Question 8.

$$2\frac{d^2x_1(t)}{dt^2} + 2\frac{dx_1(t)}{dt} + (1 + 2x_1(t)) - 2\frac{dx_2(t)}{dt} = 0. \quad \text{eq. 6.}$$

$$2\frac{d^2x_2(t)}{dt^2} + 2\frac{dx_2(t)}{dt} - 2\frac{dx_1(t)}{dt} = f(t) \quad \text{eq. 7}$$

use $\ddot{x}_1 = \frac{d^2x_1(t)}{dt^2}$ $\dot{x}_1 = \frac{dx_1(t)}{dt}$ $\ddot{x}_2 = \frac{dx_2^2(t)}{dt}$ $\dot{x}_2 = \frac{dx_2(t)}{dt}$

$$\begin{cases} 2\ddot{x}_1 + 2\dot{x}_1 + 1 + 2x_1 - 2\dot{x}_2 = 0. & ① \\ 2\ddot{x}_2 + 2\dot{x}_2 - 2\dot{x}_1 = f(t) & ② \end{cases}$$

① $\dot{z}_1 = z_2$

$2\dot{z}_2 = -2z_1 - 2z_2 + 2z_4 - 0.5$

② $\dot{z}_3 = z_4.$

$2\dot{z}_4 = 2z_2 - 2z_4 + f(t).$

Use the state space matrix.

Let $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$ $z_4 = \dot{x}_2$

we have

$\dot{z}_1 = z_2$

$\dot{z}_2 = -z_1 - z_2 + z_4 - 0.5$. — derived from ①.

$\dot{z}_3 = z_4$

$\dot{z}_4 = +z_2 - z_4 + \frac{1}{2}f(t)$ — derived from ②

use the matrix.

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} f(t) + \begin{pmatrix} 0 \\ -\frac{1}{2} \\ 0 \\ 0 \end{pmatrix}$$

put it in python code.

i) see .py file.

ii) Euler : set $h=0.05$, time = 45s.

