

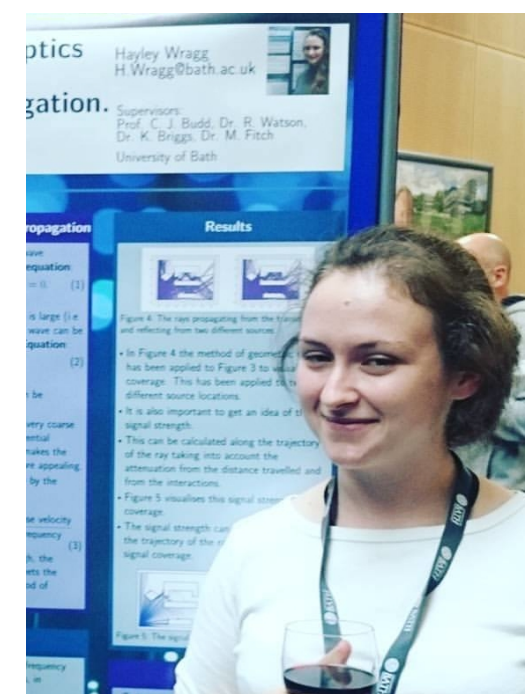
The method of geometric optics, applied to wifi propagation.



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Overview of the method of geometric optics

- The idea is to use **rays** to model waves.

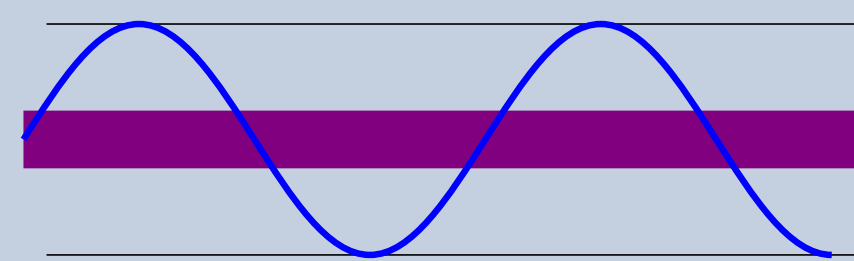


Figure 1: Straight line (ray) approximation of a wave.

- This approximation is good for small wavelengths.

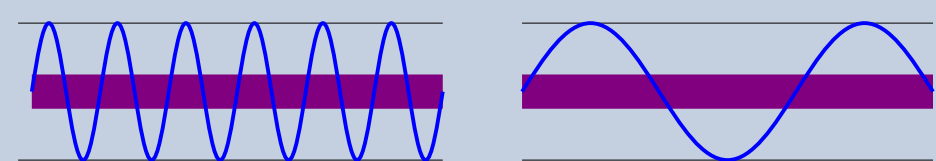


Figure 2: Ray approximation for different wavelengths.

- Map the paths of **the rays** in the environment to model the path of the waves.

