(c) $P(y=1 \mid x; \phi, \mu_0, \mu_1, \Sigma)$

$$= \frac{P(x \mid y=1)\, P(y=1)}{P(x)}$$

$$\Rightarrow P(x) = P(x \mid y=0)\, P(y=0) + P(x \mid y=1)\, P(y=1)$$

$$= \underbrace{\frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}}}_{\eta} \exp\Big(\underbrace{-\frac{1}{2}(x-\mu_0)^T \Sigma^{-1} (x-\mu_0)}_{\mathcal{L}(\mu_0)}\Big)$$

$$+ \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\Big(\underbrace{-\frac{1}{2}(x-\mu_1)^T \Sigma^{-1} (x-\mu_0)}_{\mathcal{L}(\mu_1)}\Big)$$

$$\Rightarrow P(y=1 \mid x) = \frac{\eta e^{\mathcal{L}(\mu_1)} \phi}{\eta e^{\mathcal{L}(\mu_0)} (1-\phi) + \eta e^{\mathcal{L}(\mu_1)} \phi}$$

$$= \frac{1}{1 + \left(\frac{1-\phi}{\phi}\right) \frac{e^{\mathcal{L}(\mu_0)}}{e^{\mathcal{L}(\mu_1)}}}$$

$$\rightarrow \left(\frac{1-\phi}{\phi}\right) \exp\big(\mathcal{L}(\mu_0) - \mathcal{L}(\mu_1)\big)$$

$$= \exp\left(\log\left(\left(\frac{1-\Phi}{\Phi}\right)\exp\left(\mathcal{L}(\mu_0) - \mathcal{L}(\mu_1)\right)\right)\right)$$

$$= \exp\left(\log\left(\frac{1-\Phi}{\Phi}\right) + \mathcal{L}(\mu_0) - \mathcal{L}(\mu_1)\right)$$

$$-\log\left(\frac{\Phi}{1-\Phi}\right)$$

$$\Rightarrow -\frac{1}{2}\left((x-\mu_0)^T\Sigma^{-1}(x-\mu_0) - (x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right)$$

$$\Rightarrow -\frac{1}{2}\left(\begin{array}{l} \cancel{x^T\Sigma^{-1}x} - \mu_0^T\Sigma^{-1}x - x^T\Sigma^{-1}\mu_0 + \mu_0^T\Sigma^{-1}\mu_0 \\ \cancel{-x^T\Sigma^{-1}x} + \mu_1^T\Sigma^{-1}x + x^T\Sigma^{-1}\mu_1 - \mu_1^T\Sigma^{-1}\mu_1 \end{array}\right)$$

$$\Rightarrow -\frac{1}{2}\left(\begin{array}{l} -\mu_0^T\Sigma^{-1}x - \mu_0^T\Sigma^{-T}x + \mu_0^T\Sigma^{-1}\mu_0 \\ +\mu_1^T\Sigma^{-1}x + \mu_1^T\Sigma^{-T}x - \mu_1^T\Sigma^{-1}\mu_1 \end{array}\right)$$

$$\Rightarrow -\frac{1}{2}\left(\begin{array}{l} -\mu_0^T(\Sigma^{-1}+\Sigma^{-T})x + \mu_0^T\Sigma^{-1}\mu_0 \\ +\mu_1^T(\Sigma^{-1}+\Sigma^{-T})x - \mu_1^T\Sigma^{-1}\mu_1 \end{array}\right)$$

$\{\Sigma$ is Symmetric$\}$

Date

Page

$$\Rightarrow -\frac{1}{2}\left(2(\mu_1-\mu_0)^T\Sigma^{-1}x + \left(\mu_0^T\Sigma^{-1}\mu_0 - \mu_1^T\Sigma^{-1}\mu_1\right)\right)$$

$\Rightarrow P(y=1|x)$

$$= \frac{1}{1+\exp\left(-\left(\log\left(\frac{\phi}{1-\phi}\right)+\frac{1}{2}\left(\mu_0^T\Sigma^{-1}\mu_0-\mu_1^T\Sigma^{-1}\mu_1\right)\right.\right.} \\ \left.\left. - (\mu_1-\mu_0)^T\Sigma^{-1}x\right)$$

$\Rightarrow$ Let $\theta_0 = \log\left(\frac{\phi}{1-\phi}\right) + \frac{1}{2}\left(\mu_0^T\Sigma^{-1}\mu_0 - \mu_1^T\Sigma^{-1}\mu_1\right)$

$\cancel{\theta = \Sigma^{-1}(\mu_1-\mu_0)}$ $\theta = \Sigma^{-1}(\mu_1-\mu_0)$

$$\Rightarrow \boxed{P(y=1|x) = \frac{1}{1+\exp(-(\theta^T x+\theta_0))}}$$

at decision boundary $P(y=1|x) = 0.5$

$$\Rightarrow \frac{1}{1+\exp(-(\theta^T x+\theta_0))} = 0.5$$

$$\Rightarrow \exp(-(\theta^T x+\theta_0)) = 1$$

$\Rightarrow \boxed{\theta^T x + \theta_0 = 0}$ {Linear decision boundary}