The ray-launching method applied to ultra-high frequency electromagnetic wave propagation.

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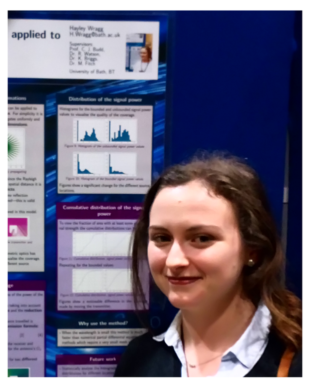

Application

- **Small wavelength wave propagation** can be used to model wifi and higher frequency electromagnetic propagation in the home.
- Developments in technologies have resulted in an increased demand for **ultra-high frequency electromagnetic propagation models**.
- This research focuses on developing a model for this ultra-high frequency propagation in a **domestic environment**.
- To get an idea of the coverage around the environment the method of ray-launching can be used.

Overview of the method of geometric optics

- The idea is to use **rays** to model electromagentic waves.
  - This approximation is good for small wavelengths.

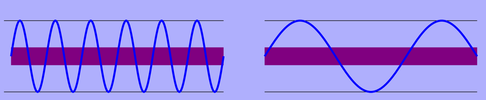

Figure 1: *Ray approximation for different wavelengths.*

- Map the paths of **the rays** in the environment to model the path of the waves.

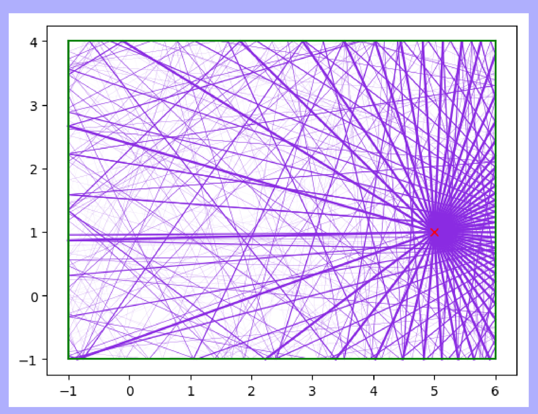

Figure 2: *Rays propagating within an environment.*

- Calculations are then made **along the rays** to determine information such as the field strength.

Mathematical motivation

- A model for electromagnetic field strength is the **Helmholtz equation**:

$$\nabla^2\phi(x) + \underbrace{k}_{\substack{\text{Wave}\\ \text{number}}}{}^2\phi(x) = 0. \qquad (1)$$

- Where k is the wave number given by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ frequency}}{\text{speed of light}}.$$

- Using the WKB ansatz $\phi = u(x)e^{ikS(x)}$,

$$\frac{1}{k^2}\Delta u - \left(|\nabla S|^2 - 1 + \frac{i}{k}\left(2(\nabla S\cdot\nabla) + \Delta S\right)\right)u = 0 \qquad (2)$$

Using the expansion $u = u_0 + \frac{1}{k}u_1 + \frac{1}{k^2}u_2 + \ldots$

$$O(k^2) \quad : \quad e^{ikS(x)}\left(1 - |\nabla S(x)|^2\right)u_0(x) \quad = \quad 0$$

$$O(k) : \quad e^{ikS(x)}\left(\left(1 - |\nabla S(x)|^2\right)u_1(x) + i\left(2i\left(\nabla S\cdot\nabla\right)u_0(x) + \Delta S(x)u_0(x)\right)\right) = 0$$

$$O(k^0) : e^{ikS(x)}\left(\Delta u_0(x) + \left(1 - |\nabla S(x)|^2\right)u_2(x) + i\left(2\left(\nabla S\cdot\nabla\right)u_1(x) + \Delta S(x)\right)\right) = 0.$$

Since $k \gg 1$, $S(x)$ satisfies the **Eikonal Equation**:

$$|\nabla S|^2 = 1 \Rightarrow |\nabla S| = 1 \qquad (4)$$

A solution to the Eikonal equation is that the path $S(x)$ is a straight line.

Trajectory approximations

- The method of geometric optics can be applied to the problem of **wifi propagation**. For simplicity it is assumed that the antenna propagates uniformly and the domain is modelled in **two dimensions**.