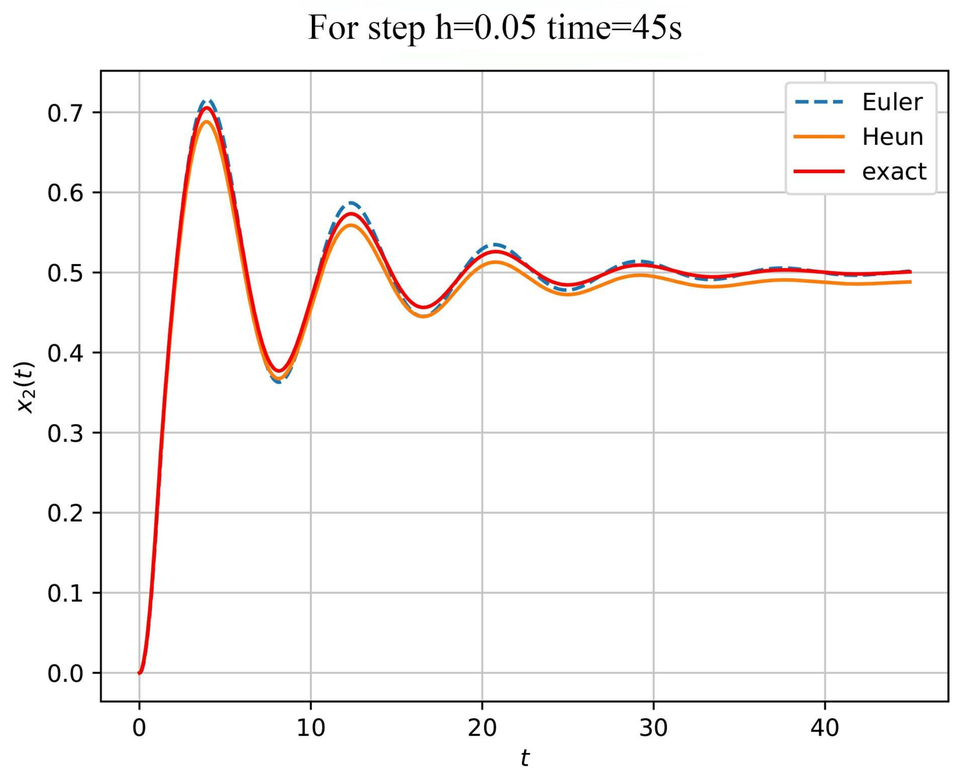
in the second plot (error in step size

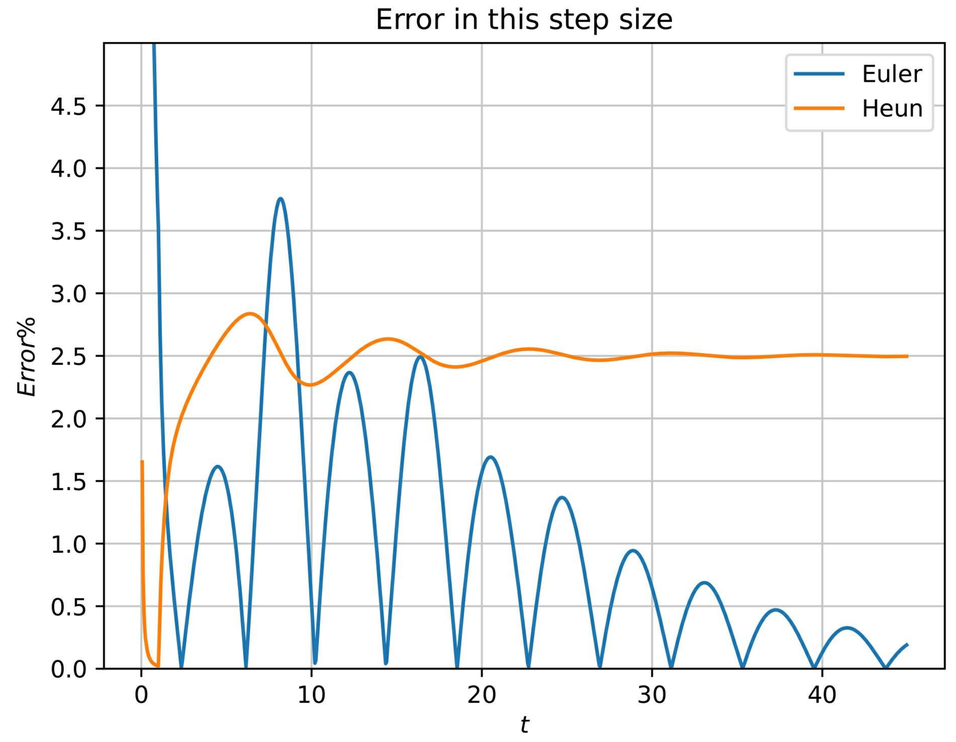
most of the error is larger than 1%

set $h=0.01$. time = 45s.

Error larger than 1% at the beginning only,
later all errors below 1%.

So, the appropriate step size $h=0.01$.





For step h=0.01 time=45s

Heun :

Set $h=0.01$, time = 45s.

All of the error are less than 1%

iii)

set $h=0.01$, time = 45s.

as we can see, when time = 30s.

