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1-1. $R_{translation}$

$$\begin{bmatrix} 1 & 0 & t \\ 0 & 1 & s \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+t \\ y+s \\ 1 \end{bmatrix},$$

$P_1 = (10, 20)$

after translation

$P_1 = (0, 0)$

$$\therefore \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -20 \\ 0 & 0 & 1 \end{bmatrix}$$

1-2. $R_{rotation}$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

$\theta = 30^\circ$

$$\therefore \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1-3. R

$R = R_{rotation} \cdot R_{translation}$

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -20 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & -5\sqrt{3}+10 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & -10\sqrt{3}-5 \\ 0 & 0 & 1 \end{bmatrix}$$

2-1. $R_1 \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi & -\sin\pi \\ 0 & \sin\pi & \cos\pi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$R_2 \Rightarrow \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R = R_2 \cdot R_1$ $\quad R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

2-2. $\underbrace{R_2 \cdot R_1}_{R} \cdot A = B$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ -20 \end{bmatrix}$$

2-3 $\overset{I}{\cancel{R_1^{-1} \cdot R_2^{-1} \cdot R_2}} \cdot R_1 \cdot A = R_1^{-1} \cdot R_2^{-1} \cdot B$

$R_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $\quad R_2^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R^{-1} \cdot R_2^{-1} = R' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

2-4 $B = [10, 0, 20]$ $\quad R' \cdot B = A$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ -20 \end{bmatrix}$$

3. Intrinsic Matrix

3-1

- focal length 4
- center (10, 20)
- unit aspect ratio
- no skew

$$\begin{bmatrix} 4 & 0 & 10 \\ 0 & 4 & 20 \\ 0 & 0 & 1 \end{bmatrix}$$

3-2

$$\begin{bmatrix} 4 & 0 & 10 & 0 \\ 0 & 4 & 20 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 12 \\ 1 \end{bmatrix} = \begin{bmatrix} 132 \\ 252 \\ 12 \end{bmatrix}$$

$$12 \begin{bmatrix} 132/12 \\ 252/12 \\ 12/12 \end{bmatrix} = 12 \times \begin{bmatrix} 11 \\ 21 \\ 1 \end{bmatrix} \quad \therefore \begin{bmatrix} 11 \\ 21 \\ 1 \end{bmatrix}$$

4. Extrinsic Matrix

4-1

Extrinsic Matrix

$$= [R \ t]$$

$$= \begin{bmatrix} 0.1 & 0.4 & 0.4 & 25 \\ 0.5 & 0.2 & 0.3 & 15 \\ 0.4 & 0.1 & 0.6 & 40 \end{bmatrix}$$

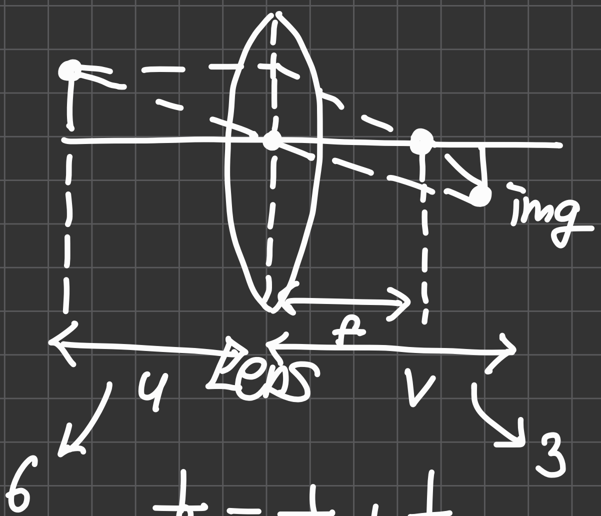
4-2

$$\begin{bmatrix} 0.1 & 0.4 & 0.4 & 25 \\ 0.5 & 0.2 & 0.3 & 15 \\ 0.4 & 0.1 & 0.6 & 40 \end{bmatrix} \begin{bmatrix} 80 \\ 40 \\ 100 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8+16+40+25 \\ 40+8+30+15 \\ 32+4+60+40 \end{bmatrix}$$

$$= \begin{bmatrix} 89 \\ 93 \\ 136 \end{bmatrix}$$

5 Focal Length



distance between an object & the lens $\Rightarrow 6$

distance between the image plane & the lens $\Rightarrow 3$

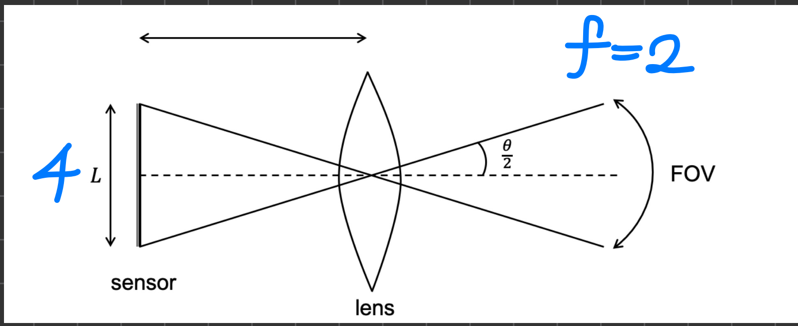
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{6} + \frac{1}{3}$$

$$= \frac{3}{6} = \frac{1}{2}$$

$\therefore f = 2$

6.



$$\text{AFOV}\ \frac{\theta}{2} = \tan^{-1}\left(\frac{4}{2\cdot 2}\right)$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

$$\text{AFOV}\ \theta = \frac{\pi}{4} \times 2 = \frac{\pi}{2}$$