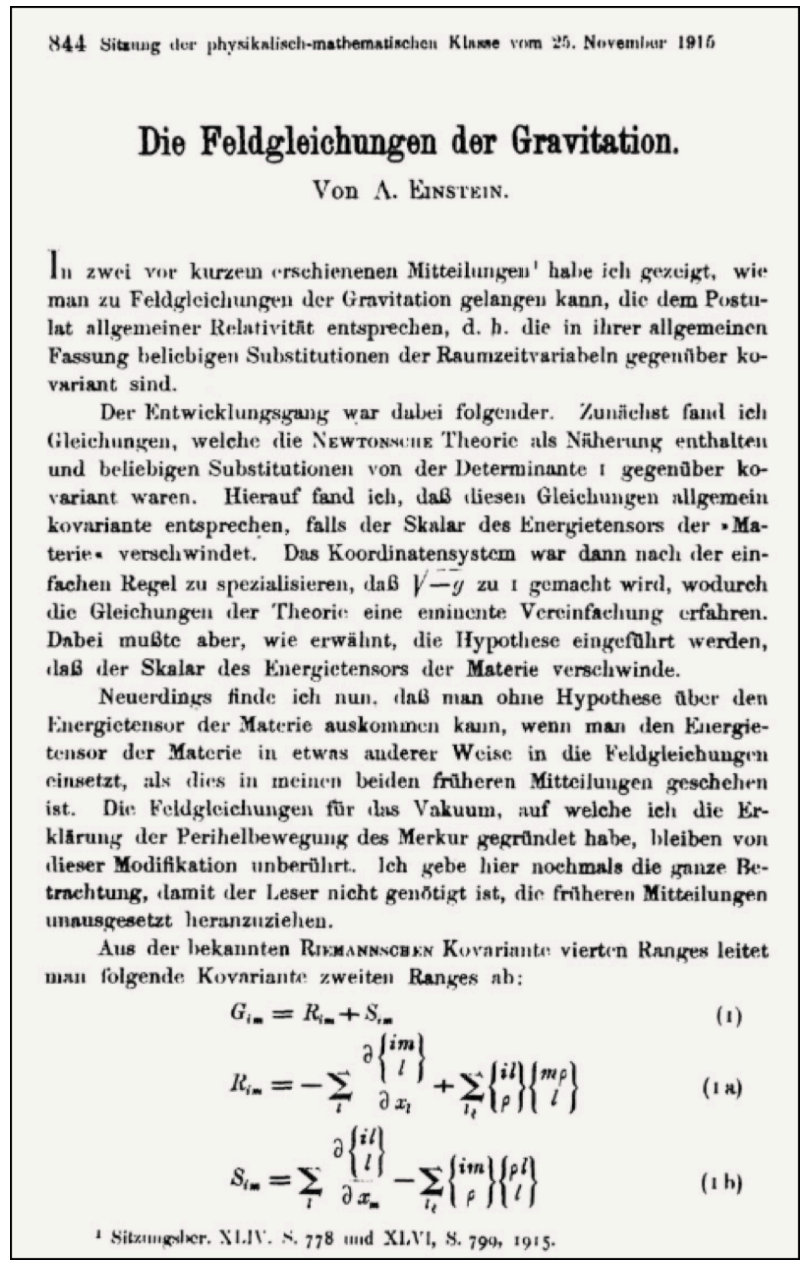
Es ist zwar hier nicht der Ort für die eingehende Behandlung dieser Frage. Da sich diese aber jedem aufdrängen muß, der die bisherigen Anwendungen des Relativitätsprinzips verfolgt hat, will ich es nicht unterlassen, zu der Frage hier Stellung zu nehmen.

Wir betrachten zwei Bewegungssysteme Σ_1 und Σ_2. Σ_1 sei in Richtung seiner X-Achse beschleunigt, und es sei γ die (zeitlich konstante) Größe dieser Beschleunigung. Σ_2 sei ruhend; es befinde sich aber in einem homogenen Gravitationsfelde, das allen Gegenständen die Beschleunigung $-\gamma$ in Richtung der X-Achse erteilt.

Soweit wir wissen, unterscheiden sich die physikalischen Gesetze in bezug auf Σ_1 nicht von denjenigen in bezug auf Σ_2; es liegt dies daran, daß alle Körper im Gravitationsfelde gleich beschleunigt werden. Wir haben daher bei dem gegenwärtigen Stande unserer Erfahrung keinen Anlaß zu der Annahme, daß sich die Systeme Σ_1 und Σ_2 in irgendeiner Beziehung voneinander unterscheiden, und wollen daher im folgenden die völlige physikalische Gleichwertigkeit von Gravitationsfeld und entsprechender Beschleunigung des Bezugssystems annehmen.

Diese Annahme erweitert das Prinzip der Relativität auf den Fall der gleichförmig beschleunigten Translationsbewegung des Bezugssystems. Der heuristische Wert der Annahme liegt darin, daß sie ein homogenes Gravitationsfeld durch ein gleichförmig beschleunigtes Bezugssystem zu ersetzen gestattet, welch letzterer Fall bis zu einem gewissen Grade der theoretischen Behandlung zugänglich ist.

Submitted
25 November
1915



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

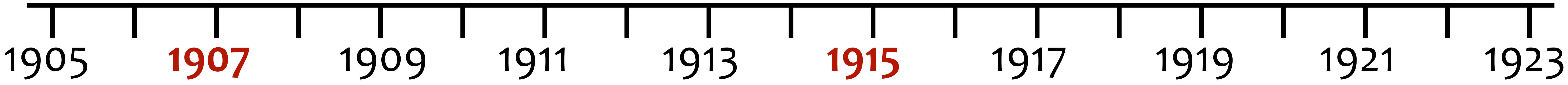
Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \tag{1}$$

$$R_{im} = -\sum_l \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \tag{1a}$$

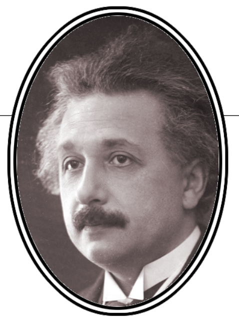
$$S_{im} = \sum_l \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \tag{1b}$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915.



Albert Einstein, "*On the relativity principle and the conclusions drawn from it*" (1907)

Albert Einstein, "*The Field Equations of Gravitation*" (1915)

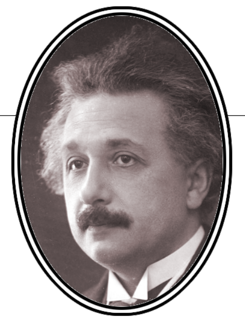


THIS TALK

I have chosen to focus on only a small part of this long and multifaceted story:

the role of the Equivalence Principle
in Einstein’s search for general relativity

In particular I want to see **how much the formulation and understanding of Einstein’s original ideas have changed** over the past century.



THIS TALK

The standard reference to the historical development and meaning of Einstein’s Equivalence Principle is

> ***John D. Norton***
> **What was Einstein's Principle of Equivalence?**
> Studies in History and Philosophy of Science, **16** (1985) 203–246
> `doi.org/10.1016/0039-3681(85)90002-0`

More works by Norton on the history of relativity can be found at

> **`sites.pitt.edu/~jdnorton`**

A WORD OF WARNING

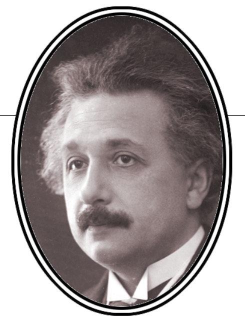
As many of you may know, …

in modern texts on general relativity one would typically distinguish between **a "weak" and a "strong" version of "the Equivalence Principle"**.

However, analysis of Einstein's original texts shows that…

the modern "weak" and "strong" Equivalence Principles are very different from the idea that Einstein proposed back in 1907 and called "the Equivalence Principle".

So to avoid confusion…



A WORD OF WARNING

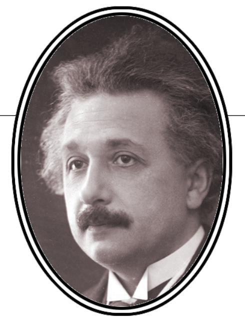
…so to avoid confusion, I will often use the German phrase

ÄQUIVALENZPRINZIP

to denote the **original 1907 idea of Einstein,** and the English phrase

EQUIVALENCE PRINCIPLE

for the various **modern principles.**

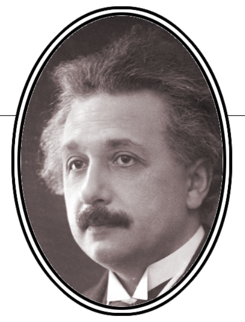


PART 1

1. A statement of **the two modern versions** of the (weak and strong) Equivalence Principles
2. **Einstein's formulation** of the original *Äquivalenz-prinzip* of 1907
3. **How Einstein made use of** the 1907 *Äquivalenz-prinzip* in his search for a relativistic theory of gravity

PART 2

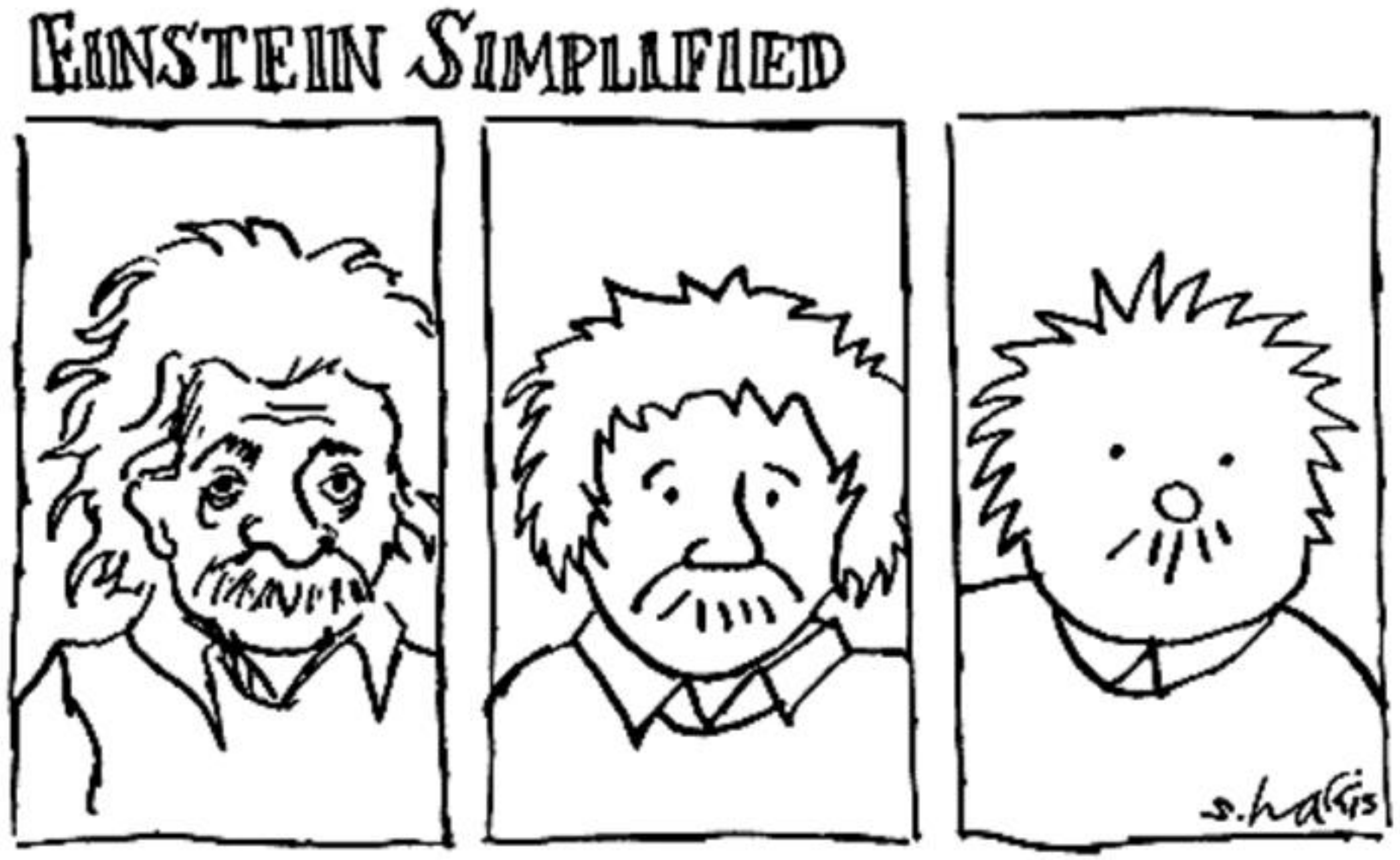
4. **The "generative" and "eliminative" observers** in different statements of the *Äquivalenzprinzip*
5. **Attempts to generalise the *Äquivalenzprinzip:*** the Infinitesimal Equivalence Principle
6. **Einstein's critique** of the generalised principles



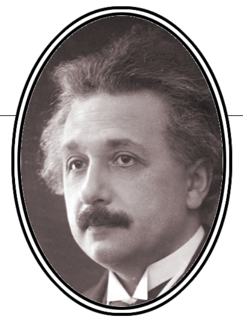
ANOTHER WORD OF WARNING

For the benefit of time **I have had to simplify the story** of what motivated Einstein to formulate his Equivalence Principle. So…

my presentation will not be historically accurate!



Cartoon by **Sidney Harris**

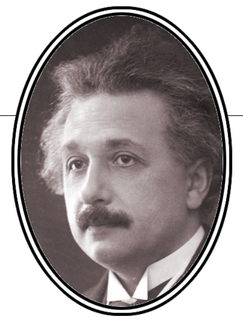


PART 1

EINSTEIN’S *ÄQUIVALENZPRINZIP*

QUESTION TO THE AUDIENCE

Who can help me give a
(roughly correct) **statement**
of the modern
"Weak Equivalence Principle",
and of the
"Strong Equivalence Principle"?



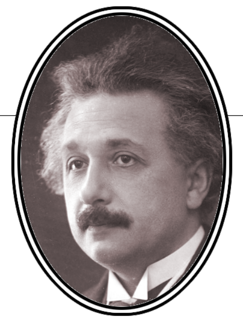
In the literature you can find MANY different variants of the definitions of WEP and SEP. These will suffice for now:

Weak Equivalence Principle (WEP)

The acceleration of a freely falling body is independent of its mass.

Strong Equivalence Principle (SEP)

In an infinitesimal spacetime region, for a freely falling observer, the laws of physics reduce to their special relativistic form.



THE MODERN EQUIVALENCE PRINCIPLES

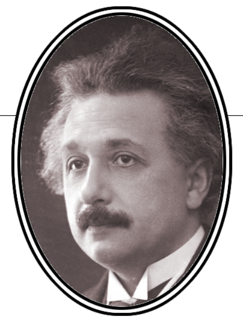
The “Equivalence Principle” has been **severely criticized** by many authors over the years:

> **I have never been able to understand this Principle** [*of Equivalence*]. Does it mean that the signature of the space-time metric is +2 [...]? **If so, it is important, but hardly a Principle** [...] Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? **If so, it is false.**

J. L. Synge, “Relativity: The General Theory” (1960)

> Despite the pivotal role played at the dawn of general relativity, **the “principle of equivalence” is now regarded as a rather vague heuristic statement**, perhaps useful for teaching purposes, but surely **unable to challenge the contemporary approach to gravity**, based on sharp and neat mathematical axioms and precision-test experiments.

Eolo Di Casola, Stefano Liberati, Sebastiano Sonego,
“Weak Equivalence Principle for Self-Gravitating Bodies” (2014)



THE MODERN EQUIVALENCE PRINCIPLES

The "Equivalence Principle" has been **severely criticized** by many authors over the years:

> Various statements identified with the principle of equivalence **are not acceptable** because they are either not generally valid or are simply the definition of an inertial coordinate system.

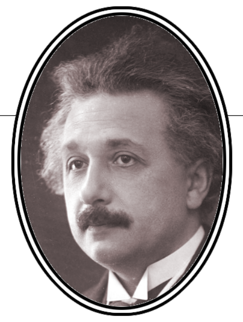
Fritz Rohrlich, "The Principle of Equivalence" (1963)

> In contrast with early statements by Einstein and others on the axiomatic basis of the theory of general relativity, **the *principle of equivalence* is not one of its underlying axioms**.

Mendel Sachs, "On the Logical Status of Equivalence Principles…" (1976)

> [*Einstein's*] formulation [*of the principle of equivalence*] is **deeply flawed** when taken literally.

E. Poisson, C. M. Will, "Gravity. Newtonian, Post-Newtonian, Relativistic" (2014)



THE MODERN EQUIVALENCE PRINCIPLES

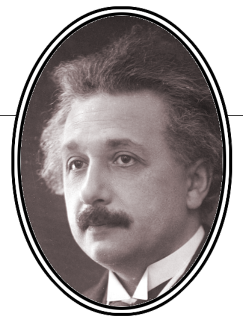
But then there are other authors who claim that the "Equivalence Principle" is indispensable, and part of the foundations of general relativity:

> **At the foundations** of Einstein's geometrodynamics [=the *theory of general relativity*] and of its geometrical structure is one of the best-tested principles in the whole field of physics: the equivalence principle.

Ignazio Ciufolini, John A. Wheeler, "Gravitation and Inertia" (1995)

> The principle of equivalence has historically played an important role in the development of gravitation theory. [...] In 1907, Einstein used the principle as a basic element in his development of general relativity (GR). **We now regard the principle of equivalence as the foundation**, not of Newtonian gravity or of GR, but of the broader idea that spacetime is curved.

C. M. Will, "The Confrontation between General Relativity and Experiment" (2014)



THE MODERN EQUIVALENCE PRINCIPLES

This fundamental disagreement between relativists on the status of the principle has led to **a vast "improved Equivalence Principles" literature**, with no signs of convergence so far:

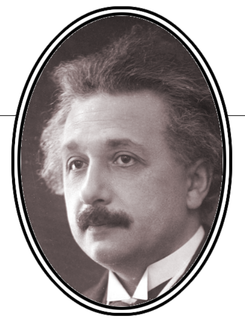
> The strict adoption of [*Einstein's*] principle has led to **a pointless literature of apparent paradoxes, debates and conundrum.**

Eric Poisson, Clifford M. Will, "Gravity. Newtonian, Post-Newtonian, Relativistic" (2014)

A selection of proposed "clarifications" of the Equivalence Principle:

*Weak Equivalence Principle (**WEP**), Universality of Free Fall (**UFF**), Equality of Inertial and Gravitational Mass, Gravitational Weak Equivalence Principle (**GWEP**), Newton's Equivalence Principle (**NEP**), Galilei's Equivalence Principle, Newstein Equivalence Principle*

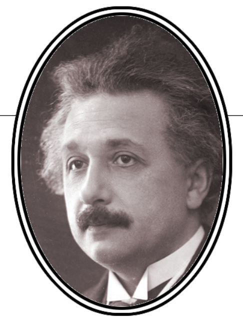
*Strong Equivalence Principle (**SEP**), Einstein Equivalence Principle (**EEP**), Medium Strong Equivalence Principle (**MSEP**), Very Strong Equivalence Principle (**VSEP**), Strict Local Equivalence Principle (**SLEP**), $\forall\delta\exists\varepsilon$ Equivalence Principle (**DEEP**):, Punctual Equivalence Principle (**PEP**), Local Lorentz Invariance (**LLI**), Local Position Invariance (**LPI**), Comma-Goes-to-Semicolon Rule*



So what was Einstein's original *Äquivalenzprinzip?*

We have to go back to 1907, when several attempts at finding a (special) relativistic theory of gravity had failed.

Then **Einstein thought he had found an argument** which promised to reveal some of the properties of a more fundamental **relativistic theory of gravity**.

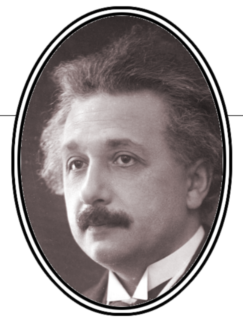


So what was Einstein’s original *Äquivalenzprinzip?*

Einstein formulated the argument **in the context of Newtonian physics**, and in **3D space** (rather than spacetime).

This is entirely textbook material, so many of you will have no problem recognising it.

Only the final step — **how Einstein actually made use of his idea to find the new theory** — might be new to you.

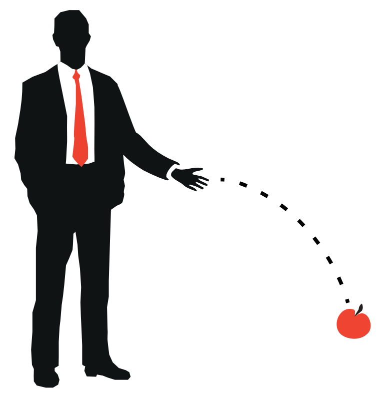


EINSTEIN’S EQUIVALENT NEWTONIAN OBSERVERS

Step 1

In an otherwise empty region of Newtonian 3D space
there is an observer performing **mechanical experiments:**

Observation



When the observer throws a ball it **accelerates** along
a **curved path.**

How would the observer explain*
the observed behaviour of the
thrown ball?

* *using only **Newtonian** physics!*

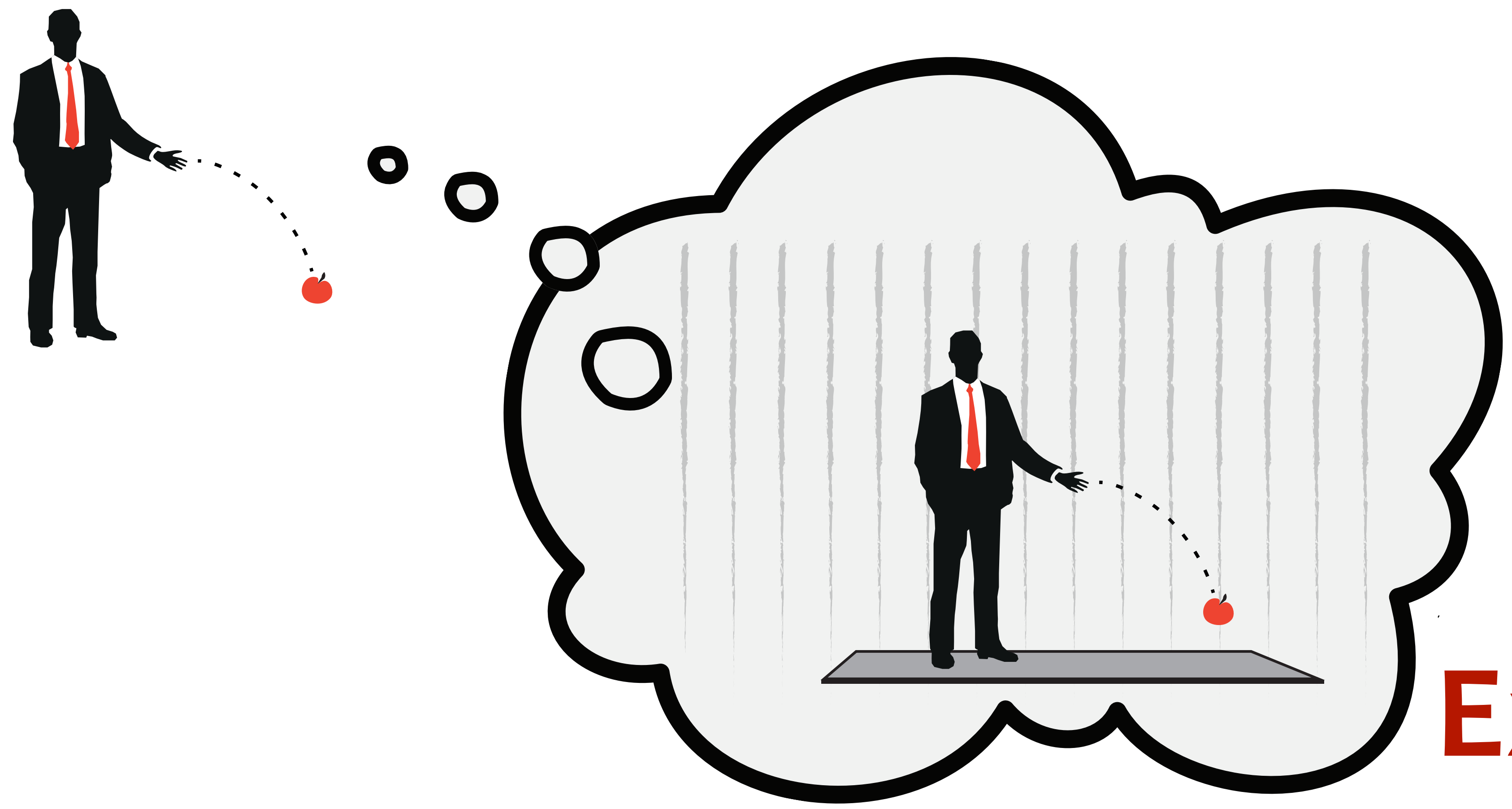
The observer might think that he is **in uniformly accelerated motion** in a region of space **without any gravitational field**…

Explanation 1.

Equations of motion for body thrown by the
observer in uniform acceleration $\boldsymbol{a_z = a}$
(in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 y^\mu}{\mathrm{d}t^2} = 0, \quad \boldsymbol{\frac{\mathrm{d}^2 z^\mu}{\mathrm{d}t^2} = -a}$$ … **a parabola**

… or the observer could claim to be **at rest in a homogeneous gravitational field.**



Explanation 2.

Equations of motion for body thrown by the **observer at rest in a homogeneous gravitational field with potential** g (in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2} = 0, \quad \boldsymbol{m_i}\frac{\mathbf{d^2}\boldsymbol{z^\mu}}{\mathbf{d}\boldsymbol{t^2}} = -\boldsymbol{m_g g}$$

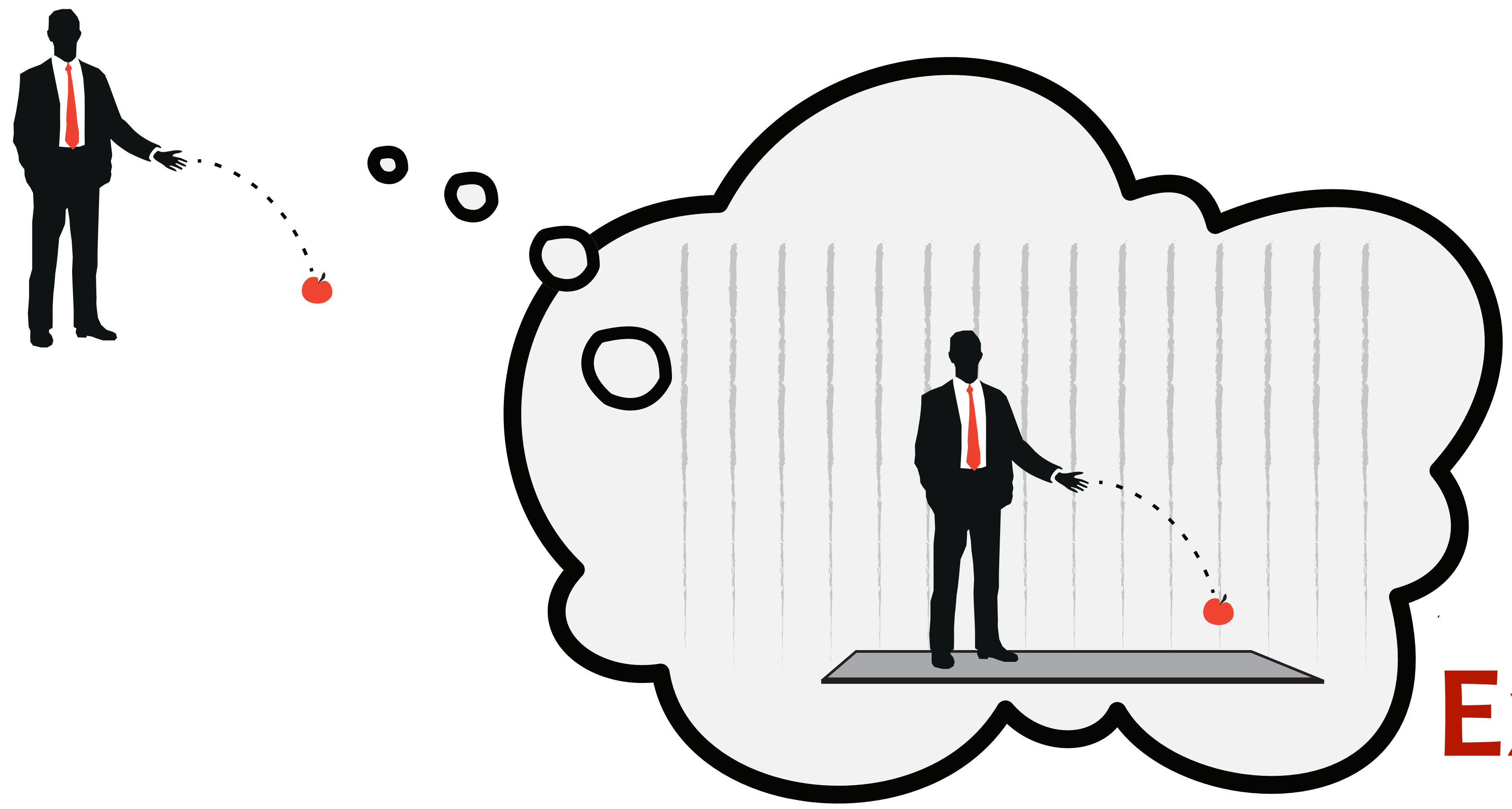
But if $\boldsymbol{m_i = m_g}$ we get:

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2y^\mu}{\mathrm{d}t^2} = 0, \quad \frac{\mathbf{d^2}\boldsymbol{z^\mu}}{\mathbf{d}\boldsymbol{t^2}} = -\boldsymbol{g}$$

… **a parabola**

EINSTEIN'S EQUIVALENT NEWTONIAN OBSERVERS

…or the observer could claim to be **at rest in a homogeneous gravitational field.**



Explanation 2.

Note that *this* result only obtains if we *combine* the **theoretical formalisms** of **Newtonian mechanics** and **Newtonian gravity** with the **empirical result** $m_i = m_g$

Equations of motion for body thrown by the **observer at rest in a homogeneous gravitational field with potential** g (in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 y^\mu}{\mathrm{d}t^2} = 0, \quad m_i \frac{\mathrm{d}^2 z^\mu}{\mathrm{d}t^2} = -m_g g$$

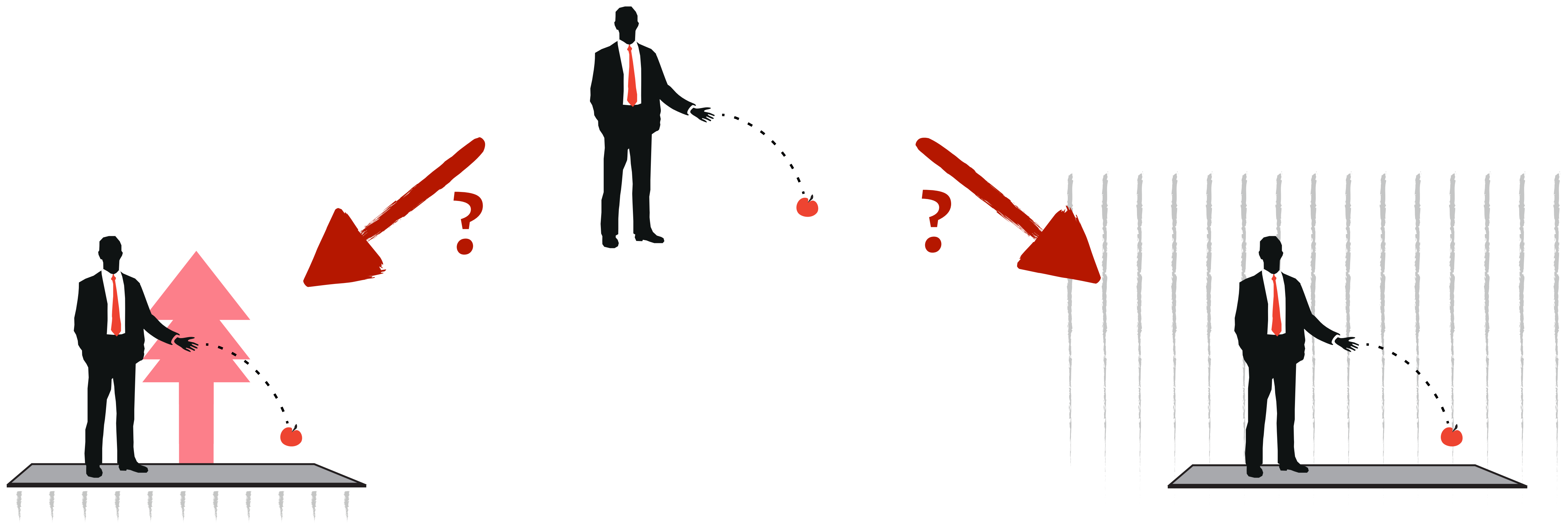
But if $m_i = m_g$ we get:

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 y^\mu}{\mathrm{d}t^2} = 0, \quad \frac{\mathrm{d}^2 z^\mu}{\mathrm{d}t^2} = -g$$

…a parabola

EINSTEIN’S EQUIVALENT NEWTONIAN OBSERVERS

Even though the observed effect is the same, the two explanations are of a **very different nature**:



In this case the observer is only **accelerating**.

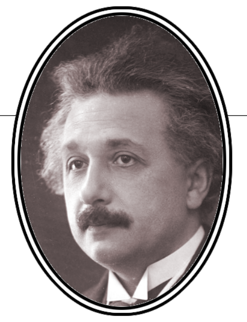
The observed curved path is a **kinematic effect** (acceleration).

The path of the body is due to its inertial motion, which is **determined only by the properties of space**.

In this case the observer is only **in a gravitational field.**

The observed curved path is a **dynamical effect** (gravitational force).

The path of the body is due to its response to an external gravitational field, which is **determined by the distribution of matter**.



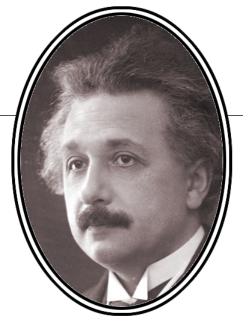
ASIDE: THE "EINSTEIN ELEVATOR"

Question from the audience:

"What about ***elevators****?*

There are always elevators involved in discussions of the Equivalence Principle.

We demand to see the actual ***Einstein elevator****!"*

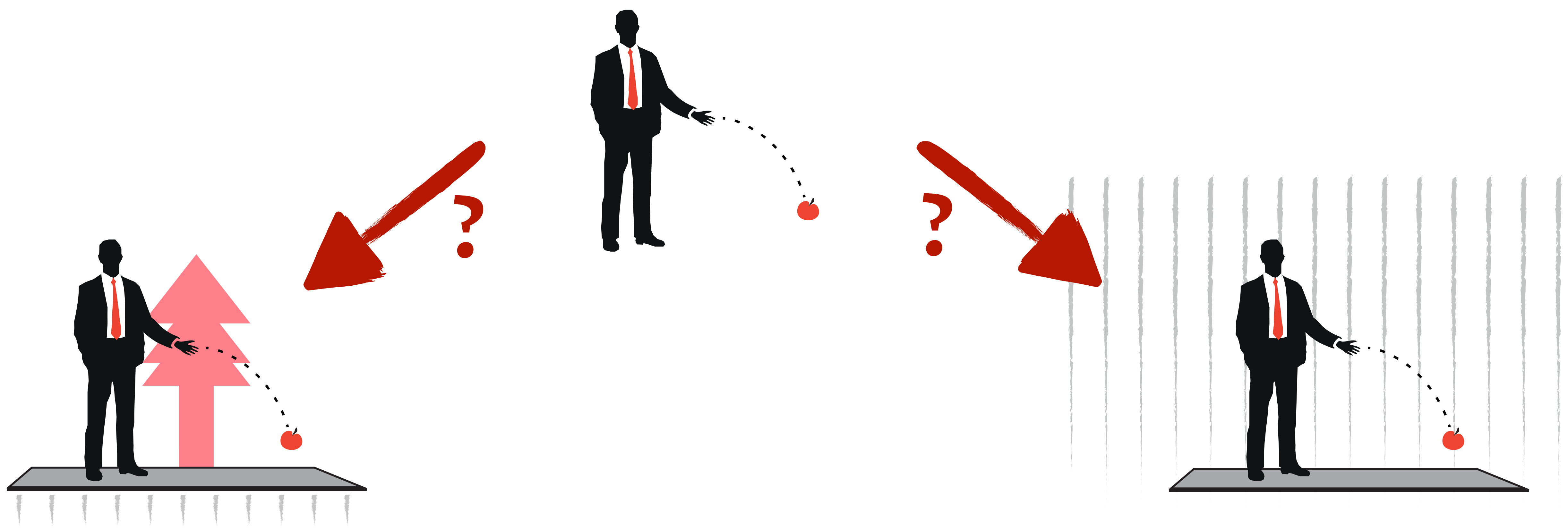


ASIDE: THE "EINSTEIN ELEVATOR"

Here is how Einstein describes the situation:

> **An observer enclosed in a box** can in no way decide whether the box is at rest in a static gravitational field, or whether it is in accelerated motion, maintained by forces acting on the box, in a space that is free of gravitational fields.