the euler's error is less than 0.5%

the Heun's error is approximately equal to 0.5%

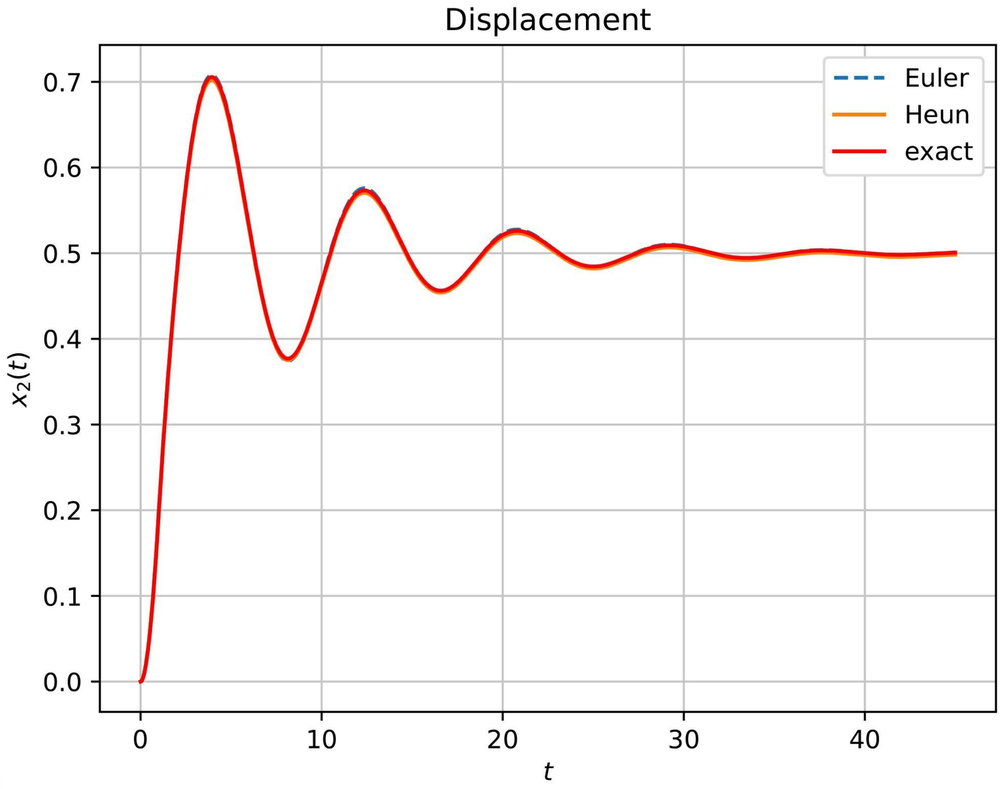
So, choose the end time which

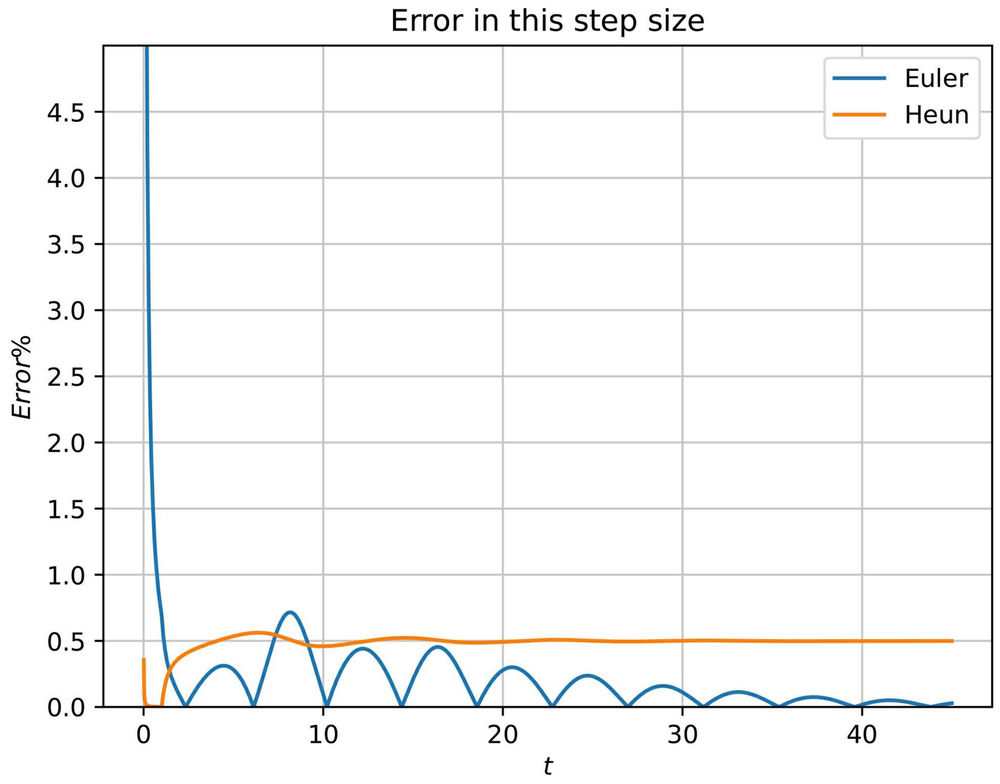
is 30s + 15s, in 45s.

