Plastic Collapse of Portal Frame Lab
Group T05

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March 25, 2023

1 Introduction

The purpose of this experiment is to investigate the behaviour of a mild steel portal frame model when subjected to increasing loads.

The rig consists of a loading system that applies a vertical load at the center of the beam and a horizontal load at the top of one column. As shown in Figure 1.

Figure 1: Experimental procedure

2 Experiment

The frame is made of mild steel and has a uniform rectangular cross-section. Also, The rig is equipped with two gauges to monitor the horizontal deflection of the beam and its central vertical deflection. The yield strenth of the steel is 250MPa.

The experimental parameters are shown in the following table Table 1.

Data type	Height	Length	Thickness	Width
Theory	200	300	3.00	12.00
Actual	200	304	3.27	12.97

(Unit: mm)

Table 1: Experiment parameters

The experiment entails the following steps:

1. Measure and record the dimensions of the frame and its cross-section.
2. Ensure the loading rig is in proper working condition and inspect cables for damage.
3. Zero the force and displacement readings while the rig is still unloaded.
4. Gradually apply increasing horizontal and vertical loads to the frame in increments of 10 N. The relationships between P and W are $y = x$.
5. Record the applied forces and corresponding deflections for each increment.
6. Continue the process until a plastic failure occurs, and observe the formation of plastic hinges and position.
7. Unload the frame and identify the locations of plastic hinges by observing permanent rotation in the joints.

Here are the recorded data Table 2.

Force(N)	Vertical	10	20	30	40	50	60	70	80	90	100	110	120	130
	Horizontal	10	20	30	40	50	60	70	80	90	100	110	120	130
Distance(mm)	Vertical	0.06	0.05	0.1	1.42	2.13	2.26	2.8	3.22	3.62	4.28	5.23	7.46	12.88
	Horizontal	0.88	1.58	2.15	2.99	4.04	4.44	5.06	6.48	7.63	8.55	9.31	15.05	23.08

Table 2: Record data

3 Theory

The method of calculating the plastic moment is introduced in the powerpoint of the Week 6 lecture. As shown in the figure below Figure 2.

The plastic moment M_p when all points in the section reached yield stress $\sigma_y(250Mpa)$ is caculated by:

$$M_p = \sigma_y \frac{bd}{2}(\frac{d}{4} + \frac{d}{4}) \tag{1}$$

Calculated from the data from Table 1, we get $M_p = 8.67N \cdot m$.

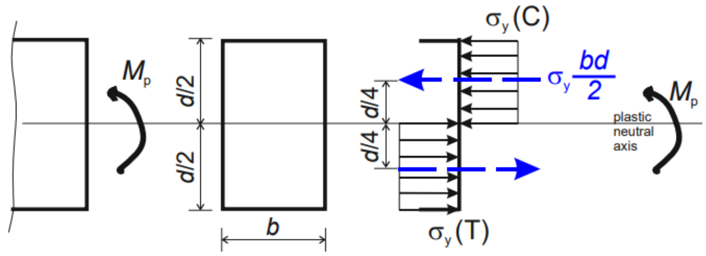

Figure 2: plastic modules of rectangular section

There are three cases of collapse of plasticity of portal frame. We use virtual work method to caculate it.

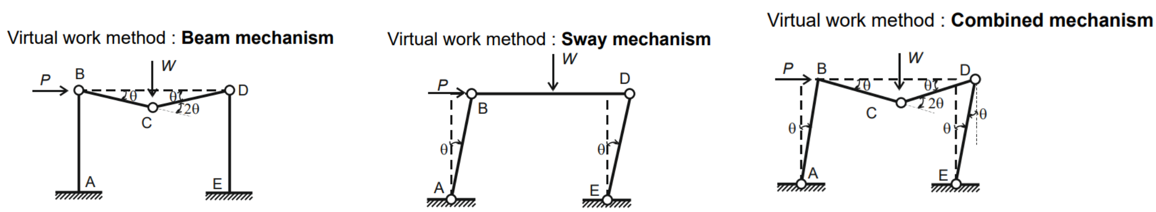

Figure 3: Three cases of collapse of plasticity of portal frame

• **Beam mechanism:** If $W >> P$, the hinges are likely to appear in B, C and D.

• **Sway mechanism:** If $P >> W$, the hinges are likely to appear in A, B, D and E.

• **Combined mechanism:** If P ≈ W, the smallest moment is at B (as moments due to P and W oppose each other), so hinges form at other possible locations. (The angle between ABC is $90°$)

The virtual work method is:

$$\sum_i P_i \delta_i = \sum_j M_j \theta_j \tag{2}$$

We noticed that the height (200mm) is equal to $\frac{2}{3}$ length (300mm). i.e. $H = \frac{2}{3}L$.