Albert Einstein, Marcel Grossmann, "Outline of a Generalized Theory of Relativity and of a Theory of Gravitation" (1913)



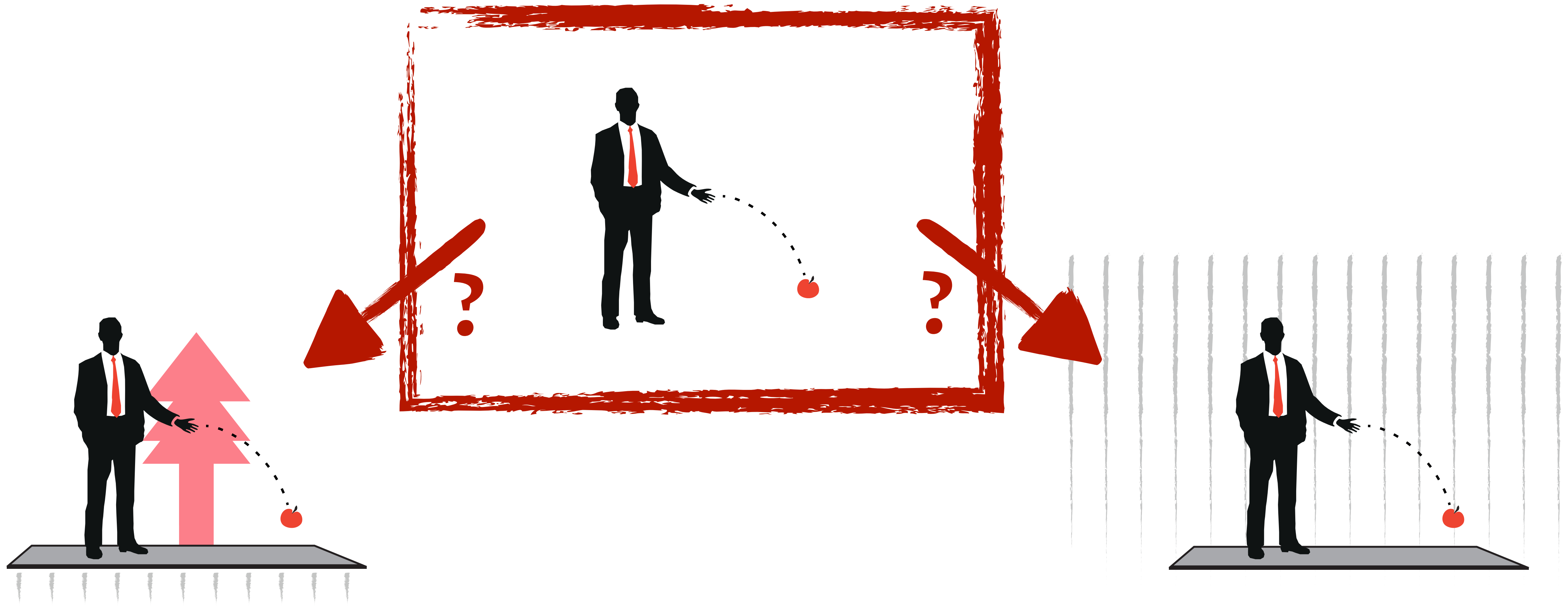
Explanation 1. **Explanation 2.**

ASIDE: THE "EINSTEIN ELEVATOR"

Here is how Einstein describes the pair of equivalent observers:

An observer enclosed in a box can in no way decide…

But why does the observer have to be **"enclosed in a box"**?



ASIDE: THE “EINSTEIN ELEVATOR”

Here is how Einstein describes the pair of equivalent observers:

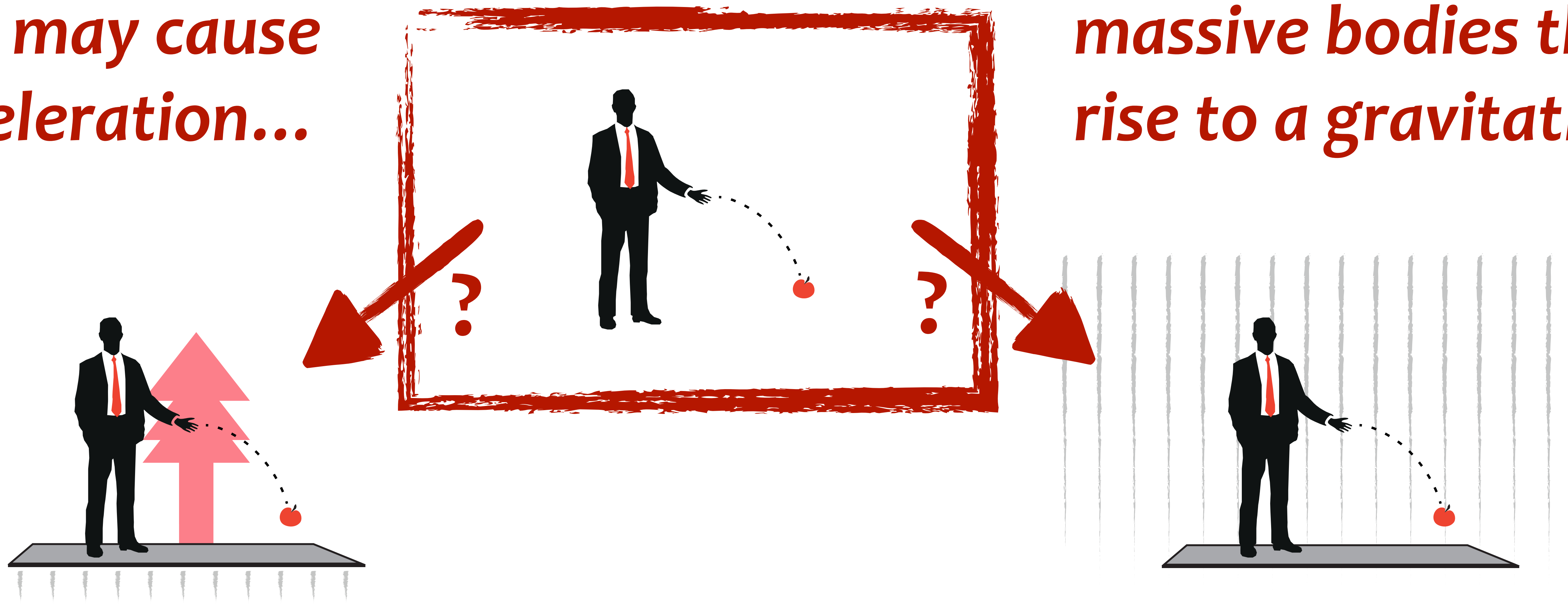
> **An observer enclosed in a box** can in no way decide…

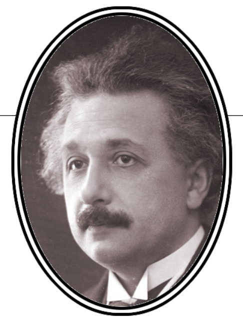
But why does the observer have to be **“enclosed in a box”**?

This is of course to prevent the observer…

…from checking for any device that may cause an acceleration…

…or from checking for any massive bodies that could give rise to a gravitational field.





ASIDE: THE “EINSTEIN ELEVATOR”

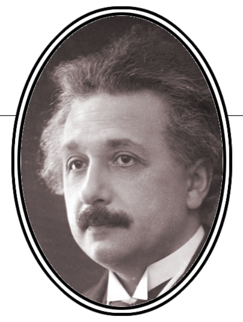
Two physicists, A and B, **awake from a narcotic sleep** and notice that **they are in a closed box that has nontransparent walls** and is equipped with all their instruments. They have no idea where the box is situated and whether it moves...

Albert Einstein, “On the Present State of the Problem of Gravitation” (1913)

Let us imagine **a spacious chest resembling a room** with an observer inside who is equipped with apparatus...

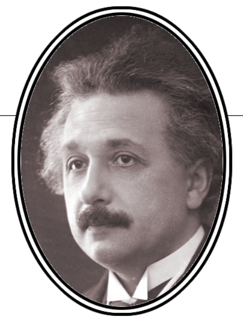
To the middle of the lid of the chest is fixed externally a hook with rope attached, and now **a ‘being’ (what kind of a being is immaterial to us)** begins pulling at this with a constant force...

Albert Einstein, “Relativity. The Special and the General Theory. A Popular Exposition” (1917)



ASIDE: THE “EINSTEIN ELEVATOR”

So **there is no mention of “elevators”** in Einstein’s writings from this time. Not even of “lifts”.



ASIDE: THE "EINSTEIN ELEVATOR"

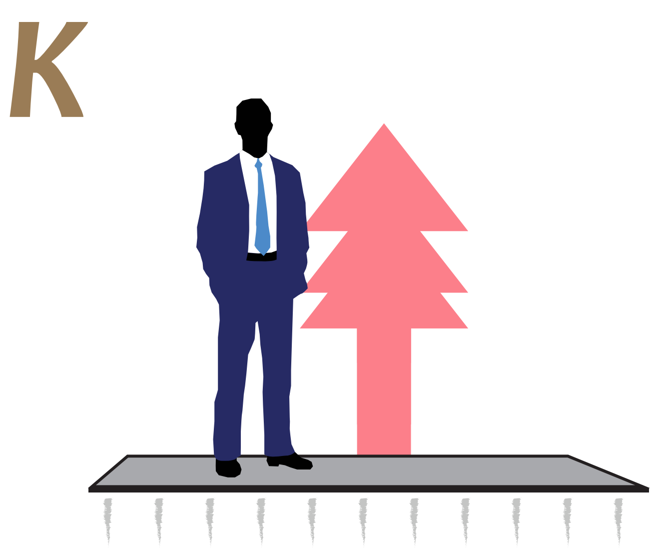
So **there is no mention of "elevators"** in Einstein's writings from this time. Not even of "lifts".

(What made Einstein daydream about drugging experimentalists and putting them in a box together with their equipment, and then have the box dragged into outer space by an alien life form, remains an unsolved problem in Einstein studies.)

EINSTEIN'S EQUIVALENT NEWTONIAN OBSERVERS

Step 2

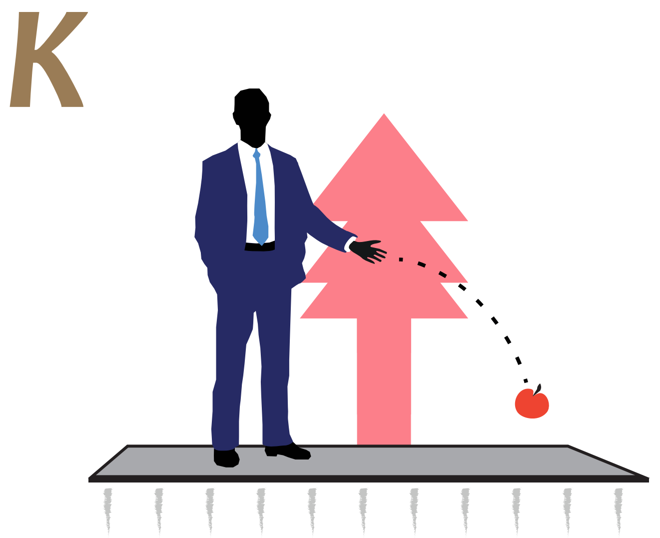
The next step is to think about the situations in the two explanations as actually being **two different observers,** ***K*** and ***K'***, in different parts of the universe:

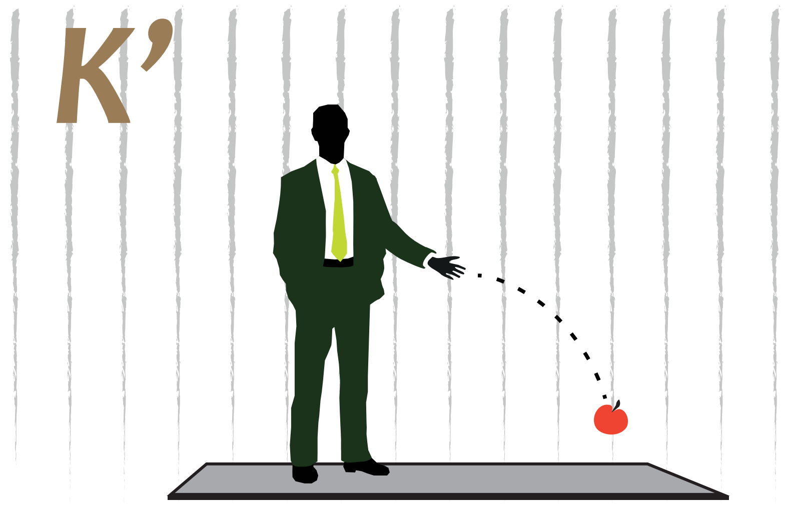


an observer ***uniformly accelerated in empty space***

an observer ***at rest in a homogeneous gravitational field.***

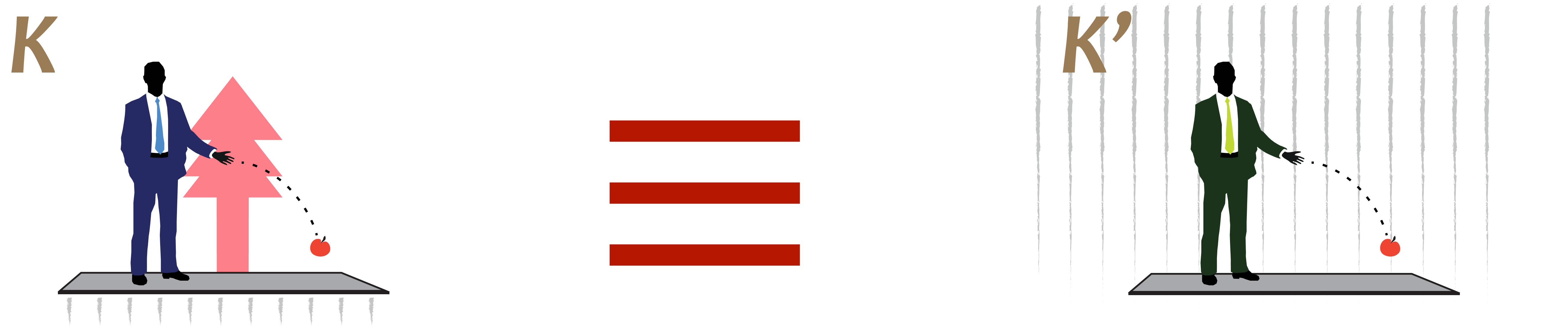
We have established that **the laws of Newtonian mechanics** (together with the observed equality of inertial and gravitational mass) **make exactly the same predictions** for any mechanical experiments the observers ***K*** and ***K'*** may make.





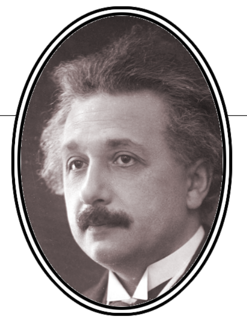
EINSTEIN’S EQUIVALENT NEWTONIAN OBSERVERS

We have established that **the laws of Newtonian mechanics** (together with the observed equality of inertial and gravitational mass) **make exactly the same predictions** for any mechanical experiments the observers ***K*** and ***K’*** may make.

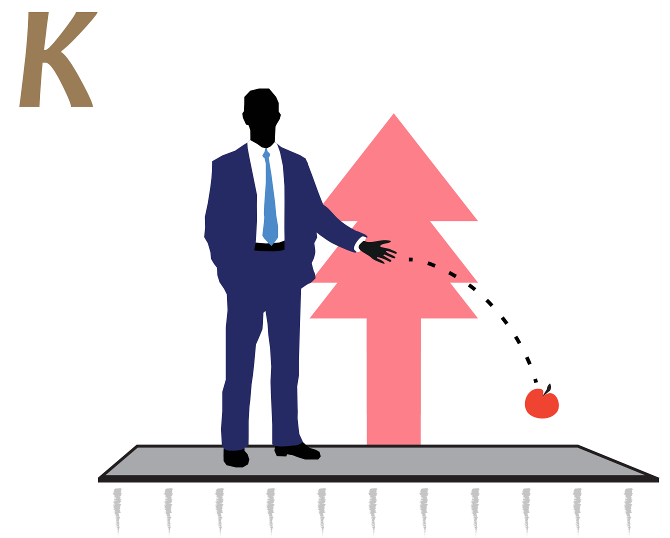


Einstein used to formulate this by saying that:

the two observers *K* and *K’* are “equivalent”



We have established that **the laws of Newtonian mechanics** (together with the observed equality of inertial and gravitational mass) **make exactly the same predictions** for any mechanical experiments the observers ***K*** and ***K'*** may make.



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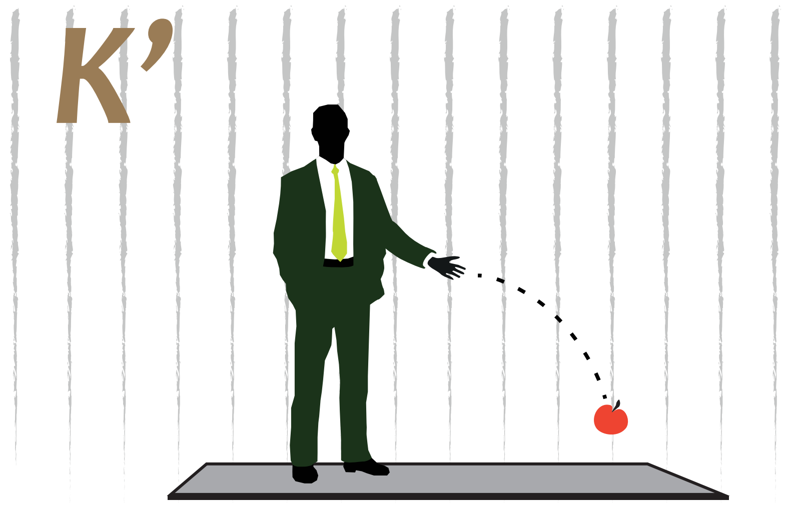
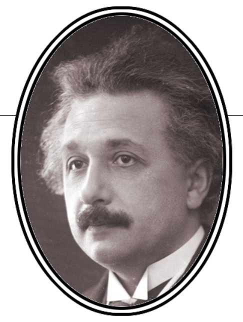


Einstein used to formulate this by saying that **the two observers *K* and *K'* are equivalent**. And here it is important to add that the **equivalence has only been established**

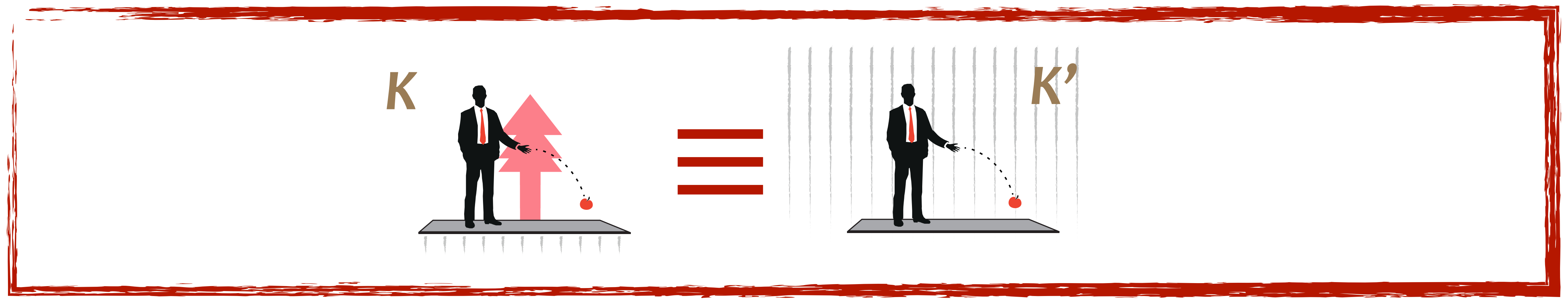
with respect to the laws of *Newtonian mechanics and Newtonian gravity*



THE ARGUMENT SO FAR…

1. THEOREM

Analysing **Newtonian physics,** and assuming that inertial and gravitational masses are equal, we have found two special observers that are **equivalent with respect to the laws of Newtonian physics**



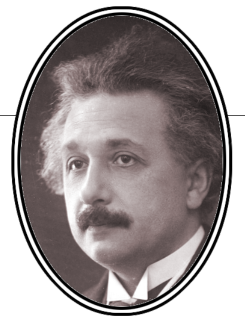
Newtonian physics

QUIZ *

What is it that is “equivalent”
in Einstein’s *Äquivalenzprinzip*?

---

* *Discuss amongst yourselves…*



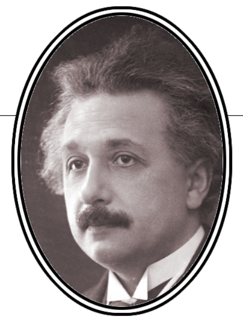
Step 3

Now let's make things a bit more interesting!

We will give the two observers ***K*** and ***K'*** **equipment to perform** **non-mechanical** **experiments.**

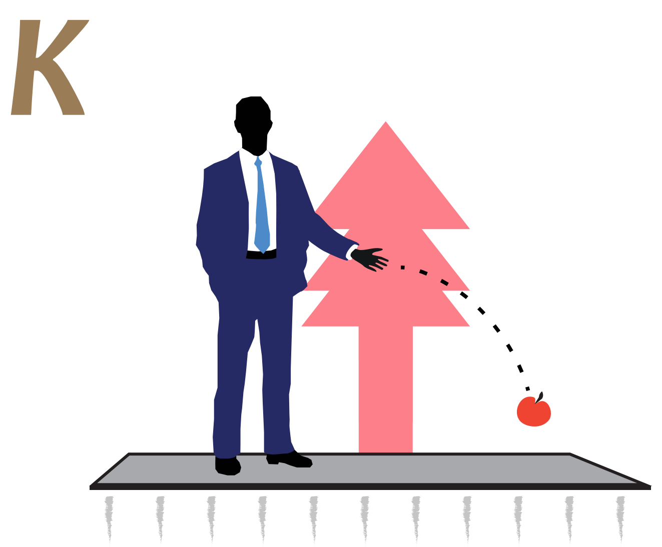
In particular: experiments involving an **electromagnetic field.**

Remember that ***Newtonian gravity does not affect electro-magnetic fields***!



We will get different predictions for the path of a light ray (in the rest frame of each observer) depending on whether or not there is a gravitational field present:

mechanical experiment



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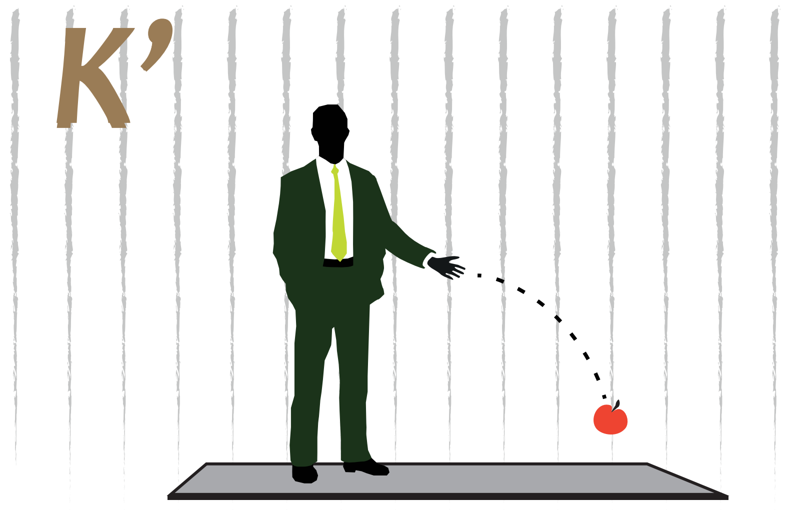
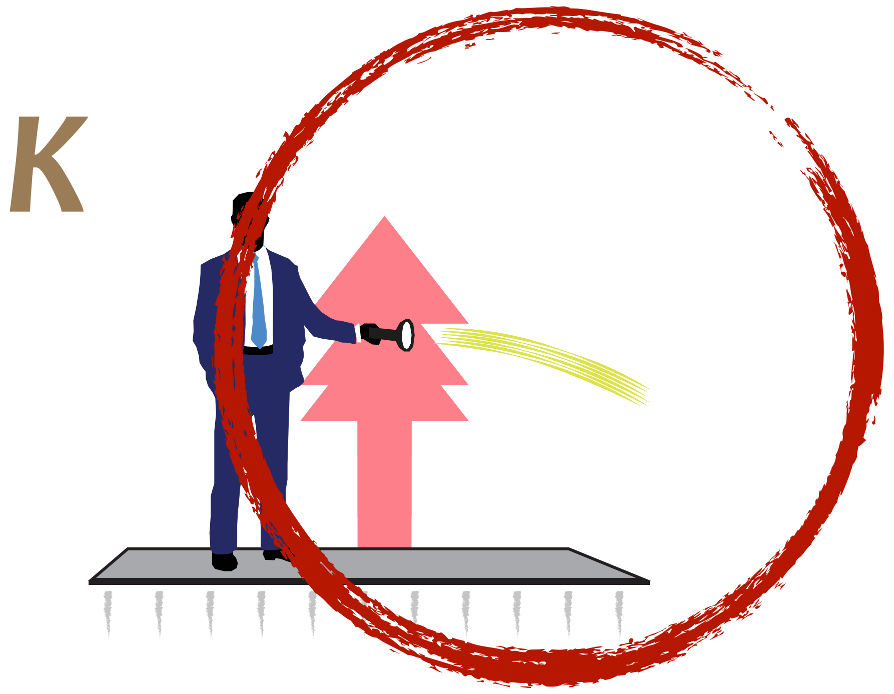


both observers get the **same** result (they are **equivalent**)

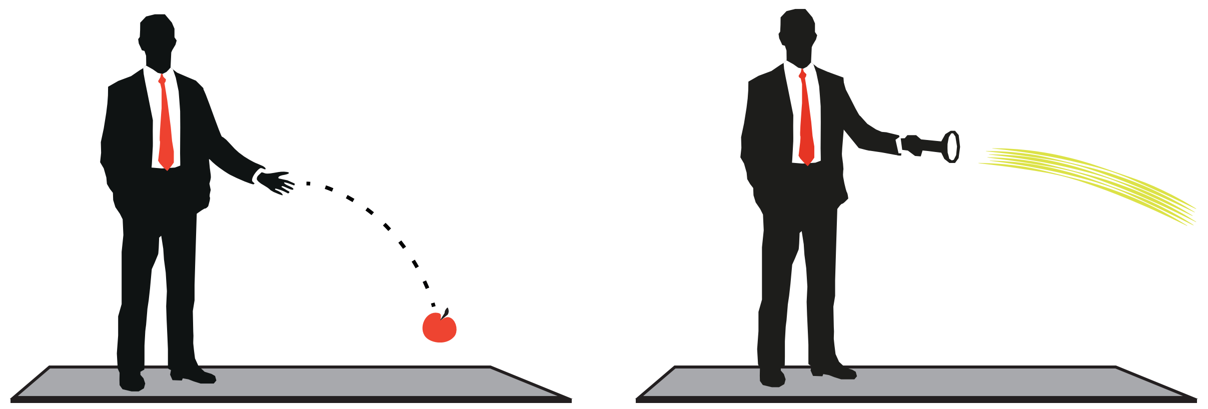
experiment with **light**



$\not\equiv$

the observers get **different** results (they are **not equivalent**)

We now realise that by using a combination of mechanical and non-mechanical (optical) experiments, **we can unambiguously determine whether or not there is a (Newtonian) gravitational field** in the region of space where the observers are:

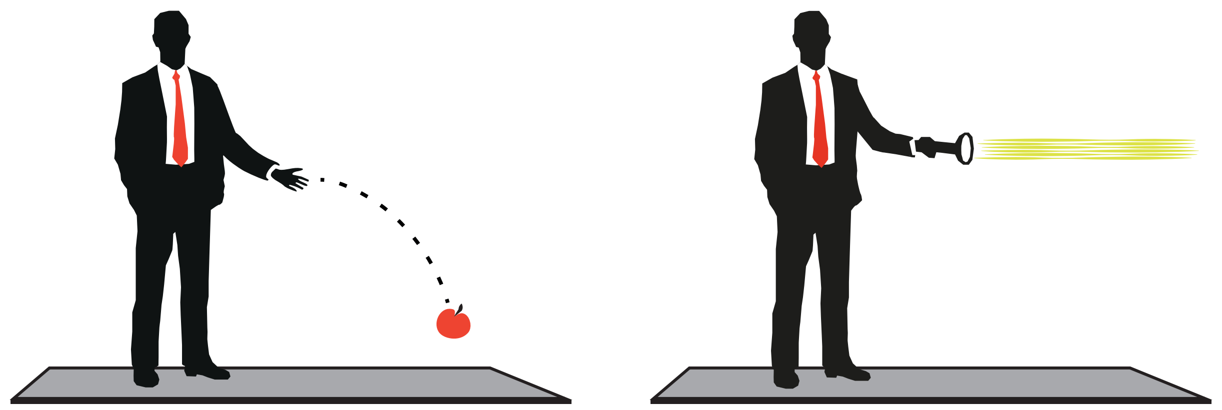


Observation

if both mechanical experiments and experiments with light give **the same outcome**

Explanation

— then **the observer is accelerated in a region without gravitational field**

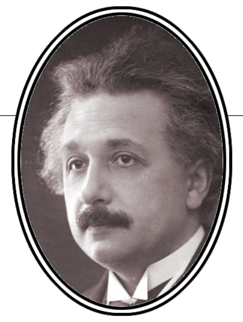


Observation

if mechanical experiments and experiments with light give **different outcomes**

Explanation

— then **the observer is at rest in a gravitational field**



THE ARGUMENT SO FAR…

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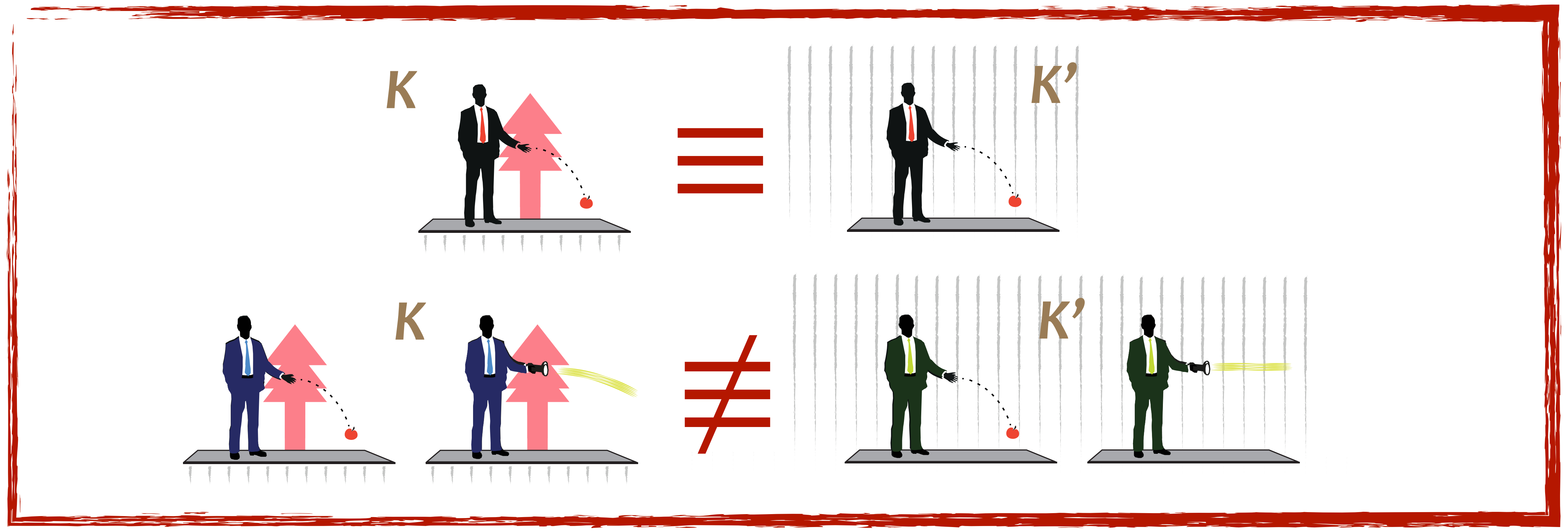
Analysing **Newtonian physics**, and assuming that inertial and gravitational masses are equal, we have found two special observers that are equivalent w.r.t the laws of Newtonian physics, but *not* equivalent w.r.t other laws, e.g. those of **electrodynamics**.



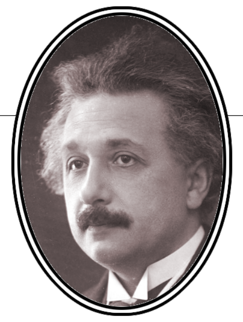
Newtonian physics

Classical physics

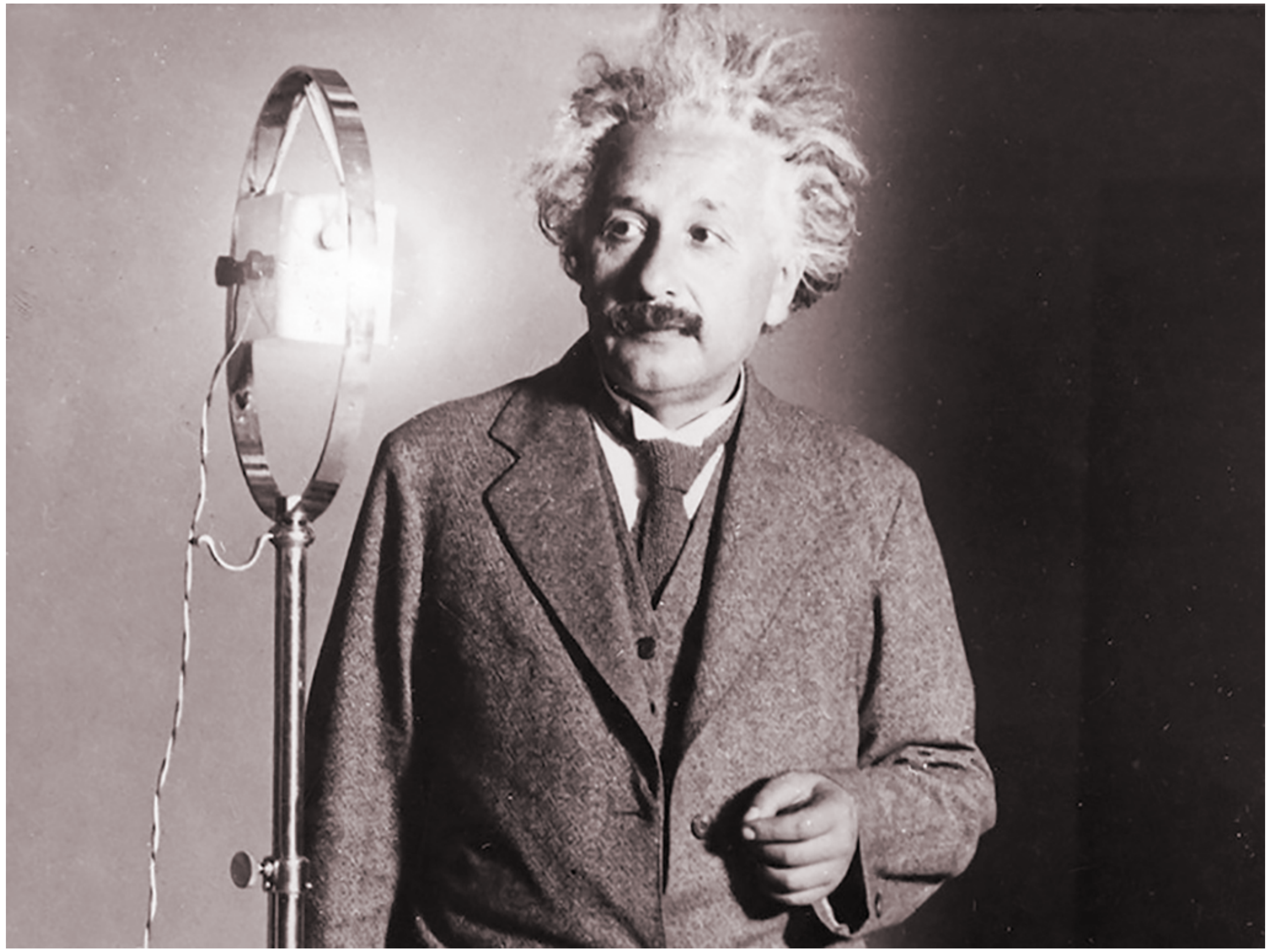
So far everything I have talked about is completely uncontroversial, and belongs to **standard Newtonian theory.**

The only addition has been the **empirically supported** assumption of numerical equality of inertial gravitational mass.

And **the only input from Einstein** has been the terminology of "**equivalent observers** (with respect to a particular theory)".



What Einstein does next
is quite unexpected…



PARADISE REGAINED — THE *ÄQUIVALENZPRINZIP*

We are now (1907) **looking for a relativistic theory of gravity.**

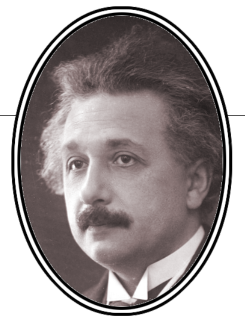
The standard SR guiding principle that

“all relativistic theories must be Lorentz invariant”

has so far **not helped anyone** to find such a theory of gravity.

What we need is **a *new* guiding principle**!

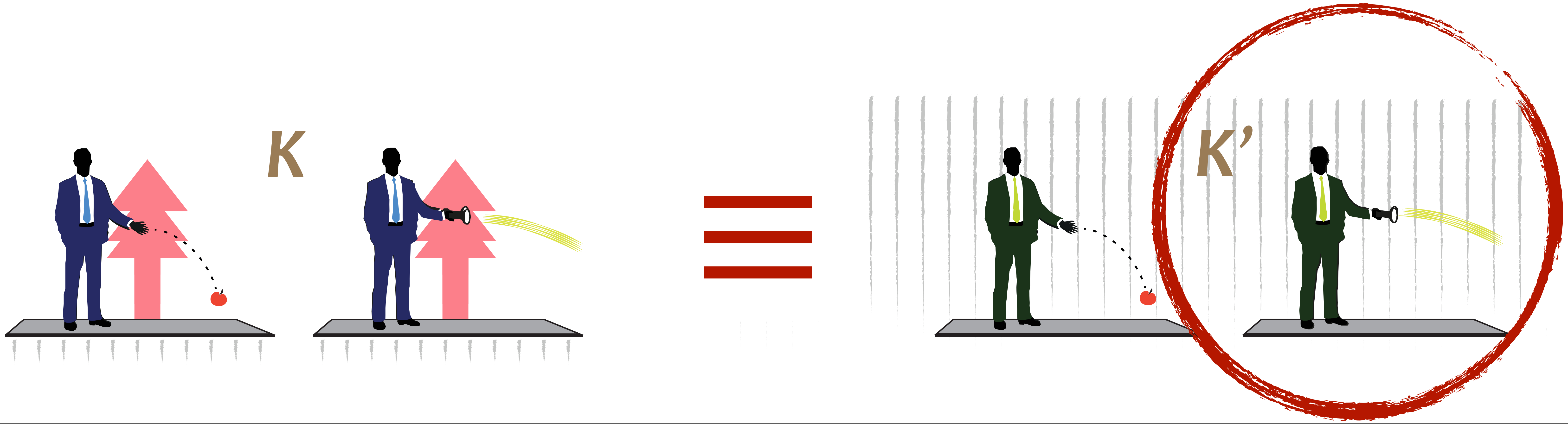
Einstein now proposes we try out the following guiding principle, and then see where it leads us:



Step 4

ÄQUIVALENZPRINZIP:

A uniformly accelerated observer in empty space and an observer at rest in a homogeneous gravitational field are **equivalent** with respect to ***all* laws of physics.**



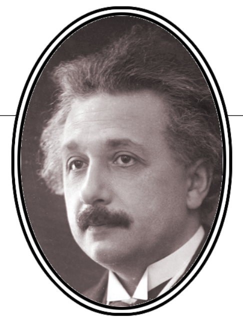
The relativistic theory must be such that this is true

The idea of postulating this equivalence between the two special observers was introduced already in 1907, but **the principle got its name *Äquivalenzprinzip* only later, in 1912.**

Here is a typical formulation of the principle:

> In a **homogeneous gravitational field** all motions take place in the same way as in the absence of a gravitational field in relation to a **uniformly accelerated coordinate system.**
>
> **If this principle held good for any events whatever** (the "principle of equivalence") [...] we were to reach **a natural theory of the gravitational fields.**

Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)

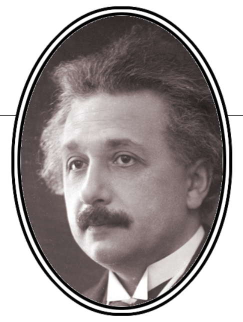


PARADISE REGAINED — THE ÄQUIVALENZPRINZIP

Another Einstein quote clarifies the logic of the "*Äquivalenz-prinzip*" argument even further:

> **As long as we confine ourselves to purely mechanical processes** within the range of validity of Newton's mechanics, **we can be sure of the equivalence of the systems *K* and *K'*.**
>
> However, for our conception [*of equivalence*] to acquire deeper significance, **the systems *K* and *K'* must be equivalent with respect to all physical processes**, *i.e.*, the natural laws with respect to *K* must coincide completely with those with respect to *K'*.
>
> If we accept this assumption, **we obtain a principle that possesses great heuristic significance…**

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



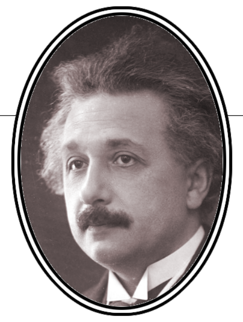
I CAN'T EMPHASIZE THIS STRONGLY ENOUGH!

