# The method of geometric optics, applied to wifi propagation.

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## Overview of the method of geometric optics

- The idea is to use **rays** to model waves.

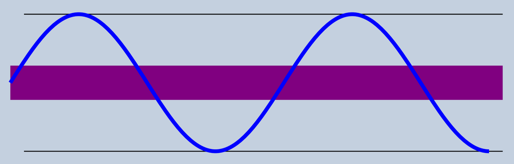

Figure 1: *Straight line (ray) approximation of a wave.*

- This approximation is good for small wavelengths.

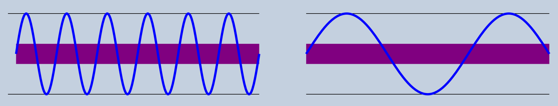

Figure 2: *Ray approximation for different wavelengths.*

- Map the paths of **the rays** in the environment to model the path of the waves.

Rayfield.png

Figure 3: *Rays propagating within an environment.*

- Calculations are then made **along the rays** to determine information such as power.

## Mathematical motivation

- A time-independent model for wave propagation is the **Helmholtz equation**:

$$\nabla^2\phi(x) + \underbrace{k}_{\text{Wave number}}{}^2\phi(x) = 0. \tag{1}$$

- Where $k$ is the wave number given by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c},$$

$f$ is the frequency of the wave, and $c$ the velocity.

- Assuming that $\phi(x)$ can be written as a function of the path $S(x)$ in the form $\phi = u(x)e^{ikS(x)}$ then $u$ satisfies,

$$\frac{1}{k^2}\nabla^2 u - u\left(|\nabla S|^2 - 1\right) = 0 \tag{2}$$

Since $k \gg 1$, $S(x)$ satisfies the **Eikonal Equation**:

$$|\nabla S|^2 = 1 \Rightarrow |\nabla S| = 1 \tag{3}$$

The constant gradient means that a ray is a valid approximation for the wave propagation when $k \gg 1$.

## Applications

- **Small wavelength propagation** can be used to model radio propagation in cities, and wifi propagation in the home.
- Developments in technologies have resulted in an increased demand for **high frequency wifi propagation models**.

Figure 4: *Template domestic environment projected into two dimensions*

- Figure (4) gives an example simulated environment.
- To get an idea of the coverage around the environment the method of geometric optics can be used.

## Trajectory approximations

- The method of geometric optics can be applied to the problem of **wifi propagation**. For simplicity it is assumed that the antenna propagates uniformly and the domain is modelled in **two dimensions**.

Figure 5: *Antenna uniformly propagating*

- Using Hewett's argument in [1] since the Rayleigh distance is much larger than the spatial distance it is valid to **ignore diffraction effects**.
- Objects are all also given the same reflection coefficient and refraction is ignored—this is valid since the loss is very high.
- Phase change has also been ignored in this model.

Figure 6: *The rays propagating from the transmitter and reflecting from two different sources.*

- In Figure (6) the method of geometric optics has been applied to Figure (4) to visualise the coverage. This has been applied to two different source locations.

## Wifi coverage - Ignoring phase

- It is also important to get an idea of the power of the **wifi coverage**.
- This is calculated along the rays taking into account the **attenuation in free space** and the **reduction at the reflections**.
- The loss resulting from the distance travelled is calculated using the **Friis transmission formula**:

$$\frac{u_r}{u_t} = G_a G_b \left(\frac{\lambda}{4\pi r}\right)^2, \quad [2] \tag{4}$$

where $u_r$, $u_t$ give the power at the receiver and transmitter relatively. The gain for the antenna's $G_a$ and $G_b$ are taken to be $1$.

- Figure 7 shows the signal power for two **different transmitter locations**.

Figure 7: *The signal power over the environment*

## Distribution of the signal power

Histograms for the bounded and unbounded signal power values to visualise the quality of the coverage.

Figure 8: *Histogram of the unbounded signal power values*

Figure 9: *Histogram of the bounded signal power values*

Figures show a significant change for the different source locations.

## Cumulative distribution of the signal power

To view the fraction of area with at least some given signal strength the cumulative distributions can be used.

Figure 10: *Cumulative distribution, signal power values unbounded*

Repeating for the bounded values:

Figure 11: *Cumulative distribution, signal power values bounded*

Figures show a noticeable difference in the coverage made by moving the transmitter.

## Why use the method?

- When the wavelength is small this method is much faster than numerical partial differential equation methods which require a very small mesh size.

## Future work

- Statistically analyse the histograms and cumulative distribtuions for different locations of transmitters in different environments.
- Account for phase change using random phases.
- Take into account different reflection coefficients.

## References


D. P. Hewett.
*Sound propagation in an urban environment.*
PhD thesis, Oxford University, 2010.

S Saunders and A Aragon-Zavala.
*Antennas and Propagation for Wireless Communication Systems.*
Manning Publications Co., Connecticut, USA, 2008.

Yingli Wang and Stephen Pettit.
*E-Logistics: Managing Your Digital Supply Chains for Competitive Advantage.*
Kogan Page Publishers, 2016.
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