

The Vector problems

Coding the Matrix, 2015

For auto-graded problems, edit the file `The_Vector_problems.py` to include your solution.

Vector Addition Practice

Problem 1: For vectors $v = [-1, 3]$ and $u = [0, 4]$, find the vectors $v + u$, $v - u$, and $3v - 2u$.

Problem 2: Given the vectors $v = [2, -1, 5]$ and $u = [-1, 1, 1]$, find the vectors $v + u$, $v - u$, $2v - u$, and $v + 2u$.

Problem 3: For the vectors $v = [0, one, one]$ and $u = [one, one, one]$ over $GF(2)$, find $v + u$ and $v + u + u$.

Expressing one $GF(2)$ vector as a sum of others

Problem 4: Here are six 7-vectors over $GF(2)$:

a =	1100000	d =	0001100
b =	0110000	e =	0000110
c =	0011000	f =	0000011

For each of the following vectors u , find a subset of the above vectors whose sum is u , or report that no such subset exists. You should be able to do this without the help of a computer.

1. $u = 0010010$
2. $u = 0100010$

Problem 5: Here are six 7-vectors over $GF(2)$:

a =	1110000	d =	0001110
b =	0111000	e =	0000111
c =	0011100	f =	0000011

For each of the following vectors u , find a subset of the above vectors whose sum is u , or report that no such subset exists.

1. $u = 0010010$
2. $u = 0100010$

Problem 6: (You should be able to solve this problem without using a computer.) Find a vector $\mathbf{x} = [x_1, x_2, x_3, x_4]$ over $GF(2)$ satisfying the following linear equations:

$$1100 \cdot \mathbf{x} = 1$$

$$1010 \cdot \mathbf{x} = 1$$

$$1111 \cdot \mathbf{x} = 1$$

Verify for yourself that $\mathbf{x} + 1111$ also satisfies the equations.

Problem 7: Consider the equations

$$2x_0 + 3x_1 - 4x_2 + x_3 = 10$$

$$x_0 - 5x_1 + 2x_2 + 0x_3 = 35$$

$$4x_0 + x_1 - x_2 - x_3 = 8$$

Your job is not to solve these equations but to formulate them using dot-product. In particular, come up with three vectors \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 represented as lists so that the above equations are equivalent to

$$\mathbf{v}_1 \cdot \mathbf{x} = 10$$

$$\mathbf{v}_2 \cdot \mathbf{x} = 35$$

$$\mathbf{v}_3 \cdot \mathbf{x} = 8$$

where \mathbf{x} is a 4-vector over \mathbb{R} .

Practice with Dot-Product

Problem 8: For each of the following pairs of vectors \mathbf{u} and \mathbf{v} over \mathbb{R} , evaluate the expression $\mathbf{u} \cdot \mathbf{v}$:

(a) $\mathbf{u} = [1, 0], \mathbf{v} = [5, 4321]$

(b) $\mathbf{u} = [0, 1], \mathbf{v} = [12345, 6]$

(c) $\mathbf{u} = [-1, 3], \mathbf{v} = [5, 7]$

(d) $\mathbf{u} = \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right], \mathbf{v} = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$