- The equivalence (**with respect to mechanics**) of the observers ***K*** and ***K'*** **is just a theorem in Newtonian physics** (with additional empirical input about $m_i = m_g$).

This tells us nothing new about gravity!

- But Einstein wants to **go beyond Newtonian physics**! He **postulates** the equivalence of the observers **with respect to *all* laws of physics** (this is the *Äquivalenzprinzip*).

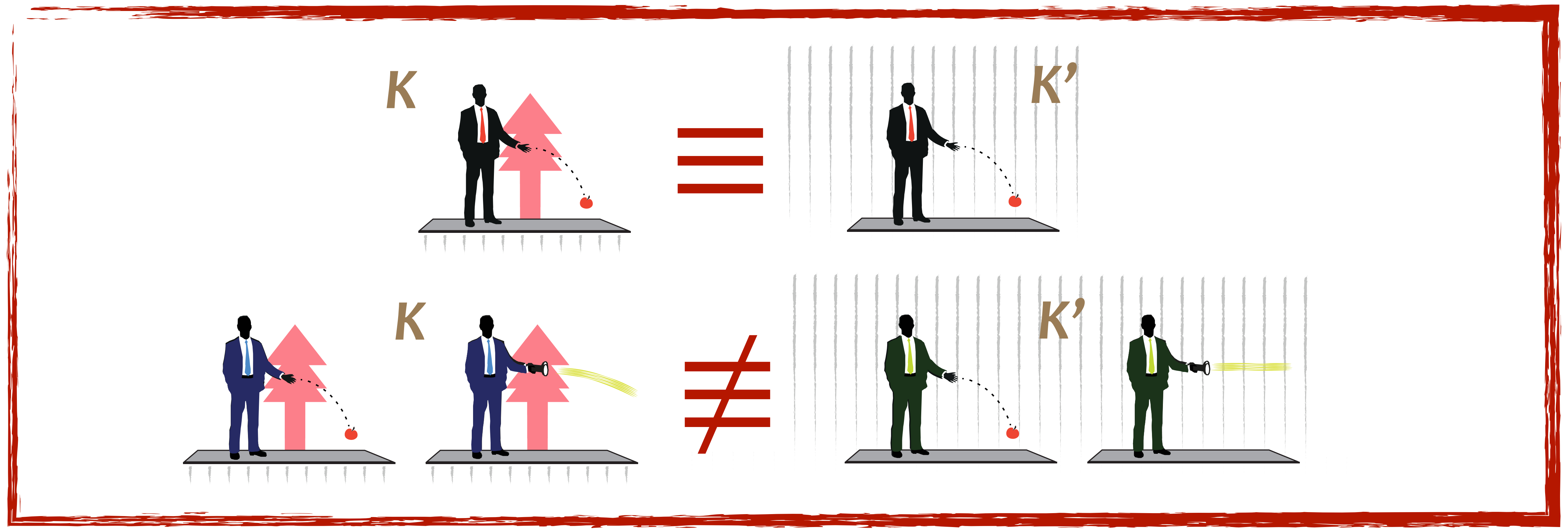
The hope is that this might give us a glimpse of the (unknown) relativistic theory of gravity.



THE ARGUMENT SO FAR…

1. THEOREM

Analysing **Newtonian physics**, and assuming that inertial and gravitational masses are equal, we have found two special observers that are equivalent w.r.t the laws of Newtonian physics, but *not* equivalent w.r.t other laws, e.g. those of **electrodynamics**.

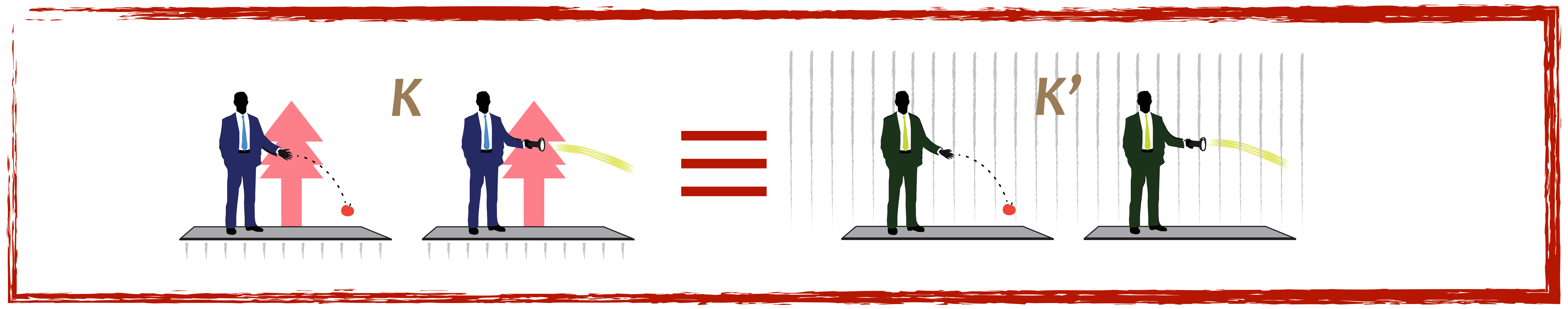


Newtonian physics

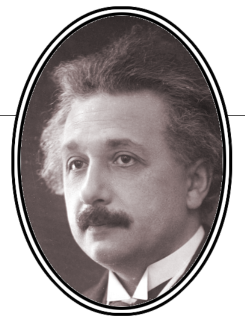
Classical physics

2. POSTULATE

Äquivalenzprinzip. In **a relativistic theory of gravitation** the special observers ***K*** and ***K'*** must be equivalent with respect to all laws of physics.



Unknown relativistic theory of gravity



Step 5

So how did Einstein think that the ***Äquivalenzprinzip*** could help him get a glimpse of the new unknown relativistic theory of gravity?

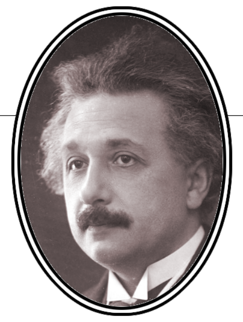
THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

"… we obtain a principle that possesses **great heuristic significance**"

The heuristic value of this assumption [*the Equivalence Principle*] rests on the fact that it permits the replacement of a homogeneous gravitational field by a uniformly accelerated reference system, **the latter case being to some extent accessible to theoretical treatment.**

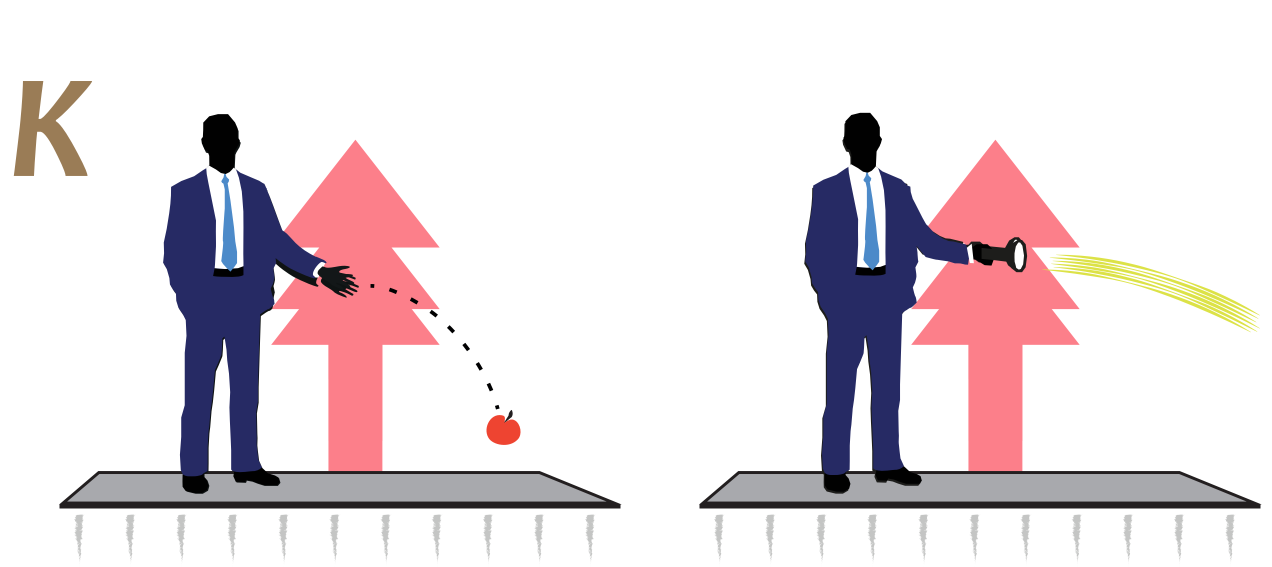
Albert Einstein, "On the relativity principle and the conclusions drawn from it" (1907)

But what does this mean?

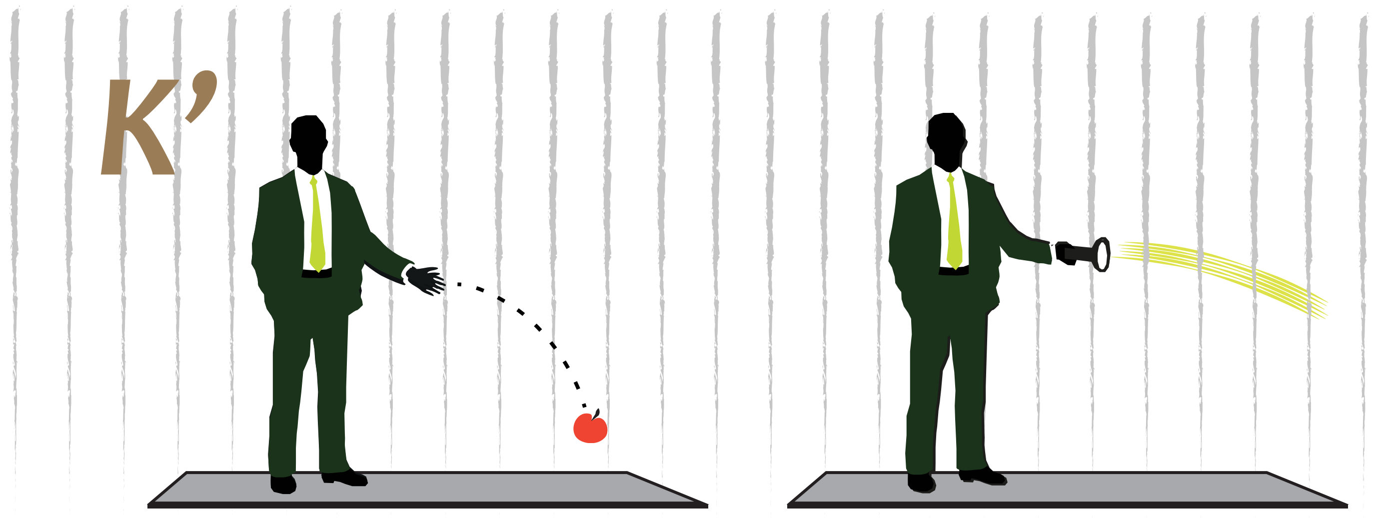


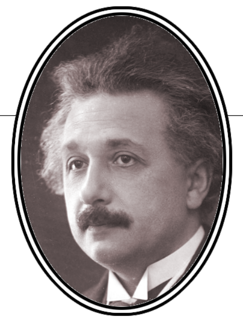
Let's take a closer look at our pair of observers, and the description of their physical experiments (assuming the *Äquivalenzprinzip*).

Non-inertial observer
in gravity-free space



Inertial observer
in a gravitational field





THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

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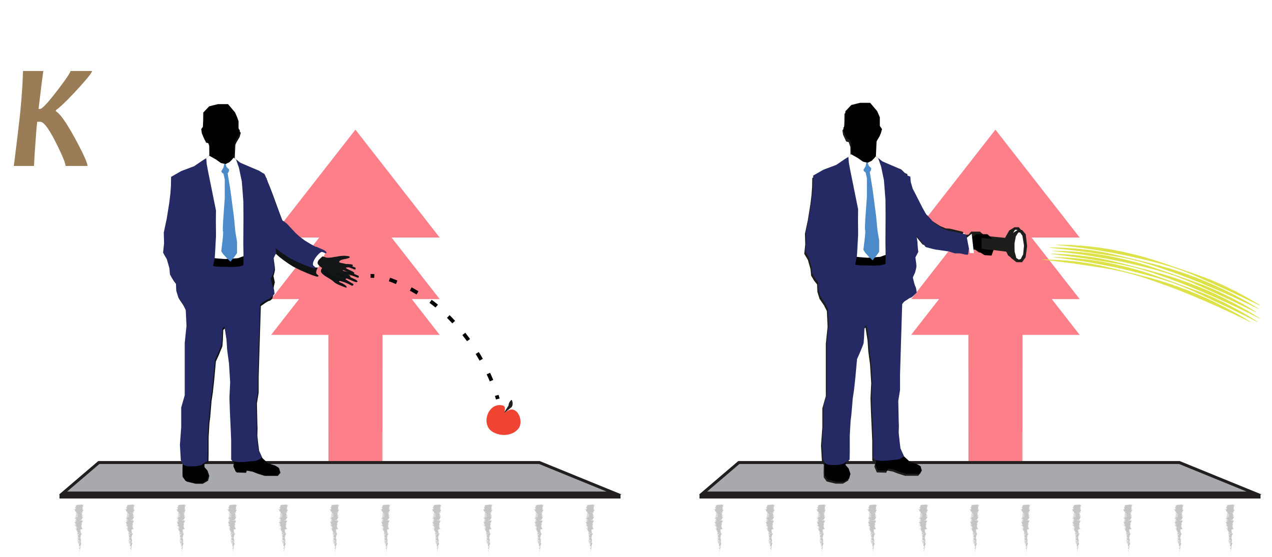
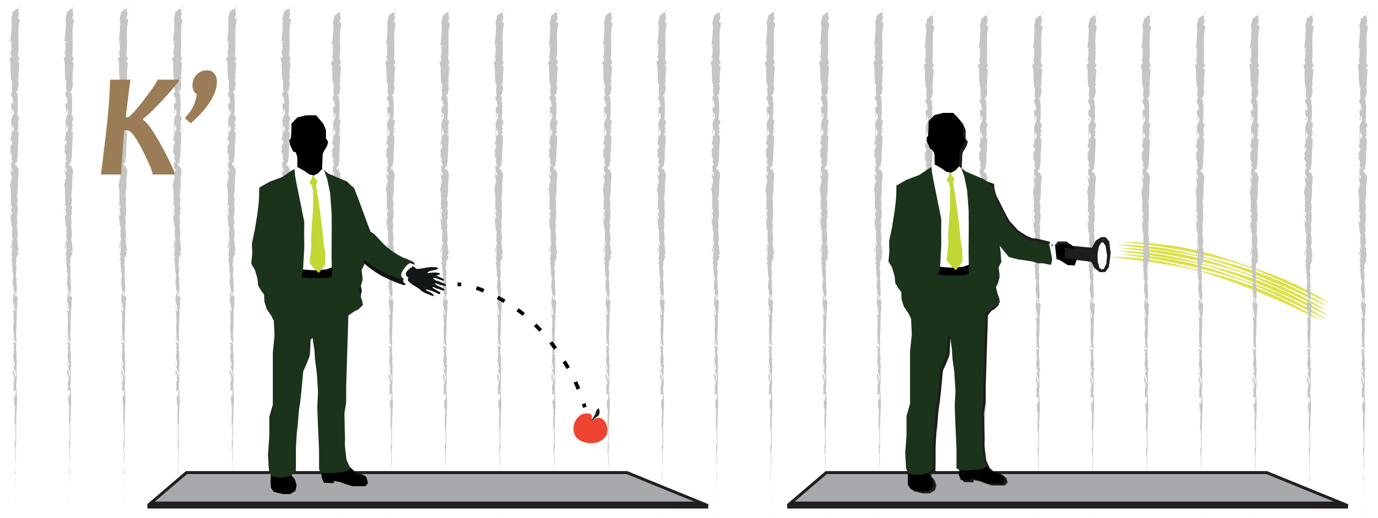
in gravity-free space



The observed curved path is a
kinematic effect
(acceleration)

Inertial observer
in a gravitational field

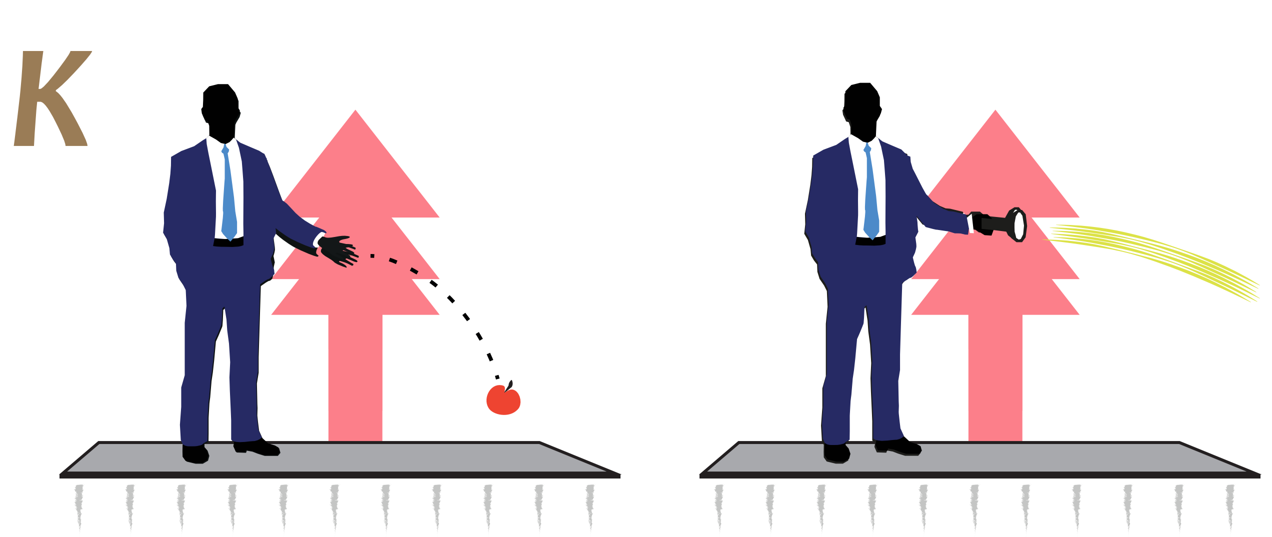


The observed curved path is a
dynamical effect
(gravitational force)

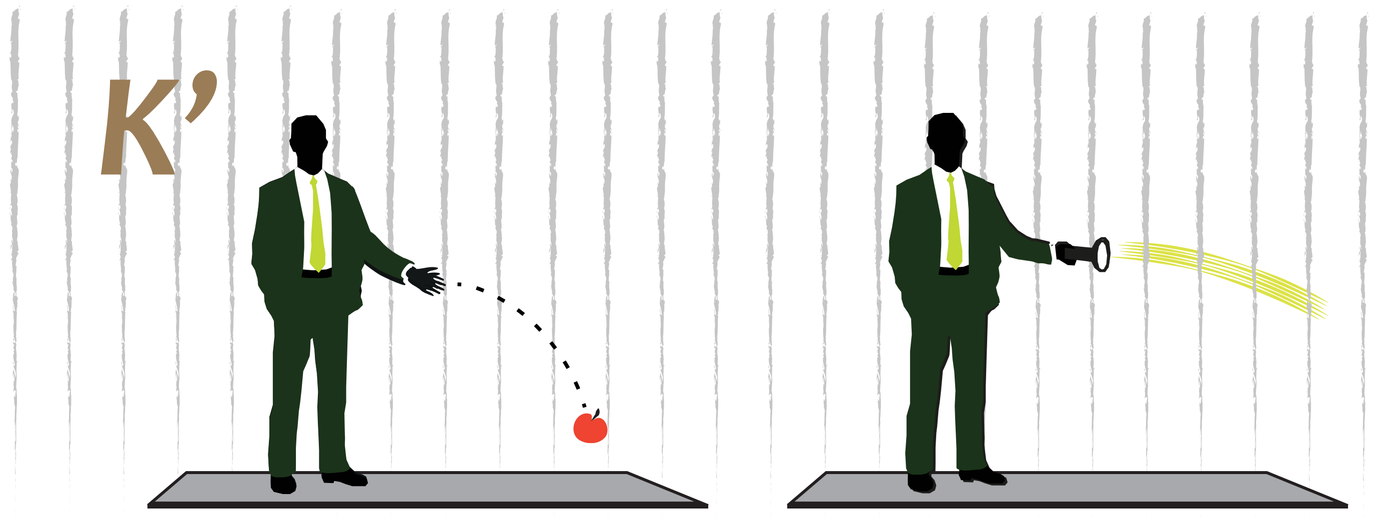
THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

Let's take a closer look at our pair of observers, and the description of their physical experiments (assuming the *Äquivalenzprinzip*).

Non-inertial observer
in gravity-free space



Inertial observer
in a gravitational field



We know how to calculate **kinematic** effects

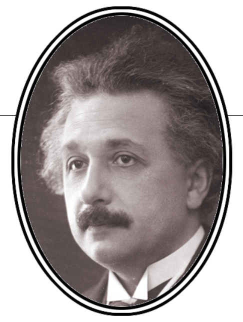
We do not know how to calculate **dynamical** effects

We can do this, *e.g.*, in **Newtonian physics** or in **special relativity**.

Kinematic effects are "**to some extent accessible to theoretical treatment**".

This is because **the theory of *relativistic* gravitation is still *unknown***.

We need help to find the laws of *relativistic* gravitation!

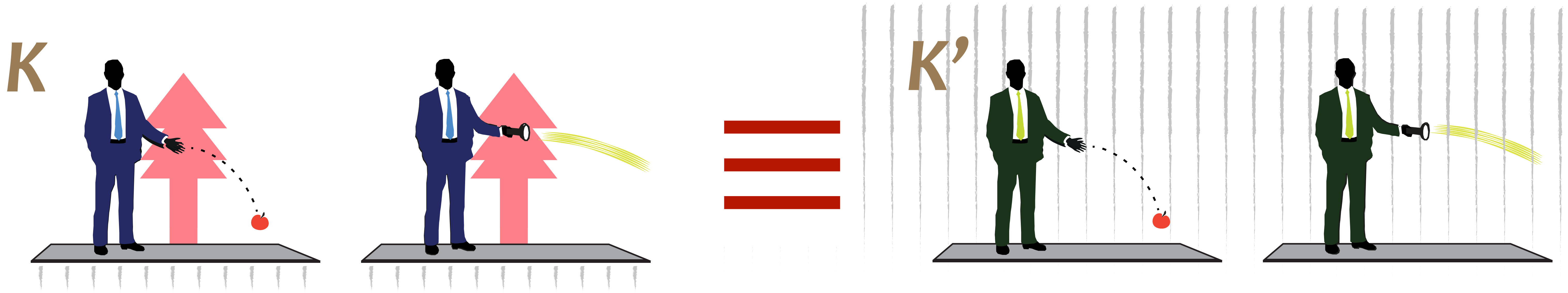


THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

Let’s take a closer look at our pair of observers, and the description of their physical experiments (assuming the *Äquivalenzprinzip*).

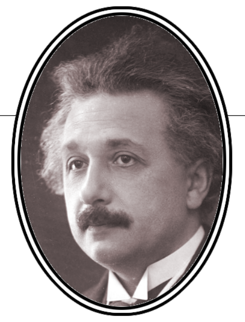
Non-inertial observer
in gravity-free space

Inertial observer
in a gravitational field



But we know — **by assumption** (this is the *Äquivalenzprinzip*) — that at least **these two special observers K and K’ are *equivalent*** in the unknown theory of gravitation.

That is: **the laws of physics** (including the unknown laws of relativistic gravitation) **must give exactly the same physical predictions in both frames of reference.**

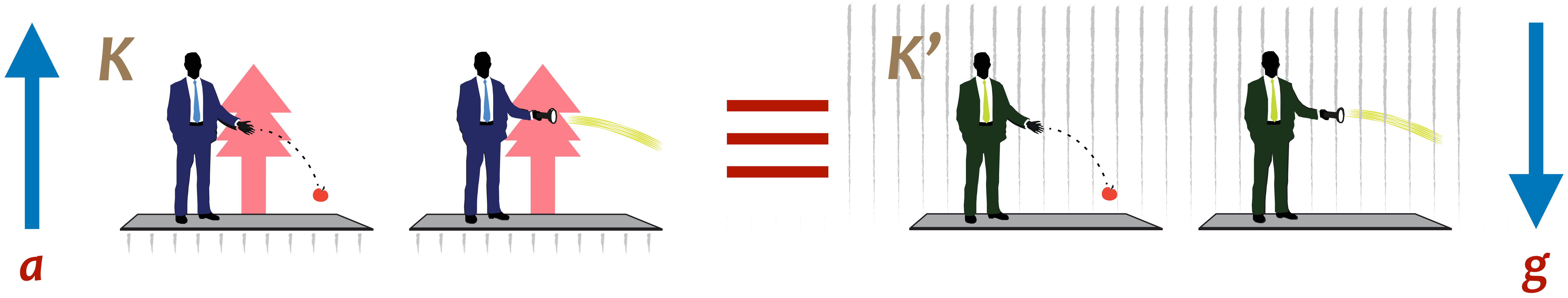


THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

This is how it could work (*in reality it is a bit more complicated…*)

observer is accelerated,

physical systems are inertial



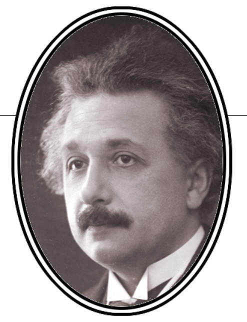
physical systems are accelerated,

observer is inertial

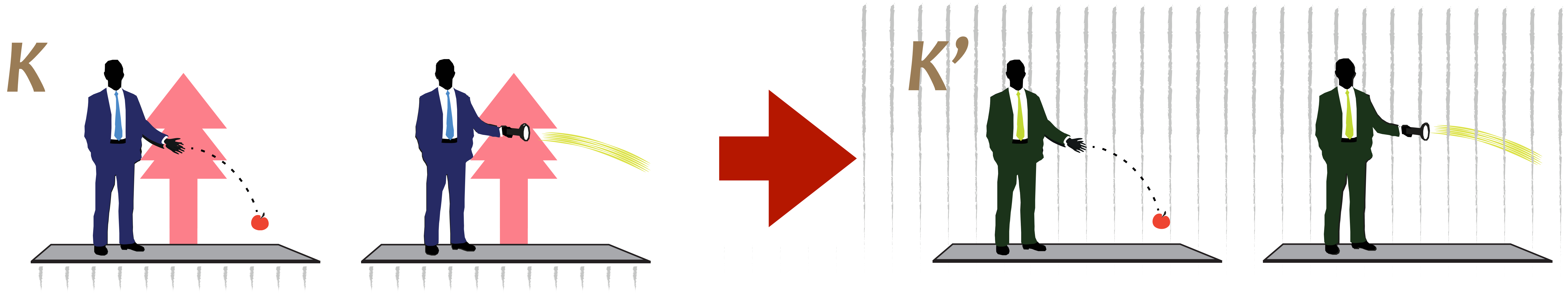
1. **Perform calculations using known laws** of (gravity-free) physics, transformed to a frame of reference which is uniformly accelerated with acceleration ***a***.

2. **Translate** the calculated results to an inertial frame in a homogeneous gravitational field by **replacing** the **kinematic acceleration** ***a*** with the **gravitational acceleration –g** in all relevant laws of physics.

3. **This gives us a clue** about the way gravity in the **unknown theory** will influence physical processes.



known physics

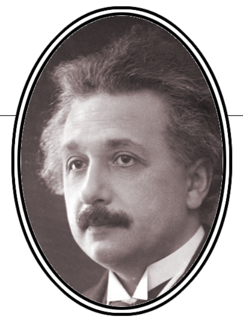


unknown physics

The **equivalence** of the two observers was established using **Newtonian physics**: we needed a theoretical context which had both kinematics and gravitational dynamics.

But note that now all actual calculations will be made **only on the kinematic side of the equivalence.**

We can therefore do all calculations using **special relativity**!



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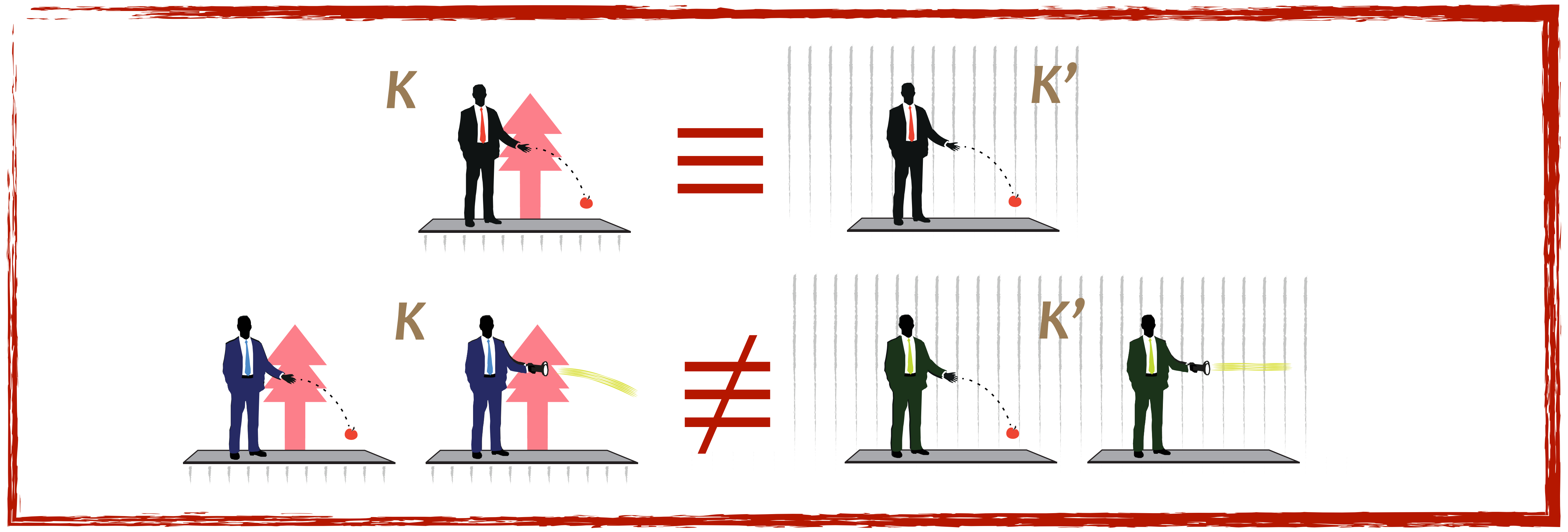
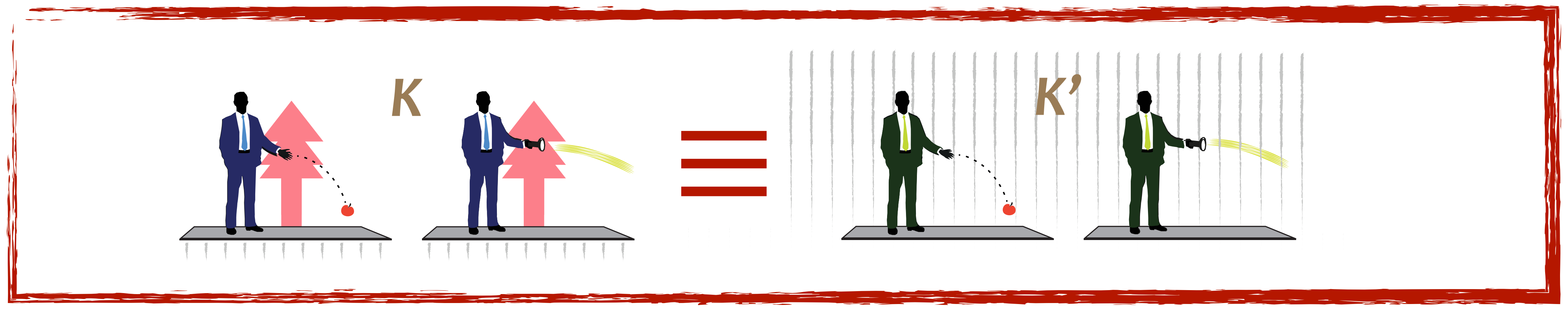


Newtonian physics

Classical physics

2. POSTULATE

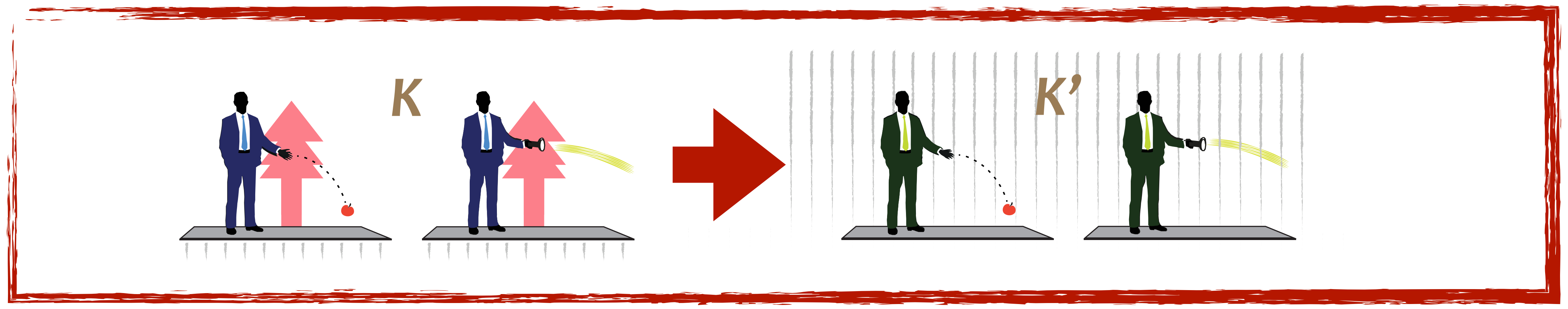
Äquivalenzprinzip. In **a relativistic theory of gravitation** the special observers ***K*** and ***K'*** must be equivalent with respect to all laws of physics.



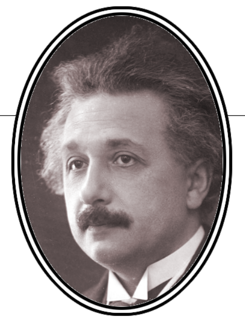
Unknown relativistic theory of gravity

3. HEURISTIC

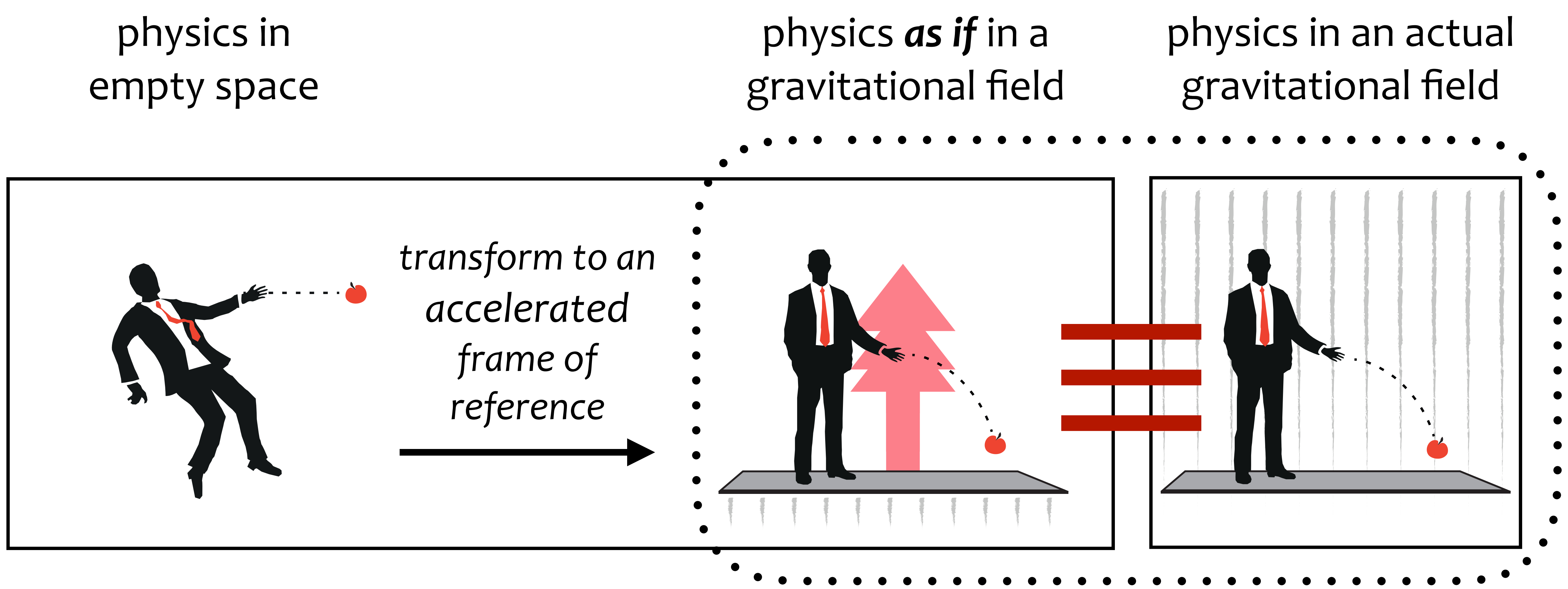
***Äquivalenzprinzip* Heuristic**. Analyse physical systems in **special relativity** w.r.t an accelerated observer. The results must also be valid for an observer in a gravitational field (in the unknown **relativistic theory of gravitation**).