in time = 45s

the euler's error is approximately equal to 0.1%

the Heun's error is approximately equal to 0.5%





(iv) Euler

$E_t = 8.926 \times 10^{-4}$

$|\epsilon_t|\% = 0.1785\%$

Heun

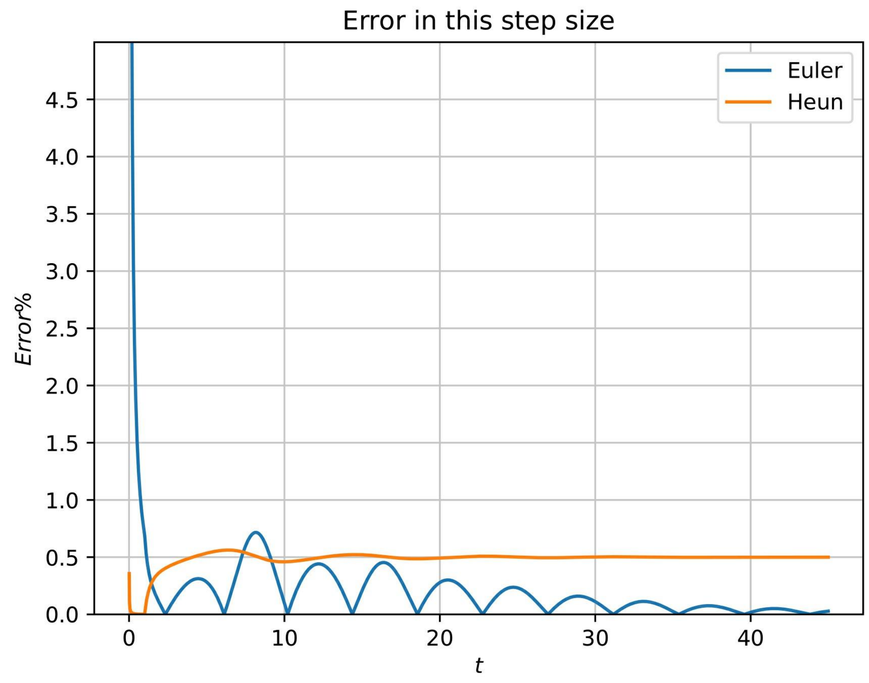
$E_t = 1.751 \times 10^{-3}$

$|\epsilon_t|\% = 0.35\%$

Discuss.

in step $h = 0.01$, time $= 45s$,

$h = 0.01$, time $= 45s$.