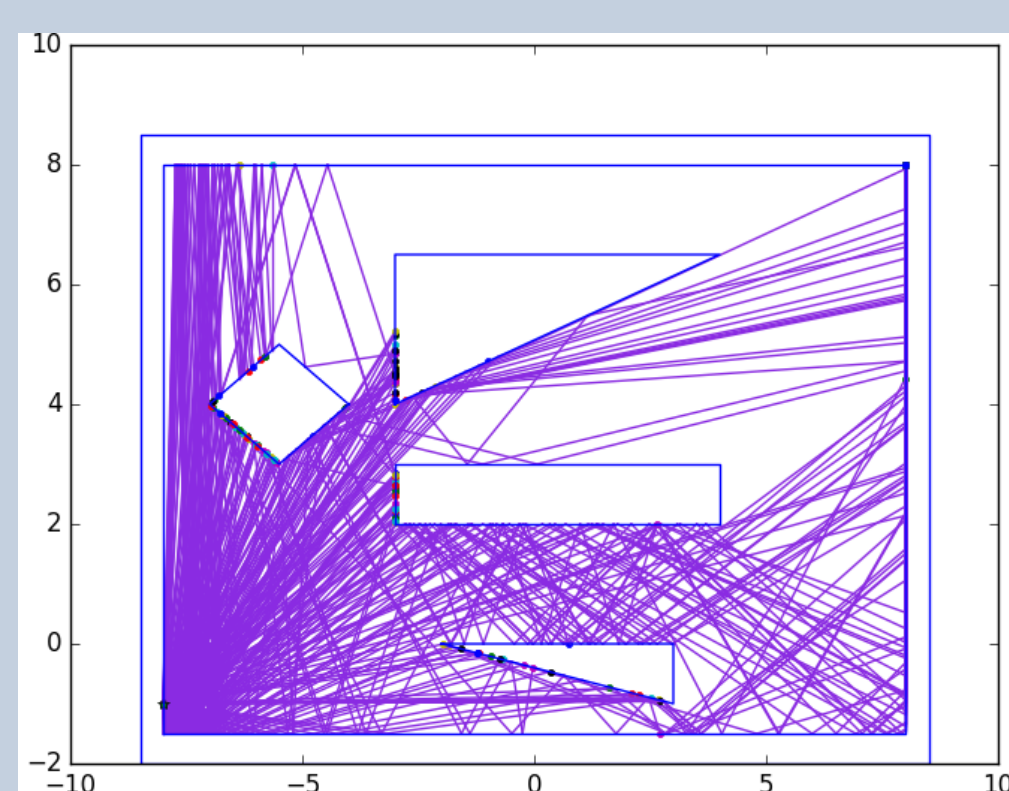


Figure 3: Rays propagating within an environment.

- Calculations are then made **along the rays** to determine information such as power.

Mathematical motivation

- A time-independent model for wave propagation is the **Helmholtz equation**:

$$\nabla^2 \phi(x) + \underbrace{k^2}_{\text{Wave number}} \phi(x) = 0. \quad (1)$$

- Where k is the wave number given by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c},$$

f is the frequency of the wave, and c the velocity.

- Assuming that $\phi(x)$ can be written as a function of the path $S(x)$ in the form $\phi = u(x)e^{ikS(x)}$ then u satisfies,

$$\frac{1}{k^2} \nabla^2 u - u (|\nabla S|^2 - 1) = 0 \quad (2)$$

Since $k \gg 1$, $S(x)$ satisfies the **Eikonal Equation**:

$$|\nabla S|^2 = 1 \Rightarrow |\nabla S| = 1 \quad (3)$$

The constant gradient means that a ray is a valid approximation for the wave propagation when $k \gg 1$.

Applications

- Small wavelength propagation** can be used to model radio propagation in cities, and wifi propagation in the home.
- Developments in technologies have resulted in an increased demand for **high frequency wifi propagation models**.

