The Method of Geometric Optics and the Application to High-Frequency Wave Propagation.

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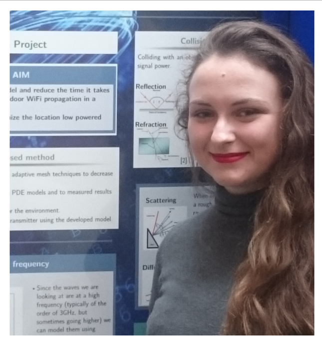
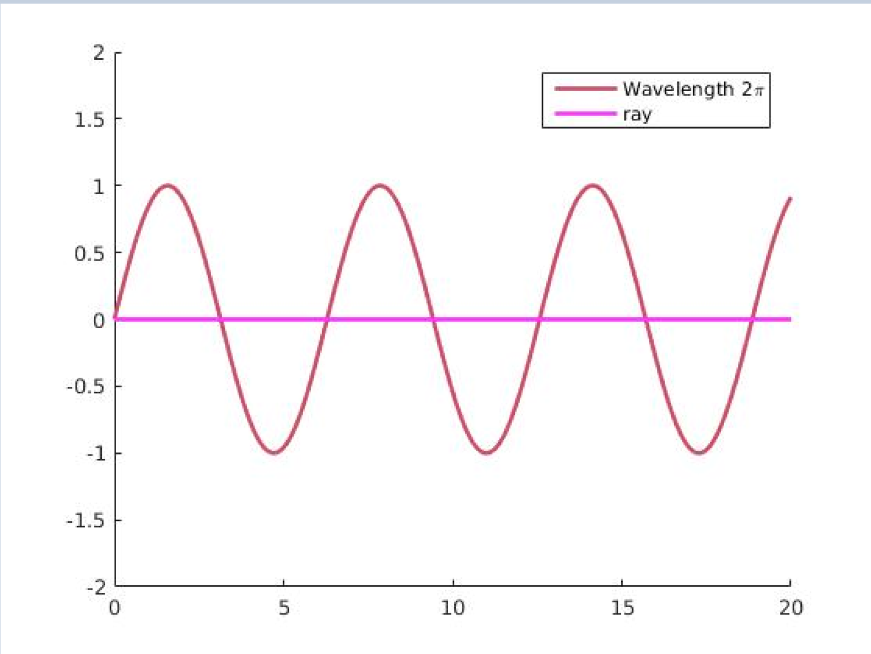


The Method of Geometric Optics

- The idea of the method is to use **straight lines (rays)** to model waves.



- **Rays** are mapped around an environment from source to visualise the path of the waves. An example is given in Figure 1.

