SMC Math Integration Club
Review of Previous Methods and Additional Basic Trigonometric Definitions

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Happy Halloween! Since it is Halloween, most of the problems you will see are reviews of previous methods. But a lot of them will be just as "scary" (hee-hee) as the integrals you encountered before. But no worries! With the power of Tangent Half-Angle Substitutions, King's rule, Loophole Logarithms, and Finding Derivatives, all of these will be cake!

Integrals before our techniques:

Integrals after our techniques:

First, let's view the new concept we are entering.
(To be honest, not necessarily new!)

$$\begin{aligned}(\cos(x)+\sin(x))^2 &= \cos^2(x)+2\cos(x)\sin(x)+\sin^2(x)\\ &= \cos^2(x)+\sin^2(x)+2\cos(x)\sin(x)\\ &= 1+\sin(2x)\end{aligned}$$

$$\begin{aligned}(\cos(x)-\sin(x))^2 &= \cos^2(x)-2\cos(x)\sin(x)+\sin^2(x)\\ &= \cos^2(x)+\sin^2(x)-2\cos(x)\sin(x)\\ &= 1-\sin(2x)\end{aligned}$$

Let's try some integrals with these new substitutions!

1) $\displaystyle \int \frac{(\cos(x)+\sin(x))^2+(\cos(x)-\sin(x))^2}{1-\sin^2(2x)}\,dx$

2) $\displaystyle\int \frac{-2\cos^2(x) + 2\sin^2(x)}{(\cos(x) - \sin(x))^2}\, dx$

3) $2\int \mathrm{e}^{x}(\cos(2x) + \frac{1}{2}(\cos(x) + \sin(x))^{2})\,dx$

4) $\int_0^{\pi/4} \frac{1-\cos^2(2x)}{x^{\frac{2\ln(\sin(x+\frac{\pi}{4})+\cos(x+\frac{\pi}{4}))}{\ln(x)}}})\,dx$

5) $\displaystyle\int \frac{x^{\frac{\ln(1-\sin(2x))}{\ln(x)}}}{(\cos(x)-\sin(x))^3}\,dx$