Special relativity



Step 6 There is **a nice intuitive way** to talk about the heuristic procedure:

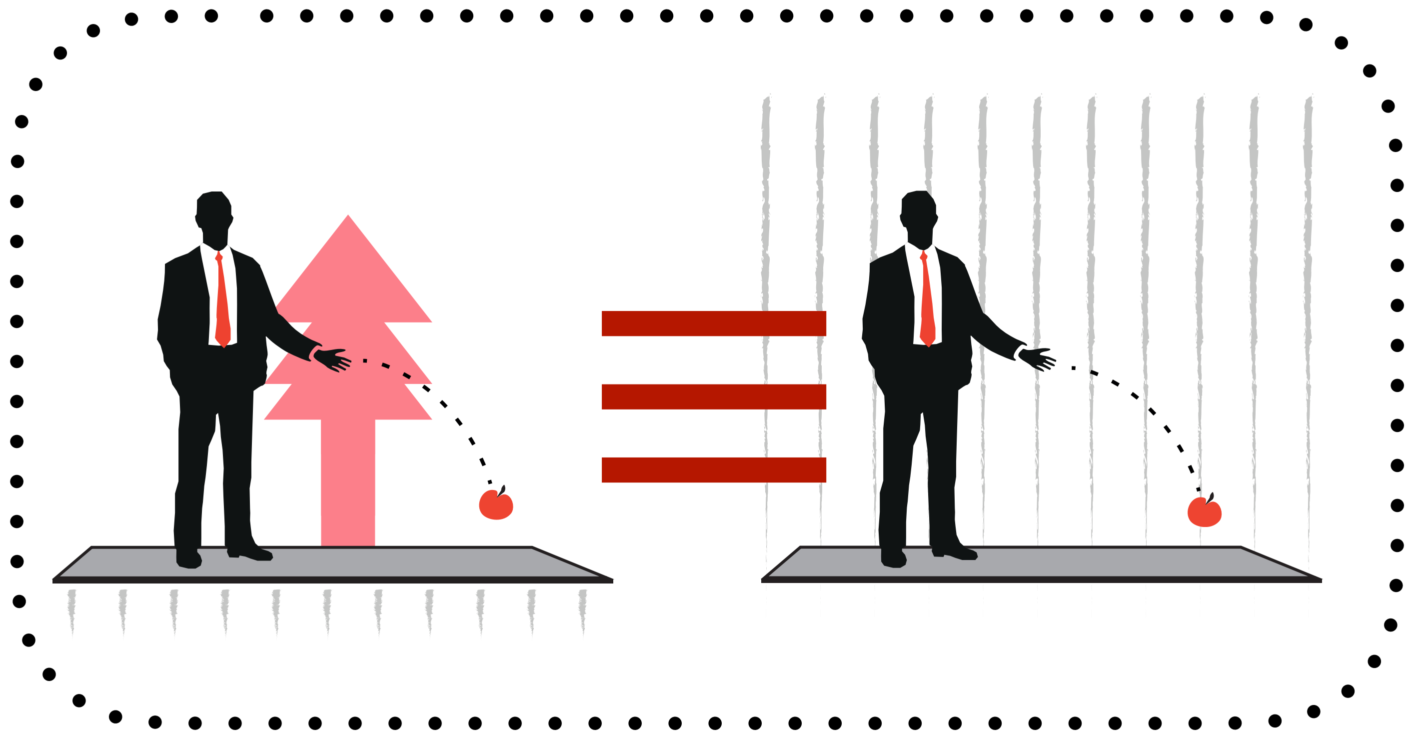


If you are an observer in free space **and start accelerating** upwards you become one of the observers that are equivalent (as far as the laws of Newtonian physics go) to an observer at rest in a gravitational field.

It is as if, **by going from rest to an accelerated frame of reference** (in empty space), **you have "created" a** (very special) **gravitational field.**

So why don't we call these two equivalent observers

the "generative" observers



since they show us how one can "create" or "generate" a gravitational field.

THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

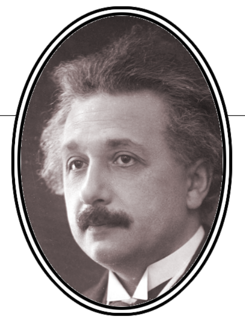
The first phase of Einstein’s search for GR (1907–1911)

These first clues to the new physical **effects** one could expect from a relativistic theory of gravity were obtained 1907–1911.

The second phase of Einstein’s search for GR (1912–1915)

Einstein’s **next step** would be to try to find the **causes** of the relativistic gravitational field (the *field equations*). He worked on this 1912–1915, with many false starts and problems.

This second phase **required new ideas,** beyond what the *Äquivalenzprinzip* could provide…



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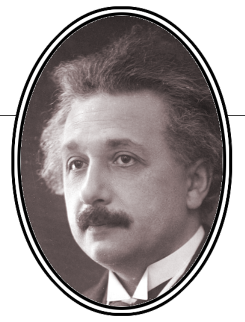
…but how Einstein found the final theory of general relativity is a different story!



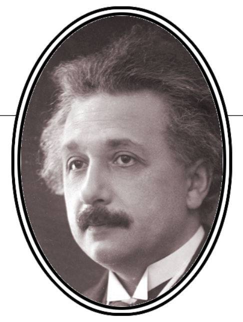
THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

"… aber leichter ist ahnen als finden."

…but premonition is easier than discovery

Albert Einstein, letter to Rudolf Förster,
16 November 1917

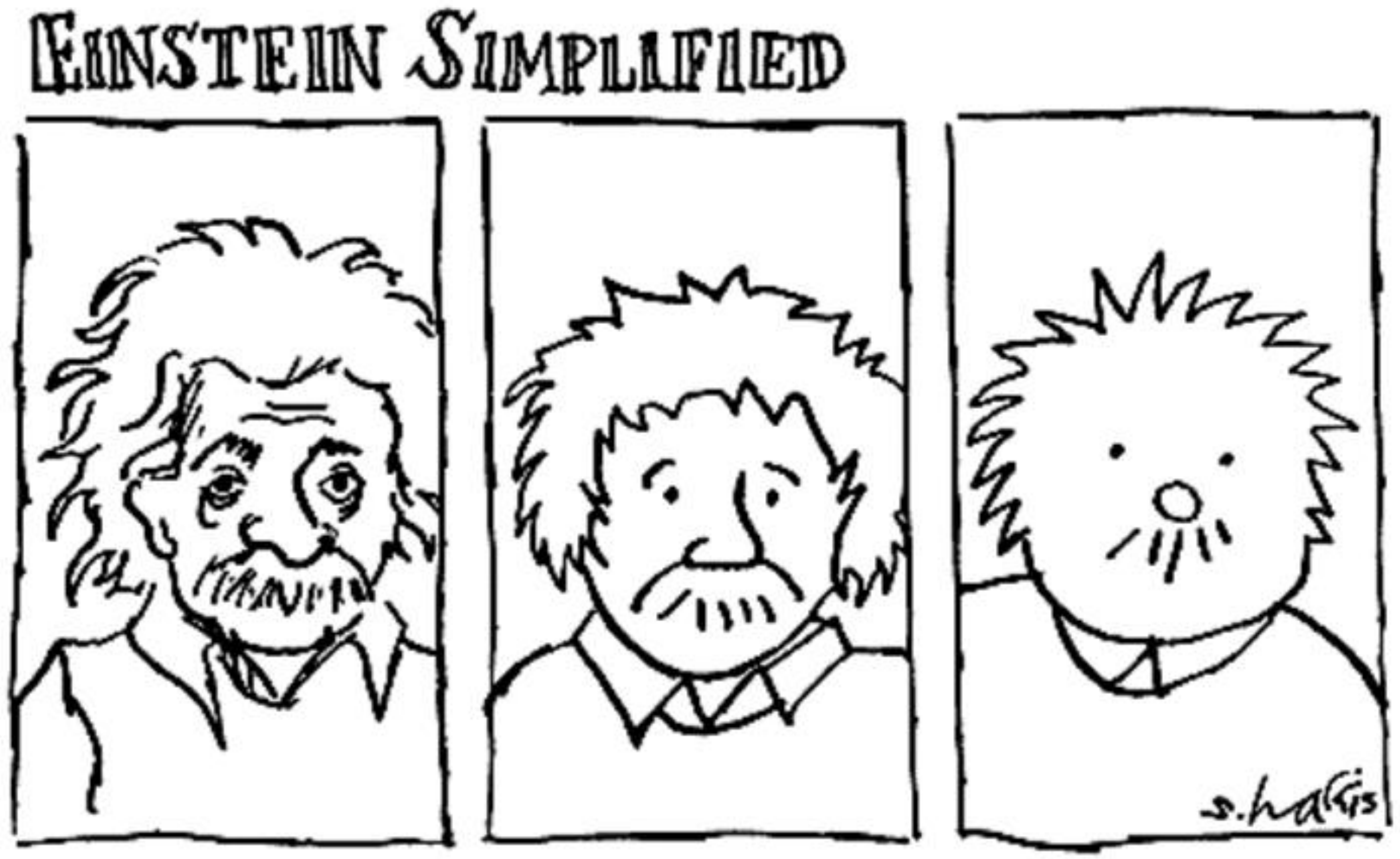
…but how Einstein found the final theory of general relativity is a different story!



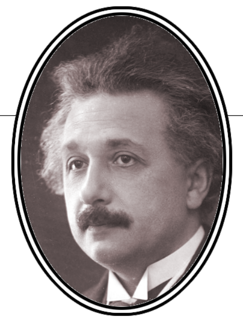
I AM NOT FOLLOWING THE HISTORICAL DEVELOPMENT

I have told the story **as if** Einstein in 1907 was motivated mainly by **the search for a new relativistic theory of gravity.**

Actually, his main motivations lay elsewhere. But this would take too long for me to to describe…



Cartoon by **Sidney Harris**



PART 2

THE MODERN EQUIVALENCE PRINCIPLES

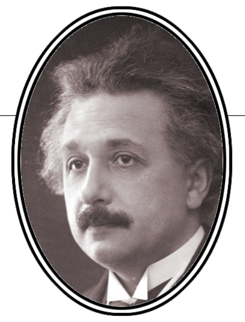
THE SECOND PAIR OF EQUIVALENT OBSERVERS

So how did **Einstein's *Äquivalenzprinzip*** argument develop into our modern **Weak and Strong Equivalence Principles**?

To understand this, we first need to explore an intermediate version of the equivalence principle:

the Infinitesimal Equivalence Principle

And just as in the case of the *Äquivalenzprinzip*, this principle emerges from **an analysis of a confused observer**…



THE SECOND PAIR OF EQUIVALENT OBSERVERS

Step 1 In an otherwise empty region of space there is an observer performing **mechanical experiments:**

Observation

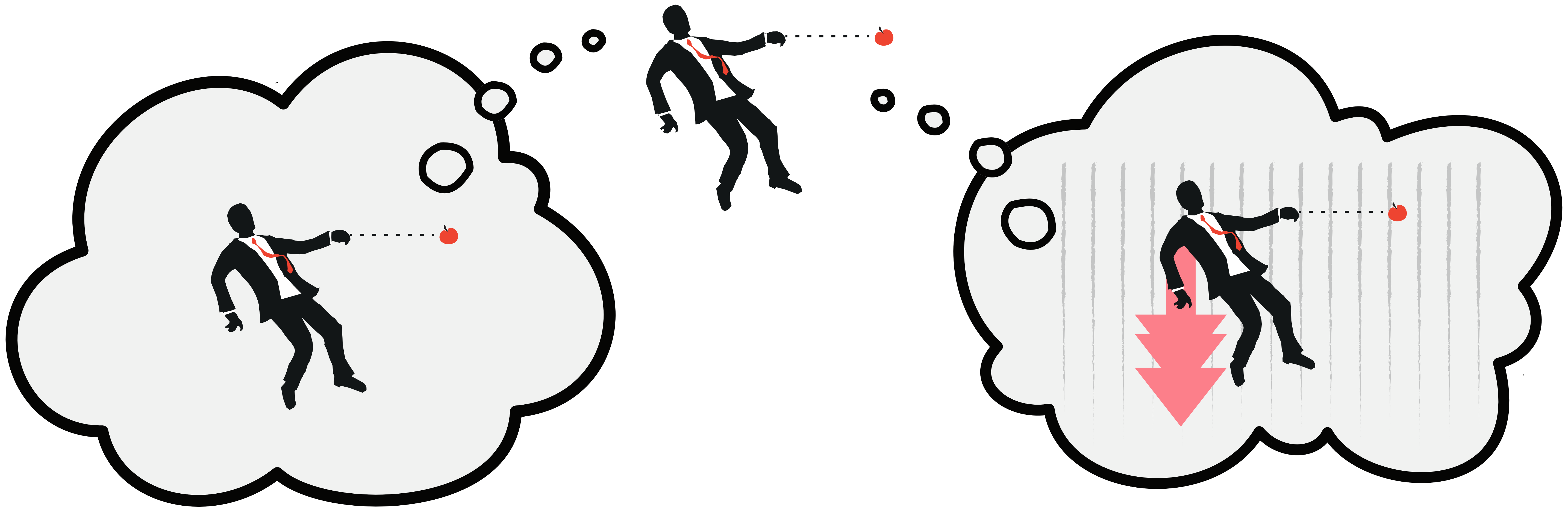
This time, when the observer throws a ball it **moves uniformly** along a **straight path**.

How would the observer explain* the observed behaviour of the thrown ball?

* *using only **Newtonian** physics!*

THE SECOND PAIR OF EQUIVALENT OBSERVERS

Again **Newtonian physics** provides us with two possible explanations, one in **empty space** and one in a **gravitational field**:



Explanation 1.

Equations of motion for body thrown by the **observer freely moving (or at rest) in a region without gravitational field** (in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 y^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 z^\mu}{\mathrm{d}t^2} = 0$$

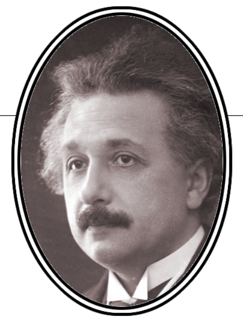
… **a straight line**

Explanation 2.

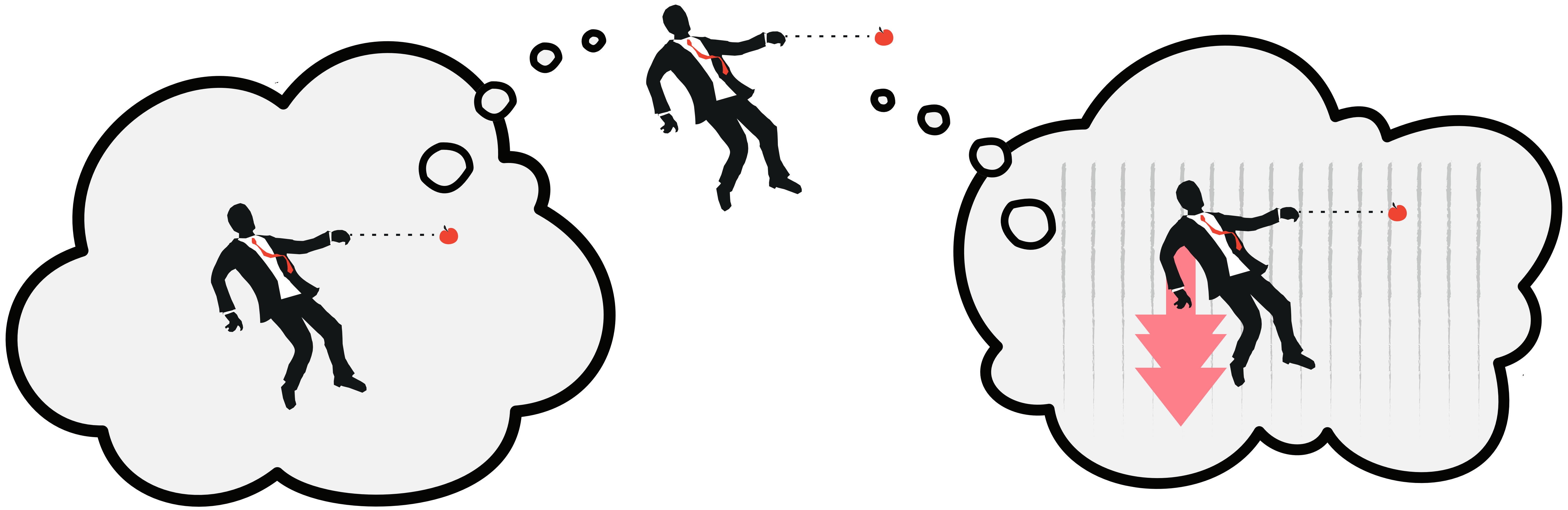
Equations of motion for body thrown by the **observer freely falling in a homogeneous gravitational field** (in coordinates adapted to the rest frame of the observer):

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 y^\mu}{\mathrm{d}t^2} = 0, \quad m_i \frac{\mathrm{d}^2 z^\mu}{\mathrm{d}t^2} = 0$$

… **a straight line**



THE SECOND PAIR OF EQUIVALENT OBSERVERS



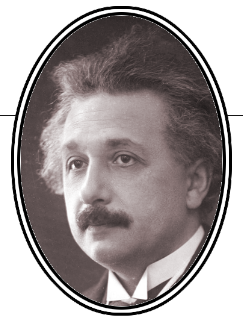
So **any mechanical process plays out exactly the same way** for **an observer in free fall** as for an **observer at rest**!

THE SECOND PAIR OF EQUIVALENT OBSERVERS

This insight made a great impression on the young Einstein — it was nothing less than ***the happiest thought of his life***:

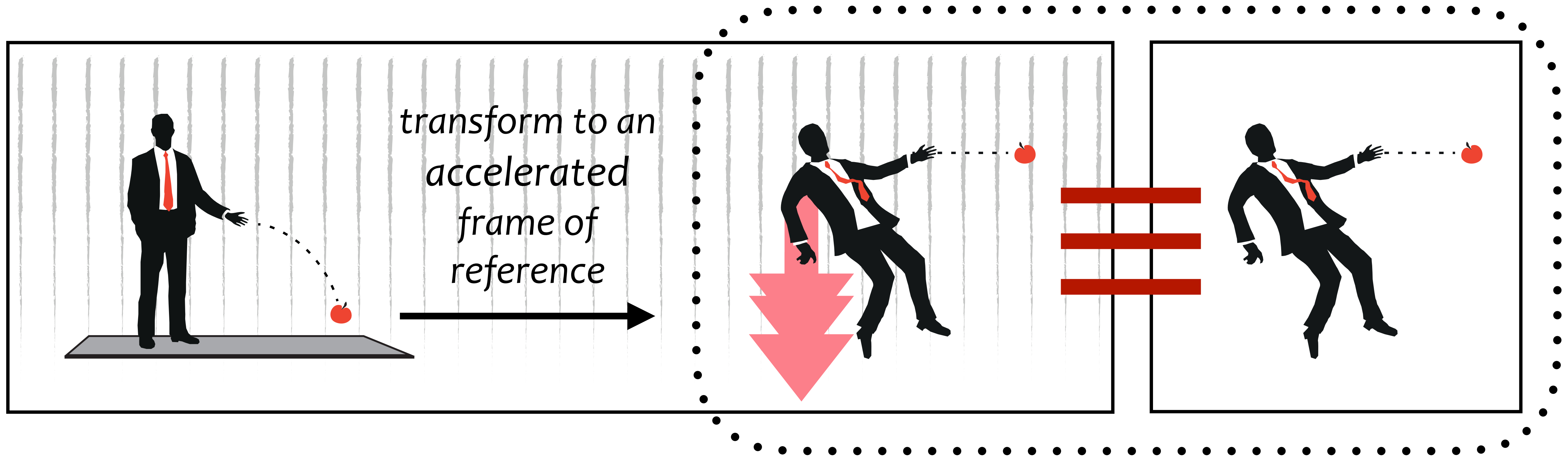
> At that moment [*in the Fall of 1907*] **I got the happiest thought of my life** in the following form:
>
> [...] **for an observer in free-fall** from the roof of a house **there is during the fall** — at least in his immediate vicinity — **no gravitational field.**
>
> Namely, if the observer lets go of any bodies, they remain relative to him in a state of rest or uniform motion, independent of their special chemical or physical nature.

Albert Einstein, Unpublished manuscript, in all probability a draft for an article in *Nature* that was never published (c.1920)



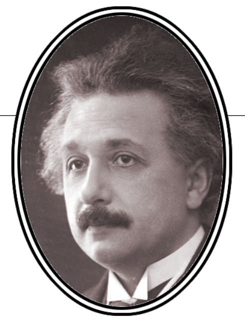
THE SECOND PAIR OF EQUIVALENT OBSERVERS

Step 2 The earlier naming convention "generative observer" inspires us to say:



If you start out as an observer at rest in a gravitational field, you could let yourself fall freely, and the laws of physics play out just as for a free equivalent observer at rest.

It is as if, **by going from rest to an accelerated frame of reference** (in a gravitational field) **you have "eliminated" the gravitational field.**

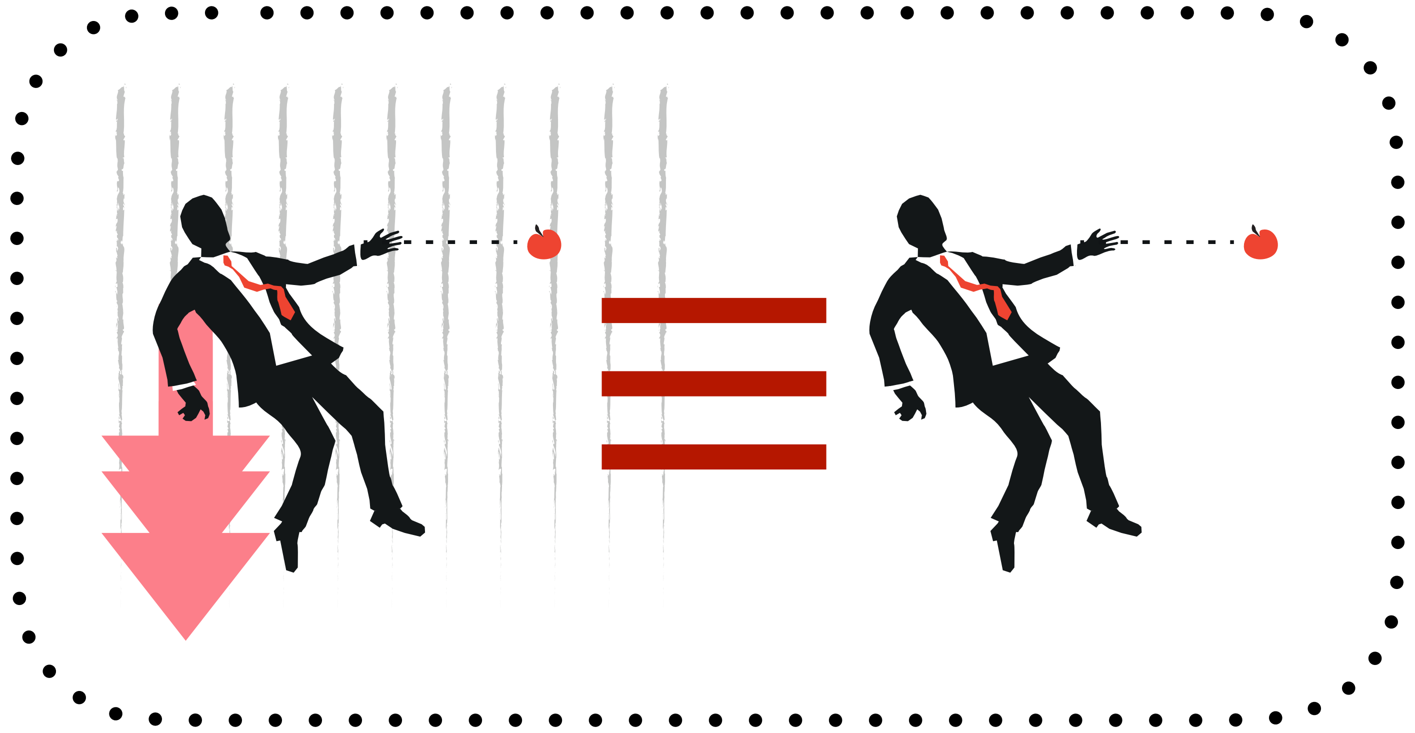


NORDITA

THE SECOND PAIR OF EQUIVALENT OBSERVERS

So let's call these two equivalent observers

the "eliminative" observers



since they show us how one can "eliminate" the effects of a gravitational field.

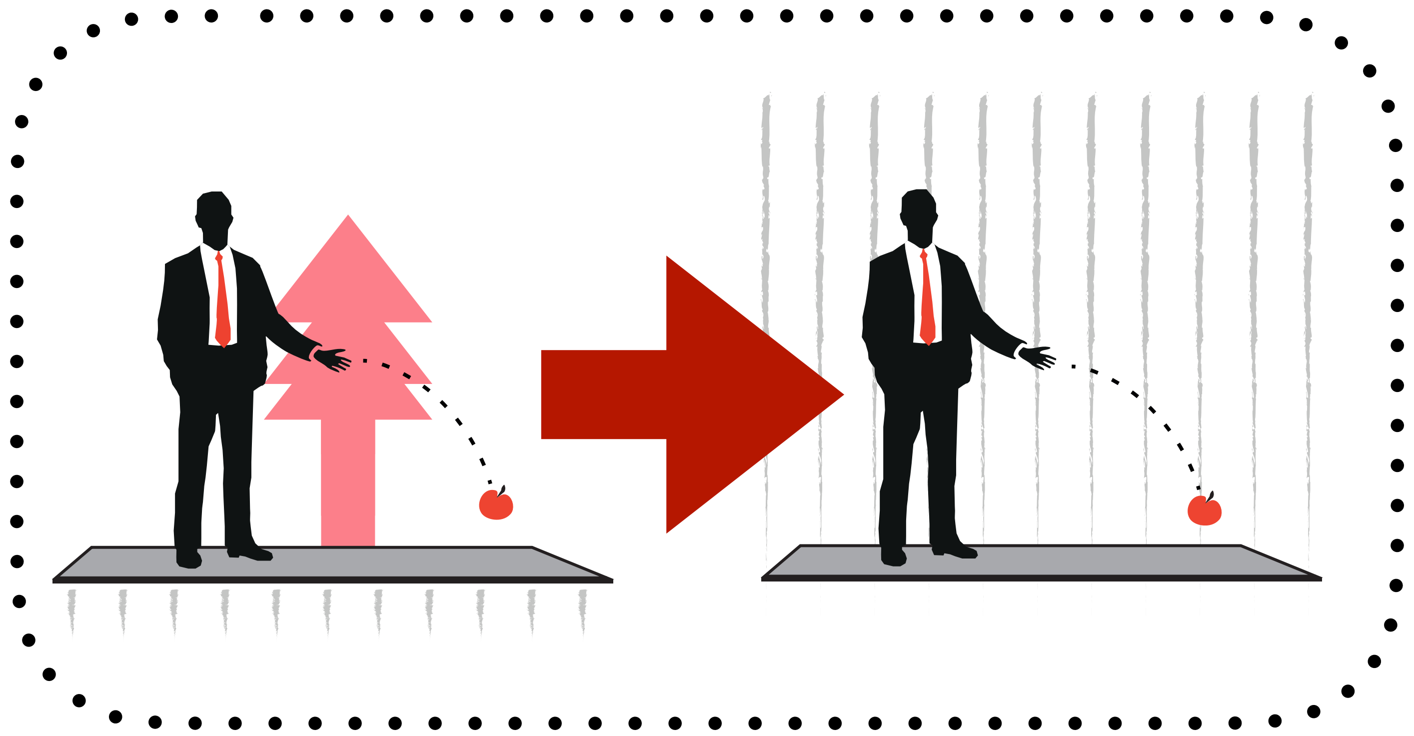
THE SECOND PAIR OF EQUIVALENT OBSERVERS

Step 3

Now —- remember how the ***Äquivalenzprinzip* heuristic** works:

We know how to calculate **kinematic** effects (in special relativity)

We do not know the properties of the unknown relativistic gravity theory

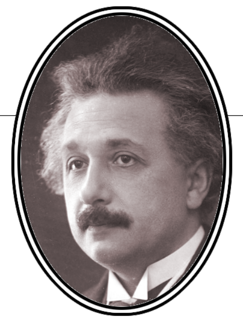


The original "**generative**" observers

Based on the "**generative**" observers:

1. We first study the (known) laws of physics for an **accelerated observer.**
2. We then translate the results to an **observer in an (unknown) gravity field.**

But — can we use the "**eliminative**" observers **in the same way,** to get a glimpse of how an (unknown) theory of relativistic gravity would look like?



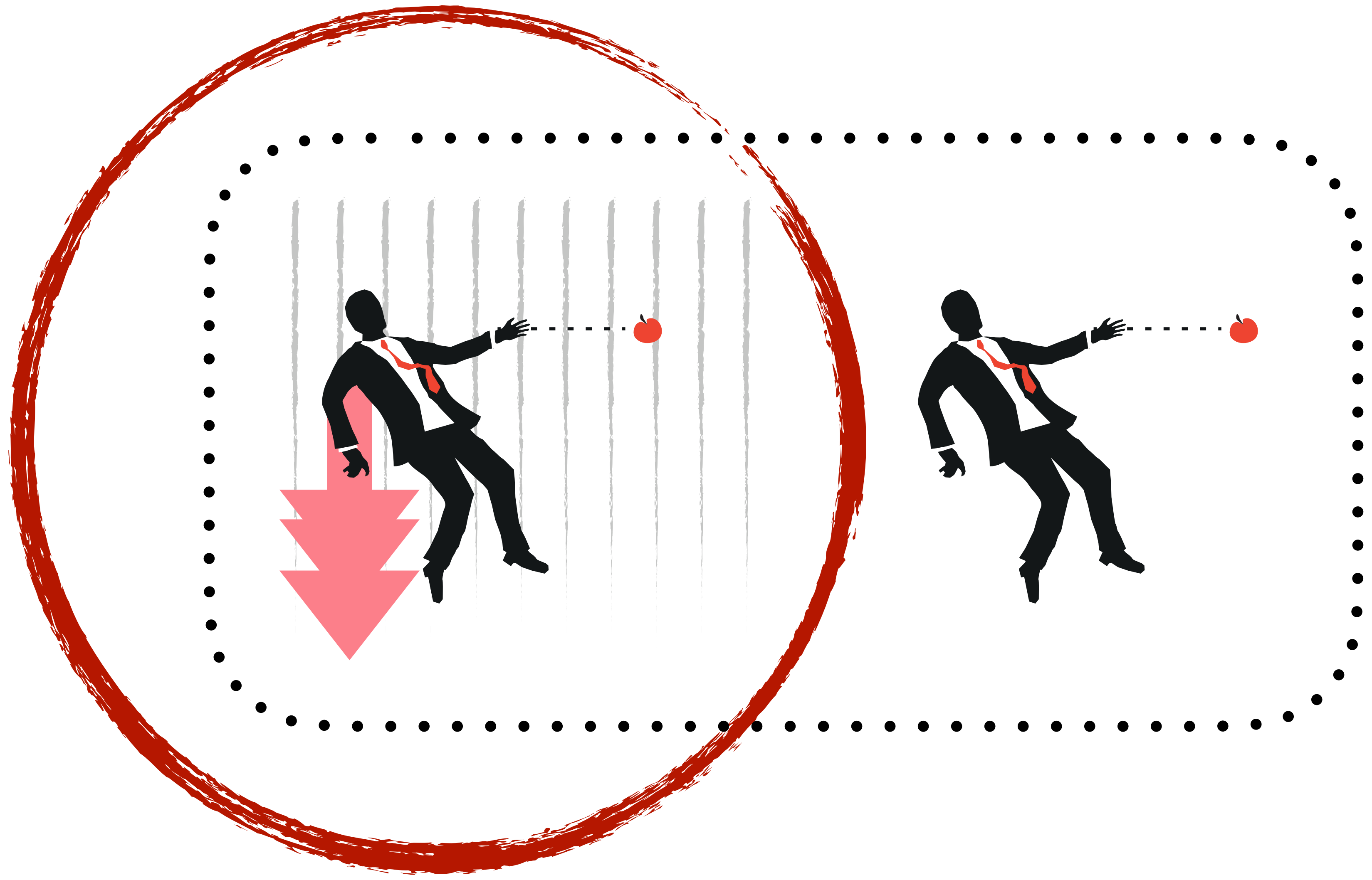
THE SECOND PAIR OF EQUIVALENT OBSERVERS

Well, not really…

The drawback is that we then would start in a frame of reference which **mixes** the kinematic effects of acceleration (which we know how to calculate) with a gravitational field (over which we have no control).

It is the clean distinction between kinematics and dynamics for the "generative" which enables the *Äquivalenzprinzip* heuristics.

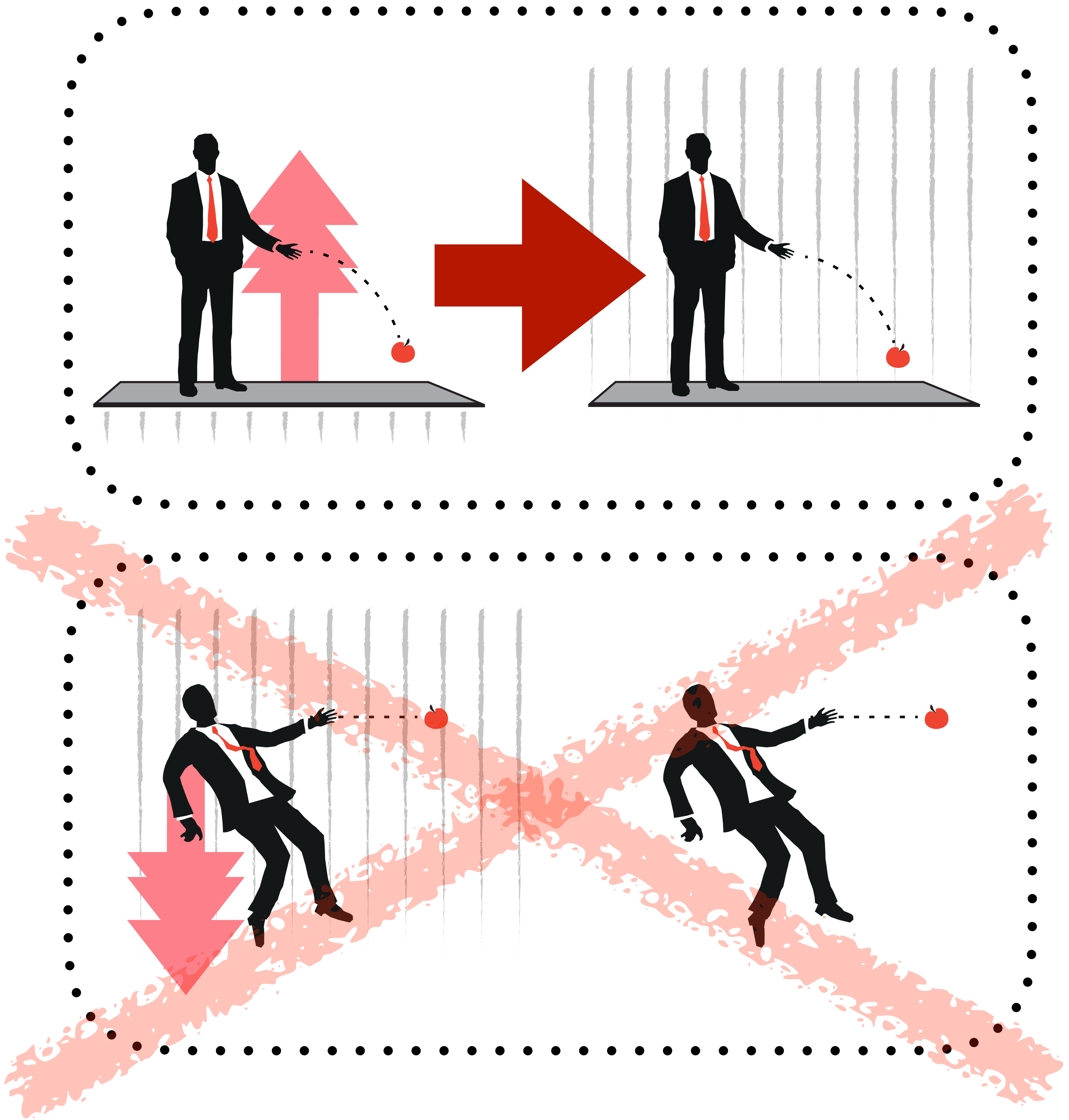
an observer
(1) in accelerated motion
(2) in a gravitational field



The new
"**eliminative**"
observers

THE SECOND PAIR OF EQUIVALENT OBSERVERS

We can now understand why **Einstein consistently used the "generative" pair of equivalent observers** in his discussion and application of the *Äquivalenzprinzip*:



The original "**generative**" observers

The new "**eliminative**" observers

SO WHAT KIND OF PRINCIPLE IS IT?

We have seen that Einstein based his 1907 ***Äquivalenzprinzip* heuristic** on an analysis of **two very special observers** (in *uniform acceleration* and in *a homogeneous gravitational field*).

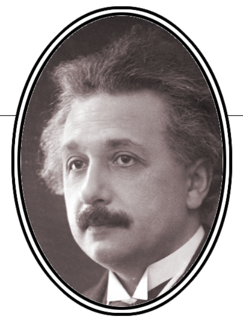
The observers were actually not very interesting in themselves!

Their only role was to reveal qualitative properties of the unknown relativistic gravitational field…

> If we accept this assumption, **we obtain a principle that possesses great heuristic significance** […]

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)

Guided by these insights, Einstein would then be able proceed **to find the actual theory** of the relativistic gravitational field (GR) **using other methods** (1912–1915).



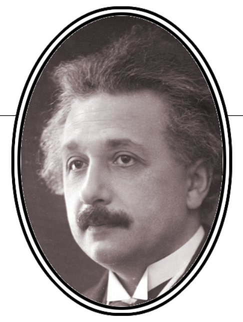
SO WHAT KIND OF PRINCIPLE IS IT?

We can conclude that:

Einstein’s 1907 *Äquivalenzprinzip* was a *heuristic* principle

> **heu·ris·tic** *n.*
> *process or method enabling persons to discover or learn something*

But *heuristics typically live dangerously*: they are essential to kickstart the discovery process, but **usually don’t make it into the final theory** (as theorems).



SO WHAT KIND OF PRINCIPLE IS IT?

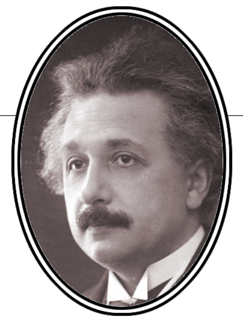
But many of Einstein's contemporaries missed this point completely.

Their concern seems to have been that **the *Äquivalenzprinzip* only talks about *a very special case*** of gravitational fields (those that can be simulated by acceleration).

Surely, anything called **a "principle" must have *general* validity**!

