

Did Einstein discover Einstein's field equations?

Hans Mühlen

Nordita Day, 20 November 2015

2015

1905 — 110 years of **Special Relativity**

1915 — 100 Years of **General Relativity**

1925 — 90 years of **Quantum Mechanics**

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You sometimes hear a different version: ***that Einstein stole the field equations from David Hilbert***, who had published a paper on the full theory one week before Einstein.

Let's have a look at the documents that supposedly support the two different versions of the events.

Einstein's papers can be found here:

einsteinpapers.press.princeton.edu

and Hilbert's papers are here:

echo.mpiwg-berlin.mpg.de/content/space/space/hilbert

Hilbert's paper, submitted 20 November 1915

Die Grundlagen der Physik.

(Erste Mitteilung.)

Von

David Hilbert.

Vorgelegt in der Sitzung vom 20. November 1915.

Die gewaltigen Problemstellungen von Einstein)

Unter Verwendung der vorhin eingeführten Bezeichnungsweise für die Variationsableitungen bezüglich der $g^{\mu\nu}$ erhalten die Gravitationsgleichungen wegen (20) die Gestalt

$$(21) \quad [\sqrt{g} K]_{\mu\nu} + \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} = 0.$$

Das erste Glied linker Hand wird

$$[\sqrt{g} K]_{\mu\nu} = \sqrt{g} (K_{\mu\nu} - \frac{1}{2} K g_{\mu\nu}),$$

Einstein's paper, submitted 25 November 1915

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

VON A. EINSTEIN.

In zwei vor kurzem erschienenen Mitteilungen¹ habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postu-

Ist in dem betrachteten Raume »Materie« vorhanden, so tritt deren Energietensor auf der rechten Seite von (2) bzw. (3) auf. Wir setzen

$$G_{im} = -\kappa \left(T_{im} - \frac{1}{2} g_{im} T \right), \quad (2a)$$

wobei

$$\sum_{i'} g^{i'} T_{i'} = \sum_{i'} T_{i'} = T \quad (5)$$

gesetzt ist; T ist der Skalar des Energietensors der »Materie«, die rechte Seite von (2a) ein Tensor. Spezialisieren wir wieder das Koordinaten-

THE “BAD EINSTEIN” TIMELINE

1912–1915 Einstein works on his theory of general relativity, but repeatedly makes simple mistakes preventing him from succeeding.

Summer
1915 Einstein gives a lecture series on his theory in Göttingen. Hilbert is in the audience.

Hilbert starts working on an axiomatization of physics [*Cf. Hilbert’s 6th problem*]. It is based on Gustav Mie’s theory of (electrodynamic) matter and Einstein’s theory of gravity.

THE “BAD EINSTEIN” TIMELINE

- 4 & 11 Nov. 1915 Einstein publishes some new field equations that are also wrong.
- 18 Nov. Hilbert sends Einstein a preprint of his first paper on the foundations of physics, containing the correct field equations.
- 20 Nov. Hilbert submits the paper for publication.
- 25 Nov. Einstein submits a paper which — finally — contains the correct field equations.

Q.E.D.

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The document was **the printer’s proof of Hilbert’s 20 November paper**, dated 6 December 1915.

Hilbert’s paper was published only on 31 March 1916.