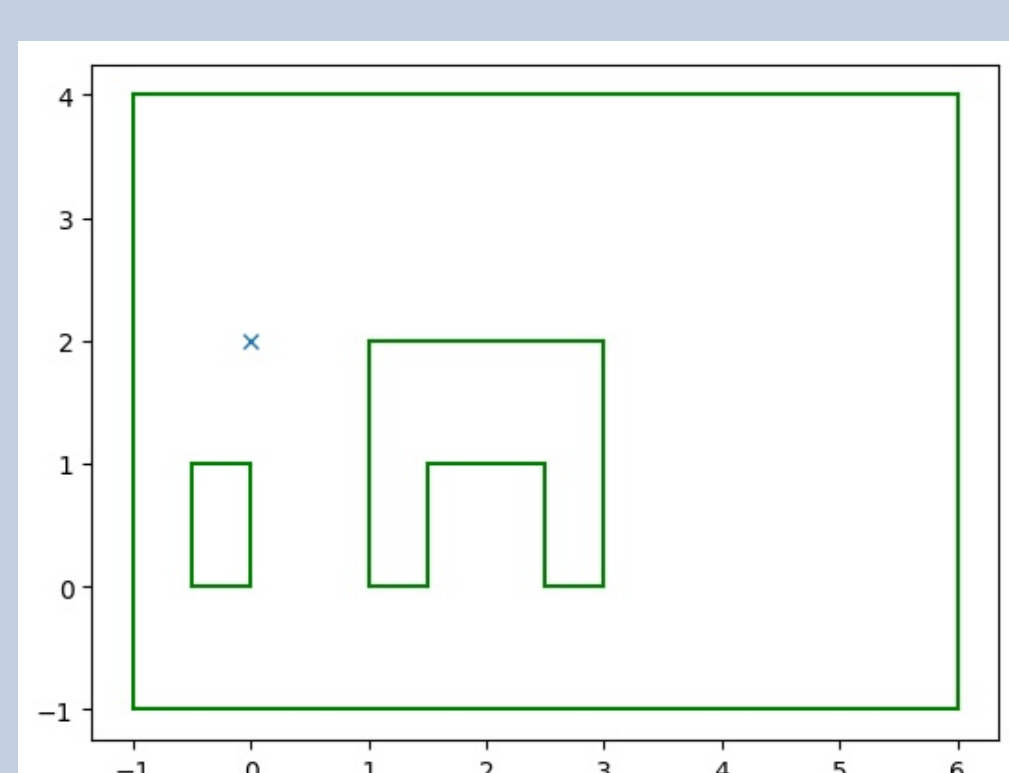


Figure 4: Template domestic environment projected into two dimensions

- Figure (4) gives an example simulated environment.
- To get an idea of the coverage around the environment the method of geometric optics can be used.

Trajectory approximations

- The method of geometric optics can be applied to the problem of **wifi propagation**. For simplicity it is assumed that the antenna propagates uniformly and the domain is modelled in **two dimensions**.