> Whatever the statement of the principle of equivalence (and there have been many) **its validity must extend beyond the framework of Newtonian gravitation or special relativity.**
>
> Otherwise, it would **hardly be justified to be called a principle.**

Fritz Rohrlich, "The Principle of Equivalence" (1963)



SO WHAT KIND OF PRINCIPLE IS IT?

Thus started the transmogrification of Einstein's *Äquivalenzprinzip:*

from

"heuristic principle"

Einstein's Äquivalenzprinzip

to

"principle of nature"

modern equivalence principles

with the understanding that a "principle of nature" is **a general fact that must be true in all theories of physics** (as theorems within the theory).

Like the laws of thermodynamics, or the conservation of energy.

SO WHAT KIND OF PRINCIPLE IS IT?

Einstein, however, did not change his mind on formulation of his "*Äquivalenzprinzip*" for the remainder of his life.

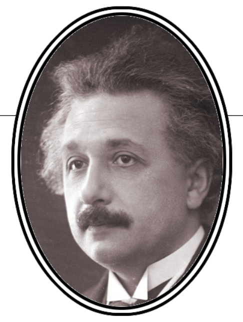
Here are quotes from the very first 1907 paper, and one of the last texts he wrote about GR (he died in 1955).

> At our present state of experience we have thus no reason to assume that the systems Σ_1 and Σ_2 differ from each other in any respect, and in the discussion that follows, we shall therefore assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system.

Albert Einstein, "*On the relativity principle and the conclusions drawn from it*" (1907)

> An inertial space without gravitational field is physically equivalent to a uniformly accelerated space, in which there is a (homogeneous) gravitational field. (Equivalence hypothesis.)

Albert Einstein, letter to Jean Becquerel, 16 August 1951



SO WHAT KIND OF PRINCIPLE IS IT?

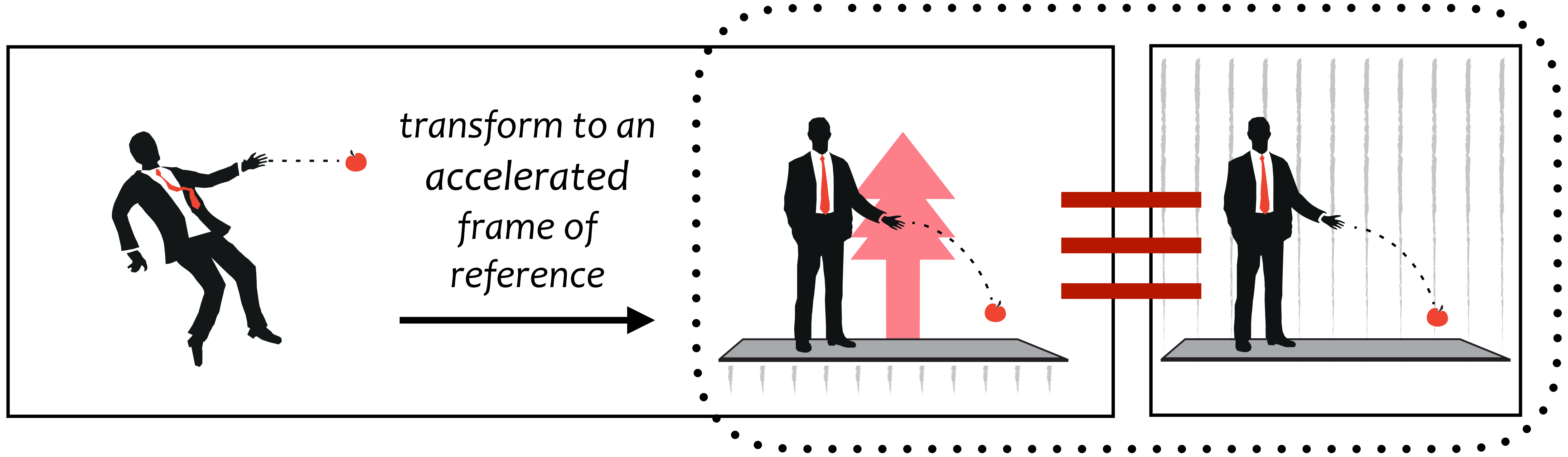
Let’s have **a quick look at** **two proposed “generalisations”** **of Einstein’s equivalence argument.**

One of the generalisations uses the “**generative**” equivalent observers, the other one starts with the “**eliminative**” equivalent observers.

And then finally we’ll have a look at **how Einstein criticised these two “generalisations”**.

GENERALISATION 1: KINEMATIC GRAVITY

We know that, in Newtonian physics, by going from rest to a state of **very special acceleration** (uniform) we can "create" a **very special gravitational field** (homogeneous).



The equivalent **"generative"** observers **create** a gravitational field

Proposed generalisation to a ***generally valid*** *law*

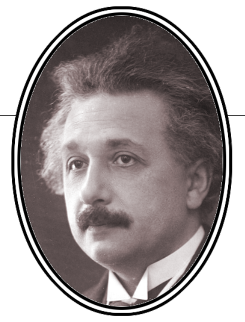
But we should be able to **"create" any *arbitrary* gravitational field** by going to appropriate states of acceleration!

GENERALISATION 1: KINEMATIC GRAVITY

Einstein had to patiently, and again several times over the years, explain to his enthusiastic followers, that **such a “generalisation” of the equivalence argument is not possible.**

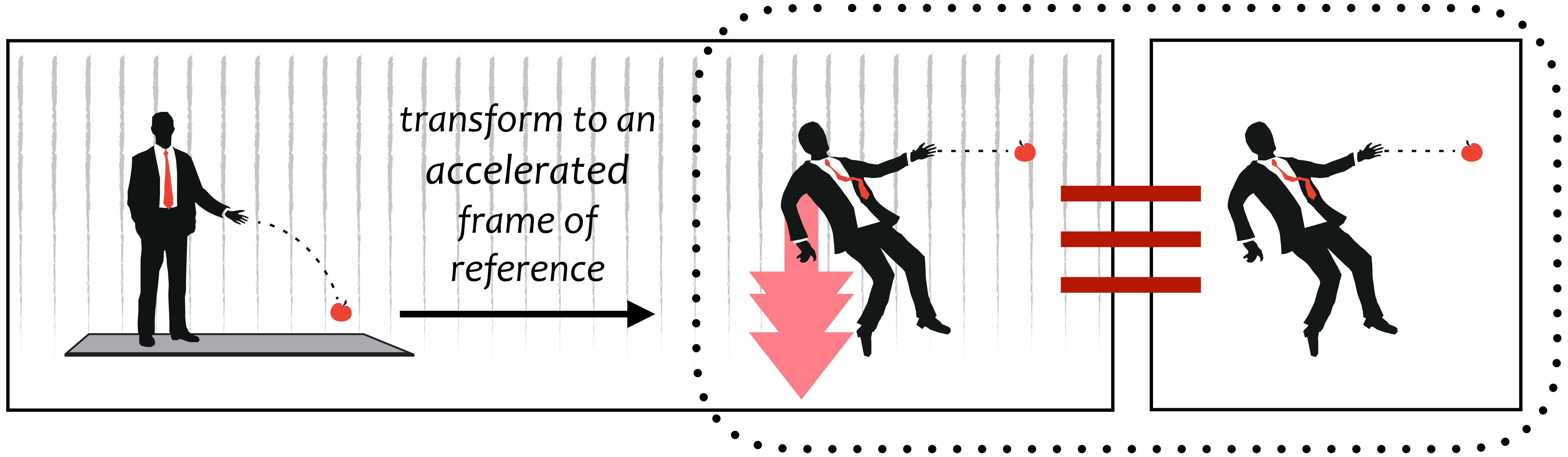
> [O]ne may never maintain that a gravitational field could be explained, so to speak, by pure kinematics; a “kinematic, nondynamic interpretation of gravitation” is not possible.
>
> By mere transformation from a Galilean [=*inertial*] system into another one by means of an acceleration transformation, **we do not learn about *arbitrary* gravitational fields but only some of a very special kind;** but these too must —- of course —- obey the same laws as all other fields of gravitation. This is again just another formulation of the principle of equivalence (specialized in its application to gravitation).

Albert Einstein, “On Friedrich Kottler's Paper: ‘On Einstein's Equivalence Hypothesis and Gravitation’ ” (1916)



GENERALISATION 2: ELIMINATE GRAVITY

We know that, in Newtonian physics, by going from rest in a **very special gravitational field** (homogeneous) to a state of **very special free fall** (uniform) we can "eliminate" the effects of gravity.



The equivalent "**eliminative**" observers **eliminate** a gravitational field

*Proposed generalisation to a **generally valid** law*

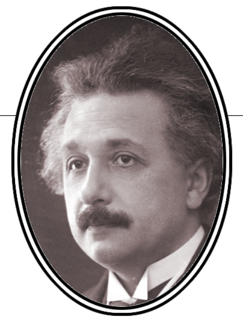
But we should be able to **"eliminate" any *arbitrary* gravitational field** by going to appropriate states of acceleration!

GENERALISATION 2: ELIMINATE GRAVITY

Again Einstein, still very patiently, had to remind his colleagues that **such a "generalisation" of the equivalence principle is not possible**:

> **However, one must be careful not to assume that** [...] **every gravitational field can be made to vanish,** *i.e.*, can be turned into a gravitation-free region, by means of a suitable choice of the coordinate system.
>
> For example, it is **impossible to make the gravitational field of the Earth vanish** by means of a suitable choice of the coordinate system. In fact, for a region of *finite extension* this is only possible with gravitational fields of a very special kind.

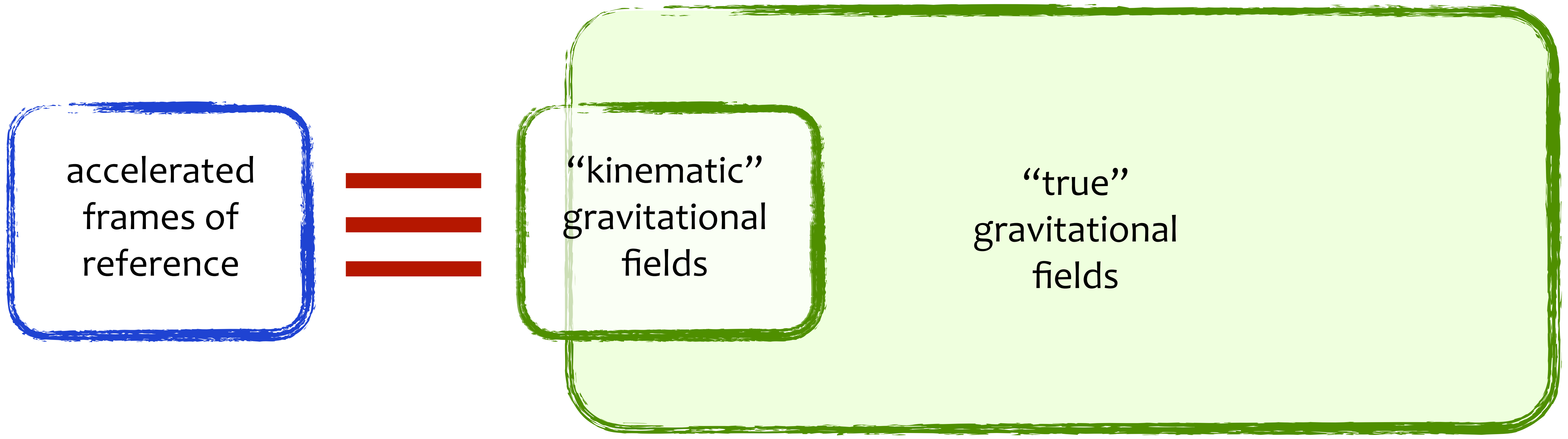
Albert Einstein, Theory of Relativity (1924)



WHY THESE GENERALISATIONS WON'T WORK

The *Äquivalenzprinzip* only establishes a kind of equivalence between **all arbitrarily** accelerated frames of reference with **a very special subset** of all gravitational fields.

(In general relativity, all such "kinematical" gravitational fields are just ***flat Minkowski spacetime*** *seen from different observers.)*



gravitation and **acceleration**
are *not* equivalent

GENERALISATION 3: INFINITESIMAL REGIONS

But there is a loophole in Einstein's objection to the idea that all gravitational fields can be eliminated!

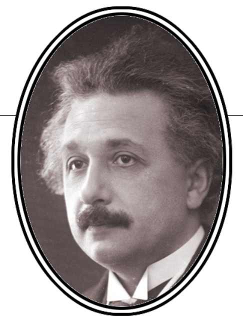
As he himself pointed out:

> **But for an infinitely small region** the coordinates can always be chosen such that no gravitational field will be present in it.

Albert Einstein, Theory of Relativity (1924)

True, **in a finite spacetime region** only **a very special kind of gravitational field** ("kinematical") can be "eliminated".

But in an infinitesimal spacetime region, you can "eliminate" **arbitrary gravitational fields**!



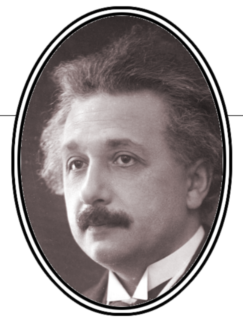
GENERALISATION 3: INFINITESIMAL REGIONS

Wolfgang Pauli seems to have been one of the first (1921) to formulate **a generalisation of the equivalence principle in terms of infinitesimal spacetime regions:**

Again this urge to **generalise** Einstein's argument

> Originally, the principle of equivalence had only been postulated for homogeneous gravitational fields. **For the general case, it can be formulated in the following way:**
>
> **For every infinitely small world region** […] there always exists a coordinate system K_o in which gravitation has no influence either on the motion of particles or any other physical processes.
>
> In short, **in an infinitely small world region every gravi-tational field can be transformed away.**

Wolfgang Pauli, "Relativitätstheorie" (1921)



GENERALISATION 3: INFINITESIMAL REGIONS

*Proposed generalisation to a **generally valid** law*

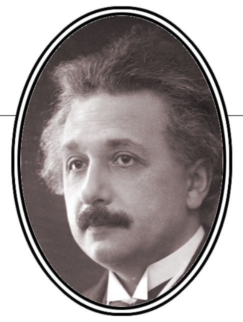
Infinitesimal Equivalence Principle

In an **infinitely small spacetime region** every gravitational field can be **transformed away.**

Intuitively, if you restrict your attention to a small enough (spatial) region *any* gravitational field will become (approximately) homogeneous.

You can now use the “eliminative” trick: by going to a freely falling frame of reference you get rid of the gravitational field.

But this of course **only works within that small region**…



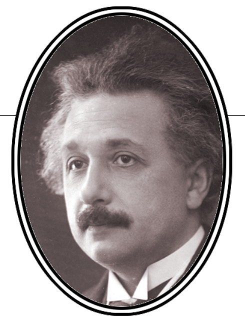
THE SR LIMIT IN INFINITESIMAL REGIONS

So **what is the point of eliminating gravitational fields** in an infinitesimal region, you might wonder.

Again Einstein comes to our help:

> **But for an infinitely small region the coordinates can always be chosen such that no gravitational field will be present in it.**
>
> **With respect to such an infinitely small region one may then assume that the special theory of relativity is valid.**
>
> That way the general theory of relativity is connected with the special theory of relativity, and the results of the latter can be utilized for the former.

Albert Einstein, Theory of Relativity (1924)



THE SR LIMIT IN INFINITESIMAL REGIONS

It seems we have stumbled across **the special-relativistic limit of general relativity**!

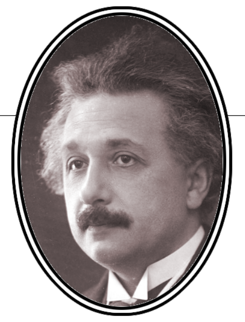
… except that the statement ***is not strictly speaking true***.

In general relativity, the curvature of the spacetime metric is expressed in the Riemann **tensor**.

And **you can't make a non-zero tensor vanish** just by transforming to a different frame of reference.

The Riemann tensor determines **tidal effects**, which would be detectable by a freely falling (infinitesimal) observer.

So what we actually have in an infinitesimal region around a spacetime point is a strange combination of special relativity (*defined* as having no spacetime curvature) with a non-zero curvature tensor…



THE SR LIMIT IN INFINITESIMAL REGIONS

Incidentally, whenever Einstein discussed the special-relativistic limit of general relativity, he always referred to **finite regions**, which he called “**Galilean regions**”.

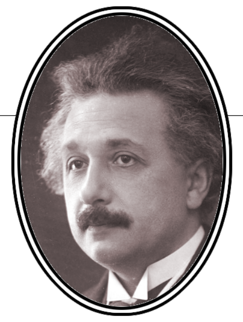
These would be regions **where the metric is approximately flat**. Here the laws of special relativity would be approximately true, and tidal effects negligible, since the curvature is almost zero.

Einstein’s SR limit:

the weak-field approximation of GR **in an extended region**

The SR limit based on the infinitesimal equivalence principle:

SR laws approximately true only **at one point**



THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

We have now come to the end of our story of the modern equivalence principles: in the early 1960's, relativist Robert Dicke

split the

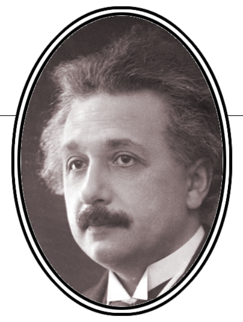
Infinitesimal Equivalence Principle

into two separate principles:

the **Weak Equivalence Principle**, and

the **Strong Equivalence Principle**

The "weak" principle was that part which got empirical support from precision experiments determining the equality of inertial and gravitational mass, while the "strong" principle was the rest.



Weak Equivalence Principle

This is Dicke's original statement of the **"Weak Equivalence Principle" (WEP):**

> The **weak principle of equivalence** states only that the local gravitational acceleration is substantially independent of the composition and structure of the matter being accelerated.

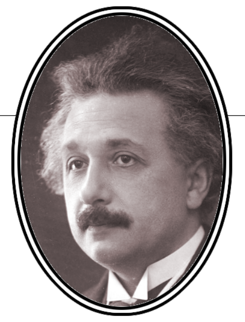
Robert Dicke, "Experimental Relativity" (1963)

The observation that all bodies fall with the same acceleration, regardless of their mass, is what is otherwise often called

the Universality of Free Fall (UFF),

usually credited to **Galileo**. But it remains a big mystery why the UFF should be confusingly renamed

the Weak Equivalence Principle



THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

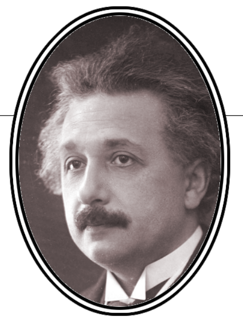
Weak Equivalence Principle

If we fast forward to **general relativity**, the WEP is usually replaced by the statement that free particles **move along geodesics of the metric**, independently of their mass.

Originally this was known as the "Geodesic Postulate". Later is was realised that it can actually be derived from the Einstein field equations (like with the **Geroch-Jang-Malament** or **Ehlers-Geroch theorems**).

But **these derivations only hold approximately**, and only for very special types of matter (non-rotating structureless test particles).

Therefore in GR the WEP is not a "**principle of nature**" (in the sense of being a strictly true theorem), but more like an **approximation scheme**.



THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

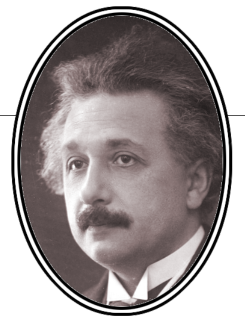
Weak
Equivalence
Principle

Einstein never made the mistake of talking about the Universality of Free Fall as a "principle", neither of "equivalence" or of anything else:

In the Newtonian context of the *Äquivalenzprinzip*, referring back to Galilei, what Einstein actually often did talk about was…

> [...] the old **experimental fact** that all bodies have the same acceleration in a gravitational field. This **law** [...]

Albert Einstein, "Notes on the Origin of the General Theory of Relativity", lecture manuscript (1933)



Strong Equivalence Principle

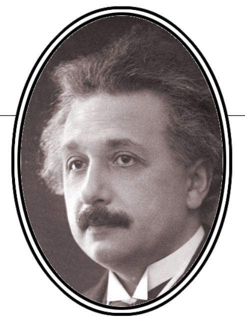
And this is Dicke's rendering of the **"Strong Equivalence Principle"** (SEP):

> The **strong equivalence principle** might be defined as the assumption that in a freely falling, non-rotating, laboratory the local laws of physics take on some standard form, including a standard numerical content, independent of the position of the laboratory in space and time.

Robert Dicke, "Experimental Relativity" (1963)

This we immediately recognise as **the special-relativistic limit of general relativity** at a spacetime point, discussed earlier.

And we already know that Einstein didn't talk about the SR limit in connection with his *Äquivalenzprinzip*, and when he did, the limit was always formulated in extended regions, not infinitesimal.

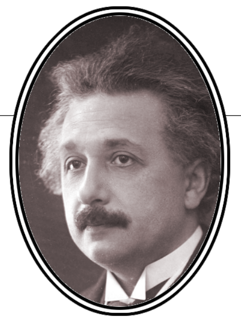


THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

Strong Equivalence Principle

We also saw earlier that the statement about the SR limit of GR in an infinitesimal region cannot be considered to be a "**principle of nature**" (in the sense of being a strictly true theorem), but that it is perfectly acceptable as an **approximation scheme.**

But it is of course trivially true that *any limit of any theory* must be formulated as an approximation scheme.



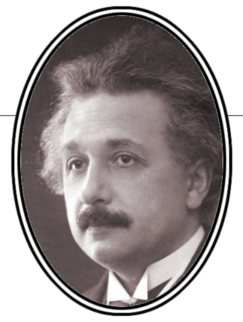
THE DECAY OF THE INFINITESIMAL EQUIVALENCE PRINCIPLE

Strong
Equivalence
Principle

Is there anything wrong with talking about the special-relativistic limit of general relativity? — **Of course not!**

The new theory of general relativity goes beyond the well-established earlier theories of Newtonian gravity and special-relativistic mechanics.

We should therefore **demand of the new theory that it recaptures the results of both old theories in a well-defined limit.**



Strong Equivalence Principle

Is there anything wrong with talking about the special-relativistic limit of general relativity? — **Of course not!**

The new theory of general relativity goes beyond the well-established earlier theories of Newtonian gravity and special-relativistic mechanics.

We should therefore **demand of the new theory that it recaptures the results of both old theories in a well-defined limit.**

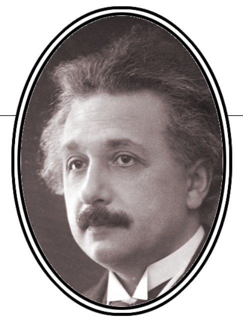
But while in modern texts the **Newtonian** limit of GR is called…

the Newtonian limit of general relativity

the **special-relativistic** limit of GR is called…

the strong equivalence principle

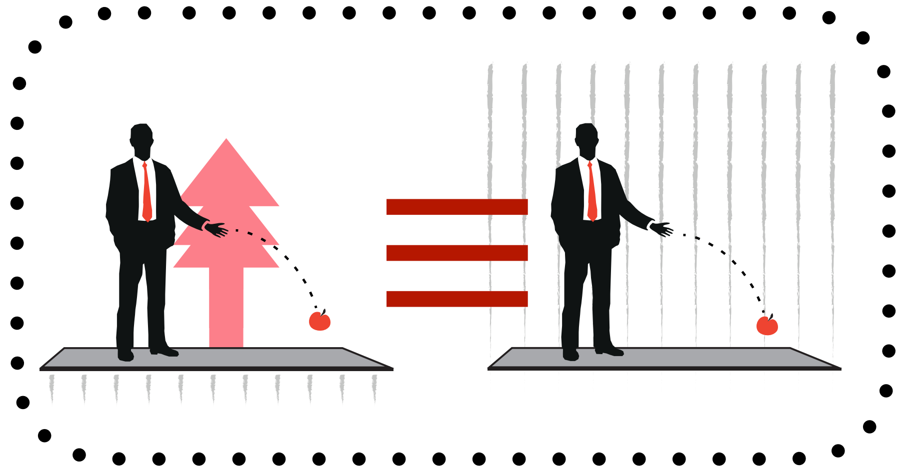
… No wonder people get confused!

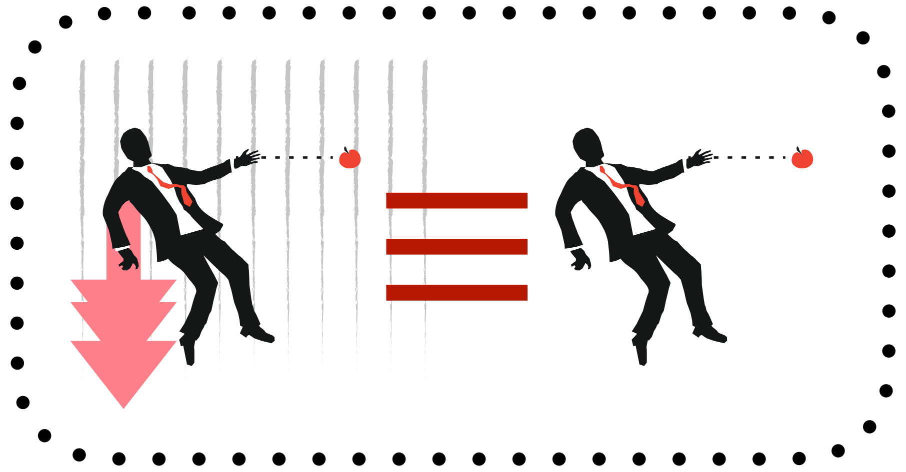


CONCLUSION

THEOREMS IN NEWTONIAN PHYSICS

The "**generative**" observers





The "**eliminative**" observers

POSTULATED HEURISTIC PRINCIPLE

Einstein's
Äquivalenzprinzip

POSTULATED PRINCIPLES OF NATURE

"Kinematic" gravitation

Infinitesimal Equivalence Principle

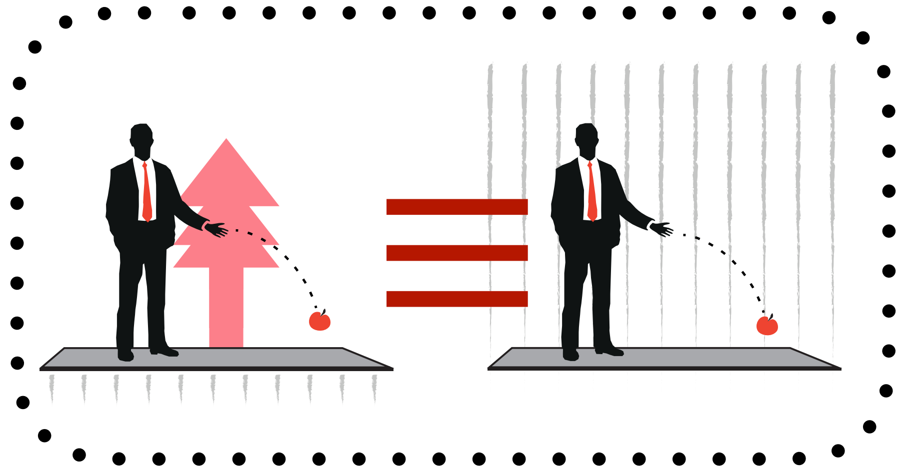
Strong Equivalence Principle

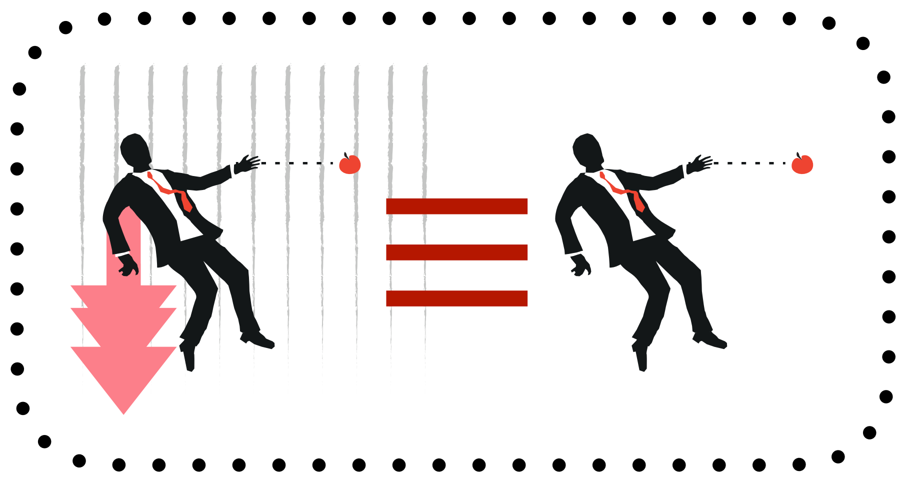
Weak Equivalence Principle

CONCLUSION

THEOREMS IN NEWTONIAN PHYSICS

The “**generative**” observers





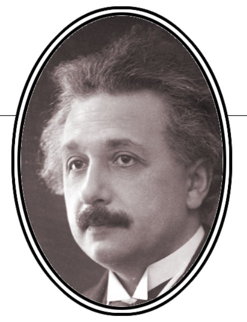
The “**eliminative**” observers

POSTULATED HEURISTIC PRINCIPLE

Einstein’s
Äquivalenzprinzip

*If **applied in special relativity** it can give hints of the unknown relativistic theory of gravity*

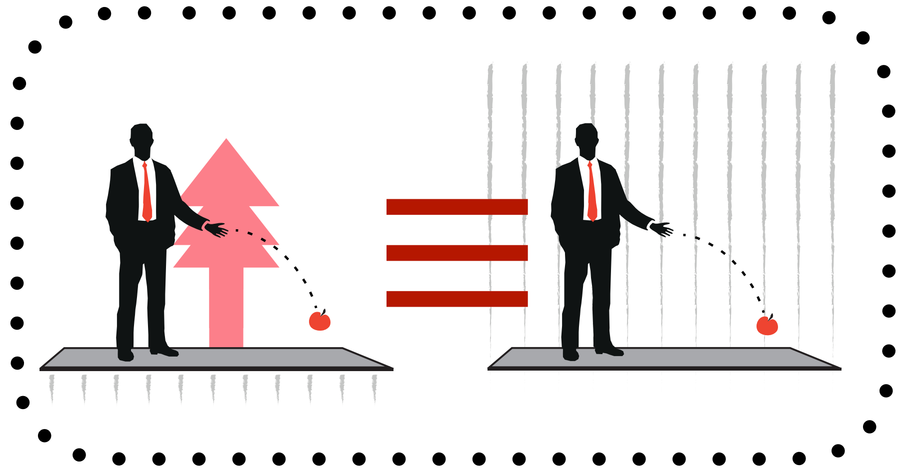
POSTULATED PRINCIPLES OF NATURE

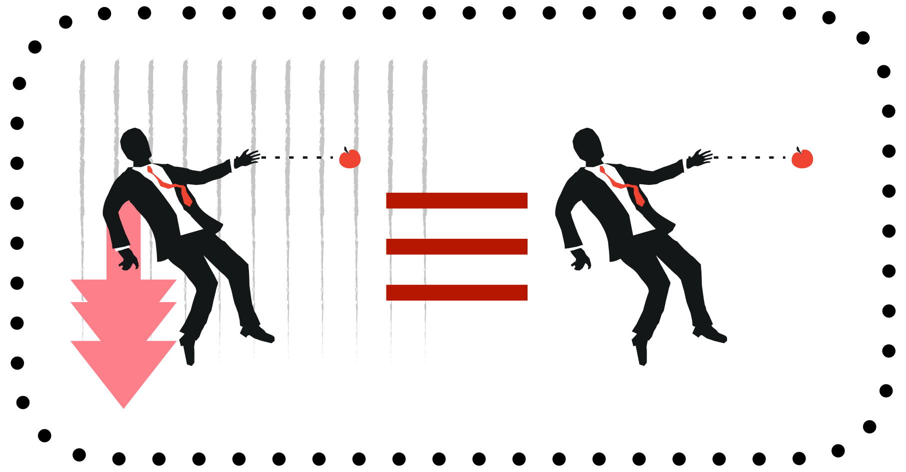


CONCLUSION

THEOREMS IN NEWTONIAN PHYSICS

The "**generative**" observers





The "**eliminative**" observers

POSTULATED HEURISTIC PRINCIPLE

POSTULATED PRINCIPLES OF NATURE

These propositions are **not strictly true in general relativity** (other than as approximation schemes)

Infinitesimal Equivalence Principle

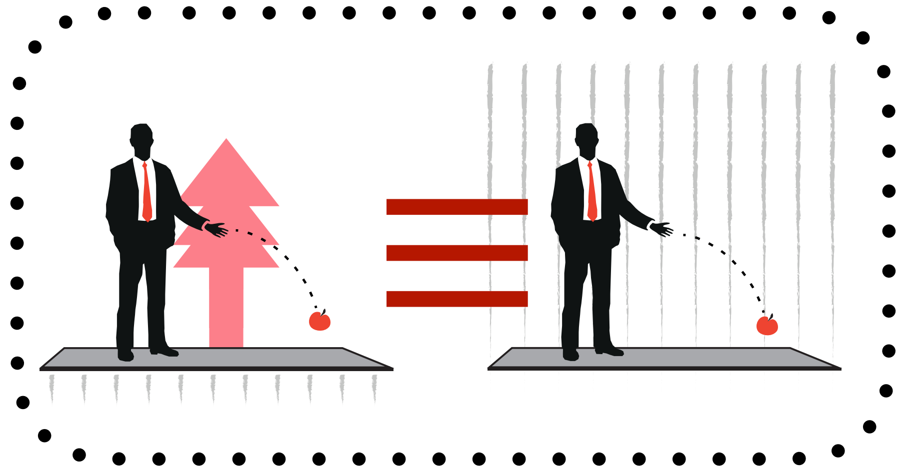
Strong Equivalence Principle

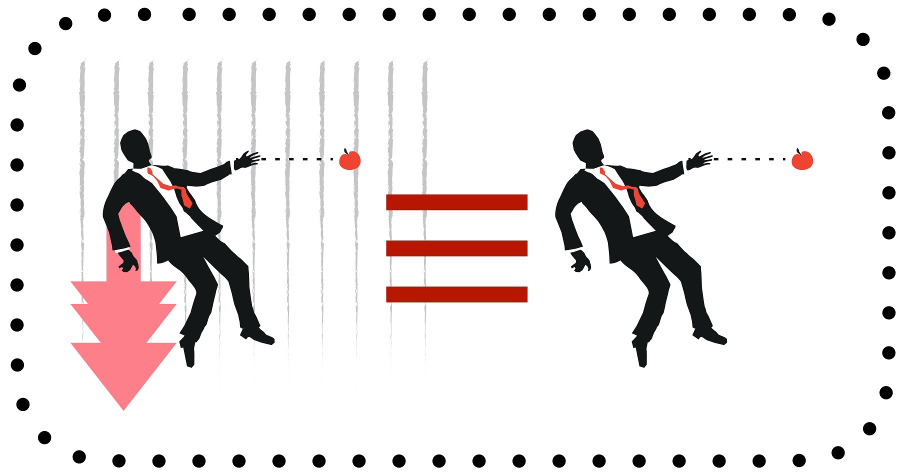
Weak Equivalence Principle

CONCLUSION

THEOREMS IN NEWTONIAN PHYSICS

The "**generative**" observers





The "**eliminative**" observers

POSTULATED HEURISTIC PRINCIPLE

POSTULATED PRINCIPLES OF NATURE

The WEP is just a formulation of the **Universality of Free Fall**

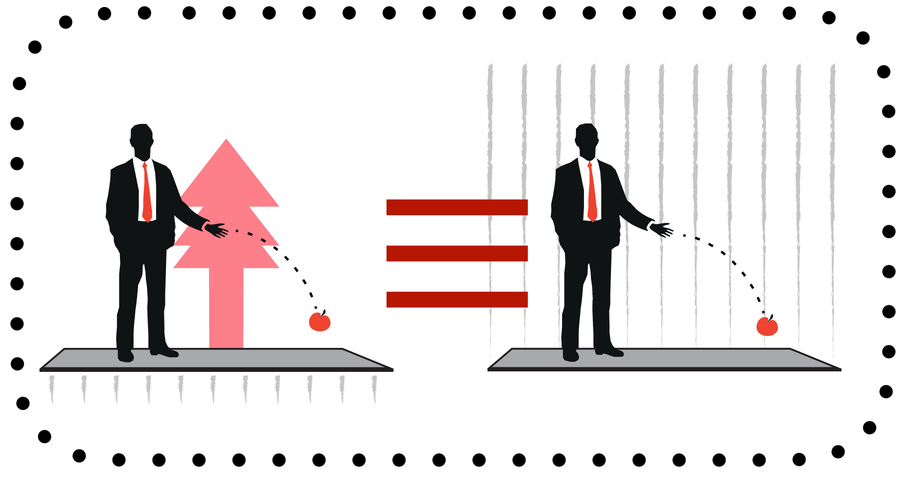
Infinitesimal Equivalence Principle

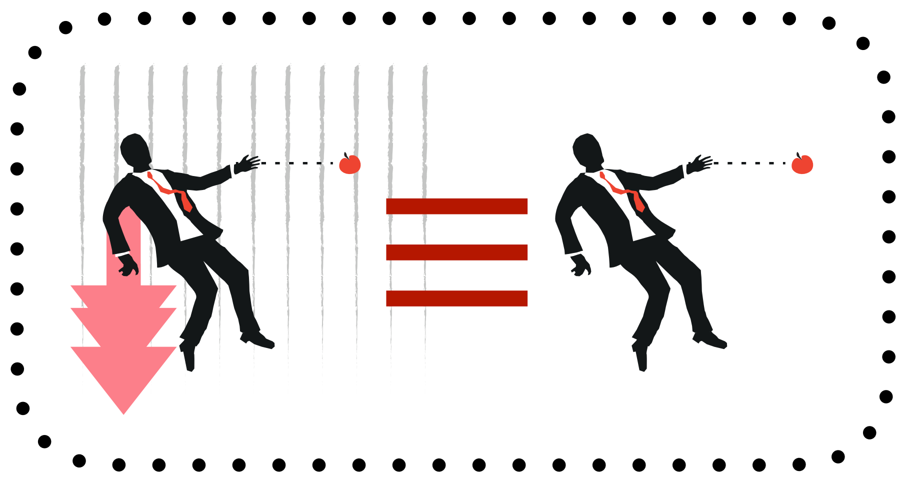
Weak Equivalence Principle

CONCLUSION

THEOREMS IN NEWTONIAN PHYSICS

The “**generative**” observers





The “**eliminative**” observers

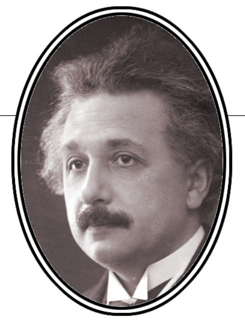
POSTULATED HEURISTIC PRINCIPLE

POSTULATED PRINCIPLES OF NATURE

The SEP should be reclassified as an implementation of **the special-relativistic limit of general relativity**

