2.1 ID3

(a) By the formula:

$$H(Y) = -\sum_{i=1}^{k} P(Y=y_i)\log_2 P(Y=y_i)$$

$P(Y=+1) = \frac{12}{21} = \frac{4}{7}$ $P(Y=-) = \frac{9}{21} = \frac{3}{7}$

$H(Y) \approx 0.461 + 0.524 \approx 0.985$

(b)

$$H(Y|X_2) = -\sum_{j=1}^{v} P(X_2=x_j)\sum_{i=1}^{k} P(Y=y_i|X_2=x_j)\log_2 P(Y=y_i|X_2=x_j)$$

$P(X_2=T) = \frac{10}{21}$ $P(Y=+|X_2=T) = \frac{7}{10}$

$P(X_2=F) = \frac{11}{21}$ $P(Y=-|X_2=T) = \frac{3}{10}$

$P(Y=+|X_2=F) = \frac{5}{11}$

$P(Y=-|X_2=F) = \frac{6}{11}$

$H(Y|X_2) \approx 0.940$

$IG(X_2) = H(Y) - H(Y|X_2) \approx 0.0447$

$$H(Y|X_1) = -\sum_{j=1}^{v} P(X_1 = x_j) \sum_{i=1}^{k} P(Y = y_i | X_1 = x_j) \log_2 P(Y = y_i | X_1 = x_j)$$

$P(X_1 = T) = \frac{8}{21}$ $\quad P(Y = + | X_1 = T) = \frac{7}{8}$

$P(X_1 = F) = \frac{13}{21}$ $\quad P(Y = - | X_1 = T) = \frac{1}{8}$

$P(Y = + | X_1 = F) = \frac{5}{13}$

$P(Y = - | X_1 = F) = \frac{8}{13}$

$H(Y|X_1) \approx 0.738$

$IG(X_1) = H(Y) - H(Y|X_1) \approx 0.246.$

(c) Decision tree learned by ID3.