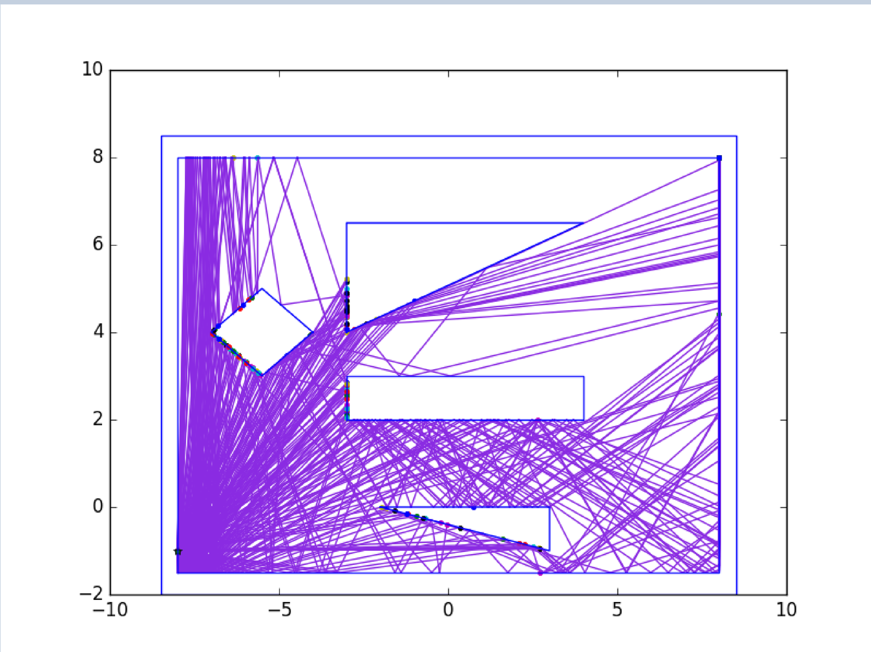
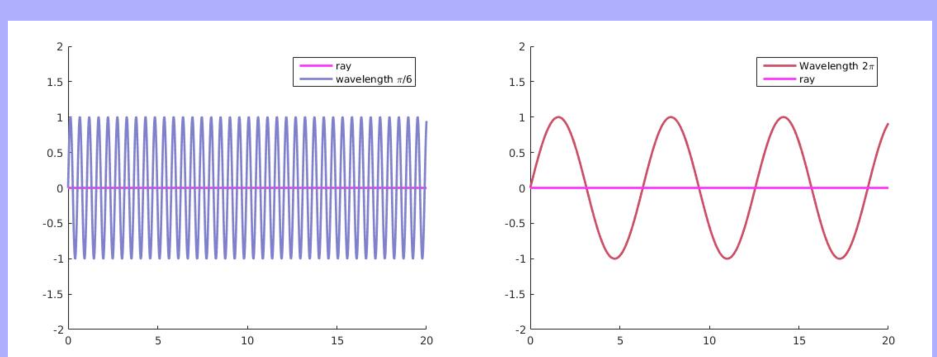


Figure 1: Rays propagating from a source and reflecting within an environment.

- **Field of rays** is then observed to give an overview of the results in the environment.
- Calculations are then made **along the ray** and hence **over the ray field** to determine information about the **properties wave**.
- **Ray-tracing** requires the wave-length to be small relative to the dimensions of the environment.

Why Use the Method?

An **advantage** is that the method is **much faster** to compute than numerical partial differential equation methods for the wave equation, especially for waves with **small wave lengths**.



A **disadvantage** is that the method requires lots of information about the environment which can be difficult to obtain.

High Frequency Wave Propagation

A time-independent model for wave propagation is the **Helmholtz equation**:

$$\nabla^2 S(x) + \underbrace{k}_{\text{Wave number}}{}^2 S(x) = 0. \qquad (1)$$

When the wave number $k = \frac{2\pi}{\lambda}$ is large (i.e λ-wavelength is small) then the wave can be approximated by the **Eikonal Equation**:

$$|\nabla S|^2 = n, \qquad (2)$$

where n is the refractive index.

Hence the wave propagation can be described with the use of rays.

High frequency waves require a very coarse mesh for numerical partial differential equation approximations. This makes the method of Geometric Optics more appealing.

Wavelength relates to frequency by the following,

$$\text{Wavelength} = \lambda = \frac{v_p}{f} = \frac{\text{phase velocity}}{\text{frequency}} \qquad (3)$$

Hence when the frequency is high, the wavelength is small, and this meets the requirements for using the method of geometric optics.

Application

- One real world example of high frequency waves are electromagnetic waves, in particular wifi waves.
- Developments in wifi-enabled technologies have resulted in an increased demand for wifi propagation models.

