Experiment A
Orifice and Free Jet Flow

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1 Introduction

The main purpose of this experiment is to investigate the Bernoulli equation for orifice and free jet flow. This experiment is divided into two parts, the first part is to determine the coefficient of velocity from the jet trajectory. the second part is to determine the coefficient of discharge under the constant head.

2 Theory

In Experiment A1, the basic equations are Bernoulli's (1) .

$$\left[\frac{p}{\gamma}+\frac{v^2}{2g}+z\right]_1=\left[\frac{p}{\gamma}+\frac{v^2}{2g}+z\right]_2 \tag{1}$$

As shown in the figure below, where 1 and 2 represent the surface of the reservoir and the water discharge point.

In Bernoulli's equation, because the position 1 is in contact with the air and the water surface is static, so the pressure force p = 0, and the water surface flow velocity v = 0. Similarly, the pressure at position 2 is p = 0. With the datum at 2, we only need to measure the head difference between

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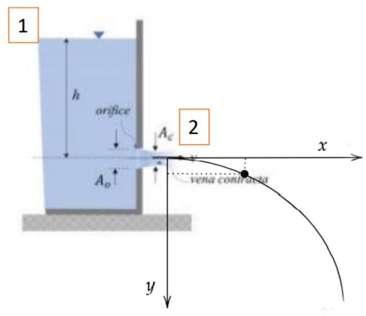

Figure 1: Experimental Demonstration

position 1 and 2, and measure the jet velocity of 2, after that put into other constants, Bernoulli's equation could be proved.

Therefore, Bernoulli's equation can be simplified.

$$v_2 = \sqrt{2gh} \tag{2}$$

Where v_2 is the water velocity in position 2, h is the head difference between 1 and 2. In fact, due to the jet vena contracta, it exists a coefficient C_v to effect the real velocity. C_v depends on the viscosity of the water, so $C_v < 1$.

The real velocity is

$$v = C_v\sqrt{2gh} \tag{3}$$

By neglecting the air resistance, the x and y position could be caculated By

$$x = vt \tag{4}$$

$$y = \frac{1}{2}gt^2 \tag{5}$$

From Equation 6, it can be caculated

$$t = \sqrt{\frac{2y}{g}} \tag{6}$$

By using equations Equation 3,Equation 4,Equation 6 we can derive

$$C_v = \frac{x}{2\sqrt{yh}} \tag{7}$$

Also can be rewrite as

$$x = 2C_v\sqrt{yh} \tag{8}$$

Hence, For a constant coefficient C_v, which can be determined from the x and y coordinates.

In Experiment A2, the continuity equation is given by

$$Q = Av \tag{9}$$

From this, we can calculate the real water flowrate.

Because the vena contracta decrease the cross-sectional area of jet flow, it also exists a coefficient C_c, which $C_c < 1$.

Hence,

$$A = C_c A_0 \tag{10}$$

$$Q = C_c A_0 v \tag{11}$$

Substitution for v from (3), the results is

$$Q_t = C_c A_0 C_v \sqrt{2gh} \tag{12}$$

Which used the discharge coefficientC_d to Substitute C_c and C_v.

$$Q_t = C_d A_0 \sqrt{2gh} \tag{13}$$

This is also a linear equation with horizontal coordinates $\sqrt{h}$ and vertical coordinates Q_t. The slope S $= C_d A_0 \sqrt{2g}$

3 Method

As can be seen above, It need to measure the difference in head between 1 and 2.

The head difference between 1 and 2 can be easily measured by tools such as a ruler. For the trajectory of the water jet of 2, adjusting the needle mounted on the back plate so that its bottom point is as close as possible to the upper edge of the jet, and tighten the screws.

Next, attach a sheet of paper to the back-board between the needle and board, and make sure it is horizontal and secure.

As you can see in the Figure 1, Set up the same coordinate system on the attachment sheet, and then measure the x and y distance between each marker point to get table 1.

In order to ensure the accuracy of the data, four data sets were measured by using two different orifice diameters and two different heights.

In experiment A2, a measuring cylinder is a good tool to measure the flow rate of water. First, prepare an empty measuring cylinder and collect the water jet from position 2. When the measuring cylinder starts to receive water, press the stopwatch to start timing. Wait for a period of time until the measuring cylinder exceeds 800 ml, then stop collecting water and stop timing.

4 Results

The experiment data is shown in the following table.

No.		X=	43	93	143	193	243	293	343	393
1	D=3	h=256	-1	-11	-24	-42	-63	-91	-123	-161
2		h=365	-1	-8	-17	-31	-44	-65	-88	-116
3	D=6	h=276	-1	-8	-21	-38	-60	-88	-122	-151
4		h=380	-1	-8	-16	-28	-44	-62	-84	-110

(unit:mm)

Table 1: attachment sheet recording table

h(mm)	V(ml)	T(s)	Q= $\frac{V}{T}$ (L/s)
256	875	76	0.0115
296	855	70	0.0122
333.5	830	64	0.0130
365	855	64	0.0134

Table 2: record of flowrate (diameter:3mm)

h(mm)	V(ml)	T(s)	Q= $\frac{V}{T}$ (L/s)
256	875	76	0.0440
296	855	70	0.0472
333.5	830	64	0.0510
365	855	64	0.0538

Table 3: record of flowrate (diameter:6mm)

5 Analysis

5.1 Regression analysis

5.1.1 ExperimentA1

Plot a scatter chart with $\sqrt{gh}$ as the horizontal coordinate and x as the vertical coordinate based on the data from Table 1. And a regression analysis and linear fit was performed on the data, as shown in Figure 2 and results in Table 4.

