Experiment A
Orifice and Free Jet Flow

Jiaqi, Yao.*

3 November, 2022

1 Introduction

The main purpose of this experiment is to investigate the Bernoulli equation for orifice and free jet flow. This experiment is divided into two parts, the first part is to determine the coefficient of velocity by different diameter of orifice from the jet trajectory. the second part is to determine the coefficient of discharge under the constant head.

2 Theory

In Experiment A1, the basic equations are Bernoulli's (1) .

$$\left[\frac{p}{\gamma}+\frac{v^2}{2g}+z\right]_1=\left[\frac{p}{\gamma}+\frac{v^2}{2g}+z\right]_2 \tag{1}$$

As shown in the figure below, where 1 and 2 represent the surface of the reservoir and the water discharge point. Because $p_1 = 0$ $p_2 = 0$ $v_1 = 0$ $z_1 - z_2 = h$, Bernoulli's equation can be simplified as

$$v_2 = \sqrt{2gh} \tag{2}$$

Where v_2 is the water velocity in position 2, h is the head difference between 1 and 2. In fact, due to the jet vena contracta, it exists a coefficient C_v to effect the real velocity. C_v depends on the

*jy431@exeter.ac.uk

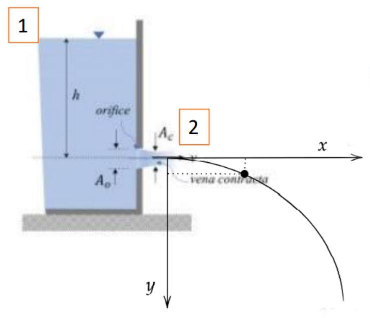

Figure 1: Experimental Demonstration

viscosity of the water, so $C_v < 1$.

$$v = C_v\sqrt{2gh} \tag{3}$$

By neglecting the air resistance, the x and y position could be caculated By

$x = vt \ y = \frac{1}{2}gt^2$

Hence,

$$C_v = \frac{x}{2\sqrt{yh}} \tag{4}$$

Also can be rewrite as

$$x = 2C_v\sqrt{yh} \tag{5}$$

Hence, For a constant coefficient C_v, which can be determined from the x and y coordinates. In Experiment A2, the continuity equation is given by

$$Q = Av \tag{6}$$

It also exists a coefficient C_c, which $C_c < 1$. Hence,

$$A = C_c A_0 \tag{7}$$

Substitution for v from (3), the results is

$$Q_t = C_c A_0 C_v\sqrt{2gh} = C_d A_0\sqrt{2gh} \tag{8}$$

Which used the discharge coefficientC_d to Substitute C_c and C_v.($C_d = C_c * C_v$) This is also a linear equation with horizontal coordinates $\sqrt{h}$ and vertical coordinates Q_t. The slope S = $C_d A_0\sqrt{2g}$.

3 Method

As can be seen above, It need to measure the trajectory of jet and different head between 1 and 2.

The head difference between 1 and 2 can be easily measured by tools such as a ruler. For the trajectory of the water jet of 2, adjusting the needle mounted on the back plate so that its bottom point is as close as possible to the upper edge of the jet, and tighten the screws.

Next, attach a sheet of paper to the back-board between the needle and board, and make sure it is horizontal and secure.

As you can see in the Figure 1, Set up the same coordinate system on the attachment sheet, and then measure the x and y distance between each marker point to get table 1.

In order to ensure the accuracy of the data, four data sets were measured by using two different orifice diameters and two different heights.

In experiment A2, a measuring cylinder is a good tool to measure the flow rate of water. First, prepare an empty measuring cylinder and collect the water jet from position 2. When the measuring cylinder starts to receive water, press the stopwatch to start timing. Wait for a period of time until the measuring cylinder exceeds 800 ml, then stop collecting water and stop timing.

4 Results

The experiment data is shown in the following table.

No.		X=	43	93	143	193	243	293	343	393
1	D=3	h=256	-1	-11	-24	-42	-63	-91	-123	-161
2		h=365	-1	-8	-17	-31	-44	-65	-88	-116
3	D=6	h=276	-1	-8	-21	-38	-60	-88	-122	-151
4		h=380	-1	-8	-16	-28	-44	-62	-84	-110

(unit:mm)

Table 1: attachment sheet recording table

h(mm)	V(ml)	T(s)	Q= $\frac{V}{T}$ (L/s)
256	875	76	0.0115
296	855	70	0.0122
333.5	830	64	0.0130
365	855	64	0.0134

Table 2: record of flowrate (diameter:3mm)

h(mm)	V(ml)	T(s)	Q= $\frac{V}{T}$ (L/s)
256	875	76	0.0440
296	855	70	0.0472
333.5	830	64	0.0510
365	855	64	0.0538

Table 3: record of flowrate (diameter:6mm)

5 Analysis

5.1 Regression analysis

For experimentA1, Plot a scatter chart with $\sqrt{gh}$ as the horizontal coordinate and x as the vertical coordinate based on the data from Table 1. And a regression analysis and linear fit was performed on the data, as shown in Figure 2 and results in Table 4.

