

[#1] ABC 행은  $x, y, z$ 를 구하라

태워

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} \text{를 계산하면}$$

$$AI + BX = 0$$

$$BX = I$$

$$CI = Z$$

$I$ 는 항등행렬이므로

$$CI = Z \quad \therefore C = Z \text{ 이다.}$$

$$\therefore Z = C$$

$BX = I$  이므로  $BX$ 는 서로 역행렬인 관계이다

$$X = -B^{-1}A$$

$$Y = B^{-1}$$

$$\therefore X = B^{-1} \text{ 이다}$$

$$\therefore Z = C$$

$$AI + BX$$

$$= -A + BX = 0 \text{ 이고}$$

$$BX = -A \quad B^{-1}BX = -B^{-1}A$$

$$\cancel{X = -B^{-1}A} \quad \therefore X = -B^{-1}A$$

[#2] LU 분해 방식을  $AX=b$ 를 풀어야

$$A = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

Lower triangular  
= L

상삼각행렬  
= U

$$[Lb] = \begin{bmatrix} 1 & 0 & 0 & 2 \\ -1 & 1 & 0 & -4 \\ 2 & 0 & 1 & 6 \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \end{matrix}$$

②에 ①+②

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 2 & 0 & 1 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix} = [I \quad Y]$$

$UX=Y$  풀면

$$[UY] = \begin{bmatrix} 4 & 3 & -5 & 2 \\ 0 & -2 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{matrix}$$

②, ③ ÷ 2

$$\sim \begin{bmatrix} 4 & 3 & -5 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 3 & -5 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 0 & -5 & -8 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow X = \begin{bmatrix} -\frac{3}{4} \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} \frac{3}{4} \\ 2 \\ 1 \end{bmatrix}$$

[#3] 주어진 행렬의 LU분해를 구하라

$$\begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}$$

A는 3x3행렬.

L=3x3행렬이다. -가 맨왼쪽 주대각.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & * & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \boxed{-5} & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix} \begin{matrix} -\textcircled{1} \\ -\textcircled{2} \end{matrix}$$

②에 ①x2+②

$$\sim \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 15 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} -5 & 3 & 4 \\ 0 & \boxed{-2} & -1 \\ 0 & 10 & 14 \end{bmatrix}$$

$$\sim \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & \boxed{9} \end{bmatrix} = U$$

□부분 원소가 A → U 3번째 행을 양으로 변경

주대각으로 □부분 원소를 만든다

$$\begin{bmatrix} -5 \\ 10 \\ 15 \end{bmatrix} \begin{bmatrix} -2 \\ 10 \end{bmatrix} \begin{bmatrix} 9 \end{bmatrix}$$

÷-5     ÷-2     ÷9

$$\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \begin{bmatrix} 1 \\ -5 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$$

↓     ↓     ↓

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & * & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix} = L.$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix}$$

∴

$$U = \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$$