

학과	소프트웨어 공학과	학번	2016039069	성명	김은지
----	-----------	----	------------	----	-----

Note: 풀이과정이 없는 답은 0점 처리됨. 간결하고 읽을 수 있도록 정자체로 쓸것.

1. (page 173) 2.7 연습문제 2

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2,1)행은 영행으로 원점에 대각화시킨.

$$\text{계산} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{2\sqrt{3}+1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}-2}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

2. (page 173) 2.7 연습문제 10

사영행렬, D=자극 행렬

$$PD = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/10 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 2 \\ 1 & 4 & 2 \\ 4 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 6 & 2 \\ 1 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ \frac{6}{10} & \frac{8}{10} & \frac{4}{10} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 6 & 2 \\ 1 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0.6 & 0.8 & 0.4 & 1 \end{bmatrix}$$

3. v_1, v_2, v_3, u 가 다음과 같을 때, u 가 $\{v_1, v_2, v_3\}$ 에 의해 생성되는 \mathbb{R}^4 부분공간에 속하는지 판단하라.

$$v_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ -7 \\ 9 \\ 7 \end{bmatrix}, v_3 = \begin{bmatrix} 5 \\ -8 \\ 6 \\ 5 \end{bmatrix}, u = \begin{bmatrix} -4 \\ 10 \\ -7 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 5 & -4 \\ -2 & -7 & -8 & 10 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 5 & -5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & -7 & -14 & 9 \\ 0 & -5 & -10 & 17 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 23 \\ 0 & 0 & 0 & 17 \end{bmatrix} \rightarrow \text{행렬가짜리 X}$$

\therefore 속하지 않음

4. (page 182) 2.8 연습문제 13

$$A = \begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \\ 3 & -6 & 6 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} = [A \ 0]$$

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 0 \\ 3x_3 + 6x_4 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 2x_2 - 3x_3 - 4x_4 = 0 \\ x_3 = -2x_4 \\ x_2, x_4 \text{는 자유변수} \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_1 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_4$$

$$= \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_4$$

이것 u, v, w 는 $\text{Nul } A$ 의 기저이다.

$$\begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

주요열 $\rightarrow \therefore \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ 은 $\text{Col } A$ 의 기저를 이룬다.

$$\text{Col } A = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

5. (page 189) 2.9 연습문제 4

~~$$X = a \begin{bmatrix} 3 \\ 0 \end{bmatrix} + b \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$~~

$$X = a \begin{bmatrix} 3 \\ 0 \end{bmatrix} + b \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$[X]_B = \begin{bmatrix} a \\ b \end{bmatrix} \text{ 일 때}$$

$$= \begin{bmatrix} 3a - b \\ 2b \end{bmatrix}$$

$$\begin{aligned} 3a - b &= 4 \\ 2b &= 1 \end{aligned}$$

$$\begin{aligned} b &= 0.5 \\ a &= 1.5 \end{aligned}$$

$$[X]_B = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}$$

~~$$X = a \begin{bmatrix} 3 \\ 0 \end{bmatrix} + b \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$~~

$$[W]_B = \begin{bmatrix} x \\ y \end{bmatrix}_B$$

$$\begin{aligned} 3x - y &= 1 \\ 2y &= -2 \end{aligned}$$

$$\begin{aligned} x &= 2 \\ y &= -1 \end{aligned}$$

$$[W]_B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

~~$$\text{2.9.1.1. } b_1, b_2 \text{ 0.3 단계를 평면의 } \frac{1}{2}x + y = \frac{3}{2}$$~~

$$1.5b_1 + 0.5b_2 = 1.5 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + 0.5 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} = X \text{ (계산 결과일지)}$$

$$\sim \begin{bmatrix} 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

6. (page 189) 2.9 연습문제 7

$$A = \begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 1 \end{bmatrix} \text{이 2차원}$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 10 & -9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

3차원

Nul A 3차원

$$\begin{bmatrix} 1 & -3 & 2 & -4 & 0 \\ 0 & 0 & 5 & -1 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 & -4 & 0 \\ 0 & 0 & 5 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^3$$

$$\begin{cases} x_1 = 3x_2 \\ x_3 = 0 \\ x_4 = 0 \end{cases} \quad x_2 \text{ 자유변수}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \text{기저는 } \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ 이 됨, } \therefore \text{Null } A = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

∴ ~~부분공간~~ 1개인 ~~Null~~ Null A를 가짐, $\frac{3}{2}$, Null A의 차원은 1이다,

$$\frac{1}{2} \text{ 차원이다, } \dim H = 1$$