Take the average of the data and take it into equation (5) to get $C_v = \frac{slope}{2}$. The final C_v is 0.955.

For experiment A2, it is the same as experiment A1, with the horizontal coordinates replaced by $\sqrt{h}$ and the vertical coordinates replaced by Q the data of regression analysis is in Figure 3 and results in Table 5.

Use equation (8) to caculate the $C_d = \frac{Slope}{A_0\sqrt{2g}}$ and the final C_d is 0.615 (3mm) and 0.8816 (6mm).

	Slope	R-square
1	1.923	0.9968
2	1.923	0.9976
3	1.855	0.9985
4	1.936	0.998

Table 4: result of A1 regression analysis

	Slope	R-square
3mm	0.00001924	0.9955
6mm	0.0001103	0.9811

Table 5: result of A2 regression analysis

5.2 Error analysis

• Inaccurate recording of water trajectory to the sheet due to not looking horizontally at the top point of the needle.

• Inaccurate calculation of the flow rate due to residual water in the measuring cylinder.

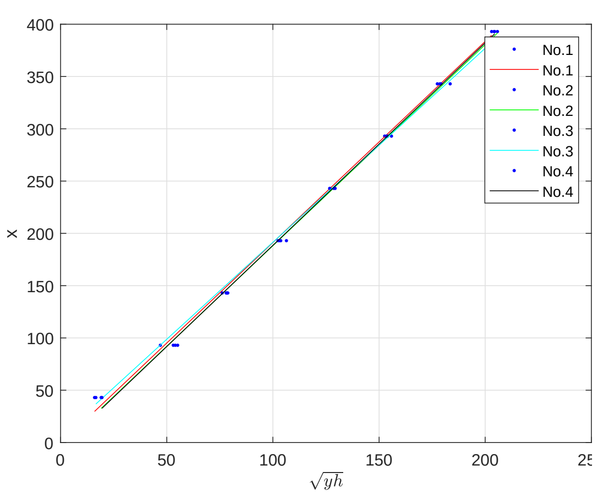

Figure 2: ExperimentA1

(Unit:mm)

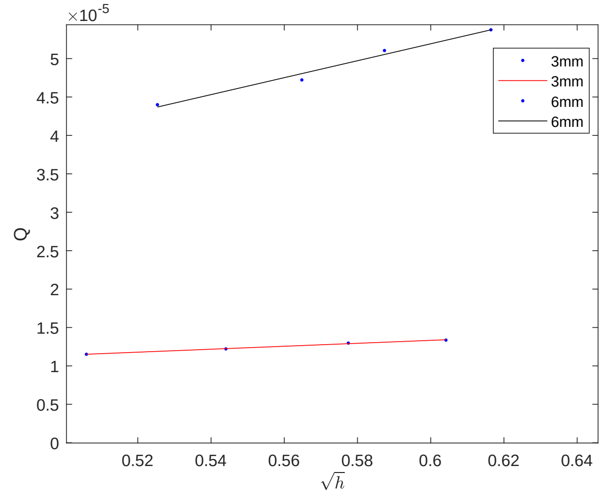

Figure 3: ExperimentA2

(Unit:$\sqrt{m}$ and m^3/s)

• When calculating the flow rate, errors caused by the stopwatch timing not being fully synchronised with the water collection time.

6 Conclusion

From the results of the regression analysis, the R-square value of both experiment A1 and A2 are larger than 0.98, which demonstrates a strong linear correlation between the x and y axes. Additionally, from the above it follows that $C_v = 0.955$,$C_d = 0.615$(3mm),$C_d = 0.8816$ (6mm). Both of the coefficient are less than 1.This satisfies the hypothetical conditions.

Therefore, the data from the regression analysis is valid.

For C_v, due to vena contracta , the real area that approximates the area of the orifice with a little smaller, and its value of 0.95 is consistent with the reality.

For C_d(3mm), the value is 0.615, whereas it should actually be between 0.8 and C_v, probably due to experimental error. This should be done several times for the 3mm diameter orifice to avoid experimental coincidence and make it more accurate.

For C_d(6mm), the value is 0.8816, which is less than C_v and indicate the $C_c < 1$(8).This data is inaccurate but should not be significant.