An adapted ray-launching method for optimising wifi coverage.

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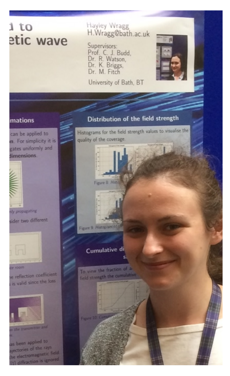


Overview of ray-launching methods

- Use **rays** to model electromagentic waves.
- **Rays** give the direction of the waves.
- Calculate the **field or power** using ray lengths.

Mathematical motivation

- A model for the field is the **Helmholtz equation**:

$$\nabla^2\boldsymbol{\phi}(\mathbf{x}) + \underbrace{k}_{\text{Wave number}}{}^2\boldsymbol{\phi}(\mathbf{x}) = 0. \qquad (1)$$

- Where k is the wave number given by

$$k = 2\pi(\lambda^{-1}) = 2\pi(\text{wavelength}^{-1})$$

- Using the WKB ansatz $\boldsymbol{\phi} = u(\mathbf{x})e^{ikS(\mathbf{x})}$,

$$k^{-2}\Delta u - \left(|\nabla S|^2 - 1 + i(k^{-1})\left(2(\nabla S\cdot\nabla) + \Delta S\right)\right)u = 0$$

Using the expansion $u = u_0 + k^{-1}u_1 + k^{-2}u_2 + \ldots$

$$O(k^2): \quad e^{ikS}\left(1 - |\nabla S|^2\right)u_0 = 0$$

$$O(k): e^{ikS}\left[\left(1-|\nabla S|^2\right)u_1 + i\left(2i\left(\nabla S\cdot\nabla\right)u_0 + \Delta S u_0\right)\right] = 0$$

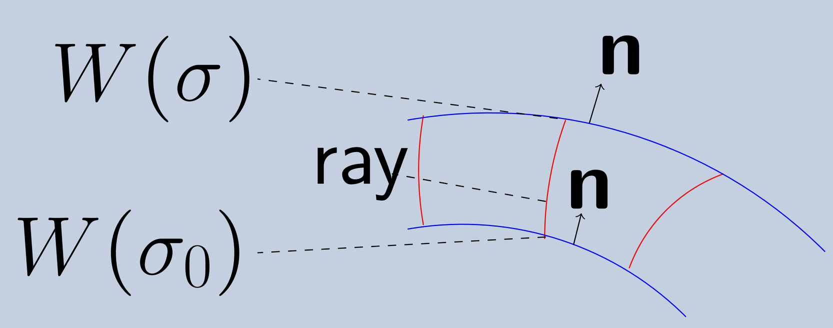
$$O(k^0): e^{ikS}\left[\Delta u_0 + \left(1-|\nabla S|^2\right)u_2 + i\left(2\left(\nabla S\cdot\nabla\right)u_1 + \Delta S\right)\right] = 0$$

Since $k \gg 1$, S satisfies the **Eikonal Equation**:

$$|\nabla S|^2 = 1 \Rightarrow |\nabla S| = 1 \qquad (3)$$

Definition (wavefront)

Wavefronts $W(\mathbf{x})$ of a solution to (1) are the surfaces with $S(\mathbf{x}) = constant$.



Definition (ray)

Rays parameterised by σ corresponding to a wavefront $W(\mathbf{x}(\sigma))$, are the curves orthogonal to $W(\mathbf{x}(\sigma))$.
i.e.Set $\gamma(\mathbf{x})$ to be an arbitrary proportionality factor. Then $S(\mathbf{x})$ and σ satisfy the orthogonality condition.

$$\frac{dx_j}{d\sigma} = \gamma(\mathbf{x})\frac{dS}{dx_j}, \quad j = 1,2,3. \qquad (4)$$

The equation of the ray is a straight line when $S(\mathbf{x})$ satisfies the Eikonal equation. The transport equations can then be used to find $u_0(\mathbf{x})$.

Why ultra-high frequency?

f / GHz			pros		cons
2.5	Wifi		well understood	penetrates walls	narrow bandwidth interference less reflection
5					
3.4	5G	4G			expensive
30			wide bandwidth low cost	reflects	doesn't penetrate walls still under study susceptible to roughness short range
60					

- New technologies enable **ultra-high frequency electromagnetic waves**.
- Old propagation models are designed for lower frequencies.
- This research focuses on a **domestic environment**.
- This model will then be used to optimise **the location of transmitters, the choice of transmitters, and the number of transmitters in the home**.

Trajectory approximations

- Take a template room and consider the ray paths from the transmitter.

