Computation Physics Homework 1

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Summary

In this homework I worked on numerical differentiation and integration in single precision. I tested the methods on simple functions and then applied them to a cosmology problem. In Problem 1, I compared forward, central, and extrapolated finite differences for $\cos(x)$ and $\exp(x)$ at $x = 0.1$ and $x = 10$. In Problem 2, I computed

$$I = \int_0^1 e^{-t}\, dt \tag{1}$$

using the midpoint, trapezoid, and Simpson rules, and looked at how the error changes with the number of bins. In Problem 3, I calculated the correlation function $\xi(r)$ from a given power spectrum $P(k)$ using Romberg integration with cubic spline interpolation, and found the baryon acoustic oscillation (BAO) peak. In all cases, the error behaved as expected: truncation error dominates when the step size is large, and round-off error shows up when the step size is very small. The BAO peak was found near $r \approx 106\,\mathrm{Mpc}/h$.

Methods

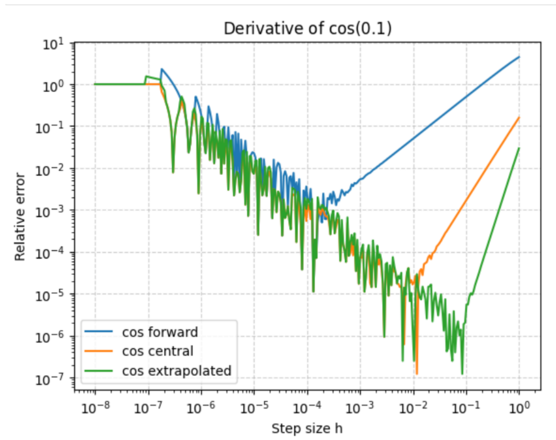
- **Problem 1:** Wrote functions for forward, central, and extrapolated finite differences in single precision. Plotted relative error against step size h on log–log axes.
- **Problem 2:** Wrote codes of midpoint, trapezoid, and Simpson rules. Evaluated the integral for a range of N values and plotted the relative error against N.
- **Problem 3:** Used a cubic spline to interpolate $P(k)$. Defined the integrand

  $$f(k, r) = k^2 P(k)\, \frac{\sin(kr)}{kr}. \tag{2}$$

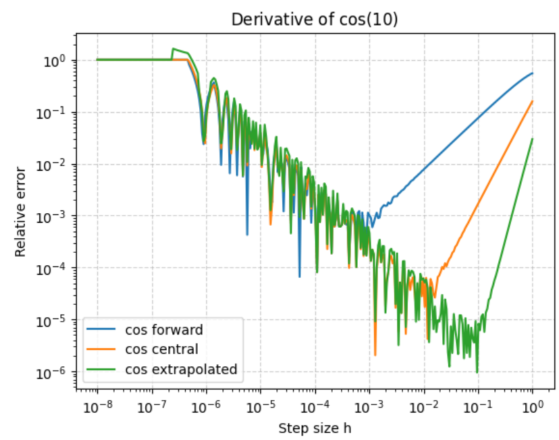
  Applied Romberg integration starting with $N_0 = 128$. Computed $\xi(r)$ for r between 50 and 120 Mpc/h. Used a peak-finding function to locate the BAO bump in $r^2\xi(r)$.

Results

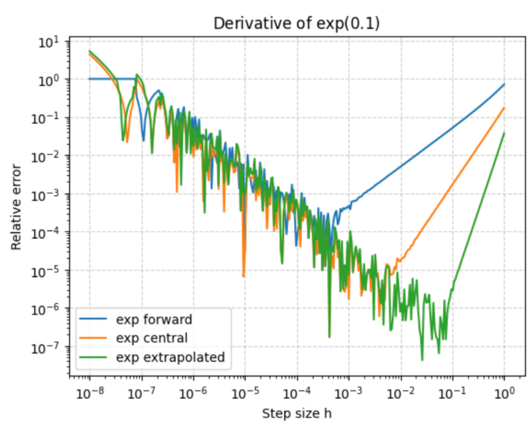
- **Problem 1:** Forward difference error scaled like $O(h)$, central like $O(h^2)$, and extrapolated like $O(h^4)$ until round-off error took over. The plots showed that there is an optimal h for different algorithm. The smallest errors at optimal h for the three algorithms are approximately $\epsilon_m^{\frac{1}{2}}, \epsilon_m^{\frac{2}{3}}, \epsilon_m^{\frac{3}{5}}$.



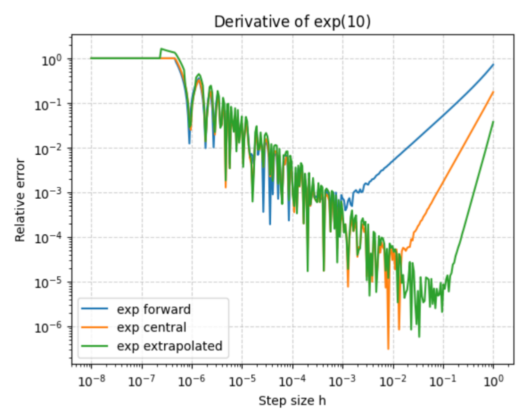
(a)



(b)



(c)



(d)

Figure 1: (a) and (b) are the derivative of cos(x) for x at 0.1 and 10 respectively. (c) and (d) are the derivative of exp(x) for x at 0.1 and 10. In all the four plots, the truncation error dominates when h is large, and the round off error dominates when h is small.

