Plastic Collapse of Portal Frame Lab
Group T05

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1 Section A

In this section, the Young's modulus of two materials (mild steel and aluminium) can be calculated from experimental data.

A total of six groups of data were obtained from the experiment. (See Table 1)

Analysis

By plotting these 6 groups of data on a scatter plot and performing regression analysis, a total of 6 groups of graphs were obtained. (See Figure 2)

The Free-body-diagram of the components is shown in Figure 1.

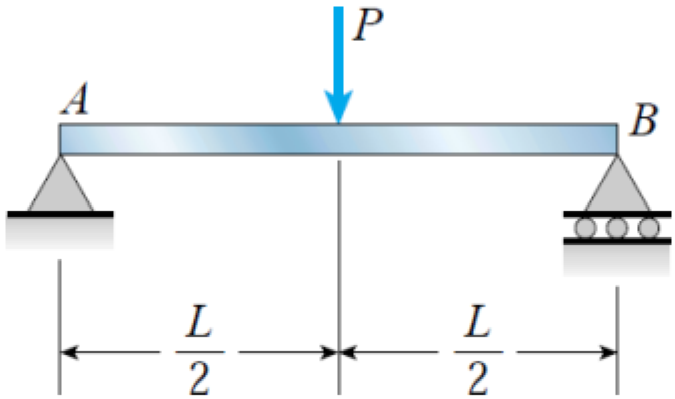

Figure 1: Free-body-diagram

No.	Material	Final load (N)	K(Slope)	R-square
1	Steel	50	370.4	0.9999
2	Steel	100	379.1	0.9998
3	Steel	150	369.4	0.9989
4	Aluminium	50	138.9	0.9999
5	Aluminium	100	139.4	0.9999
6	Aluminium	150	134.8	0.9987

(Notice: The unit of K is N/mm, it needs to multiply 10^3 to transfer to N/m)

Table 1: Experimental data and linear regression results

In order to calculate E, the moment of inertia I needs to be calculated first.

$$I = \frac{bh^3}{12} = \frac{20 * 3^3}{12} * 10^{-12} = 4.5 * 10^{-11} (mm^4) \tag{1}$$