By using Equation 2, we can get

$$\begin{cases} W\frac{\theta L}{2} = M_p\theta + M_p 2\theta + M_p\theta \rightarrow W = \frac{8M_p}{L}(Beam) \\ P\frac{2L}{3}\theta = M_p\theta + M_p\theta + M_p\theta + M_p\theta \rightarrow P = \frac{6M_p}{L}(Sway) \\ P\frac{2L}{3}\theta + W\frac{\theta L}{2} = M_p\theta + M_p 2\theta + M_p 2\theta + M_p\theta \rightarrow 4P + 3W = \frac{36M_p}{L}(Combined) \end{cases}$$

Plot the relationships P-W on a graph

Substitude data from Table 1 and Equation 1, We have $\frac{M_p}{L} = 28.9N$ and the boundary of the graph.

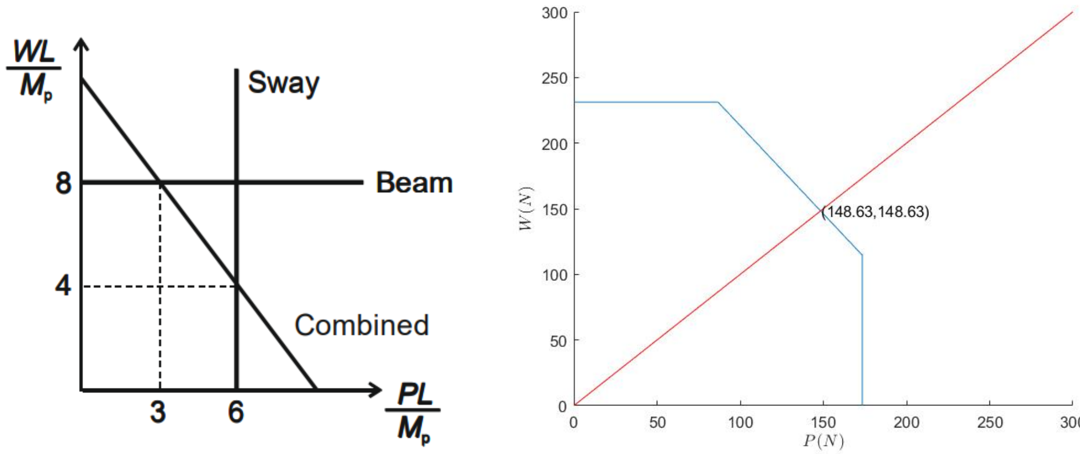

Figure 4: P-W graph

$$
\begin{cases}
W = \frac{8M_p}{L} = 231.2 \\
P = \frac{6M_p}{L} = 173.4 \\
4P + 3W = \frac{36M_p}{L} = 1040.4
\end{cases}
$$

Plotting the three boundary lines with the applied load lines ($y = x$) on the graph, the following figure is obtained.

$$
\begin{cases}
4P + 3W = 1040.4 \\
P = W
\end{cases}
$$

The solution to the equation is ($P = W = 148.63(N)$) where the coordinates of the load line and the boundary line are (148.63,148.63).

4 Analysis

A : presentation of force vs deformation relationships

Using the data in Table 2, the force vs deformation plot can be derived as Figure 4. Also, from the lecture in week 7, we can learn that the total bending moment diagram is as follows

The chart illustrates the relationship between the strength and deformation of steel, which can be discussed in two different situation.

• W<110N,P<110N: Elastic deformation: The relationship between the elastic deformation variable and the external force is linear.

• W≥110N,P≥110N: Plastic deformation: The relationship between force and deformation is

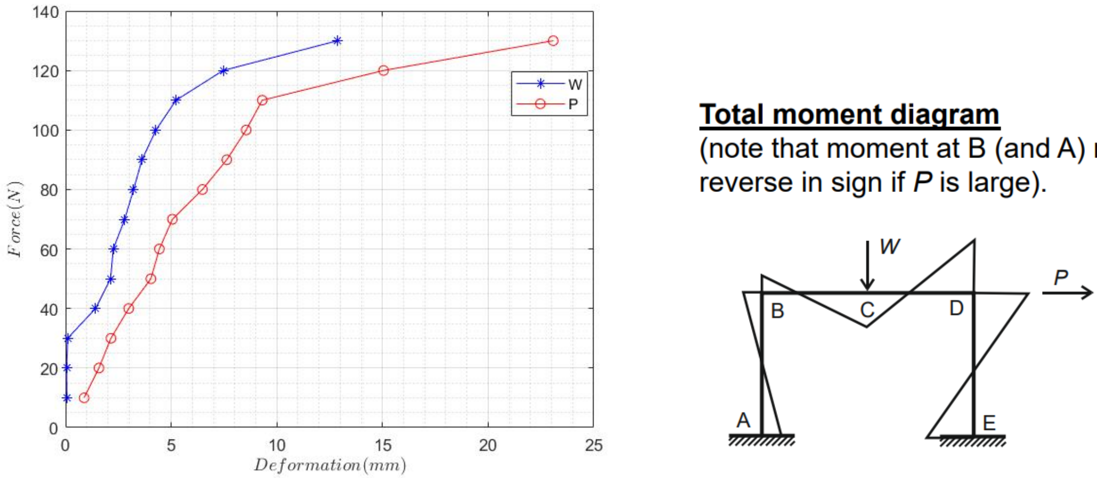

Figure 5: Force vs deformation and total moment diagram

nonlinear, i.e., the strain shows a rapid increase with the increase of stress.