	Slope	R-square
1	1.923	0.9968
2	1.923	0.9976
3	1.855	0.9985
4	1.936	0.998

Table 4: result of A1 regression analysis

Take the average of the data and take it into equation (8) to get $C_v = \frac{slope}{2}$. The final C_v is 0.955.

5.1.2 ExperimentA2

Plot a scatter chart with $\sqrt{h}$ as the horizontal coordinate and Q as the vertical coordinate based on the data from Table 1. And a regression analysis and linear fit was performed on the data, as shown in Figure 3 and results in Table 5

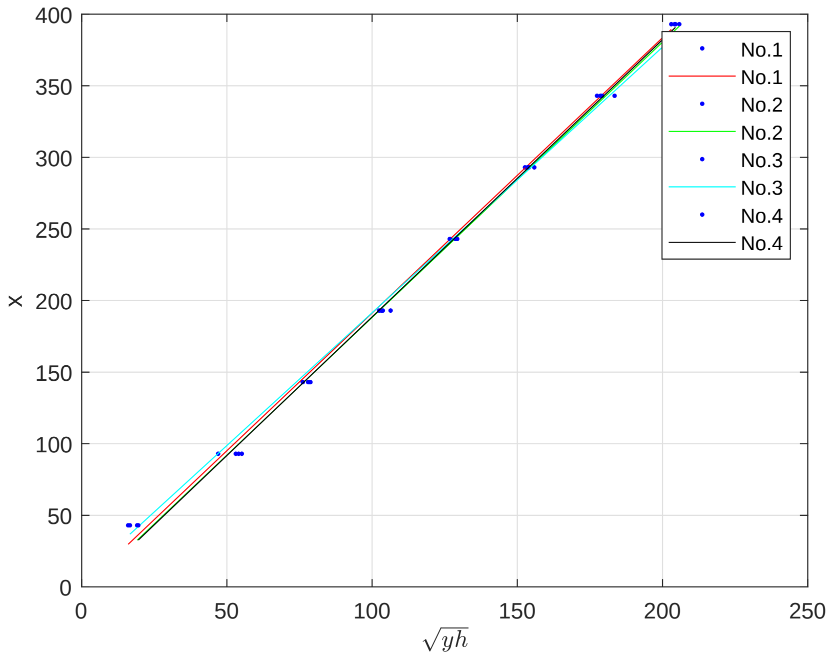

Figure 2: ExperimentA1

(Unit:mm)

	Slope	R-square
3mm	0.00001924	0.9955
6mm	0.0001103	0.9811

Table 5: result of A2 regression analysis

Use equation (13) to caculate the $C_d = \frac{Slope}{A_0\sqrt{2g}}$ and the final C_d is 0.615 (3mm) and 0.8816 (6mm)

5.2 Error analysis

• Inaccurate recording of water trajectory to the sheet due to not looking horizontally at the top point of the needle.

• Inaccurate calculation of the flow rate due to residual water in the measuring cylinder.

• When calculating the flow rate, errors caused by the stopwatch timing not being fully synchronised with the water collection time.

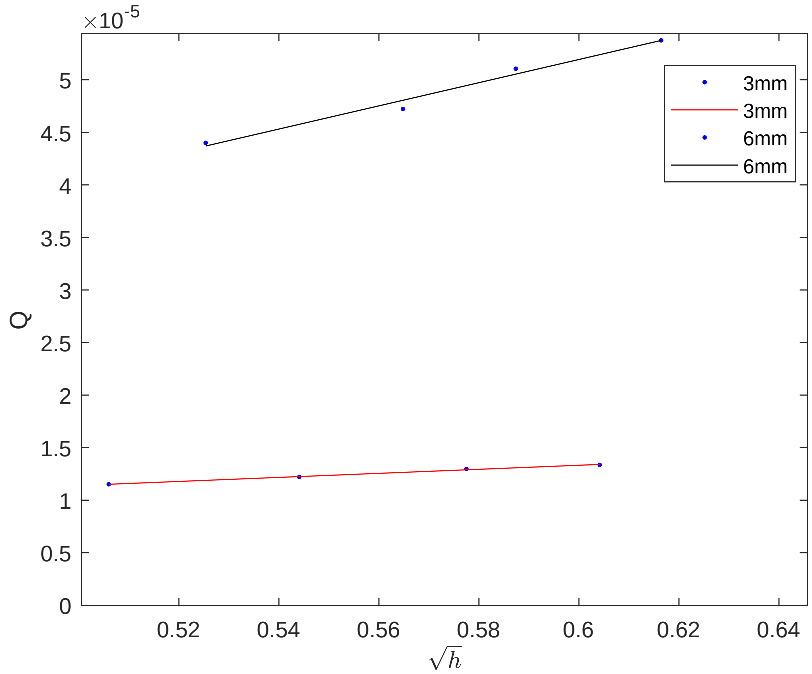

Figure 3: ExperimentA2

(Unit:$\sqrt{m}$ and m^3/s)

6 Conclusion

From the results of the regression analysis, the R-square value of both experiment A1 and A2 are larger than 0.98, which demonstrates a strong linear correlation between the x and y axes.

In addition, from the above it follows that $C_v = 0.955$,$C_d = 0.615$(3mm),$C_d = 0.8816$ (6mm). Both of the coefficient are less than 1.This satisfies the hypothetical conditions.

Therefore, the data from the regression analysis is valid.