- **Problem 2:** Midpoint and trapezoid rules showed $O(1/N^2)$ convergence. Simpson's rule showed $O(1/N^4)$ until round-off error flattened the curve. At very large N, all methods showed error growth due to single precision limits.

- **Problem 3:** The cubic spline gave a smooth $P(k)$. Romberg integration converged quickly. The correlation function $\xi(r)$ showed the BAO bump, with the peak of $r^2\xi(r)$

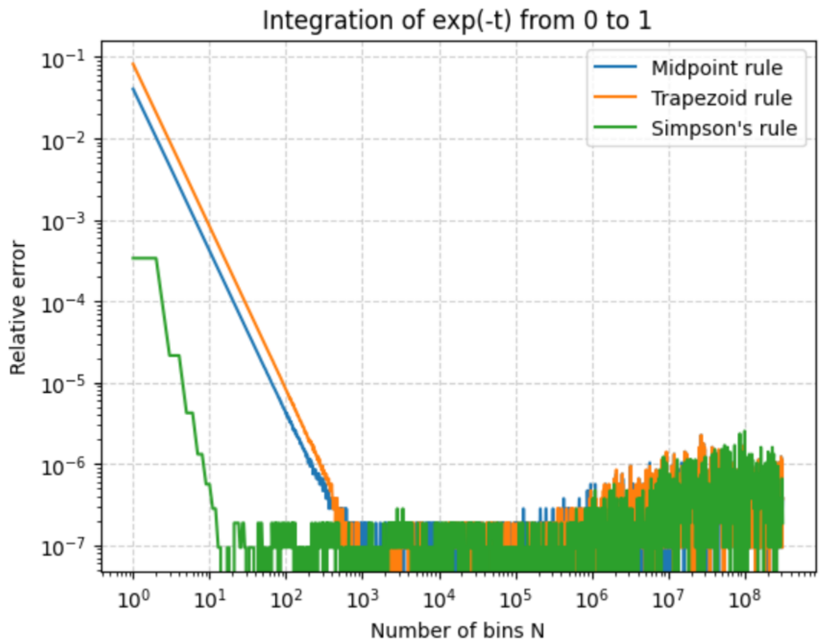


Figure 2: Relative error of numerical integrals for exp(-x) from 0 to 1. The Midpoint and Trapeaoid rules show the same scaling with N, i.e. $O(1/N^2)$. The Simpson's rule shows $O(1/N^4)$ sccaling. When N is larger than 10^6, we can see the round off error.

at $r \approx 106\,\mathrm{Mpc}/h$.

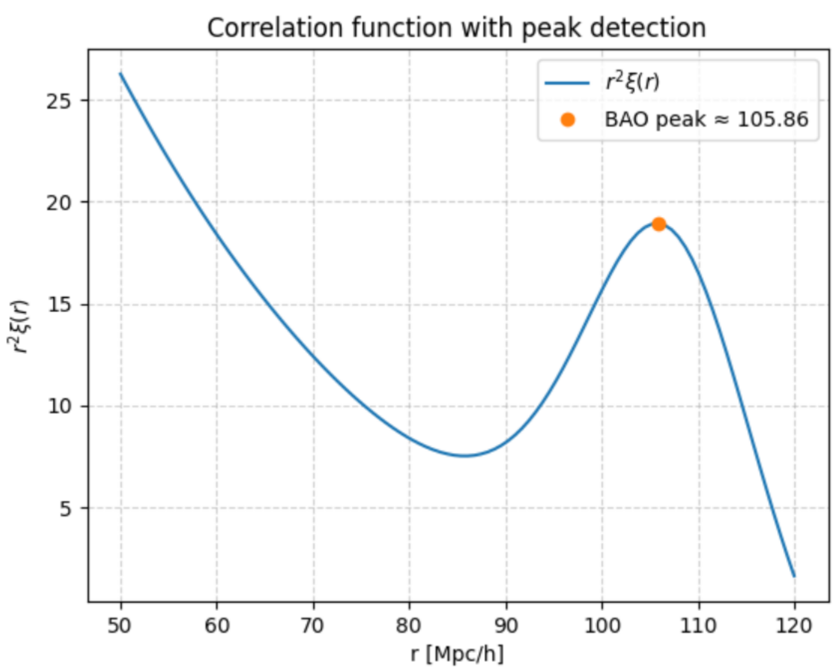


Figure 3: Correlation function. There is a clear bump at $r \approx 106\,\mathrm{Mpc}/h$