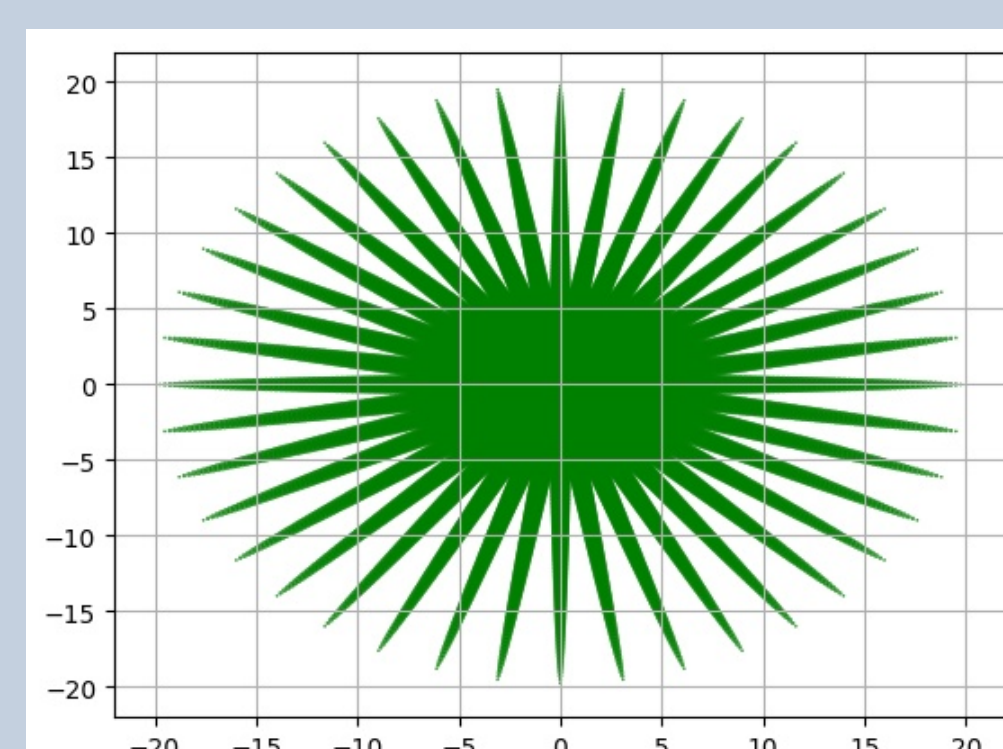


Figure 5: Antenna uniformly propagating

- Using Hewett's argument in [1] since the Rayleigh distance is much larger than the spatial distance it is valid to **ignore diffraction effects**.
- Objects are all also given the same reflection coefficient and refraction is ignored—this is valid since the loss is very high.
- Phase change has also been ignored in this model.

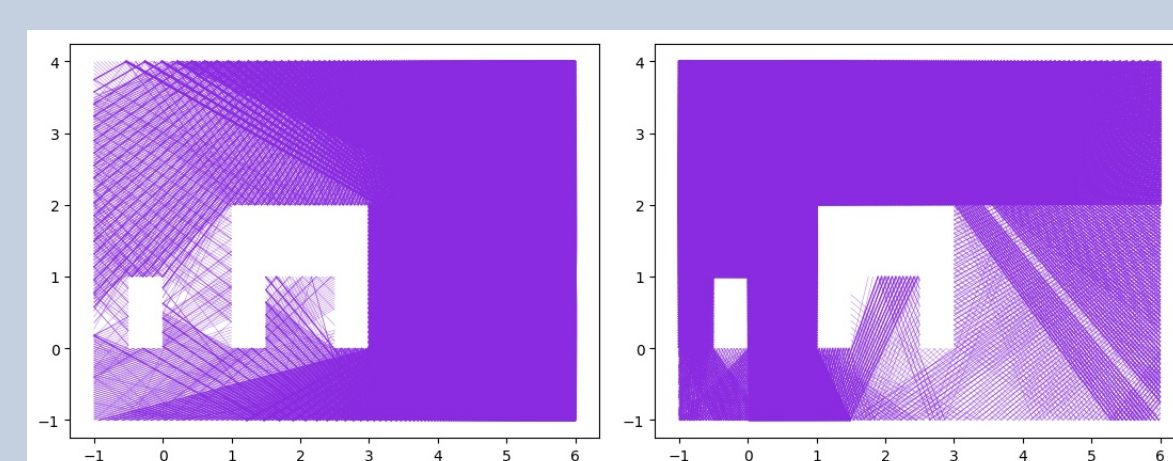


Figure 6: The rays propagating from the transmitter and reflecting from two different sources.

- In Figure (6) the method of geometric optics has been applied to Figure (4) to visualise the coverage. This has been applied to two different source locations.

Wifi coverage

- It is also important to get an idea of the power of the **wifi coverage**.
- This is calculated along the rays taking into account the **attenuation in free space** and the **reduction at the reflections**.
- The loss resulting from the distance travelled is calculated using the **Friis transmission formula**:

$$\frac{u_r}{u_t} = G_a G_b \left(\frac{\lambda}{4\pi r} \right)^2, \quad [2] \quad (4)$$

where u_r , u_t give the power at the receiver and transmitter relatively. The gain for the antenna's G_a and G_b are taken to be 1.

- Figure 7 shows the signal power for two **different transmitter locations**.