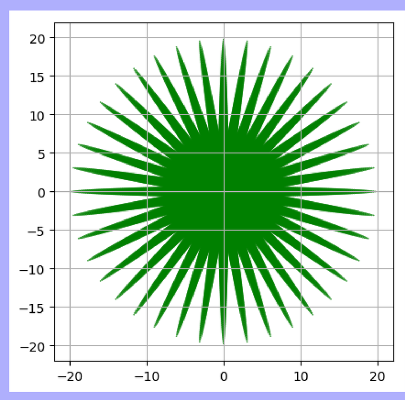

Figure 3: *Antenna uniformly propagating*

- Take a template room and consider two different source locations.

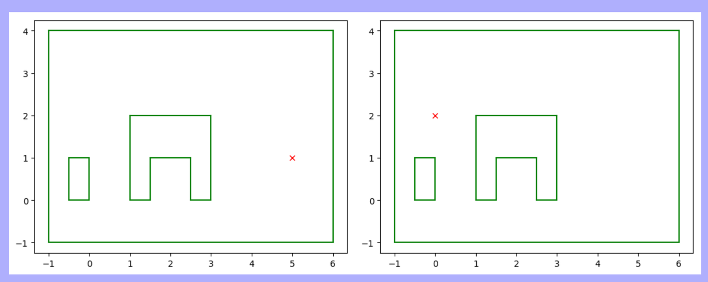

Figure 4: *Template room*

- Objects are also given the same reflection coefficient and refraction is ignored—this is valid since the loss is very high.

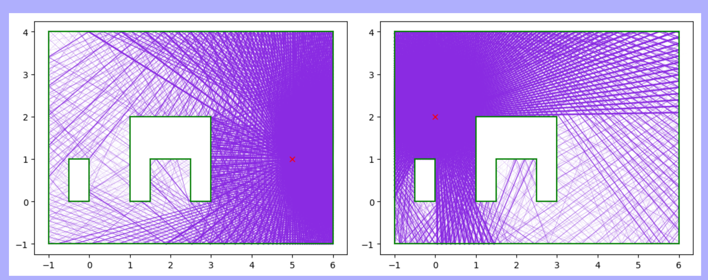

Figure 5: *The rays propagating from the transmitter and reflecting from two different sources.*

- In Figure (5) ray-launching has been applied to Figure (4) to visualise the trajectories of the rays approximating the travel of the electromagnetic field. Using Hewett's argument in [1] diffraction is ignored. This has been applied to two different source locations.

Wifi coverage - Ignoring phase

- The strength of the field is calculated in two parts accounting for the **attenuation in free space** and the **reduction at the reflections**.
- Let u_r, u_t be the field strength at the receiver and transmitter relatively. **The Friis transmission formula** gives the loss resulting from the spread from the antenna:

$$\frac{|u_r|}{|u_t|} = \sqrt{G_a G_b}\left(\frac{\lambda}{4\pi r}\right), \qquad [2] \qquad (5)$$

The antenna has no preferred direction so the gains for the antenna's G_a and G_b are 1.

- Figure 6 shows the signal strength for two **different transmitter locations**.

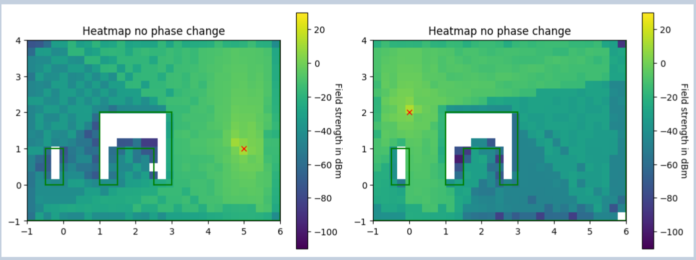

Figure 6: *The signal power over the environment*

Wifi coverage - With random phase

- To account for the destruction from phase difference the magnitude of the field is multiplied by a random complex number with magnitude 1 at reflection.