We know

$$\delta_{max} = \frac{PL^3}{48EI} \tag{2}$$

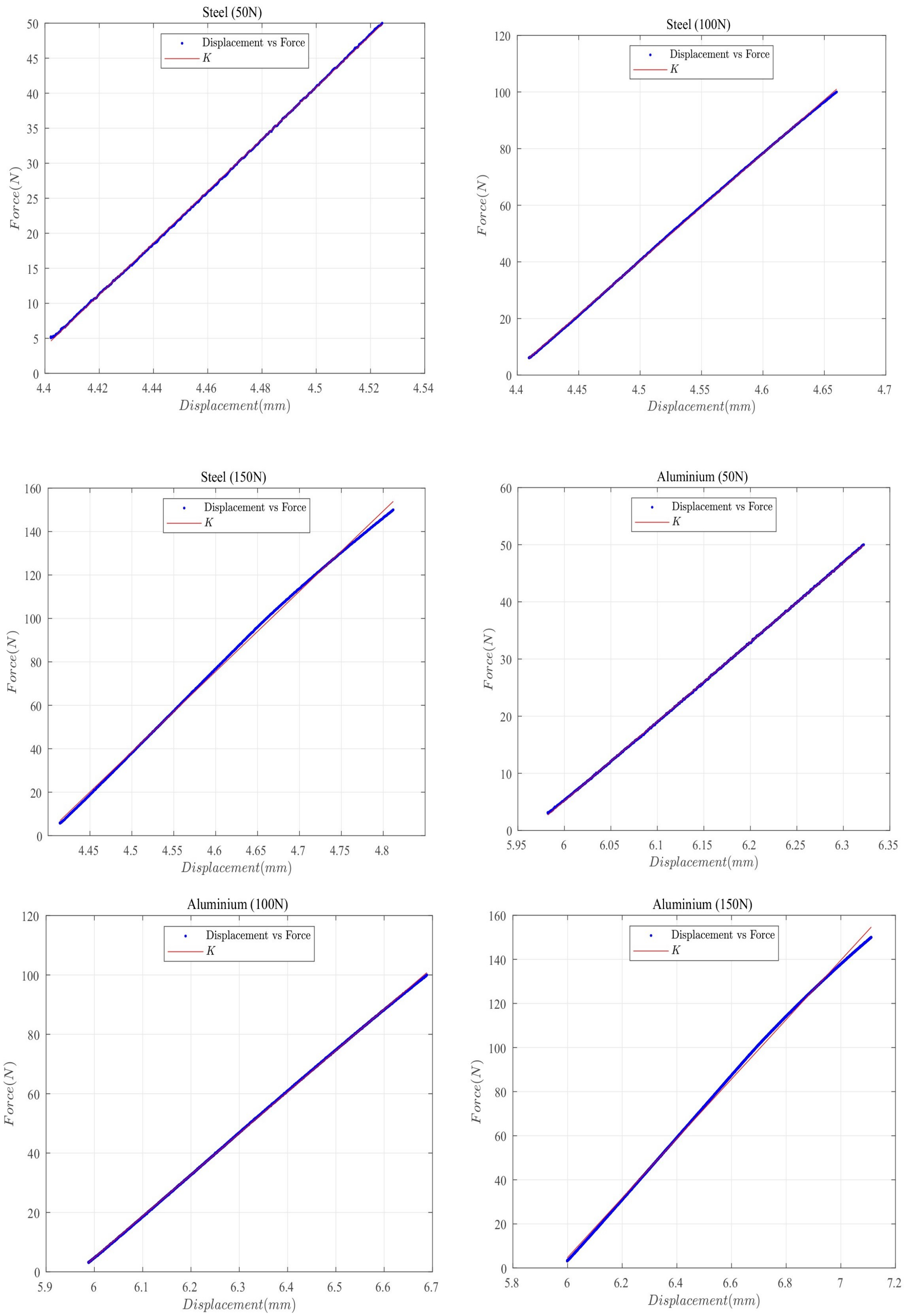

Figure 2: Figure of linear regression analysis

And the slope of the regression analysis is

$$K = \frac{P}{\delta} = \frac{48EI}{L^3} \tag{3}$$

So

$$E = (\frac{P}{\delta}) * \frac{L^3}{48I} = K * \frac{L^3}{48I} \tag{4}$$

Calculate each group of data and obtain the table 2.

Modulus of Elasticity	Mild Steel	Aluminium
$E_1(P = 50N)$	171.482	64.3056
$E_2(P = 100N)$	175.509	64.5370
$E_3(P = 150N)$	171.019	62.4074
$E_{exp} = (E_1 + E_2 + E_3)/3$	172.670	63.75

(Unit: GPa)

Table 2: Experimental results - Modulus of elasticity

Summary

As can be seen from the data in Table 2, the modulus of elasticity of mild steel (172.67GPa) is much larger than aluminium (63.75GPa). In addition, due to the difference in modulus of elasticity, the deformation of aluminium is greater than that of mild steel under the same force.

The comparison for one material in different force indicates that there is a slight difference in the modulus of elasticity, which may be due to experimental error, and the more accurate modulus of elasticity can be obtained through more experiments.

Section B

In this section, the deformation of each material under different forces can be calculated from the data obtained in section A.

Analysis

We know

$$\delta_{max} = \frac{PL^3}{48EI} = \frac{P * 0.1^3}{48 * E_{exp} * 4.5 * 10^{-11}} \tag{5}$$

Using the E_{exp}(172.67GPa and 63.75GPa) in different material(Mild Steel and Aluminium) with different force(50N, 100N, 150N) in Table 2, the data in Table 3 can be calculated.