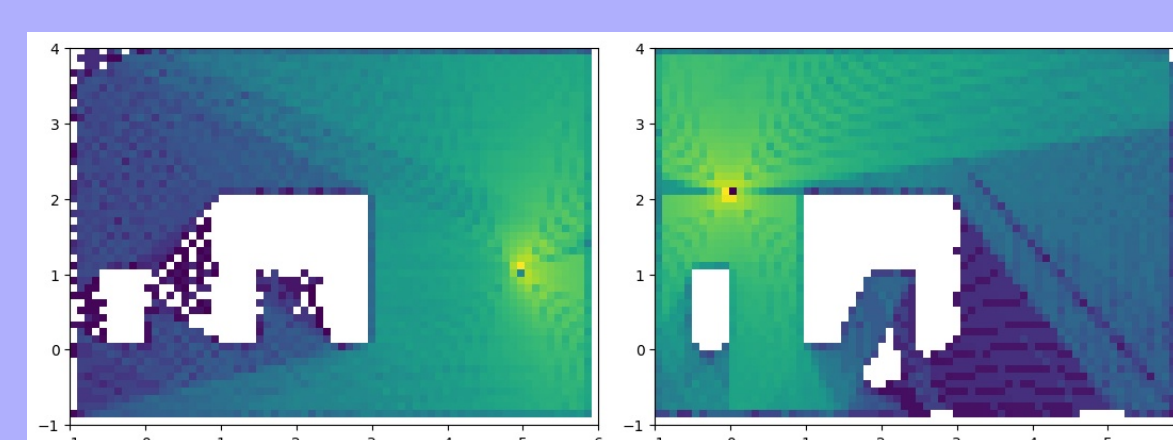


Figure 7: The signal power over the environment

- A problem with this visualisation is that **very extreme low values makes the fairly low values look better**. Practically any value below 30 dBm is unusable. Similarly for the **extreme high values**.
- To account for this the values are **bounded** within their useful range of [30, 120] dBm. The resulting coverage is then found.

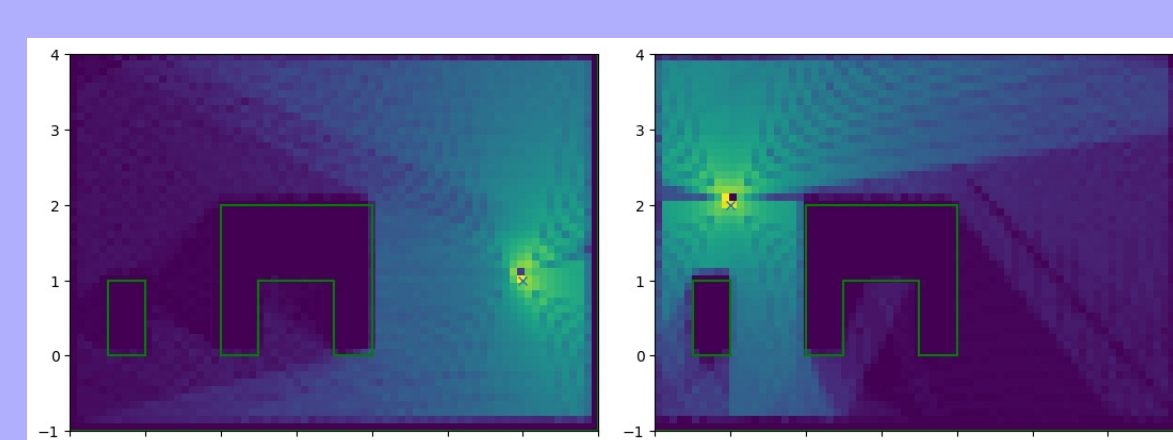


Figure 8: The signal power over the environment

Distribution of the signal power

Histograms for the bounded and unbounded signal power values to visualise the quality of the coverage.