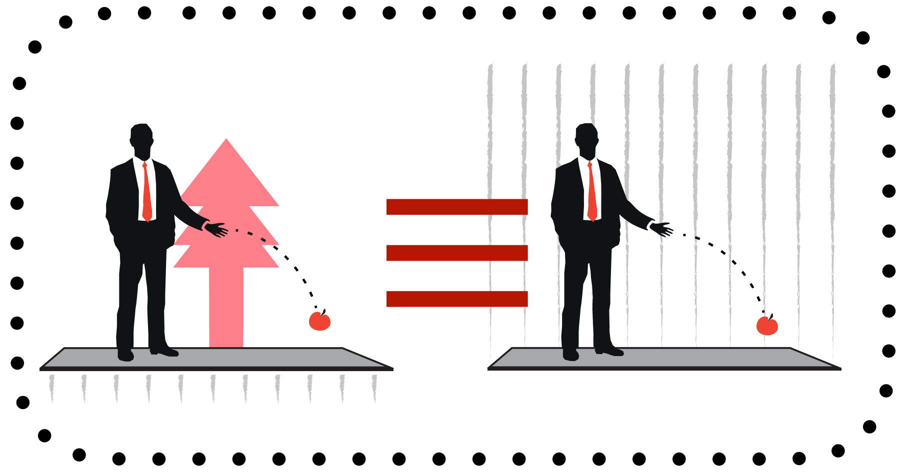
Infinitesimal Equivalence Principle

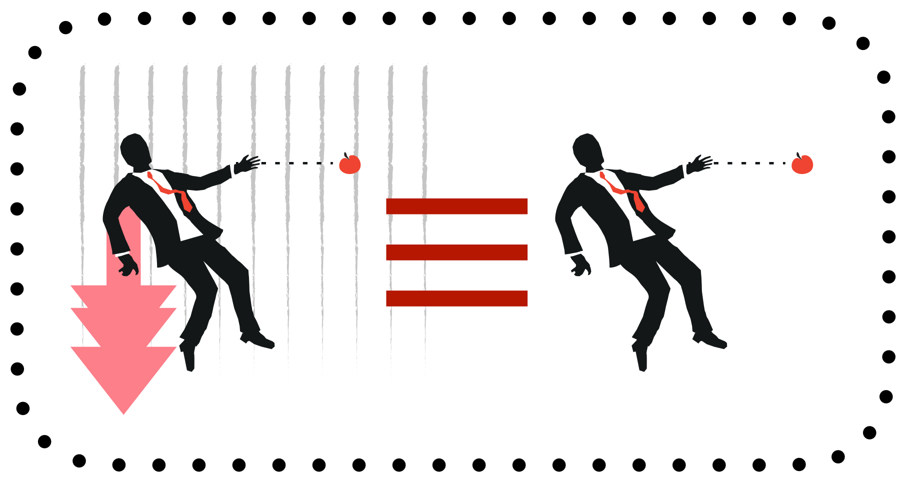
Strong Equivalence Principle



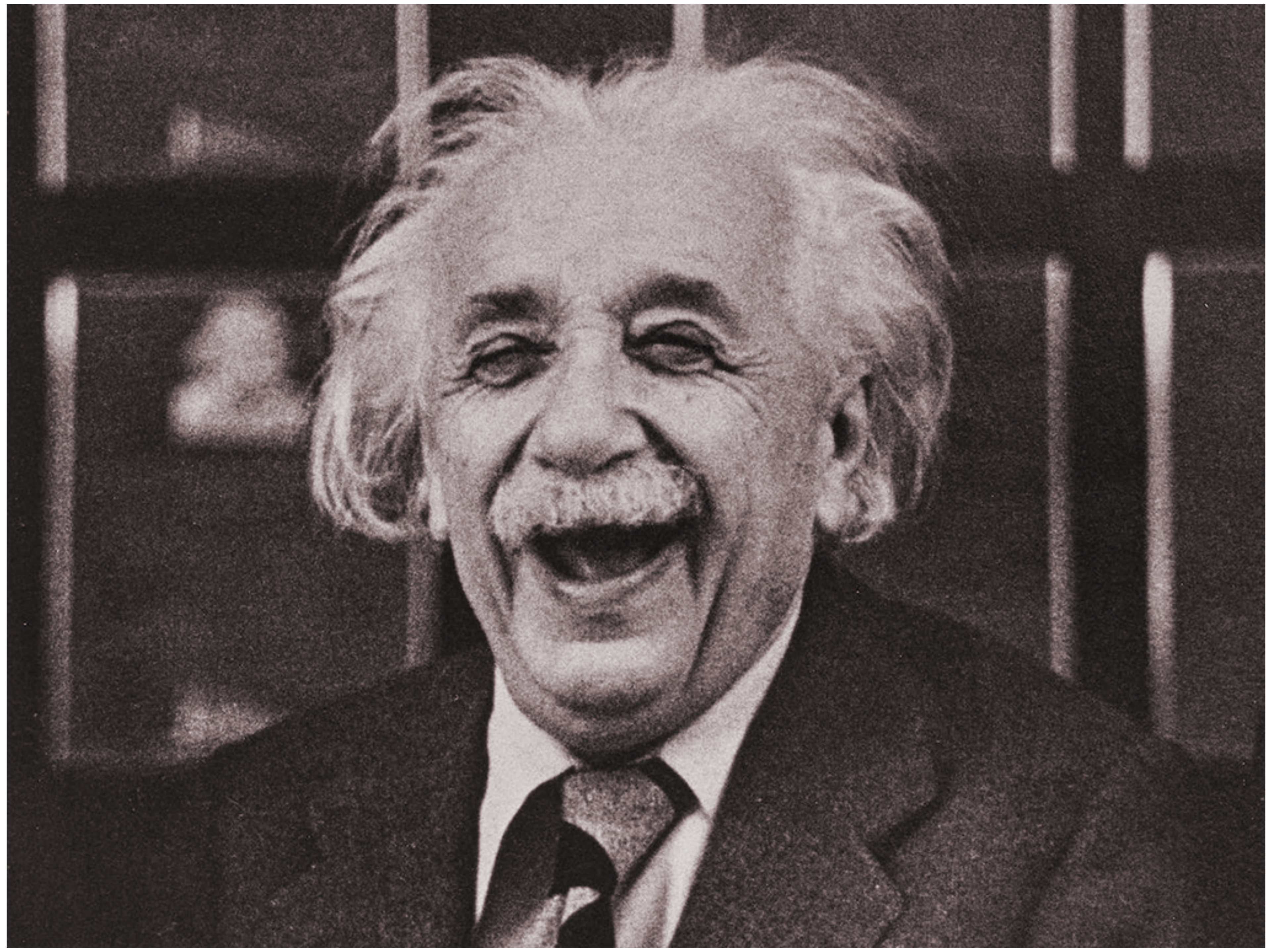
Thank you for your attention!

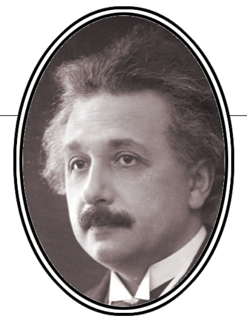
The "**generative**" observers





The "**eliminative**" observers



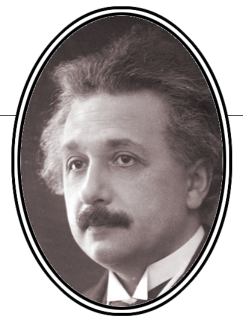


CONCLUSION

Unfortunately, Einstein's contemporaries seized upon one of Einstein's intermediate results, that in certain cases the gravitational fields […] have a relative existence, dependent on the choice of frame of reference. They sought to generalize this result from the simple cases in Minkowski spacetime which Einstein considered, to arbitrary gravitational fields. **It has rarely been acknowledged that Einstein never endorsed the principle which results, here called the "infinitesimal principle of equivalence."** […]

In recent decades there has been much criticism of "the" principle of equivalence. **But the principle under cogent attack has rarely been Einstein's version.**

John D. Norton, "What was Einstein's principle of equivalence?" (1985)



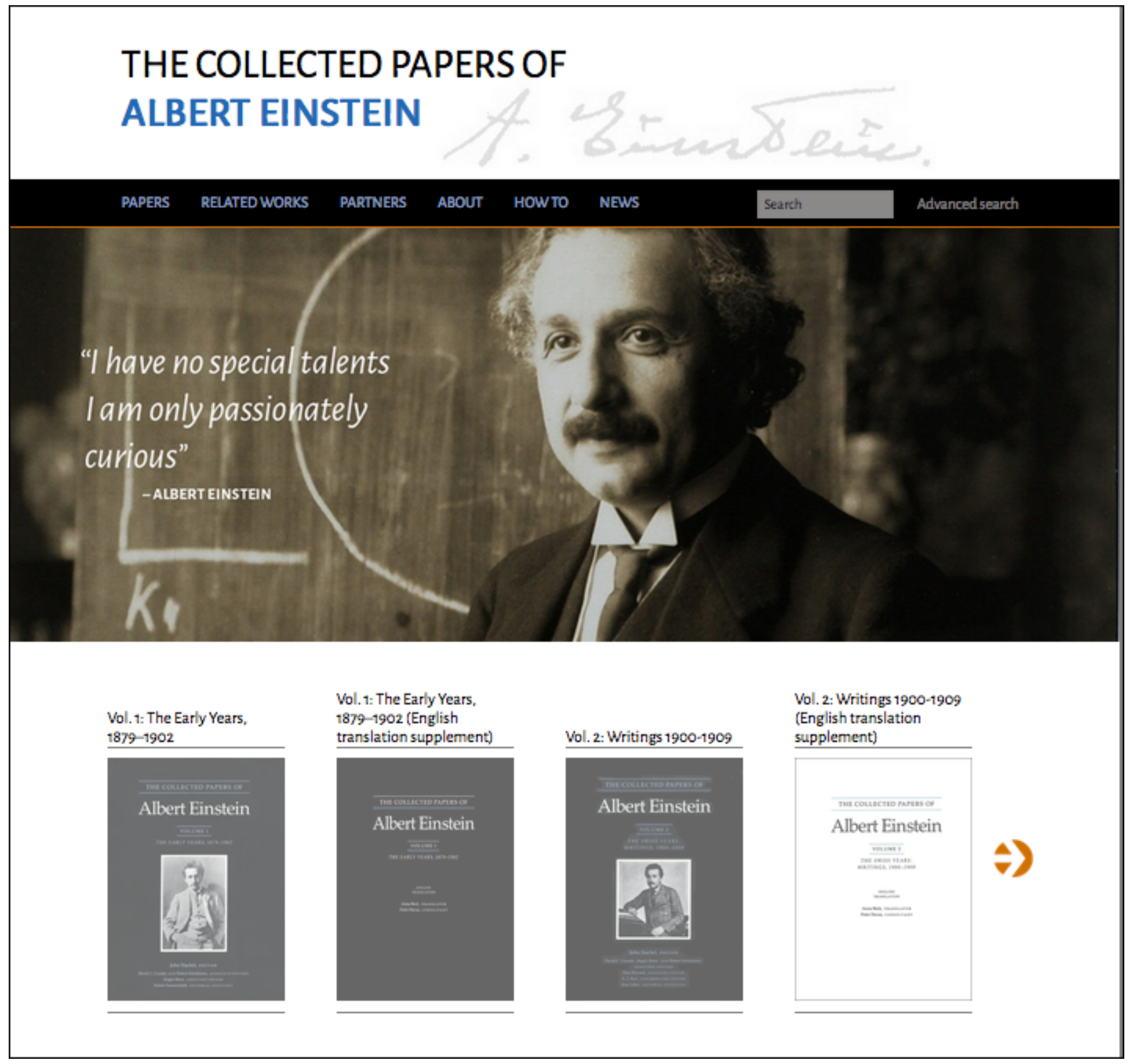
FURTHER READING

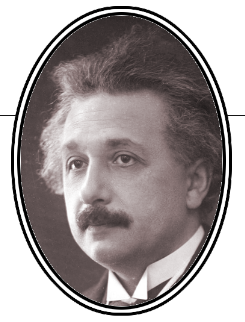
Tips for further reading…

The task of reconstructing Einstein’s views is greatly simplified by the **easy access** we have today to everything that Einstein wrote:

All original texts by Einstein (currently **16 volumes,** up until ~1929) are available in German as well as in English translation at

einsteinpapers.press.princeton.edu





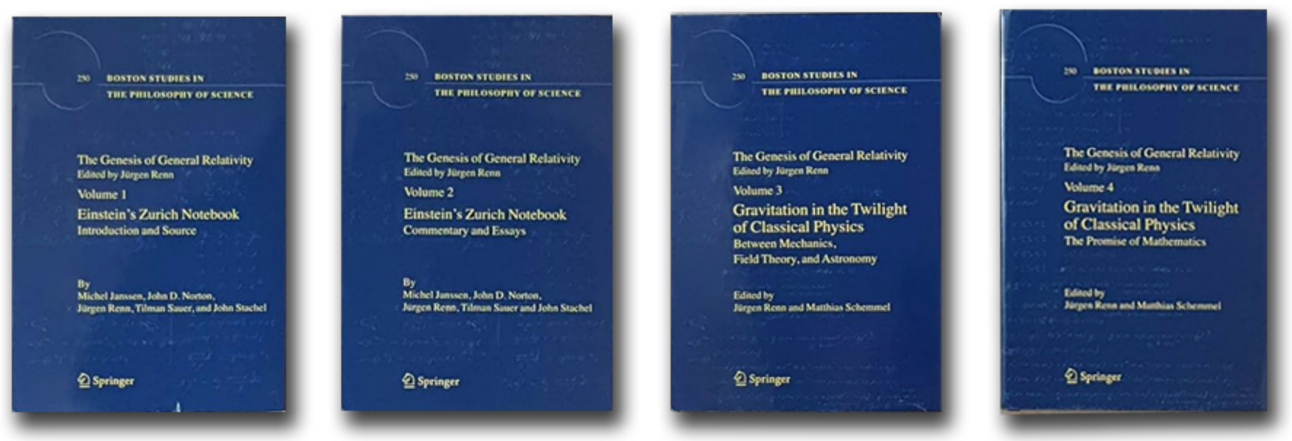
FURTHER REFERENCES

Tips for further reading…

Much of the relevant ***secondary literature*** has been collected in:

…

- the **Einstein Studies** anthology series from Springer (there are currently **16 titles** in the series)
  springer.com/series/4890



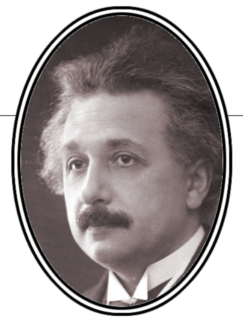
- the **The Genesis of General Relativity** anthology series (**4 volumes**) from the Max Planck Institute for the History of Science (MPIWG) in Berlin
  mpiwg-berlin.mpg.de

FURTHER REFERENCES

The standard reference to the historical development and meaning of the *Äquivalenzprinzip* is

John D. Norton
"**What was Einstein's Principle of Equivalence?**"
Studies in History and Philosophy of Science, **16** (1985) 203–246
`doi.org/10.1016/0039-3681(85)90002-0`

More works by Norton on the history of relativity can be found at
`sites.pitt.edu/~jdnorton`



Einstein’s search for general relativity

Texts important for understanding Einstein’s “*Äquivalenzprinzip*”

“On the relativity principle and the conclusions drawn from it” (Dec 1907)

“On the influence of gravitation on the propagation of light” (Sept 1911)

“On the Theory of the Static Gravitational Field” (May 1912)

“The speed of light and the statics of the gravitational field” (May 1912)

“Is There a Gravitational Effect Which Is Analogous to Electrodynamic Induction?” (July 1912)

“Relativity and gravitation. Reply to a comment by M. Abraham” (Aug 1912)

Zürich notebook (Summer 1912 – Spring 1913)

“Outline of a Generalized Theory of Relativity and of a Theory of Gravitation” (June 1913)

“On the Present State of the Problem of Gravitation” (Sept 1913)

“On the Foundations of the Generalized Theory of Relativity and the Theory of Gravitation” (Jan 1914)

“The Formal Foundation of the General Theory of Relativity” (Nov 1914)

“On the General Theory of Relativity” (4 Nov 1915)

“On the General Theory of Relativity (Addendum)” (11 Nov 1915)

“Explanation of the Perihelion Motion of Mercury from the General Theory of Relativity” (18 Nov 1915)

“The Field Equations of Gravitation” (25 Nov 1915)

“The Foundation of the General Theory of Relativity” (May 1916)

“Relativity. The Special and the General Theory. A Popular Exposition” (Dec 1916)

“On Friedrich Kottler's Paper: ‘On Einstein's Equivalence Hypothesis and Gravitation’” (Dec 1916)

“Cosmological Considerations in the General Theory of Relativity” (Feb 1917)

"On Gravitational Waves" (Feb 1918)

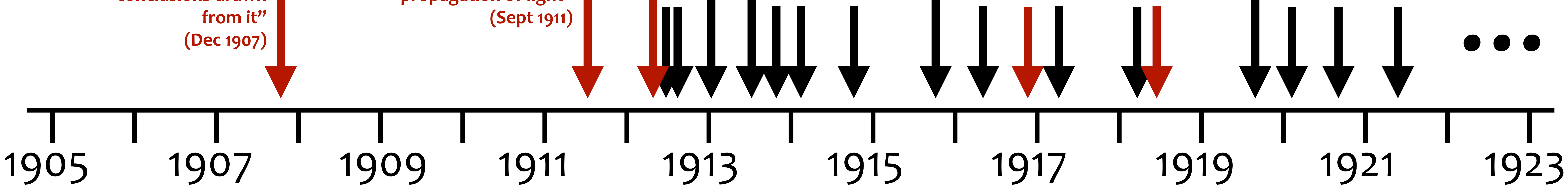
“On the Foundations of the General Theory of Relativity” (May 1918)

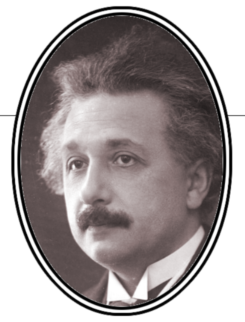
“My Theory” (Nov 1919)

Draft of “Fundamental Ideas and Methods of the Theory of Relativity, Presented in Their Development" (Jan 1920)

“Ether and Relativity” (Oct 1920)

Four Lectures on the Theory of Relativity, Held at Princeton University (May 1921)





FURTHER REFERENCES

Wolfgang Pauli, “Theory of Relativity” (1921)

John L. Synge, “Relativity: The General Theory” (1960)

Robert Dicke, “Experimental Relativity” (1963)

Fritz Rohrlich, “The Principle of Equivalence” (1963)

J. L. Anderson, “Principles of Relativity Physics” (1967)

Steven Weinberg, “Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity” (1972)

K. S. Thorne, D. L. Lee, A. L. Lightman, “Foundations for a Theory of Gravitation Theories” (1973)

Robert Geroch, J. S. Jang, “Motion of a Body in General Relativity” (1975)

Charles Misner, Kip Thorne, John A. Wheeler, “*Gravitation*” (1973)

Hans C. Ohanian, “What is the Principle of Equivalence?” (1977)

John Stachel, “Einstein and the Rigidly Rotating Disk” (1980)

Michael Friedman, “Foundations of Space-Time Theories” (1983)

John D. Norton “What was Einstein's Principle of Equivalence?” (1985)

Bruno Bertotti, Leonid P. Grishchuk, “The Strong Equivalence Principle” (1990)

Michel Ghins, Tim Budden, “The Principle of Equivalence” (2001)

Mark P. Haugan, Claus Lämmerzahl, “Principles of Equivalence: Their Role in Gravitation Physics and Experiments that Test Them” (2001)

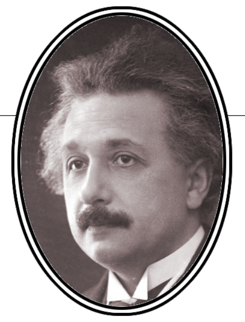
Gerardo Muñoz, Preston Jones, “The Equivalence Principle, Uniformly Accelerated Reference Frames, and the Uniform Gravitational Field” (2010)

Adán Sus, “On the Explanation of Inertia” (2014)

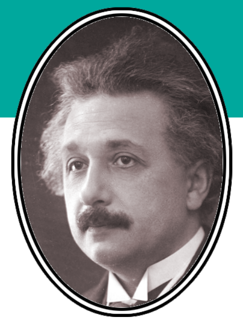
Clifford M. Will, “The Confrontation between General Relativity and Experiment” (2014)

Eolo Di Casola, Stefano Liberati, Sebastiano Sonego, “Nonequivalence of Equivalence Principles” (2015)

Dennis Lehmkuhl, “The Equivalence Principle(s)” (2017)



EINSTEIN QUOTES

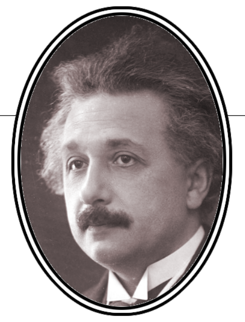


EINSTEIN QUOTES

Einstein’s path to the theory of general relativity was a difficult one, both for himself and for those who tried to follow his various arguments along the line:

> My series of gravitation papers are a chain of wrong tracks, which nevertheless did gradually lead closer to the objectives. That is why now finally the basic formulas are good, but the derivations abominable.

Albert Einstein, letter to Hendrik A. Lorentz, 17 January 1916



A recollection from 1922 on how Einstein created the theory of general relativity:

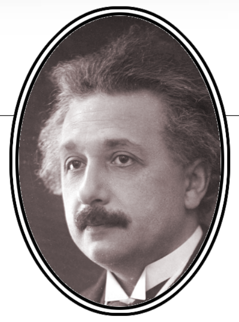
The first thought leading to the general theory of relativity occurred to me two years later, in 1907, and it did in a memorable setting.

I was already dissatisfied with the fact that the relativity of motion is restricted to motion with constant relative velocity and does not apply to arbitrary motion. I had always wondered privately whether this restriction could somehow be removed.

In 1907, while trying, at the request of Mr. Stark, to summarize the results of the special theory of relativity for the *Jahrbuch der Radioaktivität und Elektronik* of which he was the editor, I realized that, while all other laws of nature could be discussed in terms of the special theory of relativity, the theory could not be applied to the law of universal gravitation. I felt a strong desire to somehow find out the reason behind this. But this goal was not easy to reach. What seemed to me most unsatisfactory about the special theory of relativity was that, although the theory beautifully gave the relationship between inertia and energy, the relationship between inertia and weight, *i.e.*, the energy of the gravitational field, was left completely unclear. I felt that the explanation could probably not be found at all in the special theory of relativity.

…continued

Albert Einstein, "How I Created the Theory of Relativity" (1922)



EINSTEIN QUOTES

A recollection from 1921 on how Einstein created the theory of general relativity:

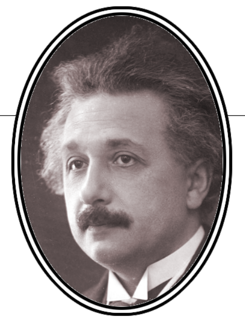
continued…

I was sitting in a chair in the Patent Office in Bern when all of a sudden I was struck by a thought: “If a person falls freely, he will certainly not feel his own weight.”

I was startled. This simple thought made a really deep impression on me. My excitement motivated me to develop a new theory of gravitation. My next thought was: “When a person falls, he is accelerating. His observations are nothing but observations in an accelerated system.” Thus, I decided to generalize the theory of relativity from systems moving with constant velocity to accelerated systems. I expected that this generalization would also allow me to solve the problem of gravitation. This is because the fact that a falling person does not feel his own weight can be interpreted as due to a new additional gravitational field compensating the gravitational field of the Earth, in other words, because an accelerated system gives a new gravitational field.

I could not immediately solve the problem completely on the basis of this insight. It would take me eight more years to find the correct relationship. In the meantime, however, I did come to recognize part of the general basis of the solution.

Albert Einstein, “How I Created the Theory of Relativity ”,
Jun Ishiwara’s Notes of Einstein’s Lecture at Kyoto University (1922)



EINSTEIN QUOTES

The first presentation of the *Äquivalenzprinzip* argument in Einstein's 1907 paper.

§17. Accelerated reference system and gravitational field

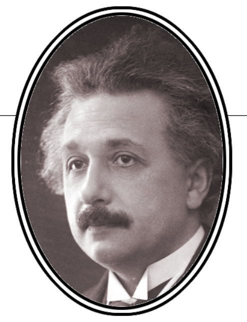
So far we have applied the principle of relativity, *i.e.*, the assumption that the physical laws are independent of the state of motion of the reference system, only to non-accelerated reference systems. Is it conceivable that the principle of relativity also applies to systems that are accelerated relative to each other?

While this is not the place for a detailed discussion of this question, it will occur to anybody who has been following the applications of the principle of relativity. Therefore I will not refrain from taking a stand on this question here.

We consider two systems Σ_1 and Σ_2 in motion. Let Σ_1 be accelerated in the direction of its X-axis, and let $\boldsymbol{G}$ be the (temporally constant) magnitude of that acceleration. Σ_2 shall be at rest, but it shall be located in a homogeneous gravitational field that imparts to all objects an acceleration $-\gamma$ in the direction of the X-axis.

…continued

Albert Einstein, *"On the relativity principle and the conclusions drawn from it"* (1907)

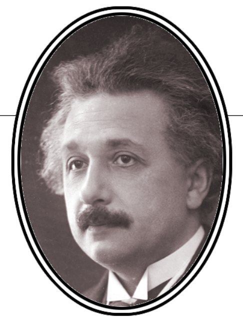


The first presentation of the *Äquivalenzprinzip* argument in Einstein's 1907 paper.

continued…

> As far as we know, the physical laws with respect to Σ_1 do not differ from those with respect to Σ_2; this is based on the fact that all bodies are equally accelerated in the gravitational field. At our present state of experience we have thus no reason to assume that the systems Σ_1 and Σ_2 differ from each other in any respect, and in the discussion that follows, we shall therefore assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system.
>
> This assumption extends the principle of relativity to the uniformly accelerated translational motion of the reference system. The heuristic value of this assumption rests on the fact that it permits the replacement of a homogeneous gravitational field by a uniformly accelerated reference system, the latter case being to some extent accessible to theoretical treatment.

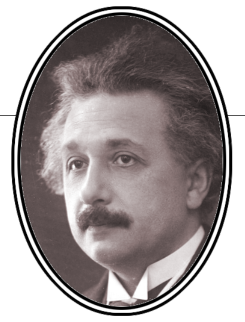
Albert Einstein, *"On the relativity principle and the conclusions drawn from it"* (1907)



Einstein's second paper from 1911 in his quest for a relativistic theory of gravitation again explores the kinematical consequences of the *Äquivalenz-prinzip*. Here is the statement of the principle (still without the name):

> **As long as we confine ourselves to purely mechanical processes** within the range of validity of Newton's mechanics, **we can be sure of the equivalence of the systems *K*** [*at rest in a homogeneous gravitational field*] **and *K'*** [*in uniform acceleration in empty space*].
>
> However, for our conception [*of equivalence*] to acquire deeper significance, **the systems K and K' must be equivalent with respect to all physical processes**, i.e., the natural laws with respect to *K* must coincide completely with those with respect to *K'*.
>
> If we accept this assumption, **we obtain a principle that possesses great heuristic significance…**

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



EINSTEIN QUOTES

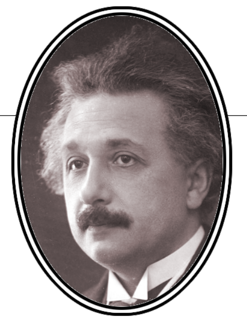
Here is the full quote form the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

§1. A Hypothesis Concerning the Physical Nature of the Gravitational Field

In a homogeneous gravitational field (acceleration due to gravity γ) let there be a coordinate system at rest *K*, which is oriented in such a way that the lines of force of the gravitational field run in the direction of the negative z-axis. In a space free of gravitational fields, let there be another coordinate system *K'* that moves with a uniform acceleration (acceleration γ) in the direction of its positive z-axis. So as not to complicate the analysis unnecessarily, we will disregard the theory of relativity for the time being, and consider, instead, the two systems according to conventional kinematics, and the motions occurring in them according to customary mechanics.

… continued

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



Here is the full quote form the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

continued…

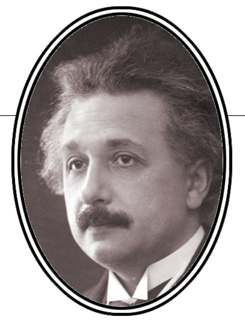
Material points not subjected to actions of other material points move relative to *K* as well as relative to *K'* according to the equations

$$\frac{d^2 x_\nu}{dt^2} = 0, \quad \frac{d^2 y_\nu}{dt^2} = 0, \quad \frac{d^2 z_\nu}{dt^2} = -\gamma$$

For the accelerated system *K'*, this follows directly from Galileo's principle, but for the system *K* at rest in a homogeneous gravitational field, this follows from the experience that all bodies undergo the same, constant, acceleration in such a field. This experience of the identical falling of all bodies in the gravitational field is one of the most universal experiences that the observation of nature has yielded to us; nevertheless, this law has not been granted a place in the foundation of our physical edifice.

…continued

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



EINSTEIN QUOTES

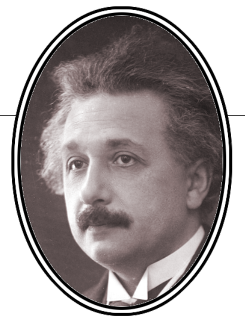
Here is the full quote form the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

continued…

> But we arrive at a very satisfactory interpretation of the empirical law if we assume that the systems K and *K'* are, physically, perfectly equivalent, *i.e.*, if we assume that the system *K* could likewise be conceived as occurring in a space free of a gravitational field; but in that case we must consider K as uniformly accelerated. Given this conception, one can no more speak of the absolute acceleration of the reference system than one can speak of a system's absolute velocity in the ordinary theory of relativity. [Footnote: Of course, one cannot replace an arbitrary gravitational field by a state of motion of the system without a gravitational field, just as one cannot transform to rest all the points of an arbitrarily moving medium by means of a relativistic transformation.] With this conception, the equal falling of all bodies in a gravitational field is self-eveident.

…continued

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)

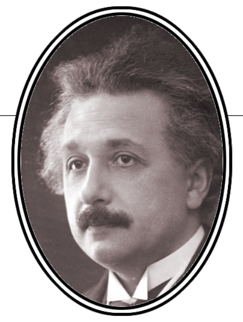


Here is the full quote form the relevant first section of the 1911 paper, giving the context of the *Äquivalenzprinzip* argument:

continued…

As long as we confine ourselves to purely mechanical processes within the range of validity of Newton's mechanics, we can be sure of the equivalence of the systems *K* and *K'*. However, for our conception to acquire deeper significance, the systems *K* and *K'* must be equivalent with respect to all physical processes, *i.e.*, the natural laws with respect to *K* must coincide completely with those with respect to *K'*. If we accept this assumption, we obtain a principle that possesses great heuristic significance, provided that it is really correct. For through a theoretical analysis of processes taking place relative to a uniformly accelerating reference system, we obtain information about the course of processes taking place in a homogeneous gravitational field. [*Footnote*: It will be shown in a subsequent paper that the gravitational field considered here is homogeneous only to first approximation.] In what follows, I shall first show that from the point of view of the ordinary theory of relativity our hypothesis has considerable probability.

Albert Einstein, "On the Influence of Gravitation on the Propagation of Light" (1911)



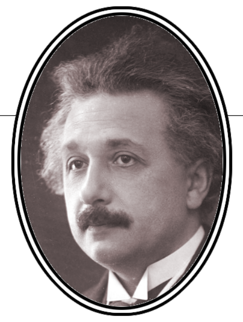
A selection of further statements of the *Äquivalenzprinzip:*

In a homogeneous gravitational field all motions take place in the same way as in the absence of a gravitational field in relation to a uniformly accelerated coordinate system. If this principle held good for any events whatever (the "principle of equivalence") [...] we were to reach a natural theory of the gravitational fields.

Albert Einstein, "Notes on the Origin of the General Theory of Relativity" (1933)

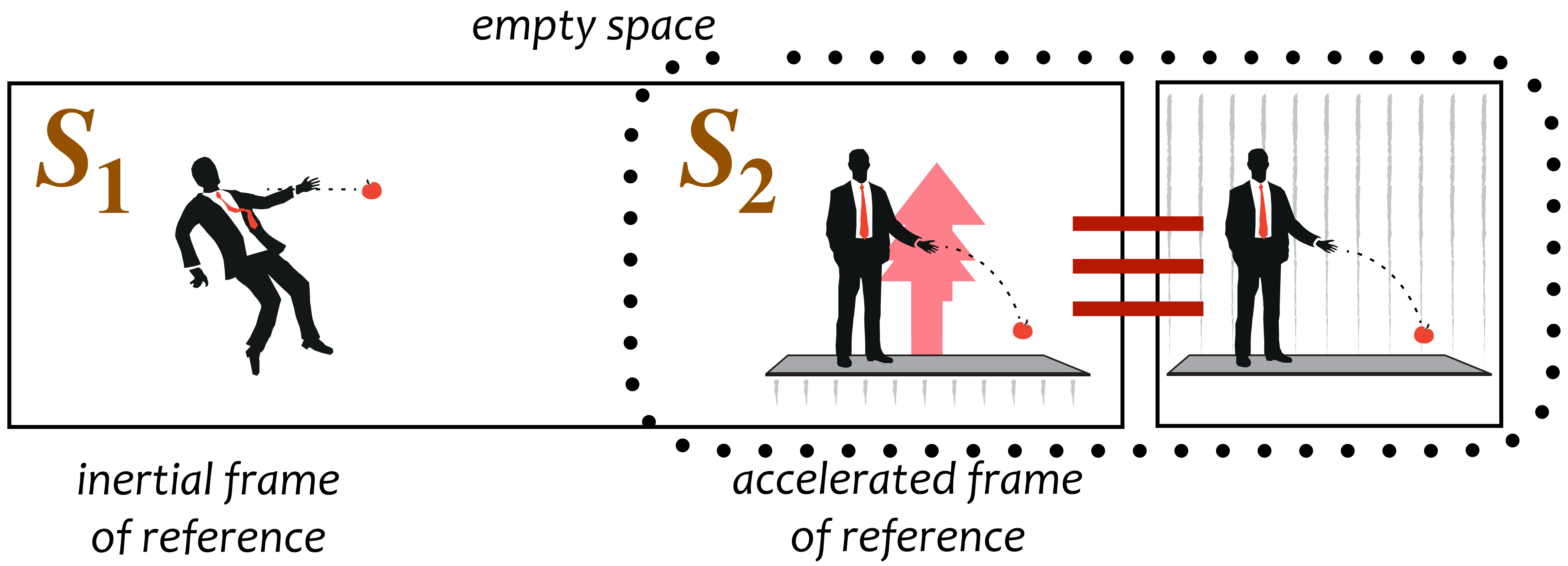
An inertial space without gravitational field is physically equivalent to a uniformly accelerated space, in which there is a (homogeneous) gravitational field. (Equivalence hypothesis.)

Albert Einstein, letter to Jean Becquerel, 16 August 1951



EINSTEIN QUOTES

In one of his last texts on GR from 1952, Einstein again presents the basic idea of the *Äquivalenzprinzip* in a way which can be illustrated like this:

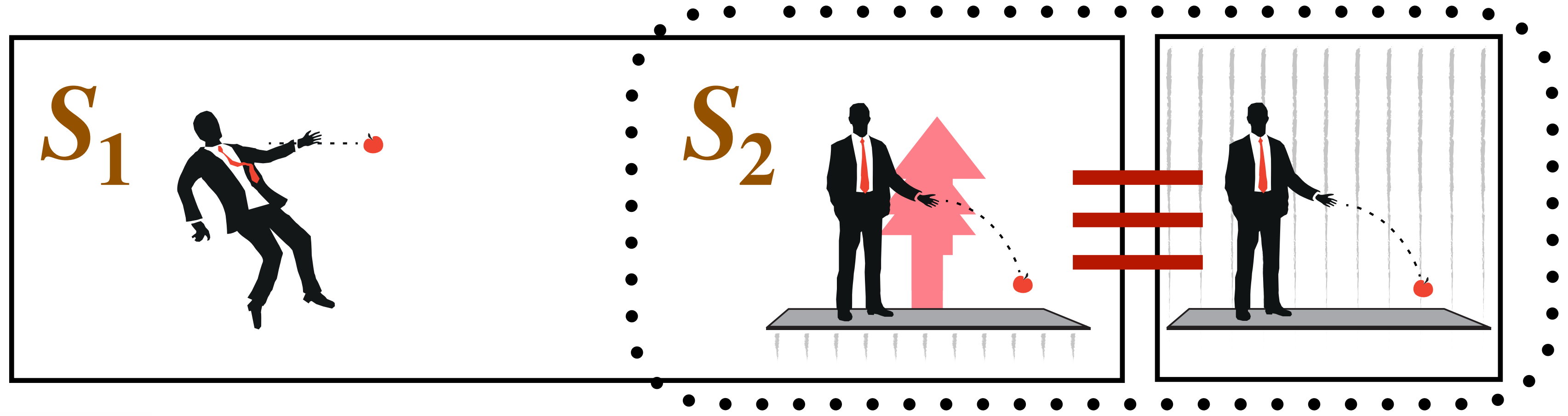


This theory arose primarily from the endeavour to understand the equality of inertial and gravitational mass. We start out from an inertial system S_1, whose space is, from the physical point of view, empty. In other words, there exists in the part of space contemplated neither matter (in the usual sense) nor a field (in the sense of the special theory of relativity).

... continued

Albert Einstein, "Relativity and the Problem of Space" (1952)

In one of his last texts on GR from 1952, Einstein again presents the basic idea of the *Äquivalenzprinzip* in a way which can be illustrated like this:



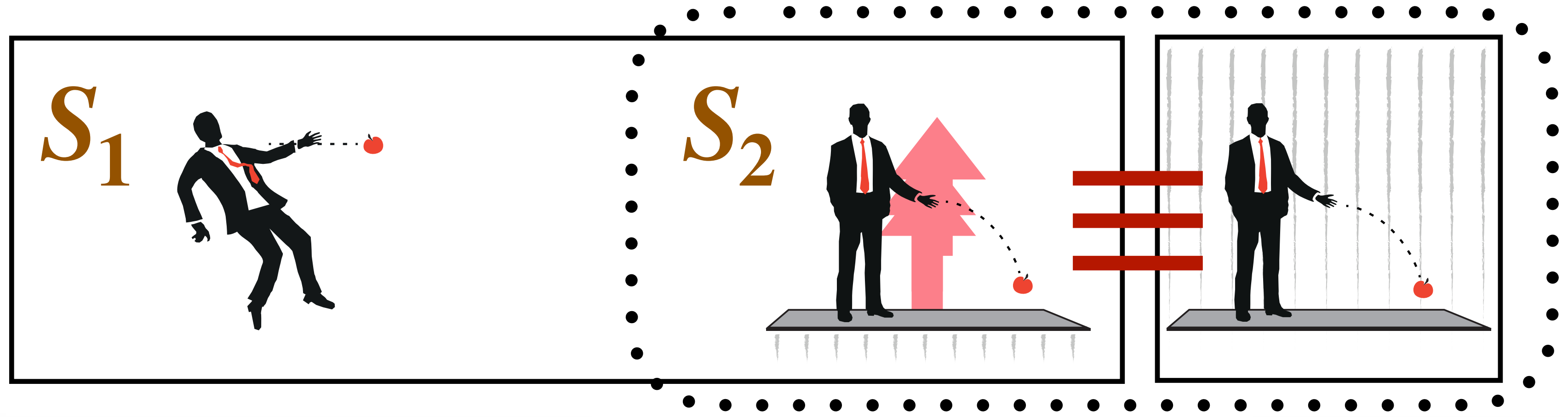
continued…

> With reference to S_1 let there be a second system of reference S_2 in uniform acceleration. Then S_2 is thus not an inertial system. With respect to S_2 every test mass would move with an acceleration, which is independent of its physical and chemical nature. Relative to S_2, therefore, there exists a state which, at least to a first approximation, cannot be distinguished from a gravitational field. The following concept is thus compatible with the observable facts: S_2 is also equivalent to an "inertial system"; but with respect to S_2 a (homogeneous) gravitational field is present (about the origin of which one does not worry in this connection).

