[#1]

$$4x_1 + x_2 = 6$$
$$3x_1 + 2x_2 = 7$$

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \qquad A_1(b) = \begin{bmatrix} 6 & 1 \\ 7 & 2 \end{bmatrix} \qquad A_2(b) = \begin{bmatrix} 4 & 6 \\ 3 & 7 \end{bmatrix}$$

$$\det A = 8 - 3 = 5$$

$$x_1 = \frac{\det A_1(b)}{\det A} = \frac{12-7}{5} = 1$$

$$x_2 = \frac{\det A_2(b)}{\det A} = \frac{28-18}{5} = 2$$

[#2]

$$\begin{array}{c|c} \begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 11 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix} & \begin{matrix} -① \\ -② \\ -③ \\ -④ \end{matrix} \end{array}$$

4열로 여인수 전개

$$0 + 0 + 6(-1)^7 \begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 4 & 2 & 4 \end{vmatrix} + 3(-1)^8 \begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 11 & 4 & 6 \end{vmatrix} = -6 \begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 4 & 2 & 4 \end{vmatrix} + 3 \begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 11 & 4 & 6 \end{vmatrix}$$

$$\underbrace{\begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 4 & 2 & 4 \end{vmatrix}}_{A\text{라하면}} \sim \begin{bmatrix} -1 & 2 & 3 \\ 0 & 10 & 12 \\ 0 & 10 & 16 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 3 \\ 0 & 10 & 12 \\ 0 & 0 & -4 \end{bmatrix} \to (-1) \begin{bmatrix} 1 & -2 & -3 \\ 0 & 10 & 12 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\det(A) = -1 \begin{bmatrix} 1 & -2 & -3 \\ 0 & 10 & 12 \\ 0 & 0 & -4 \end{bmatrix} = (-1)(1)(10)(-4) = 40$$

$$\begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 11 & 4 & 6 \end{vmatrix} \sim \begin{bmatrix} -1 & 2 & 3 \\ 0 & 10 & 12 \\ 0 & 26 & 39 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 3 \\ 0 & 10 & 12 \\ 0 & 26 & 39 \end{bmatrix}$$

$$\sim \begin{vmatrix} 1 & -2 & -3 \\ 0 & 10 & 12 \\ 0 & 2 & 3 \end{vmatrix} \sim \begin{vmatrix} 1 & -2 & -3 \\ 0 & 2 & 3 \\ 0 & 0 & -3 \end{vmatrix}$$

행렬식

$$= (-1)(13) \begin{vmatrix} 1 & -2 & -3 \\ 0 & 2 & 3 \\ 0 & 0 & -3 \end{vmatrix}$$

$$= (-13)\underbrace{(1)(2)(-3)}_{6} = 78$$

$$\therefore \underline{-6(46)} + 3(78)$$

$$-240 + 234 = \boxed{-6}$$

[#3]

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} = A$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

사다리꼴 행렬이므로

$$|A| = (1)(-3)(-2) = 6$$

$$C_{11} = (-1)^2 \begin{vmatrix} -3 & 1 \\ 0 & -2 \end{vmatrix} = 6$$

$$C_{12} = (-1)^3 \begin{vmatrix} 0 & 1 \\ 0 & -2 \end{vmatrix} = -0$$

$$C_{13} = (-1)^4 \begin{vmatrix} 0 & -3 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{21} = (-1)^3 \begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = -(-4)$$

$$C_{22} = (-1)^4 \begin{vmatrix} 1 & 4 \\ 0 & -2 \end{vmatrix} = (-2)$$

$$C_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -0$$

$$C_{31} = (-1)^4 \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} = 1(2+12) = 14$$

$$C_{32} = (-1)^5 \begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = -1(1) = -1$$

$$C_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = 1(-3) = -3$$

즉 A의 수반행렬은

$$\begin{pmatrix} 6 & 0 & 0 \\ 4 & -2 & 0 \\ 14 & -1 & -3 \end{pmatrix}^T = \begin{pmatrix} 6 & 4 & 14 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\therefore \underbrace{A^{-1}}_{\text{역행렬}} = \frac{1}{|A|} adj(A) = \frac{1}{6} \begin{pmatrix} 6 & 4 & 14 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & \frac{2}{3} & \frac{7}{3} \\ 0 & -\frac{1}{3} & -\frac{1}{6} \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} = \text{A의 역행렬}$$