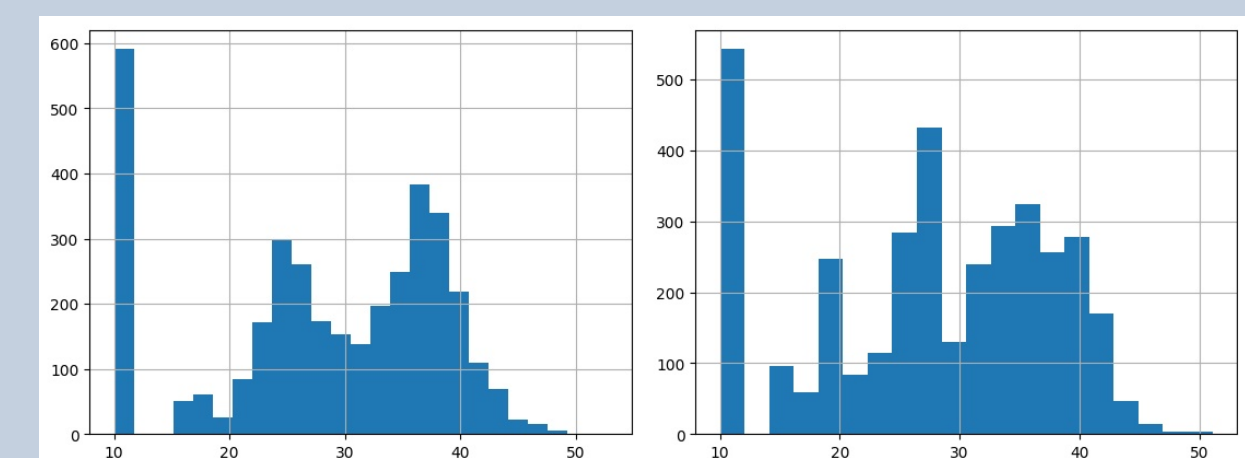


Figure 9: Histogram of the unbounded signal power values

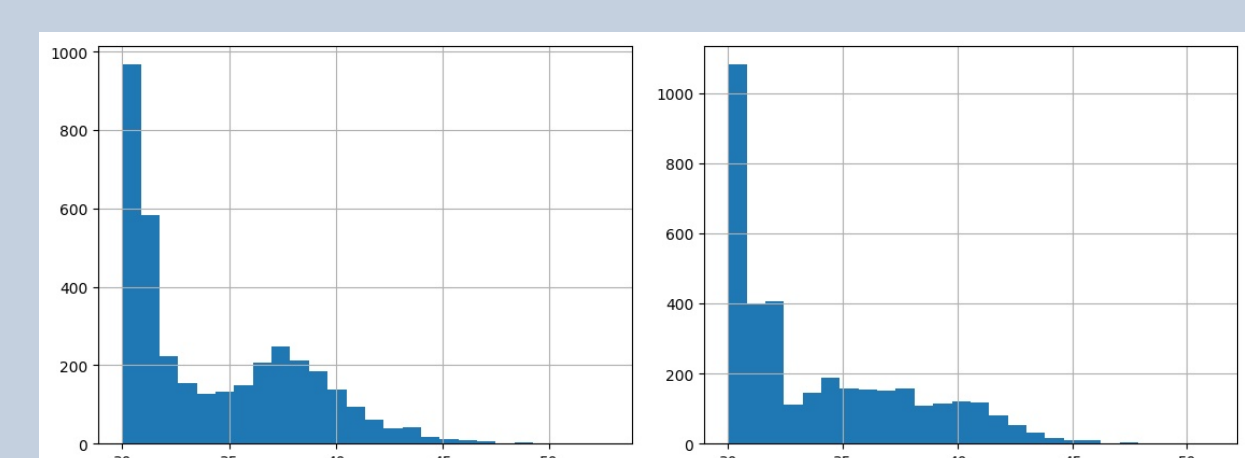


Figure 10: Histogram of the bounded signal power values

Figures show a significant change for the different source locations.

Cumulative distribution of the signal power

To view the fraction of area with at least some given signal strength the cumulative distributions can be used.