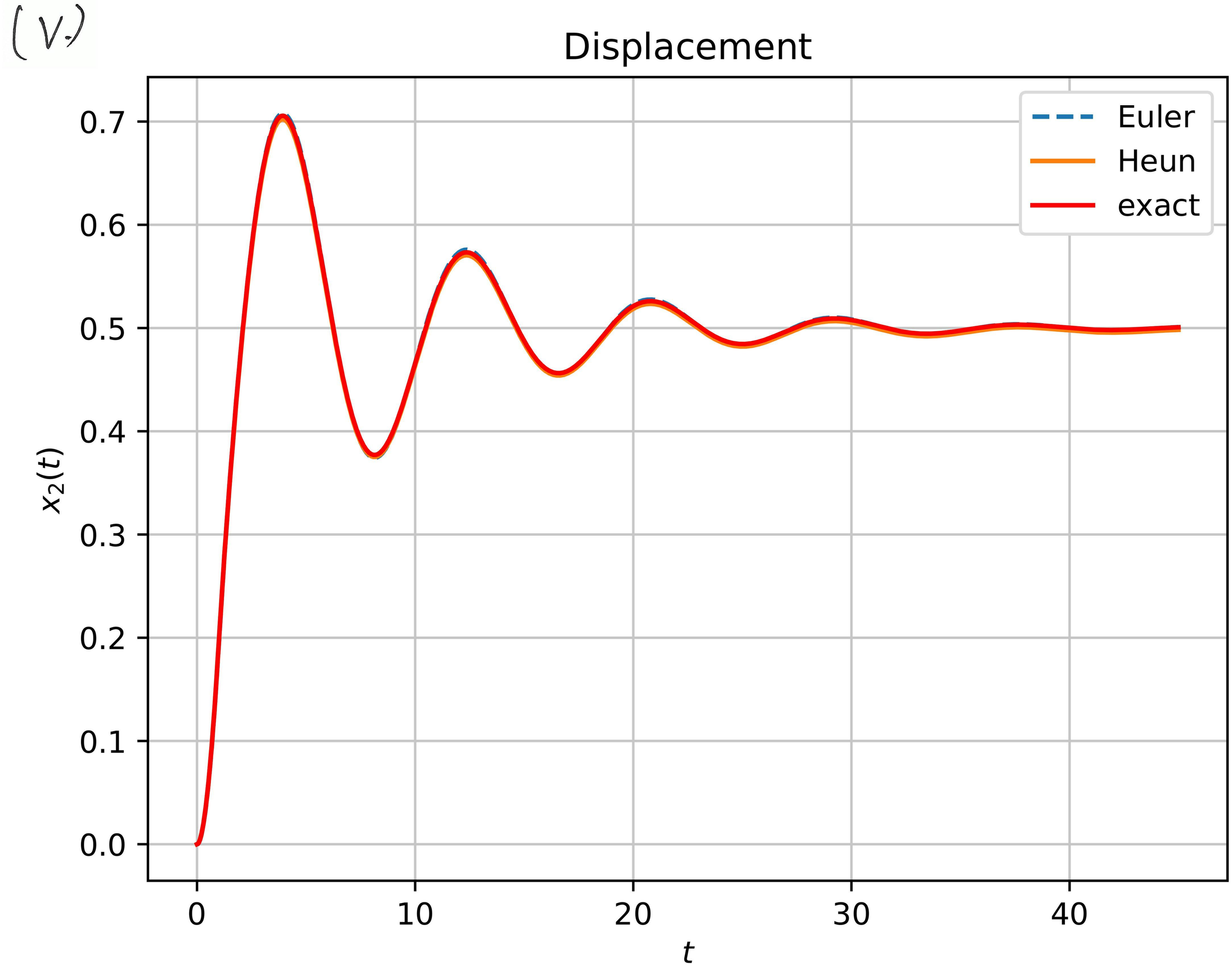
Terminal output

```
the final Euler method error is 0.0008925771159923324
the final Euler method error percentage is 0.17851542319846647
the final Heun method error is 0.0017513999471015507
the final Heun method error percentage is 0.35027998942031013
```

Euler's method are more accurate than Heun's method.

and the ~~es~~ different between euler, Heun and exact is less than 1%

(v.)



continue V.

disscuss :

The system is an underdamped system.

reason 1: It cross it's stable final value (0.5) . many times.

reason 2: It has two symmetric poles in LHP, which is $(-0.12256, \pm 0.74486)$ and the other poles is on the coordinate axis, which will effect nothing with the system.

reason 3: The figure show the system is periodly dampeed. Additionaly, the damp amplitude is reduce and it's final value is 0.5