Bending Displacement	Mild Steel	Aluminium
$\delta_{AN\_1}(P = 50N)$	0.1341	0.3631
$\delta_{AN\_2}(P = 100N)$	0.2681	0.7262
$\delta_{AN\_3}(P = 150N)$	0.4022	1.0893

(Unit: mm)

Table 3: Experimental results - maximum deformation

Summary

Bringing the average modulus of elasticity into the equation enables a more accurate calculation of the deformation of the material under different forces and helps to reduce experimental errors.

Section C

In this section, finite element analysis (ANSYS) is performed on the components.

Analysis

ANSYS analysis results are shown in Figure 3.

The finite element analysis in Figure 3 gives the deformation-position curves for different materials under different forces, where the maximum deformation of the component can be easily determined.

The data are shown in Table 4.

Bending Displacement	Mild Steel	Aluminium
$\delta_{FE\_1}(P = 50N)$	0.1150	0.3237
$\delta_{FE\_2}(P = 100N)$	0.2300	0.6473
$\delta_{FE\_3}(P = 150N)$	0.3451	0.9710

(Unit: mm)

Table 4: FEA results - maximum deformation

Summary

By using ANSYS analysis, deformation-displacement diagrams were obtained for two materials under three different forces, revealing that the maximum deformation exists at the midpoint of the element (the point where the forces are applied).

The comparison also shows that even under theoretical conditions, the deformation of aluminium is larger than that of mild steel under the same forces.

