(c)

$$l(\theta) = -\sum_{i=1}^{m} \log P(y^{(i)} | x^{(i)}; \theta)$$

OR

$$P(y|x;\theta) = \prod_{i=1}^{m} P(y^{(i)} | x^{(i)}; \theta)$$

$$-\log P(y|x;\theta) = -\sum_{i=1}^{m} \log P(y^{(i)} | x^{(i)}; \theta)$$

$$\nabla_\theta l(\theta) = \nabla_\theta \left(-\sum_{i=1}^{m} \log b(y^{(i)}) + \overset{\theta^T x^{(i)}}{\eta} y - a(\eta) \right)$$

$$= -\sum_{i=1}^{m} 0 + x^{(i)} y^{(i)} - a'(\theta^T x^{(i)}) x^{(i)}$$

$$= -\sum_{i=1}^{m} (y^{(i)} - a'(\theta^T x^{(i)})) x^{(i)}$$

$$\left(\nabla_\theta^2 l(\theta)\right)_k = H_k = \nabla_\theta \left(\nabla_\theta l(\theta)\right)_k$$

$$= \nabla_\theta \left(-\sum_{i=1}^{m} \left(y^{(i)} - a'(\theta^T x^{(i)}) \right) x_k^{(i)} \right)$$

$$= -\sum_{i=1}^{m} 0 - a''(\theta^T x^{(i)})\, x^{(i)} x_k^{(i)}$$

$$= \sum_{i=1}^{m} \underbrace{a''(\theta^T x^{(i)})}_{\mathrm{Var}(Y^{(i)} \mid x^{(i)};\theta)} x^{(i)} x_k^{(i)}$$

$$\Rightarrow \boxed{H = \sum_{i=1}^{m} \mathrm{Var}(Y^{(i)} \mid x^{(i)};\theta)\, x^{(i)} x^{(i)T}}$$

{As variance is always positive (=>0) hence H is positive semi definit.}