From the overall bending moment diagram (see Figure 5), it can be seen that point D has the maximum bending moment, making it the first point in the experiment to experience plastic deformation.

In Figure 4, The P-W graph is piecewise and if the combination of the P and W exceeds the closed figure on the left, The collapse will happen. In this case($y = x$), the collapse load for the frame was $W = 148.63N, P = 148.63N(Theoretical)$.

Based on the Figure 5 (ignoring the inaccurate data at the beginning), it can be analyzed that the first plastic hinge is formed at around 110 N, where a significant change can be observed in the slope of the W curve. The remaining plastic hinges are formed at around 120 N, where both the P and W segments exhibit drastic changes.

B : explaination of plastic hinges

There are 4 plastic hinges at the moment of the collapse of the frame. Compare Figure 3 and Figure 6 we can find:

• After the force load, the upper left corner of the frame did not rotate, and the angle between the beam and column was still degrees.

• The angle of rotation of the plastic collapse occurring at points 1,4 is θ(in Figure 3).

• The angle of rotation of the plastic collapse occurring at points 2,3 is 2θ(in Figure 3), which is double that of points 1,4.

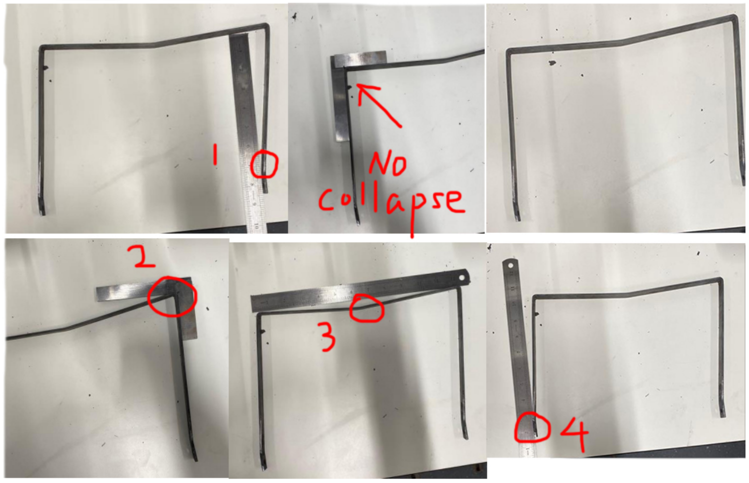

Figure 6: Experimental results

C : compare the theoretical and actual

i

The theoretical plastic collapse load derived from Section 2 is 148.63N, while the actual collapse load is in the region of $110N$-$120N$.(23.8%)

The comparison leads to the conclusion that the actual collapse load will be less than the theoretical value for these possible reasons:

• **Geometric discrepancies:** The actual dimensions of the frame may differ from the theoretical dimensions, as seen in Table 1. These discrepancies may affect the frame's overall stiffness and load-carrying capacity.(theoretical $M_p = 6.75N$ vs actual $M_p = 8.67N$)

• **Material imperfections:** The mild steel used in the experiment may contain imperfections like inclusions, voids, or uneven distribution of constituents, which could affect its mechanical properties and reduce its strength.

• **Load application:** The loading rig's accuracy in applying the horizontal and vertical loads may impact the results. Inaccurate load distribution or misalignment could cause the frame to experience additional stresses, leading to a reduced collapse load.

• **Measurement errors:** The gauges used to monitor the deflection of the frame might have inaccuracies, leading to discrepancies between the actual and recorded deflections.

ii

• There are 4 plastic hinges in theory (Figure 3-Combined mechanism). The positions of the plastic hinges are points A, C, D and E.

• There are 4 plastic hinges in experiment (Figure 6). The positions of the plastic hinges are in the position 1,2,3,4.

• Compared with that, it is easy to see that the theoretical and actual positions of the plastic hinges are roughly the same.

iii

The overall shape of the measured and predicted collapse mechanism are same. In Figure 3 and Figure 6, the upper left corner is not deformed, a rotation of angle θ occurs at positions 1,4 and a rotation of angle 2θ occurs at positions 2,3.

5 Conclusion

In conclusion, this experiment allowed us to study the behavior of a portal frame under gradually increasing loads and compare the experimental results with theoretical predictions. We identified the plastic hinges, assessed the frame's behavior, and noted a discrepancy in the collapse load between theory and practice for the deflection (23.8%), which could be attributed to geometric differences and material imperfections.

From observing Figure 5, it can be seen that the W curve changes very dramatically in the initial stage, which may be due to testing errors and a low sampling frequency. Here are some methods for improvement:

• Increase the amount of data for the experiment. The experiment should increase the sampling density to obtain a more accurate point.

• There is experimental randomness. If conditions permit, it is possible to test more identical modular frameworks, and the resulting data will be more reliable.

• Extend the experimental process. If conditions permit, it is possible to try applying larger loads to explore the material's subsequent plastic deformation behavior.