… continued

Albert Einstein, "Relativity and the Problem of Space" (1952)

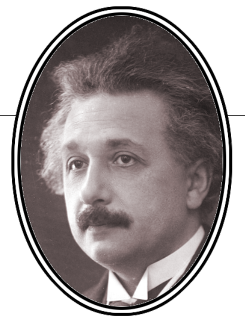
In one of his last texts on GR from 1952, Einstein again presents the basic idea of the *Äquivalenzprinzip* in a way which can be illustrated like this:



continued...

Thus when the gravitational field is included in the framework of the consideration, the inertial system loses its objective significance, assuming that this "principle of equivalence" can be extended to any relative motion whatsoever of the systems of reference. If it is possible to base a consistent theory on these fundamental ideas, it will satisfy of itself the fact of the equality of inertial and gravitational mass, which is strongly confirmed empirically.

Albert Einstein, "Relativity and the Problem of Space" (1952)



The “Einstein Box” [*in German: “Kasten”*]…

Two physicists, A and B, awake from a narcotic sleep and notice that they are in a closed box that has nontransparent walls and is equipped with all their instruments. They have no idea where the box is situated and whether it moves...

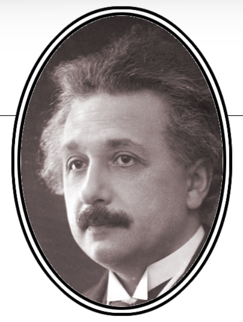
Albert Einstein, “On the Present State of the Problem of Gravitation” (1913)

An observer enclosed in a box can in no way decide whether the box is at rest in a static gravitational field, or whether it is in accelerated motion, maintained by forces acting on the box, in a space that is free of gravitational fields.

A. Einstein, M. Grossmann, “Outline of a Generalized Theory of Relativity and of a Theory of Gravitation” (1913)

Let us imagine a spacious chest resembling a room with an observer inside who is equipped with apparatus […] To the middle of the lid of the chest is fixed externally a hook with rope attached, and now a ‘being’ (what kind of a being is immaterial to us) begins pulling at this with a constant force...

Albert Einstein, “Relativity. The Special and the General Theory. A Popular Exposition” (1917)



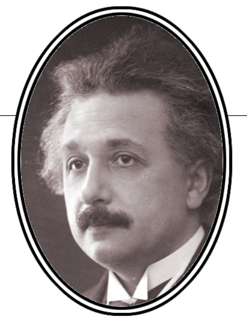
Einstein’s 1916 reply to Friedrich Kottler (who proposed a “kinematic, nondynamic interpretation of gravitation”) is a concise summary of the *Äquivalenzprinzip* argument.

Kottler claims I had abandoned in my later papers the “principle of equivalence” which I did introduce in order to unify the concepts of “inertial mass” and “gravitational mass.” This opinion must be based upon the fact that we both do not denote the same thing as “{the principle of equivalence#; because in my opinion my theory rests exclusively upon this principle. Therefore I repeat the following:

1. The Limiting Case of the Special Theory of Relativity. Let a finite space-time-like domain be without a gravitational field; i.e., let it be possible to introduce a system of reference *K* (“Galilean system”) relative to which the following is true for the domain. As is usually presupposed in the special theory of relativity, let the coordinates be directly measurable in known manner by means of a unit measuring stick, and the times by a unit clock. Relative to this system an isolated material point shall move uniformly in a straight line, just as it was postulated by Galileo.

…continued

Albert Einstein, “On Friedrich Kottler's Paper: ‘On Einstein's Equivalence Hypothesis and Gravitation’ ” (1916)



EINSTEIN QUOTES

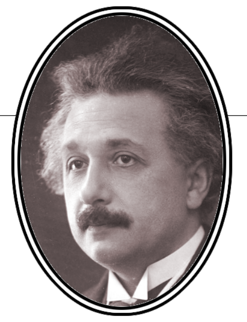
Einstein's 1916 reply to Friedrich Kottler (who proposed a "kinematic, nondynamic interpretation of gravitation") is a concise summary of the *Äquivalenzprinzip* argument.

continued…

2. The Principle of Equivalence. Starting from the limiting case of the special theory of relativity, one may ask if in the domain under consideration an observer, who is uniformly accelerated relative to *K*, must necessarily judge his state as accelerated, or whether he has an option left — according to the (approximately) known laws of nature — to interpret his state as "at rest" Or, to phrase it more precisely: Do the laws of nature, known to us in some approximation, allow us to consider a reference system *K'* as being at rest if it is in uniform acceleration with respect to *K*? Or, somewhat more generally: Can the principle of relativity be extended such as to encompass reference systems that are in (uniform) accelerated motion relative to one another? The answer is: insofar as we really know the laws of nature, nothing prevents us from considering a system *K'* as at rest, provided we assume a gravitational field (homogeneous in first approximation) relative to *K'*. Because in a homogeneous gravitational field, as with regard to our system *K'*, all bodies fall with the same acceleration independent of their physical nature. I call "principle of equivalence" the assumption that *K'* can be treated with all rigor as being at rest, such that no law of nature fails to be satisfied relative to *K'*.

…continued

Albert Einstein, "On Friedrich Kottler's Paper: 'On Einstein's Equivalence Hypothesis and Gravitation' " (1916)



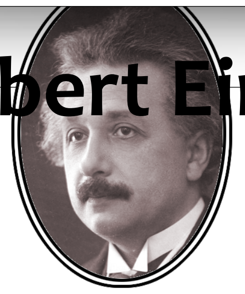
EINSTEIN QUOTES

Einstein's 1916 reply to Friedrich Kottler (who proposed a "kinematic, nondynamic interpretation of gravitation") is a concise summary of the *Äquivalenzprinzip* argument.

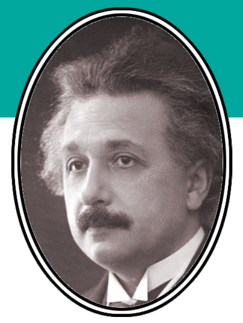
continued...

3. The Gravitational Field Not Only Kinematically Caused. The previous consideration can also be inverted. Let the system *K'* with the gravitational field, which we considered above, be the original system. One can then introduce a new system *K* that is accelerated with respect to *K'* such that (isolated) masses move uniformly in straight lines (within the domain of consideration). But one must *not* go beyond this and say: If *K'* is a reference system with an *arbitrary* gravitational field, then one can always find a system *K* relative to which isolated masses move uniformly in straight lines, i.e., relative to which no gravitational field exists. The absurdity of such a hypothesis is plainly obvious. If the gravitational field relative to *K'* is, for example, that of a mass point at rest, then not even the most refined trick of transformation can transform the field away in the entire neighborhood of the mass point. Therefore, one may never maintain that a gravitational field could be explained, so to speak, by pure kinematics; a "kinematic, nondynamic interpretation of gravitation" is not possible. By mere transformation from a Galilean system into another one by means of an acceleration transformation, we do not learn about *arbitrary* gravitational fields but only some of a very special kind; but these too must — of course — obey the same laws as all other fields of gravitation. This is again just another formulation of the principle of equivalence (specialized in its application to gravitation).

Albert Einstein, "On Friedrich Kottler's Paper: 'On Einstein's Equivalence Hypothesis and Gravitation' " (1916)



BRIEF SUMMARY OF GENERAL RELATIVITY



SUMMARY OF GENERAL RELATIVITY

If you are curious about General Relativity, here is
a short but comprehensive summary of the theory:

> **“General Relativity —**
> *A reconciliation between* ***gravitational structure*** *and* ***chronogeometry***
> *which excludes any* ***prior geometry.”***

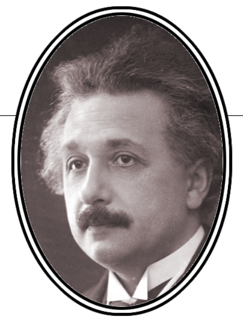
Jürgen Renn, Tilman Sauer, “Heuristics and Mathematical Representation in Einstein’s Search for a Gravitational Field Equation” (1999)

SUMMARY OF GENERAL RELATIVITY

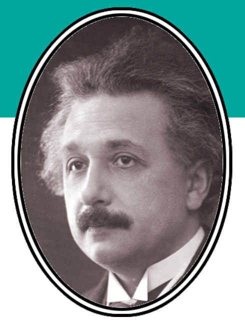
For completeness, here is the original quote for the snappy characterisation of general relativity:

> In spite of the mathematical and conceptual difficulties that he encountered, [Einstein's] heuristic principles guided **a reconciliation between gravito-inertial structure and chronogeometry which excluded any prior geometry,** thus effectively determining the characteristic features of general relativity, even as we understand it today.

Jürgen Renn, Tilman Sauer, "Heuristics and Mathematical Representation in Einstein's Search for a Gravitational Field Equation" (1999)



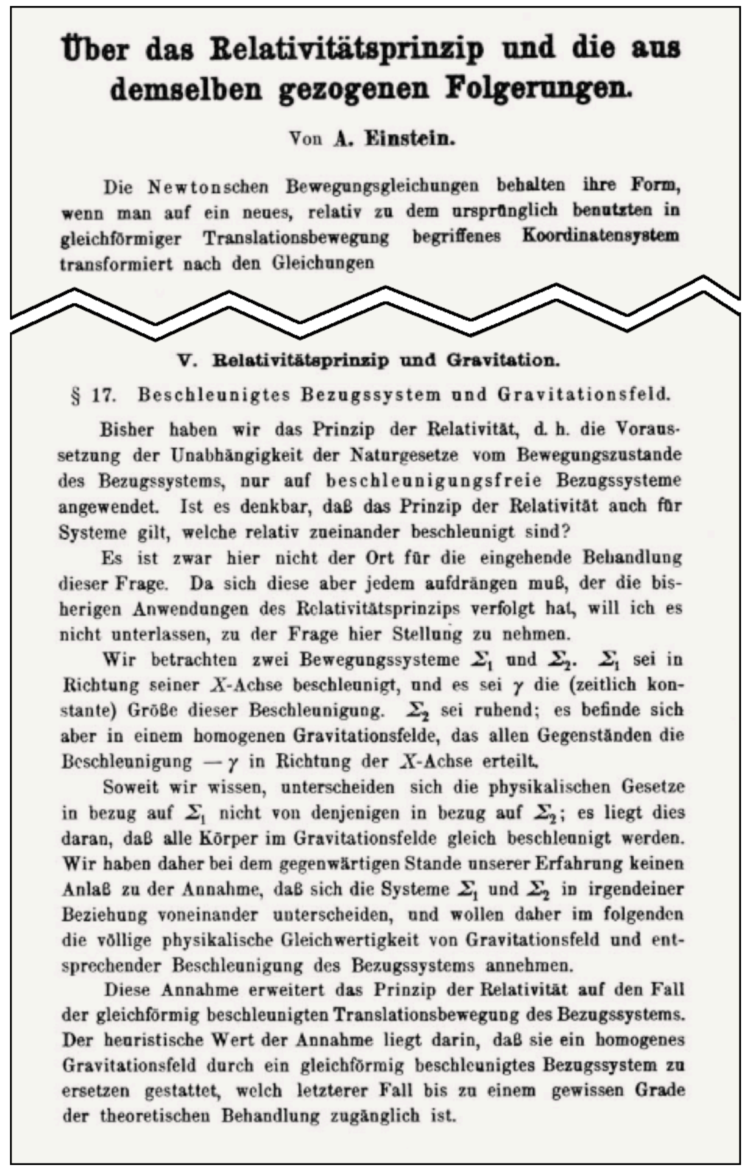
A TIMELINE FOR EINSTEIN'S SEARCH FOR GENERAL RELATIVITY



EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

It took Einstein eight long years to find the theory of general relativity:

Submitted
4 December
1907



Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen.

Von **A. Einstein.**

Die Newtonschen Bewegungsgleichungen behalten ihre Form, wenn man auf ein neues, relativ zu dem ursprünglich benutzten in gleichförmiger Translationsbewegung begriffenes Koordinatensystem transformiert nach den Gleichungen

V. Relativitätsprinzip und Gravitation.

§ 17. Beschleunigtes Bezugssystem und Gravitationsfeld.

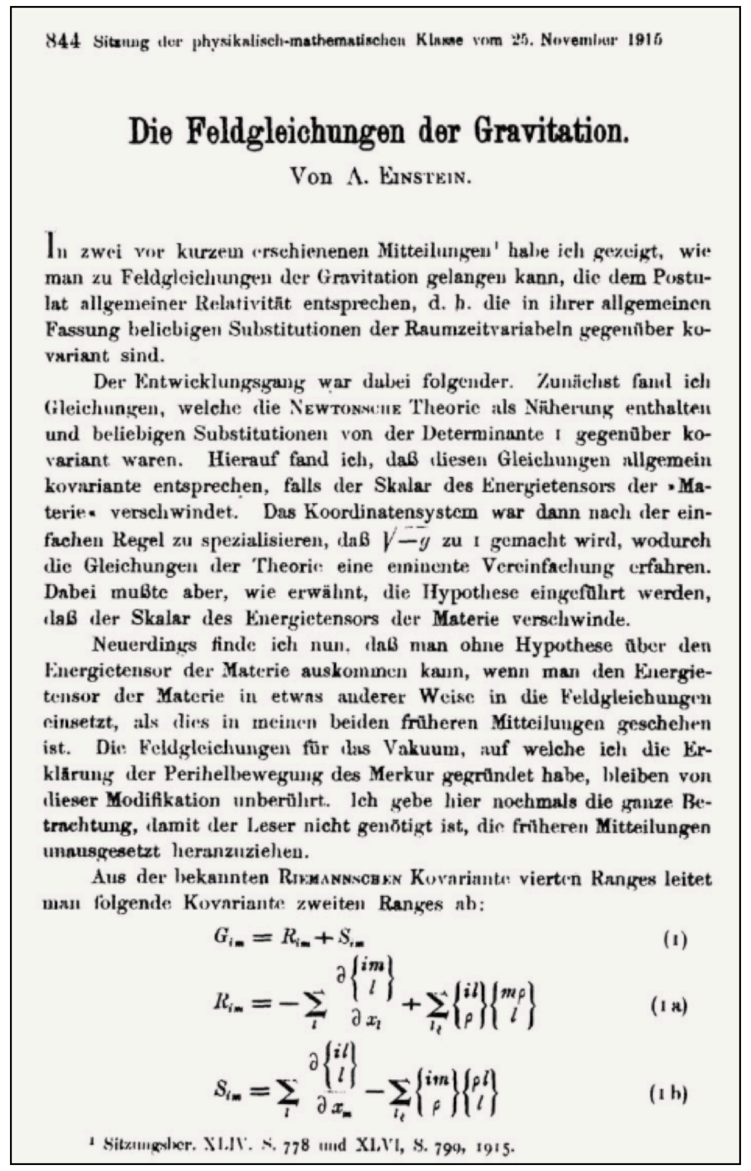
Bisher haben wir das Prinzip der Relativität, d. h. die Voraussetzung der Unabhängigkeit der Naturgesetze vom Bewegungszustande des Bezugssystems, nur auf beschleunigungsfreie Bezugssysteme angewendet. Ist es denkbar, daß das Prinzip der Relativität auch für Systeme gilt, welche relativ zueinander beschleunigt sind?

Es ist zwar hier nicht der Ort für die eingehende Behandlung dieser Frage. Da sich diese aber jedem aufdrängen muß, der die bisherigen Anwendungen des Relativitätsprinzips verfolgt hat, will ich es nicht unterlassen, zu der Frage hier Stellung zu nehmen.

Wir betrachten zwei Bewegungssysteme Σ_1 und Σ_2. Σ_1 sei in Richtung seiner X-Achse beschleunigt, und es sei γ die (zeitlich konstante) Größe dieser Beschleunigung. Σ_2 sei ruhend; es befinde sich aber in einem homogenen Gravitationsfelde, das allen Gegenständen die Beschleunigung $-\gamma$ in Richtung der X-Achse erteilt.

Soweit wir wissen, unterscheiden sich die physikalischen Gesetze in bezug auf Σ_1 nicht von denjenigen in bezug auf Σ_2; es liegt dies daran, daß alle Körper im Gravitationsfelde gleich beschleunigt werden. Wir haben daher bei dem gegenwärtigen Stande unserer Erfahrung keinen Anlaß zu der Annahme, daß sich die Systeme Σ_1 und Σ_2 in irgendeiner Beziehung voneinander unterscheiden, und wollen daher im folgenden die völlige physikalische Gleichwertigkeit von Gravitationsfeld und entsprechender Beschleunigung des Bezugssystems annehmen.

Diese Annahme erweitert das Prinzip der Relativität auf den Fall der gleichförmig beschleunigten Translationsbewegung des Bezugssystems. Der heuristische Wert der Annahme liegt darin, daß sie ein homogenes Gravitationsfeld durch ein gleichförmig beschleunigtes Bezugssystem zu ersetzen gestattet, welch letzterer Fall bis zu einem gewissen Grade der theoretischen Behandlung zugänglich ist.



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \tag{1}$$

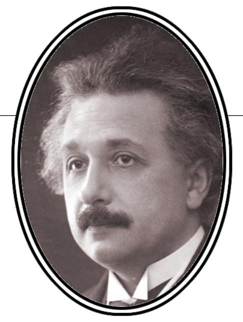
$$R_{im} = -\sum_{l} \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \tag{1a}$$

$$S_{im} = \sum_{l} \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \tag{1b}$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915.

Submitted
25 November
1915

1905 **1907** 1909 1911 1913 **1915** 1917 1919 1921 1923

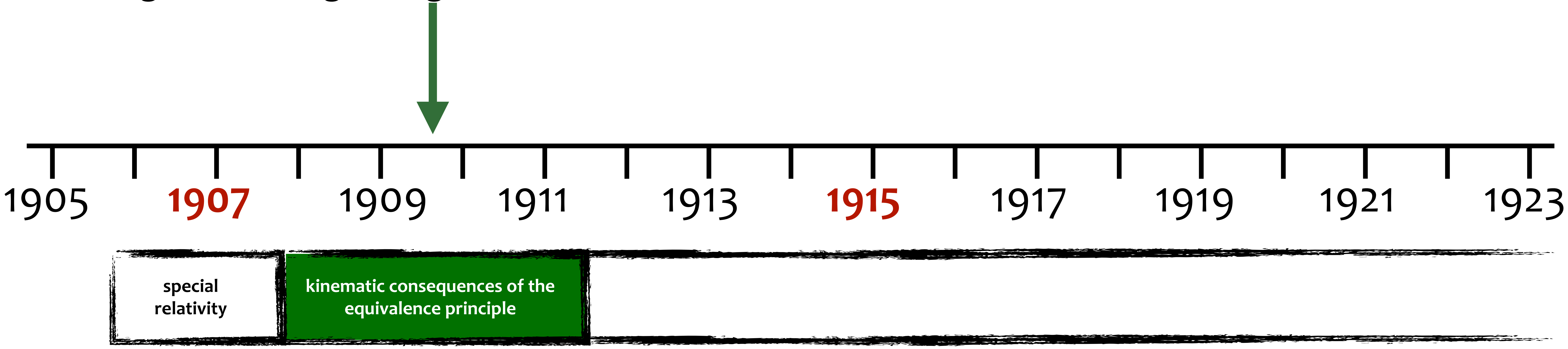


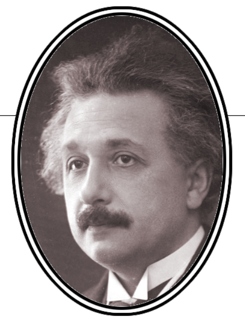
Einstein's Critique of the Equivalence Principle

NORDITA

In 1907 Einstein introduced the idea that he would later (in 1912) would call the "*Equivalence Principle*".

Exploring the kinematical consequences of this *heuristic principle* he obtained a rough idea of the properties one should expect in a relativistic theory of gravity — such as light bending and gravitational redshift.

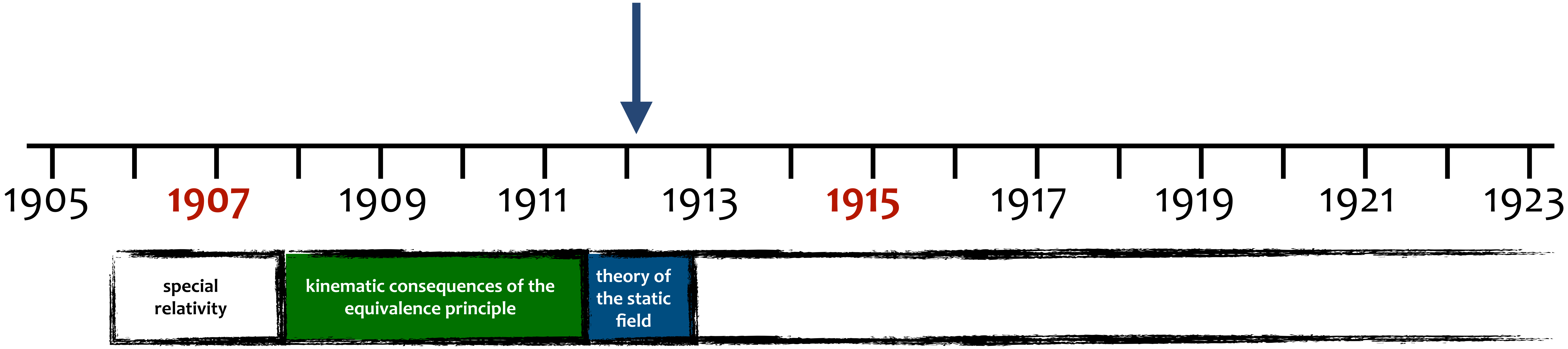


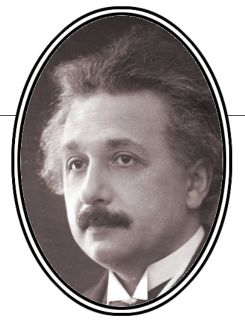


EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

When Einstein got a professorship in physics in Prag 1911–1912, he started to work on the dynamics of a simpler case, static gravitational fields.

In this theory the constant speed of light c of SR became a field in 3-space $c(x, y, z)$ representing the *scalar* gravitational potential.



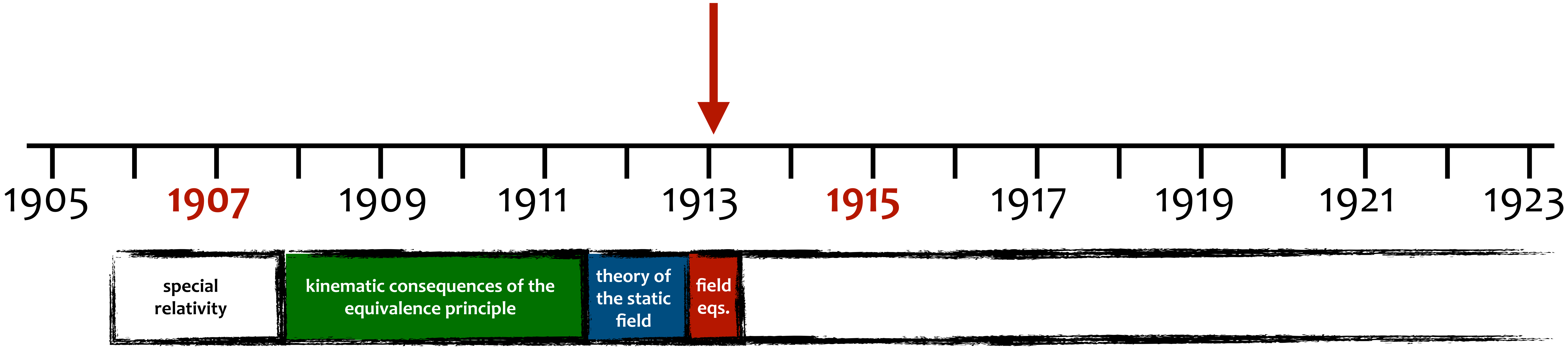


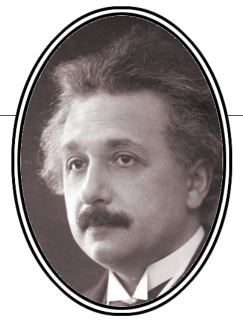
EINSTEIN’S SEARCH FOR GENERAL RELATIVITY

By the time he came back to Zürich in August 1912, Einstein had understood that the full theory of relativistic gravity must be a theory of the spacetime metric, governed by generally covariant field equations sourced by the energy-momentum content of spacetime.

He asked his friend Marcel Grossmann to help him find the appropriate mathematics for such a theory: tensor calculus in a Riemannian geometry.

We can follow Einstein’s struggle to find the correct field equations in the “Zürich notebook”.



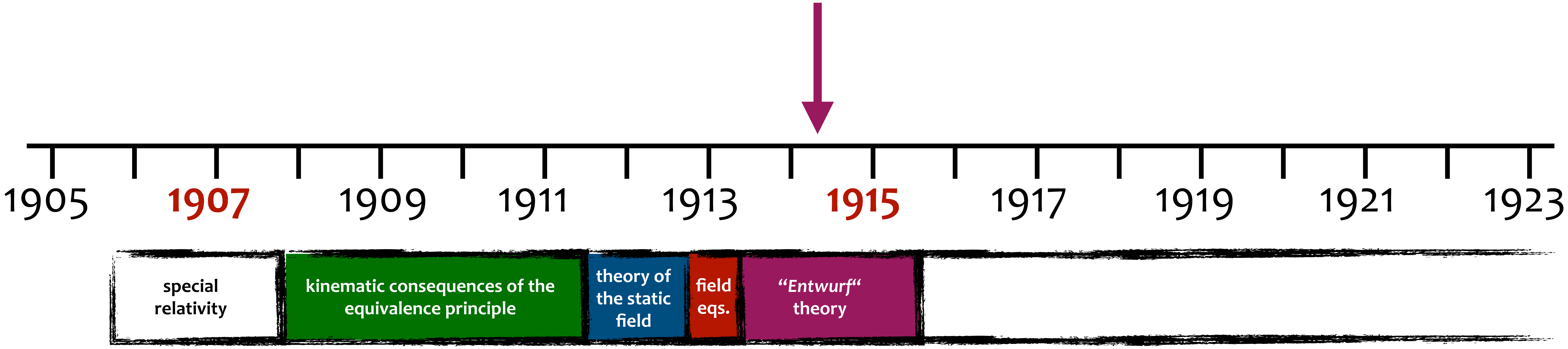


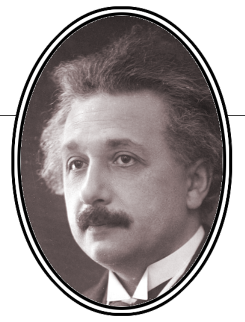
EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

But they failed to find field equations that would satisfy all requirements that Einstein had set up.

They finally settled on field equations that were *not* generally covariant: the Einstein-Grossmann (or "*Entwurf*") theory.

Einstein came up with clever arguments why the equations could not be generally covariant, the most famous being the "*hole argument*".

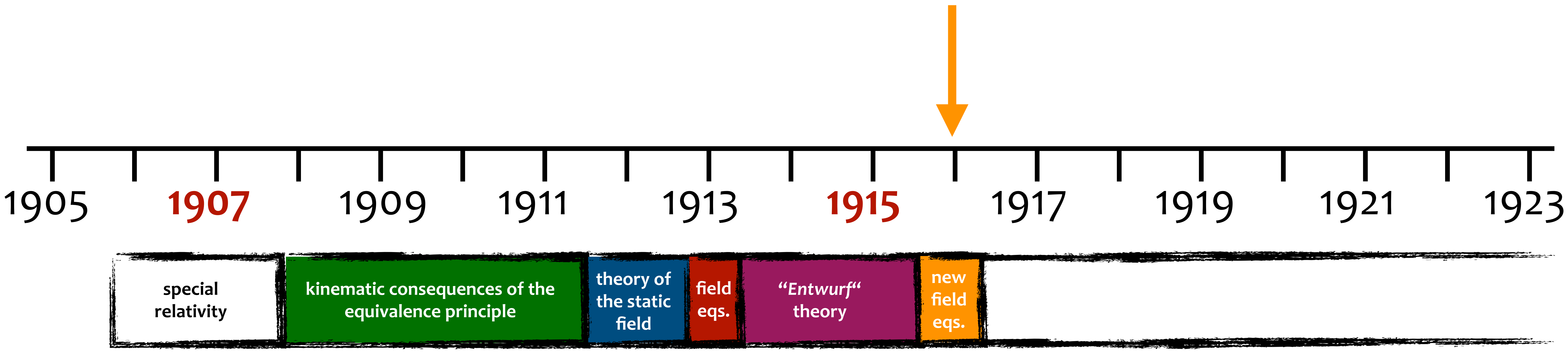


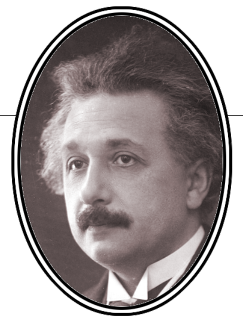


EINSTEIN’S SEARCH FOR GENERAL RELATIVITY

By November 1915 Einstein had concluded that the *Entwurf* theory could not be correct, and that his arguments against general covariance were wrong.

After some failed attempts he finally found acceptable generally covariant field equations — completing the theory of General Relativity.

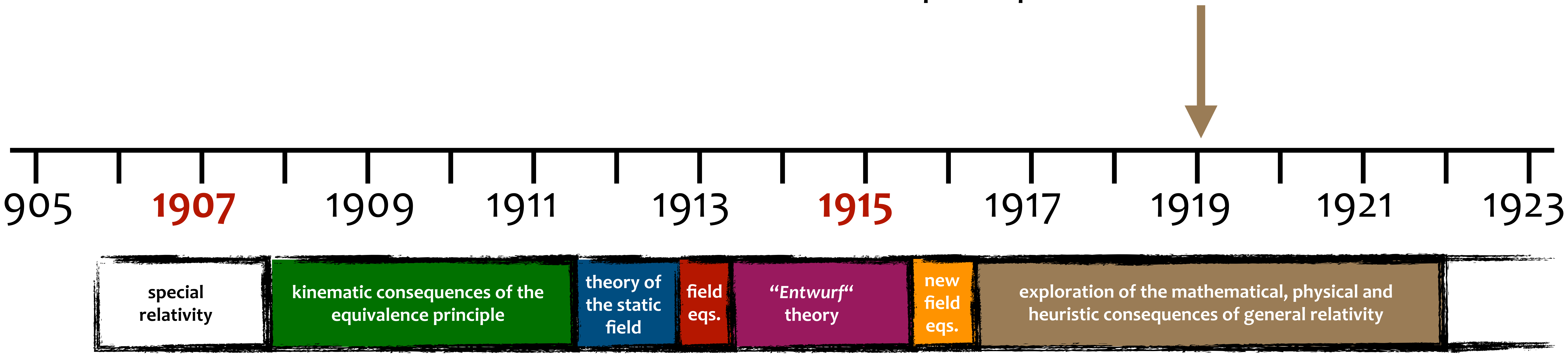


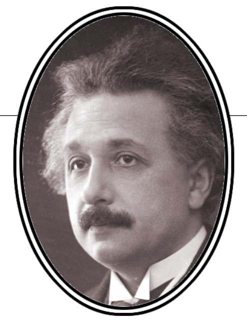


EINSTEIN'S SEARCH FOR GENERAL RELATIVITY

The new theory fulfilled all physical requirements, but did not agree with some of the intuitive heuristics that had guided Einstein during his search — in particular "Mach's Principle".

Einstein spent the next couple of years exploring the properties of the new theory (cosmological constant, gravitational waves), and very reluctantly accepted that he had to give up some of his heuristic principles.

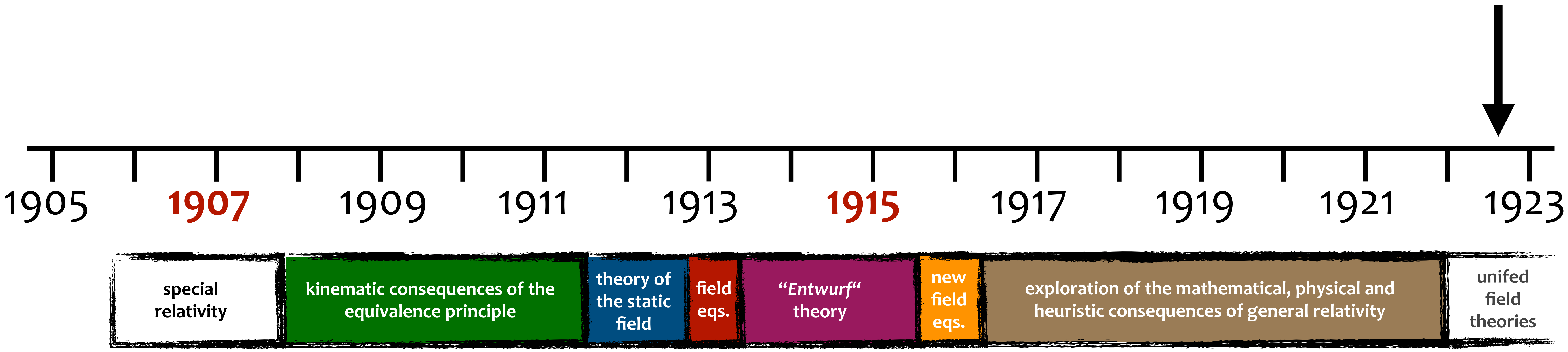


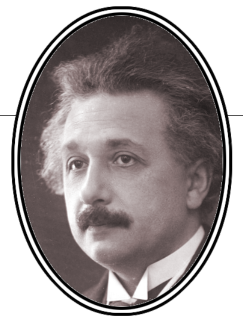


EINSTEIN’S SEARCH FOR GENERAL RELATIVITY

From the 1920s until his death in 1955 Einstein turned to what he saw as the next step — finding a “*unified field theory*” that would unite gravitation and electromagnetism, and explain the structure of matter.

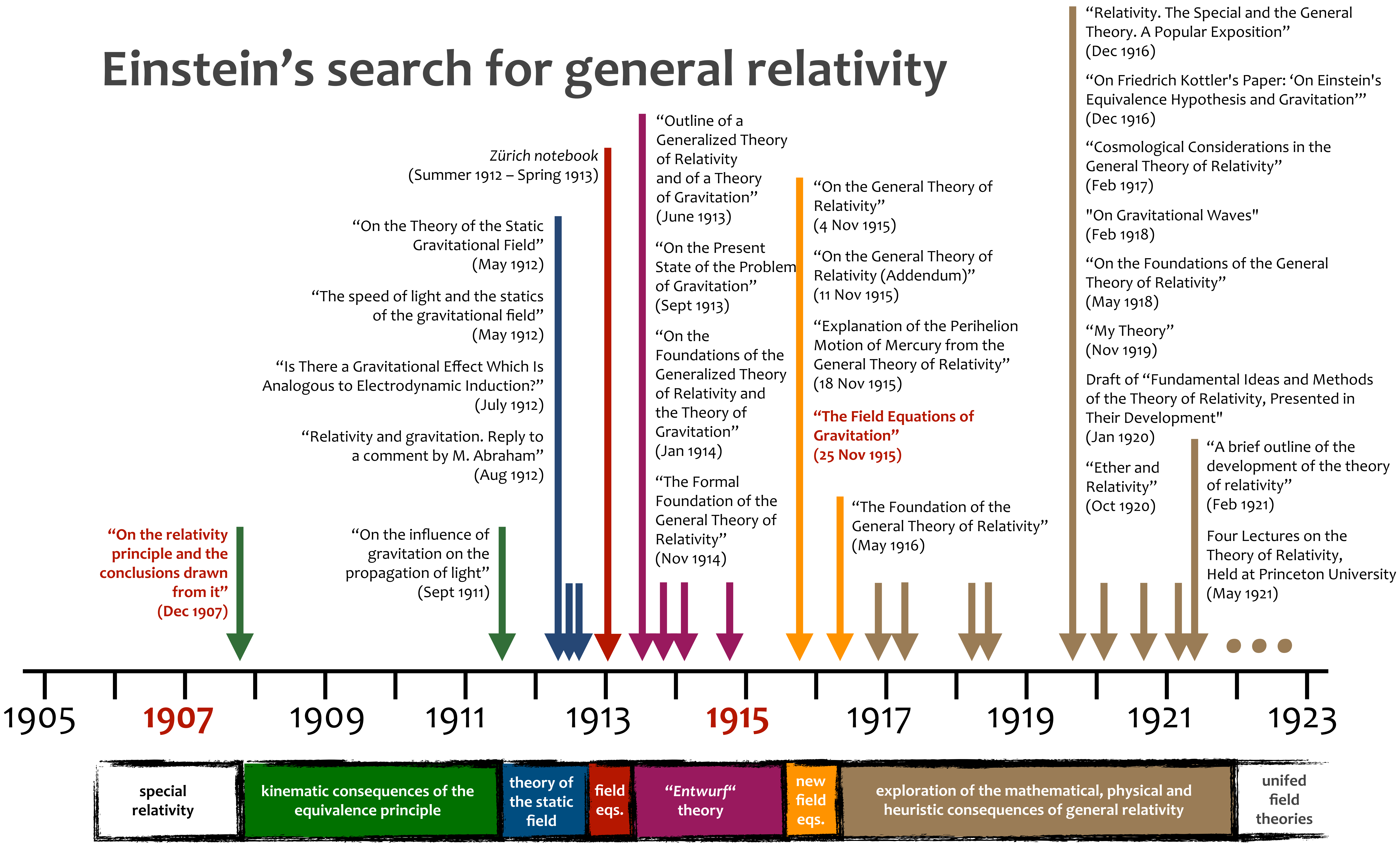
He would not succeed.