Section D

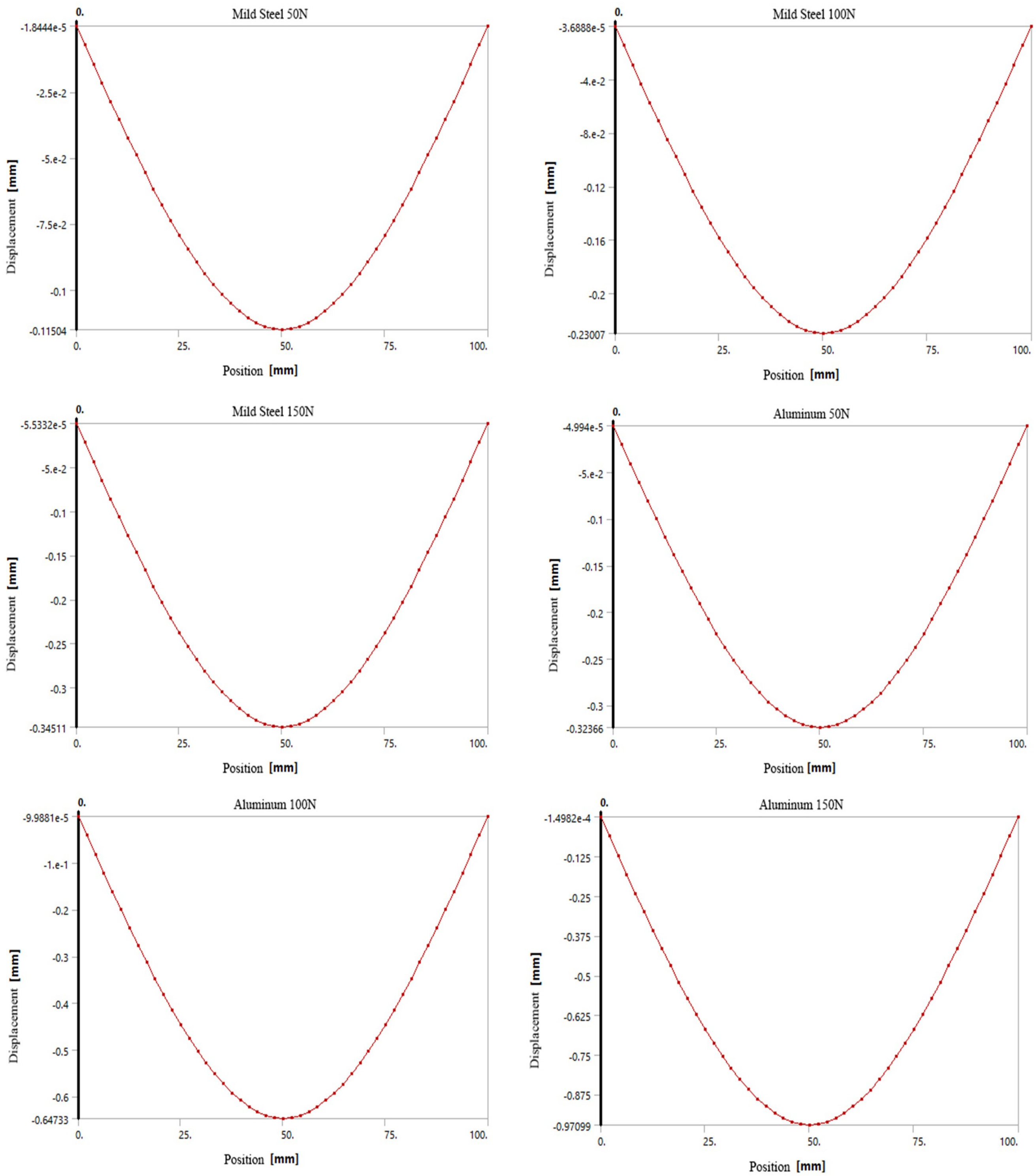

Figure 3: Figure of ANSYS analysis

Comparison of results

By comparing the experimental data(See Table 3) with the results of the ANSYS analysis(See Table 4), it can be found that all six groups of data measured in the laboratory have larger deformation values than the ANSYS analysis. This is due to the fact that the modulus of elasticity of the laboratory material is 172.67GPa(Mild Steel) and 63.75GPa(Aluminium), both of which are smaller than the theoretical value of the material(210GPa and 71GPa).

In addition, the percentage deformation of mild steel is larger than that of aluminium (i.e. $\frac{\delta_{AN}-\delta_{FE}}{\delta_{FE}} * 100\%$) and this may also be due to the larger modulus of elasticity of mild steel. Result shows in Table 5.

Force	Mild Steel(mm)	Error(%)	Aluminium(mm)	Error(%)
$P = 50N$	0.0190	16.534	0.0363	11.219
$P = 100N$	0.0381	16.539	0.0789	12.186
$P = 150N$	0.0571	16.537	0.1183	12.187
Average		16.54		11.86

Table 5: Difference deformation in Mild Steel and Aluminium

Possible reasons in error analysis

• Insufficient material purity or inhomogeneous density resulting in a modulus of elasticity less than the theoretical value.

• The memory effect of the metal caused by repeated use of the original experimental piece resulted in inaccurate deformation of the measurement.

• Errors caused by machines not being calibrated before measurement or by the machine's own measurement issues.

Experimental improvements and optimisation

• Replacement of materials with new materials which are not affected by the memory effects.

• Choose higher purity and more homogeneous density with higher precision components for measurement.

• Change the machine and take several measurements to avoid experimental chance.

2 Conclusion

From the results of the regression analysis, the R-square value of both experiment A1 and A2 are larger than 0.98, which demonstrates a strong linear correlation between the x and y axes. Additionally, from the above it follows that $C_v = 0.955$,$C_d = 0.615$(3mm),$C_d = 0.8816$ (6mm). Both of the coefficient are less than 1.This satisfies the hypothetical conditions.

Therefore, the data from the regression analysis is valid.

For C_v, due to vena contracta , the real area that approximates the area of the orifice with a little smaller, and its value is 0.95, which does not deviate much from the theoretical value..

For C_d(3mm), the value is 0.615, whereas it should actually be between 0.8 and 0.95(C_v) (equation **??**), probably due to experimental error. This should be done several times for the 3mm diameter orifice to avoid experimental coincidence and make it more accurate.

For C_d(6mm), the value is 0.8816, which is less than 0.95(C_v). This data is valid.