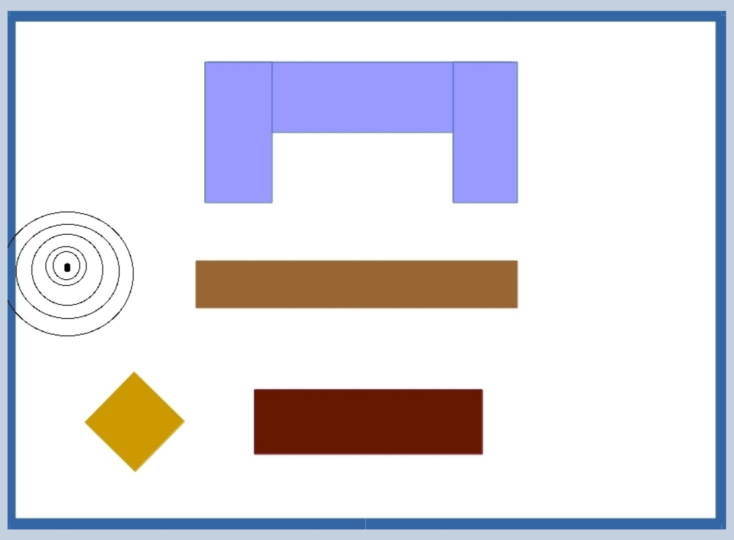


Figure 3: Template environment in two dimensions.

- Figure 3 gives an example environment with a source radiating wifi waves.
- To get an idea of the coverage around the environment the method of geometric optics can be used.

Results

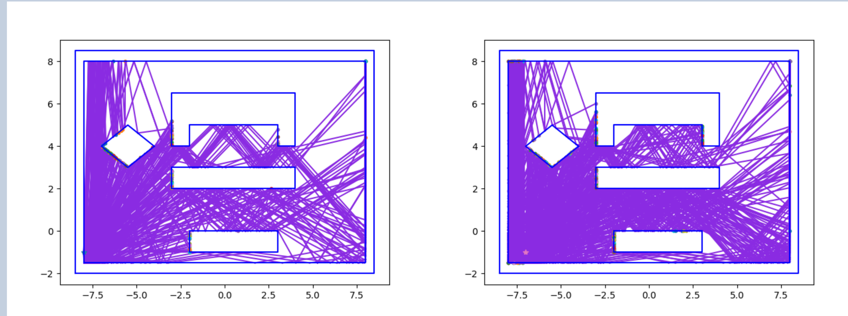


Figure 4: The rays propagating from the transmitter and reflecting from two different sources.

- In Figure 4 the method of geometric optics has been applied to Figure 3 to visualise the coverage. This has been applied to two different source locations.
- It is also important to get an idea of the signal strength.
- This can be calculated along the trajectory of the ray taking into account the attenuation from the distance travelled and from the interactions.
- Figure 5 visualises this signal strength coverage.
- The signal strength can be calculated along the trajectory of the ray to determine the signal coverage.

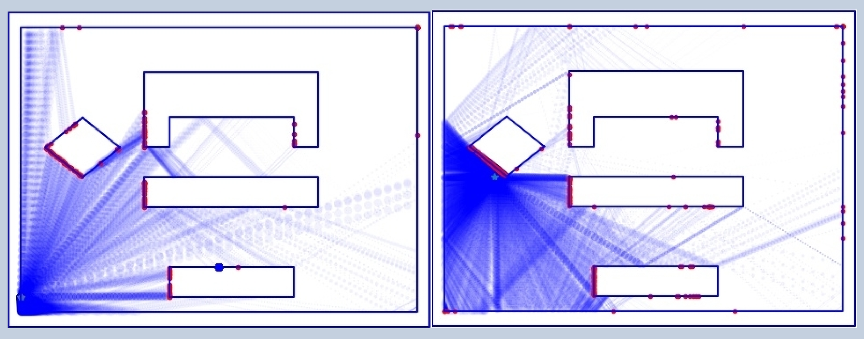


Figure 5: The signal strength along the ray trajectories.

Future Work

To improve the estimations further, consider other wave interactions such as diffraction, refraction and scattering.

Refraction often has a high attenuation [1], diffraction can be modelled using the geometric theory of diffraction [2], and scattering is the most expensive interaction to compute [3].

Another future consideration would be to extend the model from the 2 dimensional example to 3 dimensions.

References

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