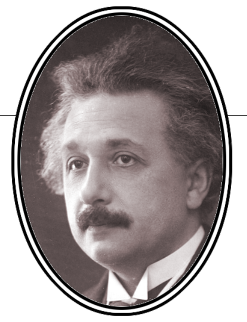




THIS TALK

Einstein’s search for general relativity





Einstein’s search for general relativity

Texts important for understanding Einstein’s “*Äquivalenzprinzip*”

“On the relativity principle and the conclusions drawn from it” (Dec 1907)

“On the influence of gravitation on the propagation of light” (Sept 1911)

“On the Theory of the Static Gravitational Field” (May 1912)

“The speed of light and the statics of the gravitational field” (May 1912)

“Is There a Gravitational Effect Which Is Analogous to Electrodynamic Induction?” (July 1912)

“Relativity and gravitation. Reply to a comment by M. Abraham” (Aug 1912)

Zürich notebook (Summer 1912 – Spring 1913)

“Outline of a Generalized Theory of Relativity and of a Theory of Gravitation” (June 1913)

“On the Present State of the Problem of Gravitation” (Sept 1913)

“On the Foundations of the Generalized Theory of Relativity and the Theory of Gravitation” (Jan 1914)

“The Formal Foundation of the General Theory of Relativity” (Nov 1914)

“On the General Theory of Relativity” (4 Nov 1915)

“On the General Theory of Relativity (Addendum)” (11 Nov 1915)

“Explanation of the Perihelion Motion of Mercury from the General Theory of Relativity” (18 Nov 1915)

“The Field Equations of Gravitation” (25 Nov 1915)

“The Foundation of the General Theory of Relativity” (May 1916)

“Relativity. The Special and the General Theory. A Popular Exposition” (Dec 1916)

“On Friedrich Kottler's Paper: ‘On Einstein's Equivalence Hypothesis and Gravitation’” (Dec 1916)

“Cosmological Considerations in the General Theory of Relativity” (Feb 1917)

"On Gravitational Waves" (Feb 1918)

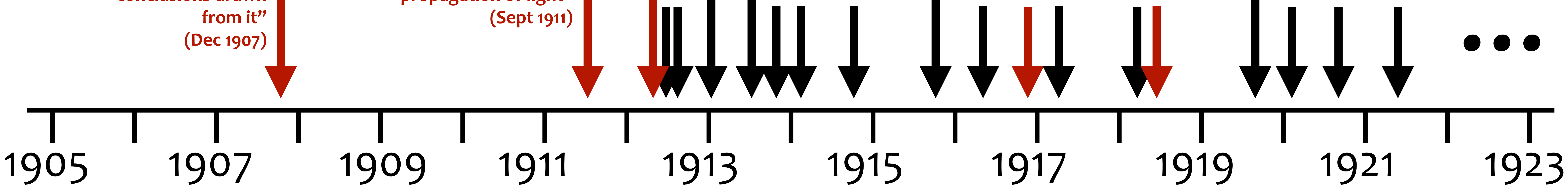
“On the Foundations of the General Theory of Relativity” (May 1918)

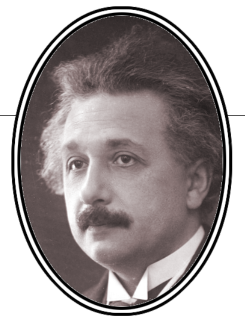
“My Theory” (Nov 1919)

Draft of “Fundamental Ideas and Methods of the Theory of Relativity, Presented in Their Development" (Jan 1920)

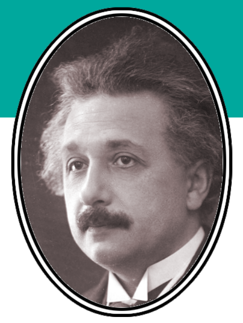
“Ether and Relativity” (Oct 1920)

Four Lectures on the Theory of Relativity, Held at Princeton University (May 1921)





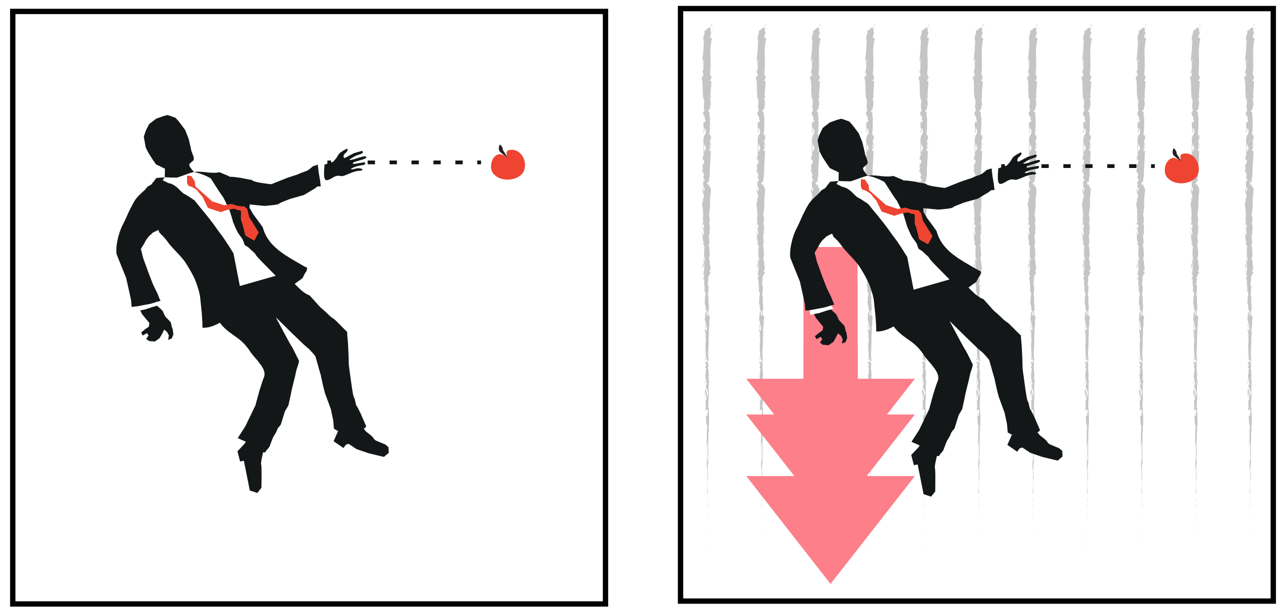
A QUIZ FOR THE GR EXPERTS



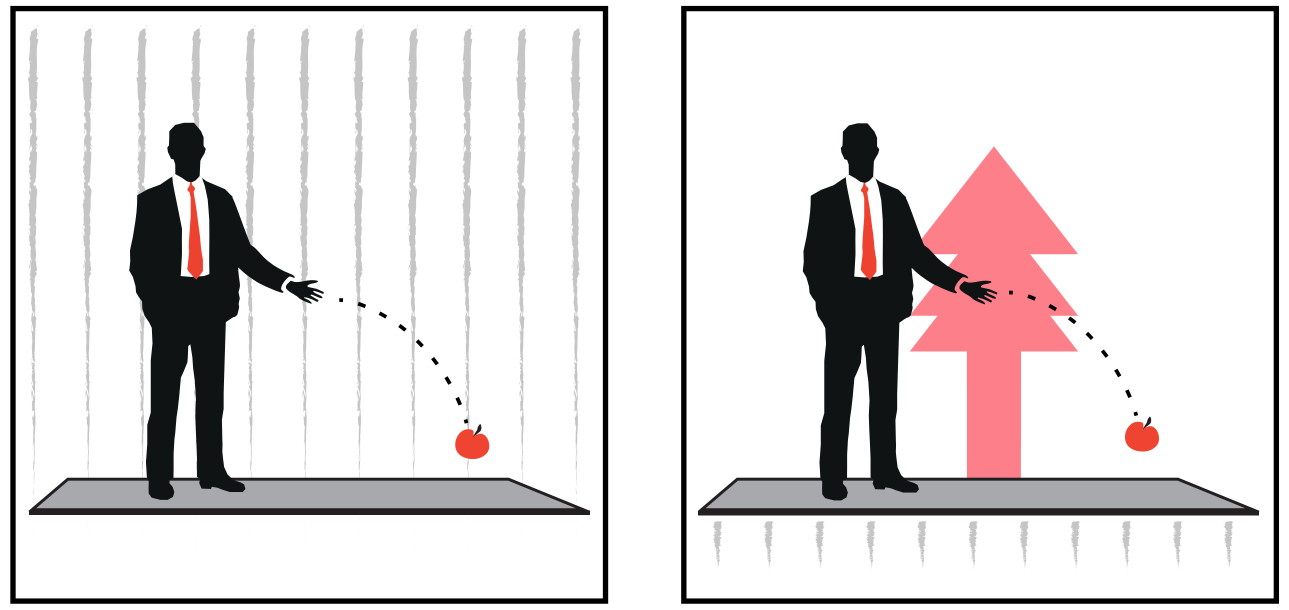
A quiz for the experts:

1) In NM, which of these four observers are **inertial observers**?

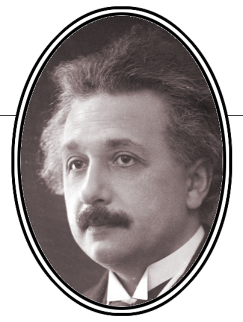
“happiest thought” observers



“Einstein elevator” observers



Newtonian mechanics	NM	?	?	?	?



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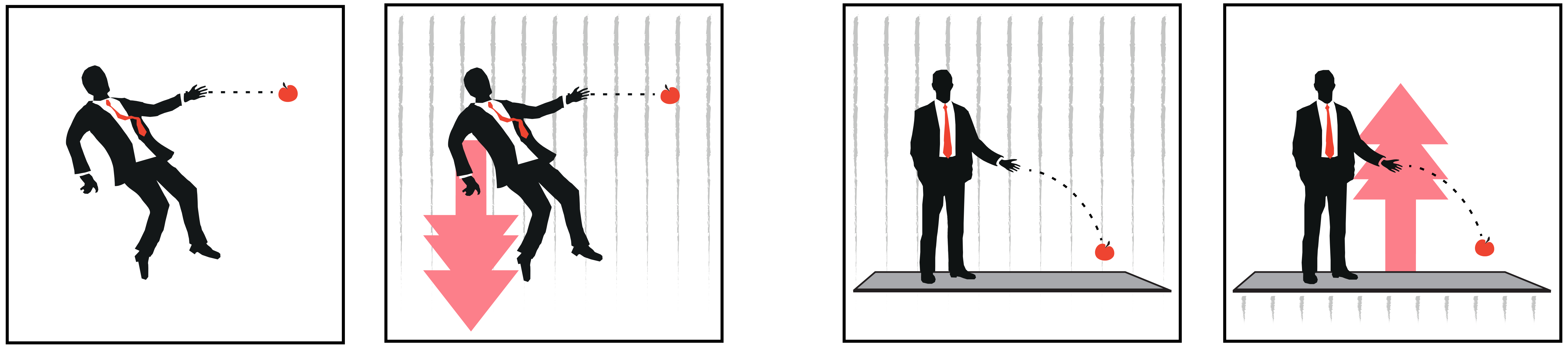
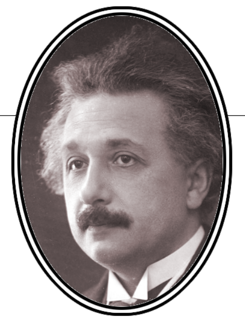
"happiest thought" observers — "Einstein elevator" observers



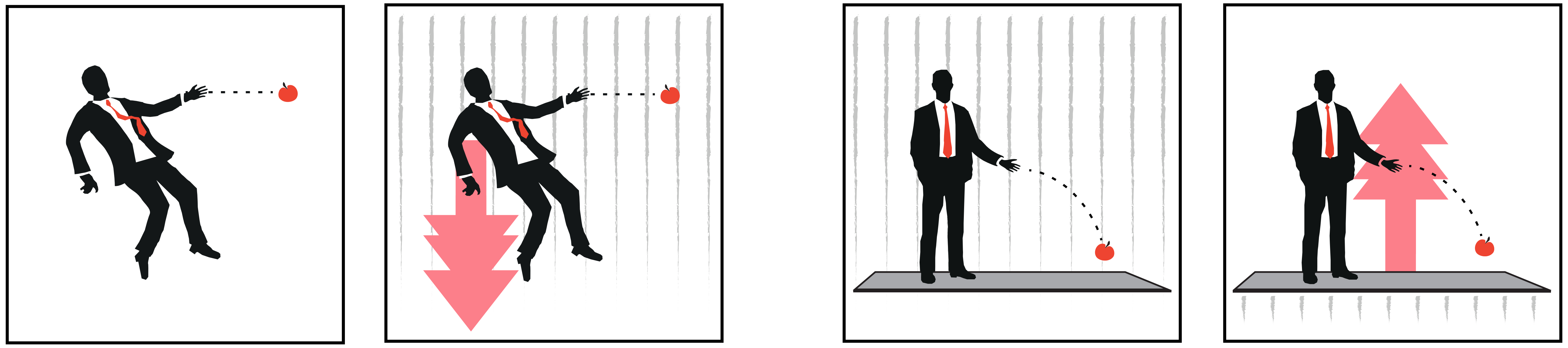
		"happiest thought" observers		"Einstein elevator" observers	
Newtonian mechanics	NM	**Inertial** observer	**Non-inertial** observer	**Inertial** observer	**Non-inertial** observer



***A quiz for the experts*:**

1) In NM, which of these four observers are **inertial observers**?
2) In GR, which of the four observers are **moving on a geodesic**?

"happiest thought" observers — "Einstein elevator" observers



Newtonian mechanics	NM	**Inertial** observer	**Non-inertial** observer	**Inertial** observer	**Non-inertial** observer
General relativity	GR	?	?	?	?

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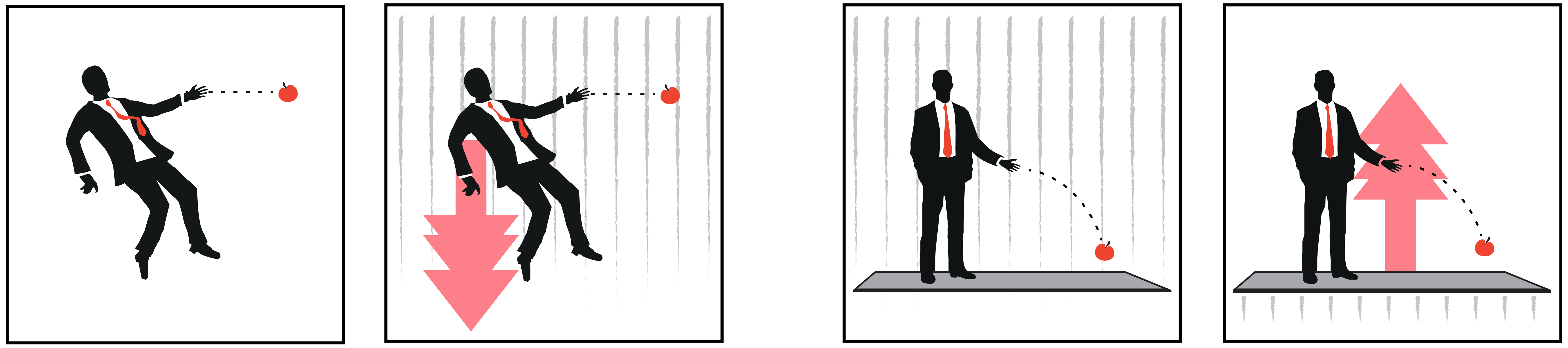
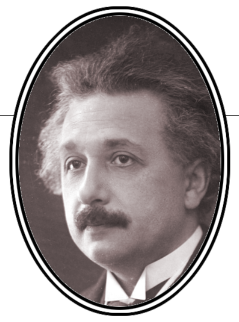
		"happiest thought" observers		"Einstein elevator" observers	
					
Newtonian mechanics	NM	**Inertial** observer	**Non-inertial** observer	**Inertial** observer	**Non-inertial** observer
General relativity	GR	Observers **moving on geodesics** of the space-time metric		Observers **in non-geodesic motion**	



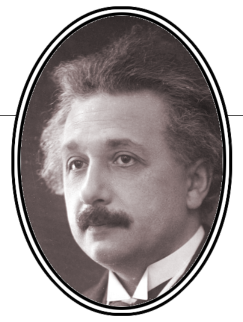
THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

We have seen that the “*Äquivalenzprinzip* Heuristic” was originally formulated for **uniform linearly accelerated motion.**

From it Einstein could deduce **a number of properties that the new relativistic theory of gravity ought to have:**

- *rate of clocks depends on gravitational potential*
  — gravitational redshift
- *light rays bend in a gravitational field*
  — electromagnetism couples to gravity
  — no global Lorentz invariance (need to break out of SR)
- *energy has weight (not just inertia as in SR)*

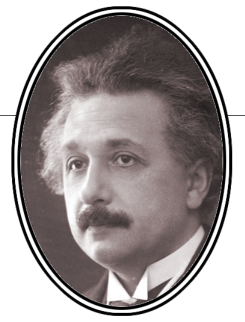


THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

Later Einstein applied the "*Äquivalenzprinzip* Heuristic" also to **uniform rotational motion.**

From this follow **a few more tentative properties of relativistic gravitational fields**:

- *3-geometry need not be Euclidean*
  — space-time geometry not flat
- *may not be possible to be define global coordinate time*
- *gravitational induction*



THE HEURISTIC POWER OF THE *ÄQUIVALENZPRINZIP*

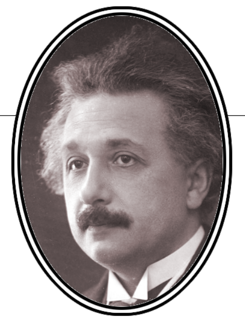
TO AVOID ANY MISUNDERSTANDINGS…

…it is important to remember that

all the results obtained from the
“*Äquivalenzprinzip* Heuristic”
must be seen as *purely qualitative*,

They are just ***hints*** at what we ***might expect*** form the still elusive complete theory of relativistic gravity.

After all: these results **have only been obtained for a *very special* gravitational field,** the perfectly homogeneous one.



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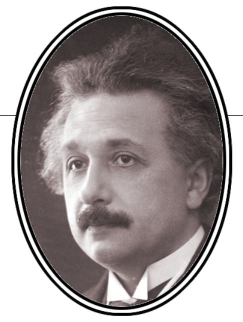
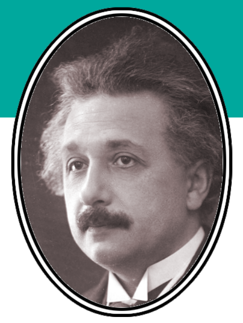
After all: these results **have only been obtained for a *very special* gravitational field,** the perfectly homogeneous one.

An example: the value obtained this way for **the bending of light** is **a factor 1/2 off** from the correct GR prediction.

…But hey! That’s not bad for a rough heuristic!



THE MIDWIFE EUOLOGY



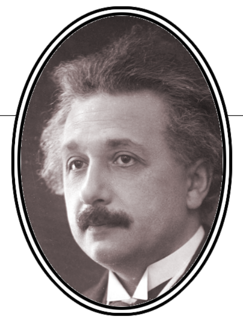
THE MIDWIFE EULOGY

The perhaps most quoted **critique of the Equivalence Principle** comes from American relativist J. L. Synge.

In the introduction to his influential textbook on relativity from 1960 he concludes that what his relativist colleagues call “the principle of equivalence” is either **trivial** or **wrong**.

And as a statement about the modern formulations of the principle I suppose he is right in being critical.

I like to think of Synge’s elegant dismissal of (what people then believed to be) Einstein’s central heuristic as “The Midwife Eulogy”:



THE MIDWIFE EULOGY

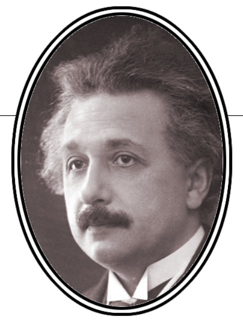
I have never been able to understand this Principle [*of Equivalence*]. Does it mean that the signature of the space-time metric is +2 (or -2 if you prefer the other convention)? **If so, it is important, but hardly a Principle…**

... Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? **If so, it is false.** In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property …

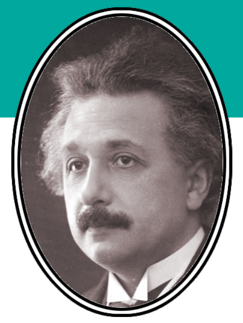
… The Principle of Equivalence performed the essential office of midwife at the birth of general relativity ...

I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced.

J. L. Synge, "Relativity: The General Theory" (1960)

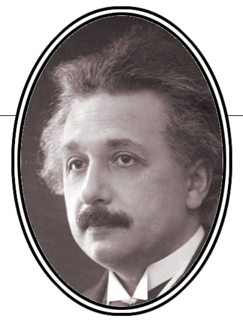


INFINITESIMAL REGIONS IN CHICAGO

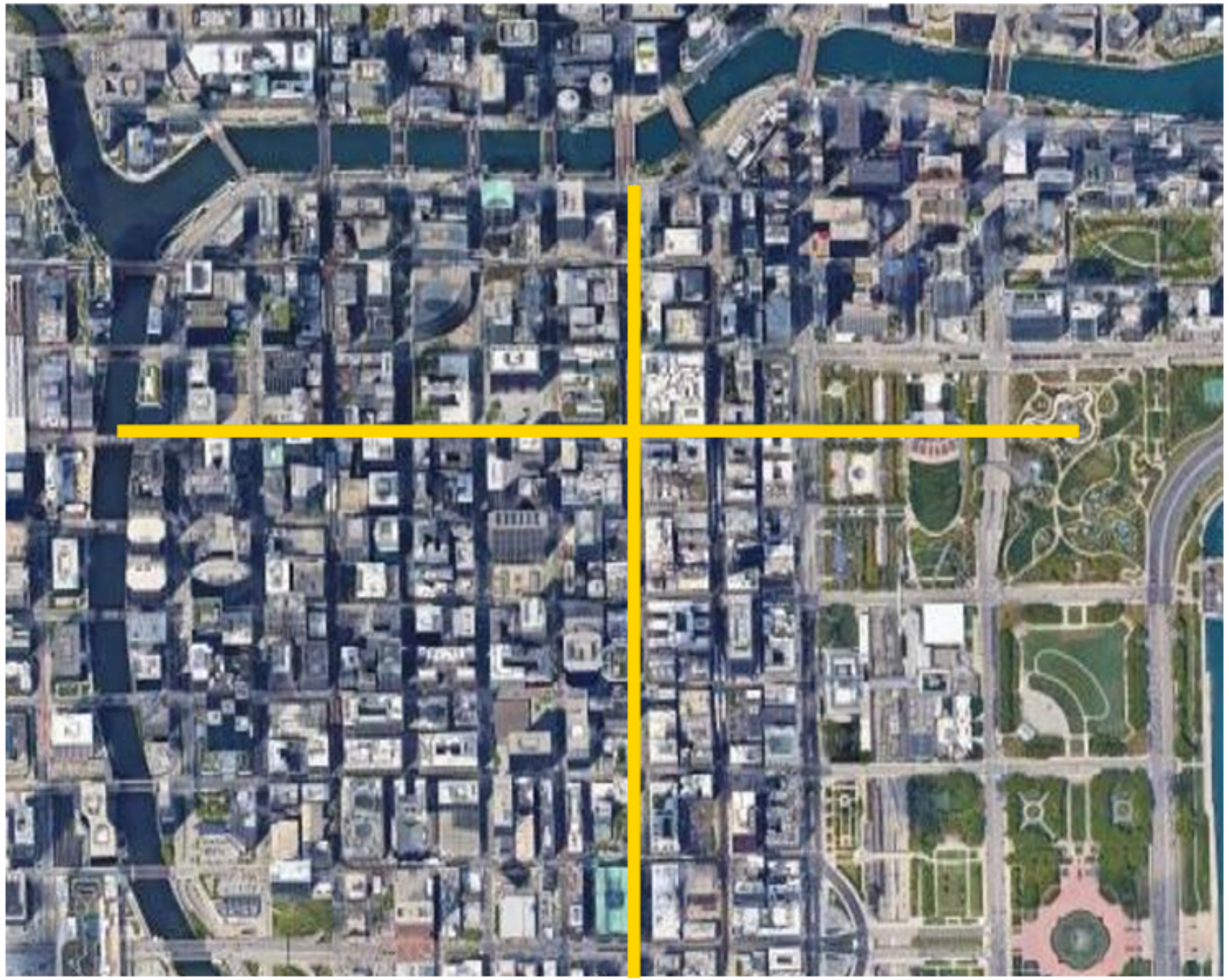


INFINITESIMAL REGIONS IN CHICAGO

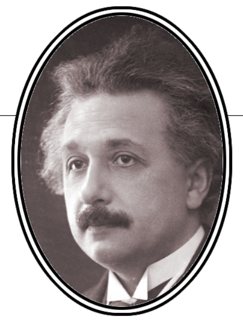
Downtown Chicago the streets follow a rectangular grid.



INFINITESIMAL REGIONS IN CHICAGO



North-South **meridians** (part of great circles) are geodesics on the sphere, East-West **parallels** are not.



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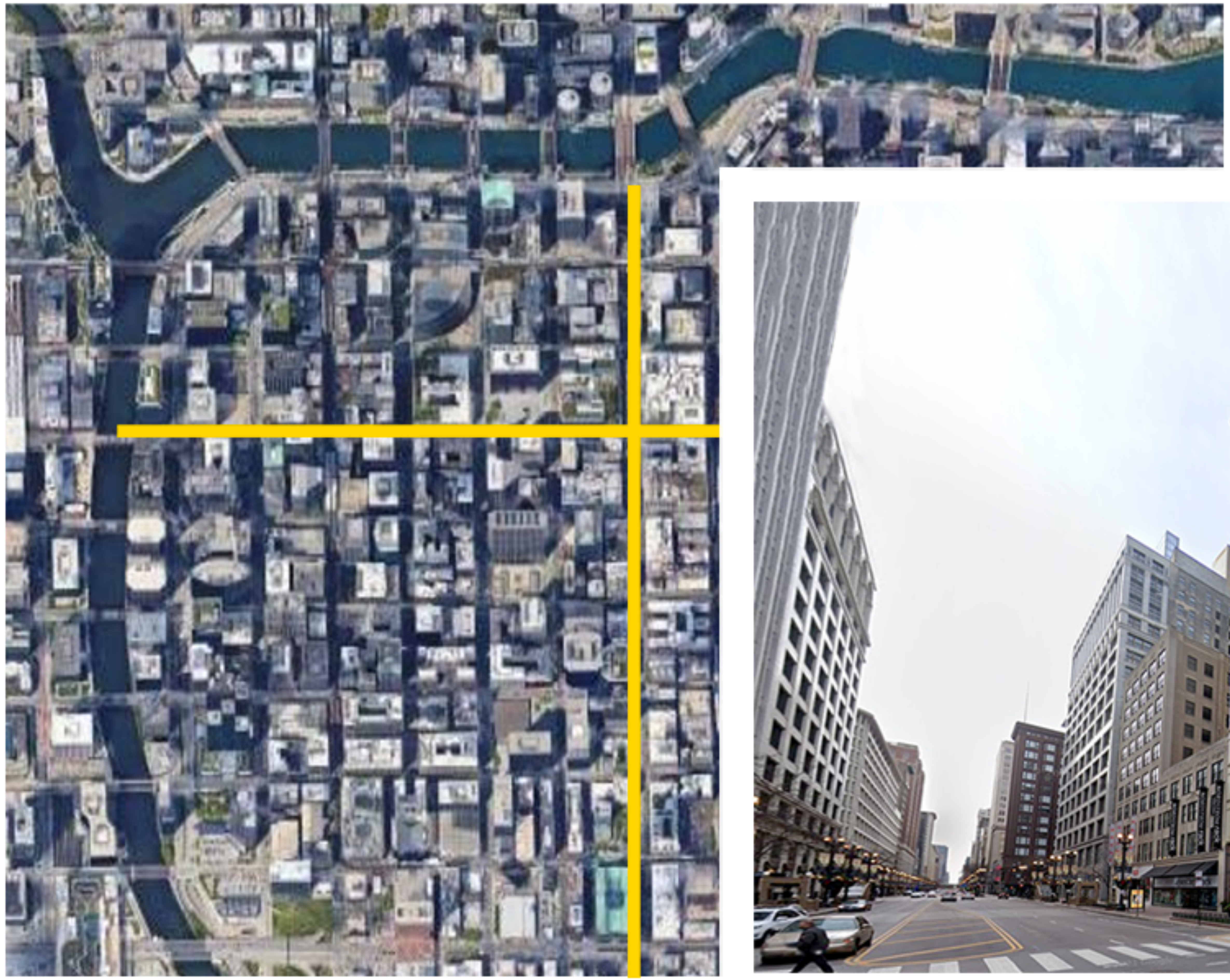
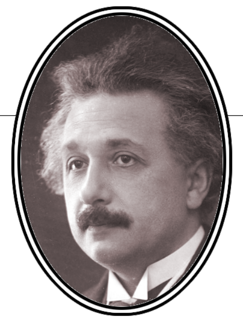


Can you tell which of the Chicago streets in the photo runs along a geodesic?



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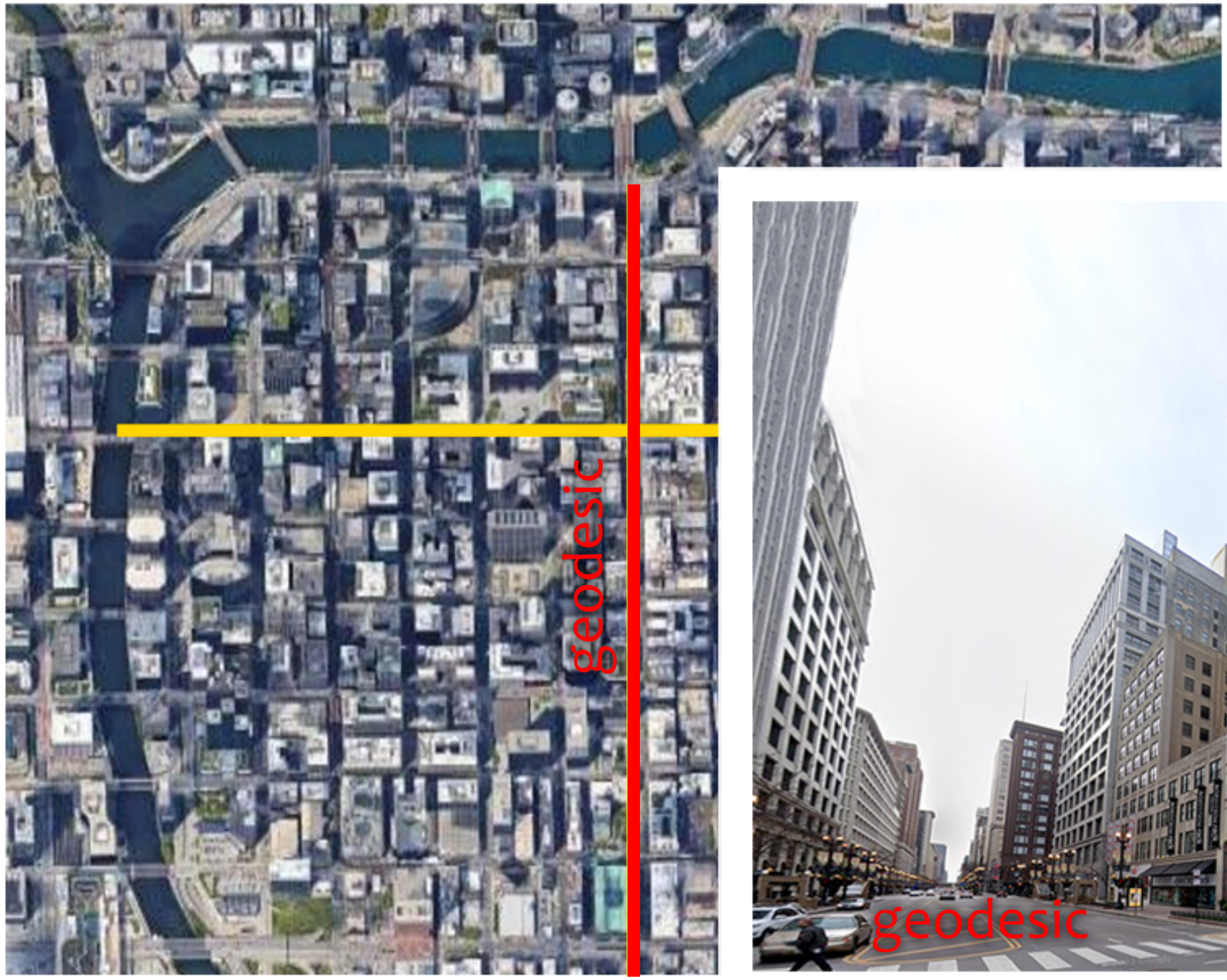


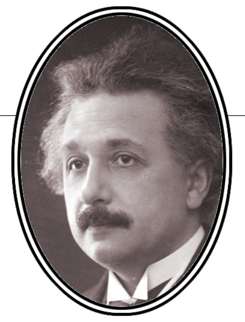
Conclusion: **in infinitesimal regions, geodesics cannot be distinguished from non-geodesics.**

INFINITESIMAL REGIONS IN CHICAGO

This argument comes from a footnote in a book by relativist Roberto Torretti:

> To suggest as some have done that the worldline of a freely falling particle must be a geodesic because, according to the EP, each line segment of it is mapped onto a straight in R^4 by a suitable local Lorentz chart, is like arguing that the parallels of latitude on the surface of the Earth are no less geodesic than the meridians, because they agree, for instance, with the avenues that cross Chicago from East to West, which are no less straight than those leading from North to South.
>
> **In a Riemannian manifold *every* curve is "straight in the infinitesimal".**

Roberto Torretti, "Relativity and Geometry" (1984)



RECONSIDERING THE WEAK EQUIVALENCE PRINCIPLE

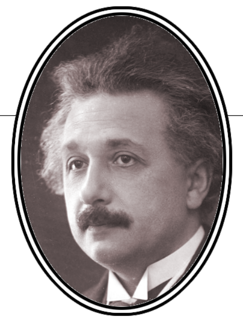
RECONSIDERING THE WEP

The **standard statement** of the WEP is due to Clifford Will:

Weak equivalence principle (WEP):

The property of a body called "mass" is proportional to the "weight". Or, the trajectory of a freely falling "test" body is independent of its internal structure and composition.

Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)



RECONSIDERING THE WEP

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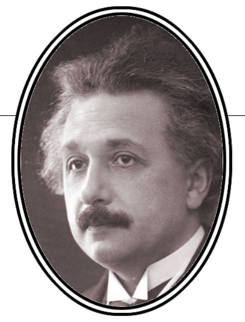
Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)

Notice that the last part of Will's definition is what is otherwise often called **the Universality of Free Fall (UFF)**.

> **Universality of Free Fall (UFF):**
>
> Two bodies dropped in a gravitational field fall with the same acceleration (simplest application of WEP).

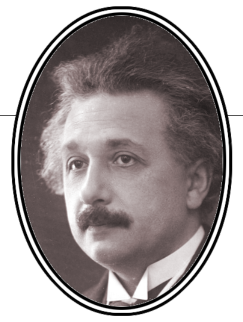
Clifford M. Will, "The Confrontation between General Relativity and Experiment" (2014)



→ *Is the Weak Equivalence Principle valid in NM?*

In Newtonian physics, the equation of motion for a freely falling body is $\boldsymbol{m_i}\,\ddot{x} = -\,\boldsymbol{m_g}\,g$, so the acceleration will be independent of the mass only if $\boldsymbol{m_i} = \boldsymbol{m_g}$. Experimentally this is true to high precision, but Newtonian theory cannot explain why.

So, as a general statement, **WEP/UFF remains unexplained in NM**.



RECONSIDERING THE WEP

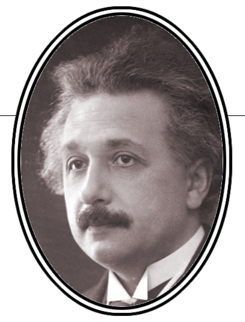
→ ***Is the Weak Equivalence Principle valid in NM?*** ***— no***

→ ***Is the Weak Equivalence Principle valid in GR?***

In GR the claim is that **free particles move along geodesics of the metric**, independently of their mass. This is not a postulate but can actually be derived from the Einstein field equations (like with the **Geroch-Jang-Malament** or **Ehlers-Geroch theorems**).

But **these derivations only hold approximately**, and only for very special types of matter (non-rotating structureless test particles).

So, as a general statement, **WEP/UFF is not strictly valid in GR**.



RECONSIDERING THE WEP

→ ***Is the Weak Equivalence Principle valid in NM?*** — ***no***

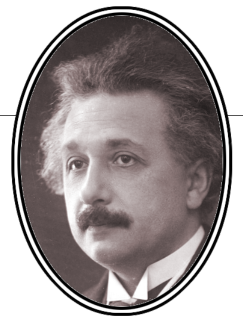
→ ***Is the Weak Equivalence Principle valid in GR?*** — ***no***

→ ***Is the Weak Equivalence Principle valid in QM?***

The Schrödinger equation for a freely falling quantum particle is $-\hbar^2 \nabla^2 \Psi / 2 \boldsymbol{m_i} + \boldsymbol{m_g} \, \varphi \, \Psi = i\hbar \, \dot{\Psi}$, but unlike in the Newtonian case the masses don't drop out if $\boldsymbol{m_i} = \boldsymbol{m_g}$. However, they do drop out in the Ehrenfest-theorem equation of motion.

But fluctuations around the Ehrenfest-theorem path of a quantum particle are mass-dependent $\sim \hbar / m_i$, so UFF is only approximately valid (and only in the limit of heavy quantum particles).

So, as a general statement, **WEP/UFF is not strictly valid in QM**.



→ ***Is the Weak Equivalence Principle valid in NM?*** ***— no***

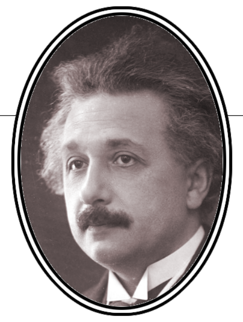
→ ***Is the Weak Equivalence Principle valid in GR?*** ***— no***

→ ***Is the Weak Equivalence Principle valid in QM?*** ***— no***

Conclusion:

In GR the proposition **WEP/UFF must be formulated in terms of an approximation scheme.**

But what justification is there for calling **an approximation scheme** a “principle”?



Einstein never made the mistake of talking about the Universality of Free Fall as a “principle”, neither of “equivalence” or of anything else:

In the Newtonian context of the *Äquivalenzprinzip*, referring back to Galilei, what Einstein actually did talk about was…

> […] the old **experimental fact** that all bodies have the same acceleration in a gravitational field. This **law** […]

Albert Einstein, “Notes on the Origin of the General Theory of Relativity”, lecture manuscript (1933)