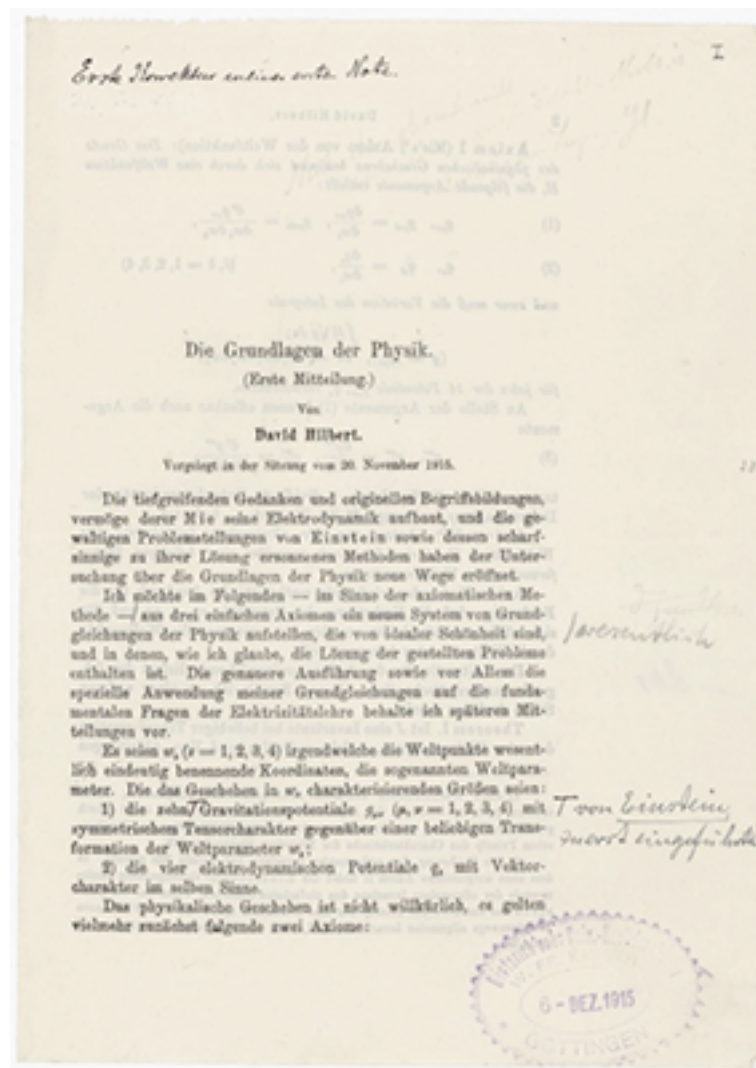
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If you **compare the proof version of the paper with the published version**, you see that large portions of the paper had been completely rewritten before publication.

Printer's proofs of Hilbert's paper, dated 6 December 1915



Three differences between proof and published version have a bearing on the “bad Einstein” story:

- In the proof version, Hilbert repeats Einstein’s (mistaken) arguments from the summer of 1915 that the **field equations cannot be generally covariant**. By November 1915, Einstein had already realized his mistake.
- In the proof version, the **field equations are not written out explicitly** (only as an action principle).
- In the published version, **Hilbert cites Einstein’s paper** from 25 November, and acknowledges that his published field equations are essentially the same as Einstein’s.

No explicit field equations in the proof version:

Unter Verwendung der vorhin eingeführten Bezeichnungsweise für die Variationsableitungen bezüglich der $g^{\mu\nu}$ erhalten die Gravitationsgleichungen wegen (17) die Gestalt

$$(26) \quad [\sqrt{g} K]_{\mu\nu} + \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} = 0.$$

Bezeichnen wir ferner allgemein die Variationsableitungen von $\sqrt{g} J$ bezüglich des elektrodynamischen Potentials q_λ mit

$$[\sqrt{g} J]_{\lambda} = \frac{\partial \sqrt{g} J}{\partial q_\lambda}$$

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THE “GOOD EINSTEIN” TIMELINE

- 1912–1915 Einstein (together with Marcel Grossmann) searches for gravitational field equations that are (1) generally covariant, and (2) have the correct Newtonian limit. They discuss the Ricci tensor but reject it.
- Summer 1913 Publication of the “Entwurf” theory with non-covariant field equations.
- Fall 1915 Einstein eventually realizes that the “Entwurf” equations are wrong. Back to investigating generally covariant equations.

THE “GOOD EINSTEIN” TIMELINE

- 4 Nov. 1915 Einstein publishes paper with “November tensor” field equations, still non-covariant but with correct Newtonian limit.
- 11 Nov. Paper with Ricci tensor based field equations but constraints on energy-momentum tensor.
- 18 Nov. Einstein acknowledges receipt of Hilbert’s preprint, notes that it contains nothing new compared to Einstein’s recent papers, and criticizes Hilbert for not checking the Newtonian limit.

THE “GOOD EINSTEIN” TIMELINE

20 Nov. 1915 Hilbert submits his paper for publication.

25 Nov. Einstein submits a paper which corrects the energy-momentum term in the field equations. These are now generally covariant and have the correct Newtonian limit. The theory of general relativity is complete.

2 Dec. Einstein's paper is published.

6 Dec. Printer's proofs of Hilbert's paper.

THE “BAD EINSTEIN” TIMELINE

Winter 1915/16	Hilbert rewrites most of his paper, removing the limitations on covariance and adding the explicit form of the field equations.
31 March 1916	Revised version of Hilbert’s paper published, but the old submission date is left unchanged.

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...and don't believe any of the Einstein anecdotes you might find on the web.

References:

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