The Vector Space problems

Coding the Matrix, 2015

`For auto-graded problems, edit the file The_Vector_Space_problems.py to include your solution.`

Warning: Avoid using the default arguments in the constructor for `Vec`.

Vectors in containers

Problem 1:

1. Write and test a procedure `vec_select` using a comprehension for the following computational problem:
   - *input:* a list `veclist` of vectors over the same domain, and an element k of the domain
   - *output:* the sublist of `veclist` consisting of the vectors `v` in `veclist` where `v[k]` is zero
2. Write and test a procedure `vec_sum` using the built-in procedure `sum(·)` for the following:
   - *input:* a list `veclist` of vectors, and a set `D` that is the common domain of these vectors
   - *output:* the vector sum of the vectors in `veclist`.

   Your procedure must work even if `veclist` has length 0.

   Hint: Recall from the Python Lab that `sum(·)` optionally takes a second argument, which is the element to start the sum with. This can be a vector.

   Disclaimer: The `Vec` class is defined in such a way that, for a vector `v`, the expression `0 + v` evaluates to `v`. This was done precisely so that `sum([v1,v2,... vk])` will correctly evaluate to the sum of the vectors when the number of vectors is nonzero. However, this won't work when the number of vectors is zero.
3. Put your procedures together to obtain a procedure `vec_select_sum` for the following:
   - *input:* a set `D`, a list `veclist` of vectors with domain `D`, and an element k of the domain
   - *output:* the sum of all vectors `v` in `veclist` where `v[k]` is zero

Problem 2: Write and test a procedure `scale_vecs(vecdict)` for the following:

- *input:* A dictionary `vecdict` mapping positive numbers to vectors (instances of `Vec`)
- *output:* a list of vectors, one for each item in `vecdict`. If `vecdict` contains a key k mapping to a vector $\boldsymbol{v}$, the output should contain the vector $(1/k)\boldsymbol{v}$

Constructing the span of given vectors over $GF(2)$

Problem 3: Write a procedure `GF2_span(D, S)` with the following spec:

- *input:* a set D of labels and a set S of vectors over $GF(2)$ with label-set D
- *output:* the set of all linear combinations of the vectors in S

(Hint: use a loop (or recursion) and a comprehension. Be sure to test your procedure on examples where S is an empty set. This problem is a bit challenging but there is a short solution. If you find it too difficult; don't worry. Just move on. It's not really a linear-algebra problem; it's more of a programming puzzle.

Vector spaces

Problem 4: Is the following statement true or false? "$\{[x, y, z] : x, y, z \in \mathbb{R}, x + y + z = 1\}$ is a vector space."

Problem 5: Is the following statement true or false? "$\{[x, y, z] : x, y, z \in \mathbb{R}$ and $x + y + z = 0\}$ is a vector space."

Problem 6: Is the following statement true or false? "$\{[x_1, x_2, x_3, x_4, x_5] : x_1, x_2, x_3, x_4, x_5 \in \mathbb{R}, x_2 = 0$ or $x_5 = 0\}$ is a vector space."

Problem 7:

1. Let $\mathcal{V}$ be the set of 5-vectors over $GF(2)$ that have an even number of 1's. Is the following statement true or false? "$\mathcal{V}$ is a vector space."
2. Let $\mathcal{V}$ be the set of 5-vectors over $GF(2)$ that have an odd number of 1's. Is the following statement true or false? "$\mathcal{V}$ is a vector space."