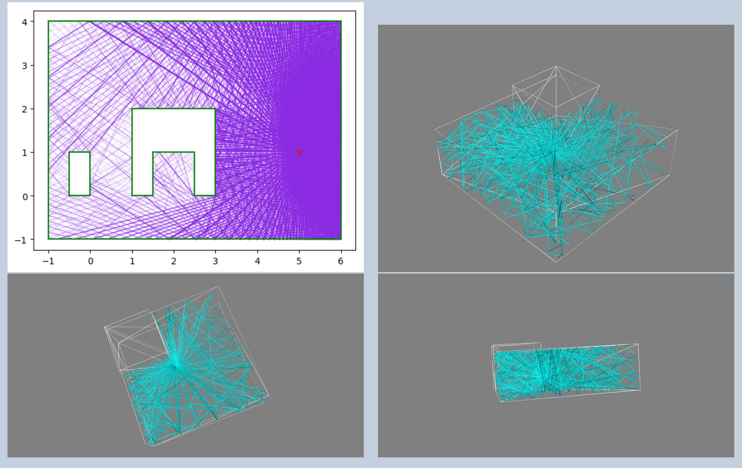
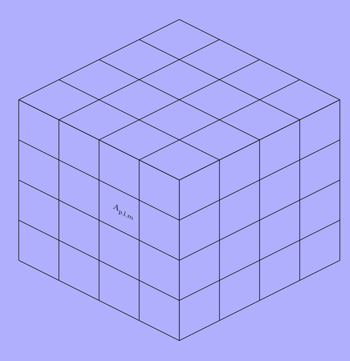


Figure 1

- In Figure (1) ray-launching has been applied to a 2D and 3D environment. The rays show the directions the electromagnetic wavefronts travel in.

Adapted ray-launching method

- **Storage structure**
  - Discretise the environment into N_x, N_y, and N_z pieces in the x, y and z directions so that the spacing in each direction is δ.
  - Each term in the mesh corresponds to a $(\underbrace{N_{Re}}_{\text{number of reflections}} * \underbrace{N_{Ra}}_{\text{number of rays}} + 1) \times (\underbrace{N_{Ob}}_{\text{number of obstacles}} * N_{Re} + 1)$ sparse matrix.



$$A_{p,l,m} = \begin{array}{cc} \begin{array}{ccccc} 0 & \cdots & N_{Re} & \cdots & N_{Re}*N_{Ra} \end{array} & \\ \begin{pmatrix} \ddots & \cdots & \cdots & \cdot^{\cdot^{\cdot}} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \cdot^{\cdot^{\cdot}} & \cdots & \cdots & \ddots \end{pmatrix} & \begin{array}{c} 0 \\ \vdots \\ N_{Ob} \\ \vdots \\ N_{Ob}*N_{Re} \end{array} \end{array}$$

- Each term in this matrix corresponds to a possible ray (N_{Ra}), obstacle (N_{Ob}) interaction after N_{Re} reflections.

- **Storing each ray**

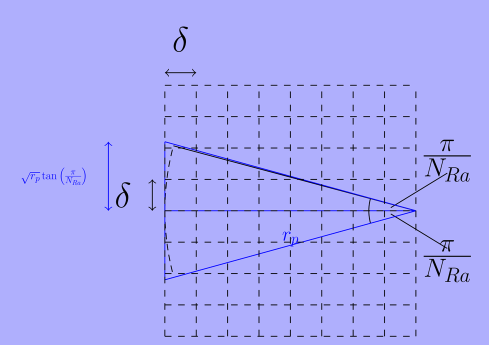


Figure 2: *Cone on mesh*

  - Form the ray cone.
  - Store the **ray reflection history** and the **distance** the ray travels to the centre point of the elements in the cone.

- **Storing the ray history**
  - Initialise a temporary vector $\boldsymbol{v}$ as empty with datatype complex and length $N_{Ob} * N_{Re} + 1$.
  - The obstacle number n_{Ob}, reflection angle θ_i, reflection number $n_{Re} \geq 1$ are stored by putting $(e^{j\theta_i})$ in the row $N_{Ob} * (n_{Re} - 1) + n_{ob}$ of $\boldsymbol{v}$.
  - If $n_{Re} = 0$ (i.e line of sight) then 1 is put in the 1^{st} row but is removed before the next reflection.
  -
$$\boldsymbol{v} = \begin{array}{c} 0 \\ \vdots \\ N_{Ob}*(n_{Re}-1)+n_{ob} \\ \vdots \\ N_{Ob}*N_{Re} \end{array} \begin{pmatrix} \vdots \\ \\ e^{j\theta_i} \\ \\ \vdots \end{pmatrix}$$

  Multiply $\boldsymbol{v}$ by the distance the ray has traveled. Then store in column $\underbrace{N_{Re}}_{\text{Maximum number of reflections}} * \underbrace{n_{Ra}}_{\text{current ray number}} + n_{Re}$ in the mesh element the ray has gone through.

- **Output**

$$A_{p,l,m} = \begin{array}{cc} \begin{array}{ccccc} 0 & \cdots & N_{Re}*n_{Ra}+n_{Re} & \cdots & N_{Re}*N_{Ra} \end{array} & \\ \begin{pmatrix} \ddots & & \vdots & & \cdot^{\cdot^{\cdot}} \\ & & \vdots & & \\ & \cdots & re^{j\theta_i} & \cdots & \\ & & \vdots & & \\ \cdot^{\cdot^{\cdot}} & & & & \ddots \end{pmatrix} & \begin{array}{c} 0 \\ \vdots \\ N_{Ob}*n_{Re}+n_{ob} \\ \vdots \\ N_{Ob}*N_{Re} \end{array} \end{array}$$

  - If $n_{Re} > 0$ the column $N_{Re} * n_{Ra} + n_{Re}$ must be either empty or contain n_{Re} terms.
  - If $n_{Re} = 0$ (i.e line of sight path) then then column is either empty or contains 1 term in the first row.
  - The grid of sparse matrices is returned by the algorithm.
  - To get the field or power combine this output with the parameters: dielectric properties of the obstacles, the antenna pattern, and the frequency.