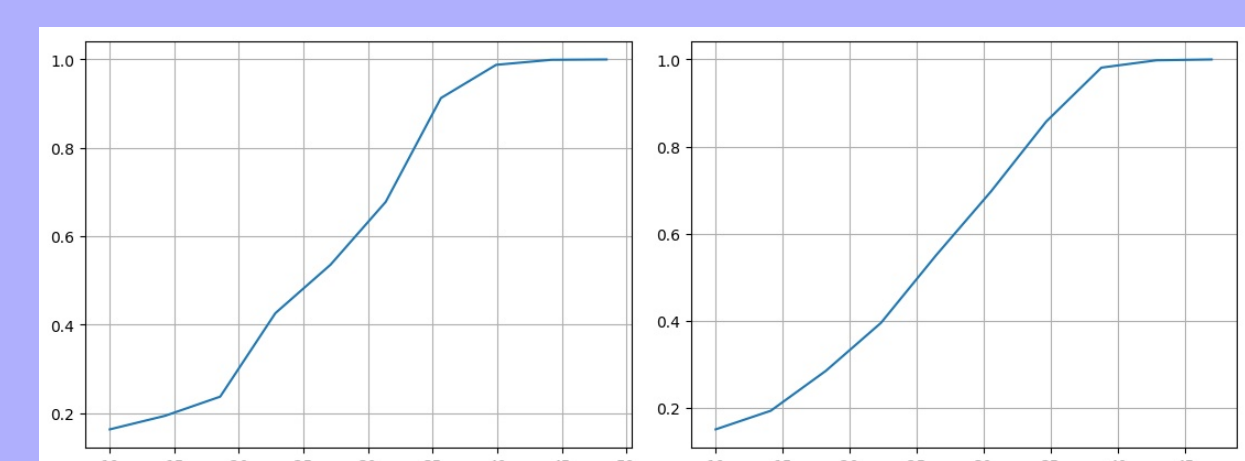


Figure 11: Cumulative distribution, signal power values unbounded

Repeating for the bounded values:

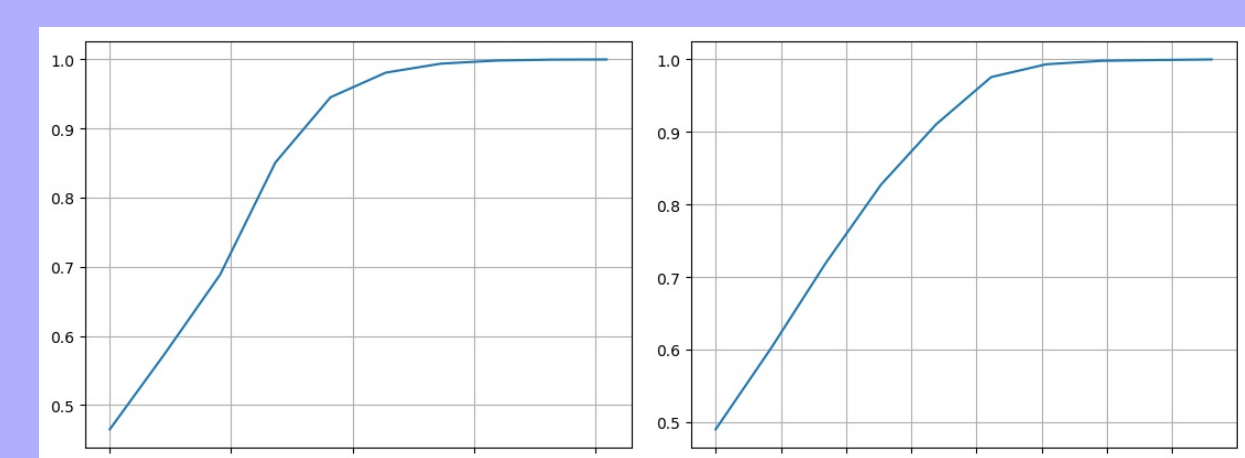


Figure 12: Cumulative distribution, signal power values bounded

Figures show a noticeable difference in the coverage made by moving the transmitter.

Why use the method?

- When the wavelength is small this method is much faster than numerical partial differential equation methods which require a very small mesh size.

Future work

- Statistically analyse the histograms and cumulative distributions for different locations of transmitters in different environments.
- Account for phase change using random phases.
- Take into account different reflection coefficients.

References

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- S Saunders and A Aragon-Zavala. *Antennas and Propagation for Wireless Communication Systems*. Manning Publications Co., Connecticut, USA, 2008.
- Yingli Wang and Stephen Pettit. *E-Logistics: Managing Your Digital Supply Chains for Competitive Advantage*. Kogan Page Publishers, 2016.
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