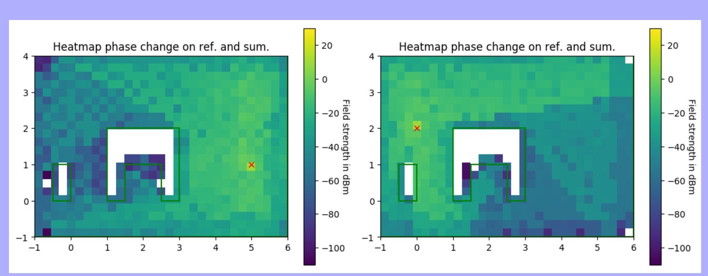

Figure 7: *The field strength with random phase*

Distribution of the field strength

Histograms for the field strength values to visualise the quality of the coverage.

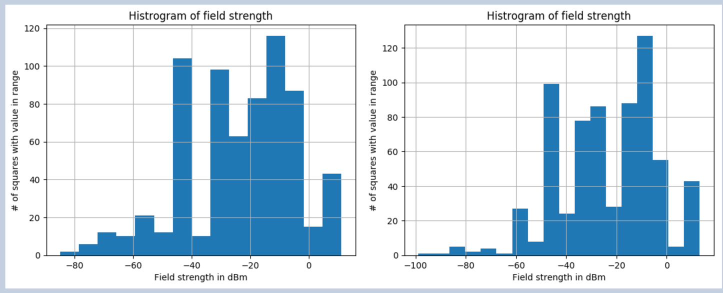

Figure 8: *Histogram of the field strength values*

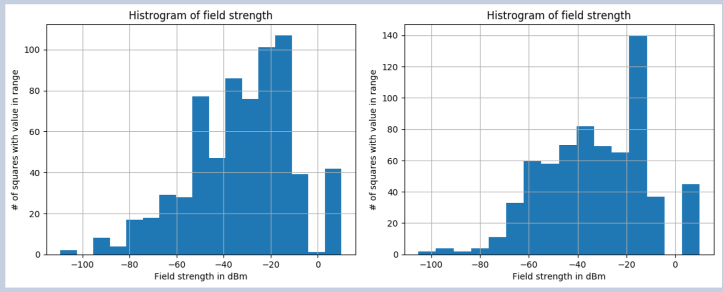

Figure 9: *Histogram of the field strength values with random phase*

Cumulative distribution of the field strength

To view the fraction of area with at least some given field strength the cumulative distributions can be used.

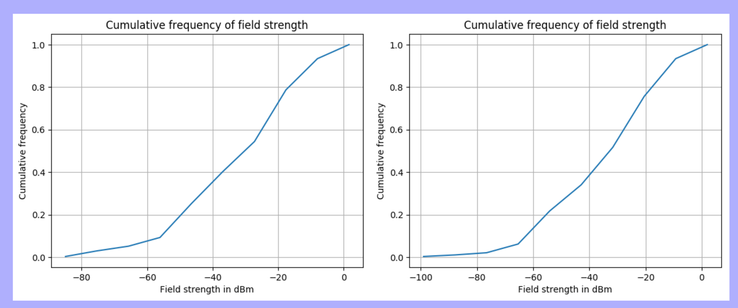

Figure 10: *Cumulative distribution, signal power values unbounded*

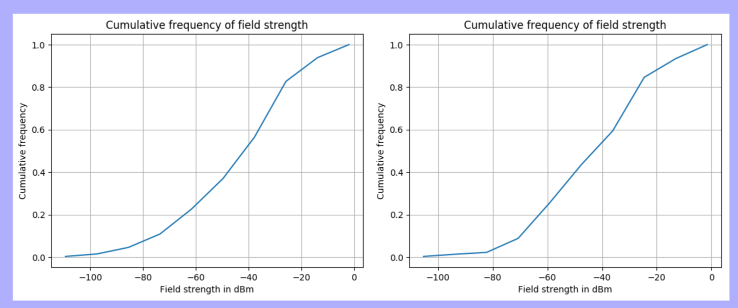

Figure 11: *Cumulative distribution, field strength values with random phase*

Figures (10) & (11) show a difference in the area of medium coverage made by moving the transmitter.

Why use the method?

- When the wavelength is small this method is much faster than numerical partial differential equation methods which require a very small mesh size.

Future work

- Analyse the distribution results for different transmitter locations and environments.
- Consider the probability distribution for the phase.
- Take into account different reflection coefficients and consider a probability distribution for these.

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