Wifi coverage – Power evaluation

- Calculate the electric/magnetic field from rays using **free-space attenuation** and **loss at reflections**.
- **The Friis transmission formula** gives the field at the receiver ϕ_r in terms of the field at the transmitter ϕ_t, in free-space. A receiver distance r from the transmitter has field:

$$\underbrace{\phi_r}_{\text{Field at receiver}} = |\underbrace{\phi_0^*}_{\text{Field emitted at source}}|\sqrt{\underbrace{G_T G_R}_{\text{Gains of the antennas}}}\underbrace{\left(\lambda r^{-1}\right)}_{\text{Wavelength divided by distance}}\frac{e^{ikr}}{4\pi}.$$

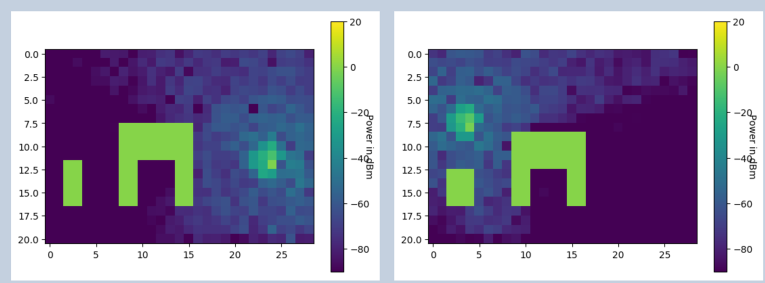
- The **Fresnel reflection coefficients[1]** gives the field after reflection[a],

$$\underbrace{\begin{bmatrix}\phi_{||}^{\text{ref}} \\ \phi_{\perp}^{\text{ref}}\end{bmatrix}}_{\text{Field strength after reflection}} = \underbrace{\begin{bmatrix}\Gamma_{||} & 0 \\ 0 & \Gamma_{\perp}\end{bmatrix}}_{\text{Fresnel reflection coefficients}}\underbrace{\begin{bmatrix}\phi_{||}^{\text{in}} \\ \phi_{\perp}^{\text{in}}\end{bmatrix}}_{\text{Field strength into reflection}}.$$

  $\Gamma_{||}$ & $\Gamma_{\perp}$ depend on the **permittivity (ϵ) & permeability (μ)** of the media and the angle of incidence and transmission; ϵ and μ vary with frequency.

- **Compute the power** from the mesh[b]:

1: AnglesMesh=angle(Mesh[nonzero terms])
2: RefMesh=Fresnel(AnglesMesh)
3: CombineRefMesh=$\prod_{\substack{row=r,\\ nonzero}}$ RefMesh$[r,c]$
4: FieldMesh= $\sum_{col=c}\frac{\sqrt{G_{T_c}G_R}}{|\text{Mesh}[c]|}$CombineRefMesh
5: FieldMesh=$|\boldsymbol{\phi}_0^*|\frac{\lambda}{4\pi}$ FieldMesh
6: PowerMesh=C_λ|FieldMesh|^2



(a) *First source location* (b) *Second source location*

Figure 3: *The power over the environment for two source locations.*

[a] $||$ & $\perp$ refer to the terms parallel to the polarisation & perpendicular to the polarisation respectively.
[b] $\boldsymbol{\phi}$ =the electric field $C_\lambda = \epsilon_0|\boldsymbol{\phi}|^2$, or $\boldsymbol{\phi}$ = the magnetic field $C_\lambda = \mu_0|\boldsymbol{\phi}|^2$

Comparison to other methods

	Standard ray-launching methods	Adapted ray-launching method	Wave solvers, e.g FDTD, FEM, BEM
Inputs	All environment information	Just the obstacle locations	All environment information in the form of boundary conditions
Finds reflection points	Yes	Yes	No
Mesh size	Independent of wavelength	Independent of wavelength	Proportional to the wavelength.
Iteration using rays	Calculate the field along the rays. Convert to power at the end.	Store the history of the ray	N/A
Output	Mesh of power values corresponding to the environment.	Mesh of sparse matrices corresponding to the ray histories.	Mesh of power values corresponding to the environment.
Vary the inputs	Repeat entire process.	Use same output, sub in new values on the power evaluation.	Repeat entire process.
Investigate sensitivity to objects	Many iterations of the whole method.	Apply a function for the derivative of the obstacle coefficient with respect to either ϵ or μ to the output grid.	Many iterations of the whole method.

References

[1] S Saunders and A Aragon-Zavala.
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[2] Yingli Wang and Stephen Pettit.
E-Logistics: Managing Your Digital Supply Chains for Competitive Advantage.
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-Background.