Random Numbers in R

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Random Numbers

Random Number Generators (RNGs)

- (*Pseudo*-)Random number generators in Base R

```
RNGkind(kind = "default", normal.kind = NULL)
set.seed(seed)  # i.e., seed <- .Random.seed

runif(n)        # or: rnorm(n); rexp(n)
sample(x, size, replace = FALSE, prob = NULL)
```

Wichmann-Hill: $6.9 \cdot 10^{12}$; Marsaglia-Multicarry: $1.1 \cdot 10^{18}$
Super-Duper: $4.6 \cdot 10^{18}$; **Mersenne-Twister**: $\approx 10^{6000}$
Knuth-TAOCP-2002: $6.8 \cdot 10^{38}$; L'Ecuyer-CMRG: $3.1 \cdot 10^{57}$

- Recommended 'help' pages:

```
?Random       # details on RNG in R, 'kinds', 'seeds',
?Random.user  # user-supplied random number generation
```

dqrng and qrng Packages

- **dqrng**: Fast pseudo-random number generator

  ```
  dqrunif()`, `dqrnorm()`, `dqrexp()
  dqset.seed()`, `dqRNGkind(kind = "Mersenne-Twister")
  ```

  64-bit Mersenne-Twister, pcg64,
  Xoroshiro128+, Xoshiro256+ (defaults in Erlang and Lua),
  Threefry (64 bit engine provided by **sitmo**)

- **qrng**: *Quasi*-random numbers in high dimensions

  ```
  korobov(n, d = 1, generator, randomize = FALSE)
  ghalton(n, d = 1, method = c("generalized", "halton"))
  sobol  (n, d = 1, randomize = FALSE, skip = 0)
  ```

  Developed specifically for Monte-Carlo applications

Pseudo, quasi, and true RNGs

- *Pseudo-random numbers*
  are sequences of numbers whose statistical properties approximate the properties of theoretical random number sequences.
- *Quasi-random numbers*
  are 'low-discrepancy sequences', that is the proportion of numbers falling into an arbitrary subset is close to the measure of that subset.
- *True random numbers*
  are generated from physical processes that are known to behave like statistically random 'noise' signals.

True Random Number Generators

- **random**
  RANDOM.ORG "samples atmospheric noise via radio tuned to an unused broadcasting frequency together with a skew correction algorithm by John von Neumann."

```
library(random); N = 10000  # maximum request
rn <- randomNumbers(n = N, min = 0, max = N, col = 2)/N
```

- **qrandom**
  ANU Quantum Random Number Generator "generates true random numbers in real-time by measuring the quantum fluctuations of the vacuum."

```
library(qrandom); N = 10000  # maximum request: 10^5 [1
rn <- qrandomunif(n = N, a = 0, b = 1)
```

Generate Random Distributions

If u are uniformly distributed random numbers (in $[0, 1]$) and F is a *cumulative distribution function*, then the numbers $F^{-1}(u)$ are random numbers in this statistical distribution.

Example: Normal (Gaussian) distribution
(with mean = 0.0 and sd = 1.0)

```
x  <- runif(1000)
xn <- qnorm(x)       # qnorm() is the inverse of pnorm(
summary(xn)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -2.83681 -0.68305  0.06580  0.04031  0.72586  2.87980
```

Alternative: Ziggurat algorithm

More RNGs in Packages

- **randaes** (2012)
  cryptographic random number generator, based on AES
- **rngwell19937** (2014)
  long period linear random number generator WELL19937a
- **rstream** (2017)
  streams of random numbers from different sources
- **Tinflex** (2017)
  generator for arbitrary distributions with piecewise twice differentiable densities
- **UnivRNG** and **MultiRNG** (2018)
  uni-/multivariate random number generation for quite a number of different distributions

User-defined RNGs and Tools

- *?Random.user*
  "Function `RNGkind()` allows user-coded uniform and normal random number generators to be supplied."

  ```
  dyn.load("<user.lib>")
  RNGkind(kind = "user-supplied")
  ```

- **randtoolbox**
  Toolbox for pseudo and quasi random number generation
- **rngtools**
  Utility functions for working with RNGs
- **setRNG**
  for compatibility with former S versions

How to Write your own RNG in R?

- Congruential random number generation

$$x_{i+1} = (ax_i + c) \bmod m$$

```
e.g., m = 2^32,     a = 1103515245, c = 12345
or    m = 2^31 - 1, a = 48271,      c = 0     (Lehmer R
```

- Knuth-TAOCP-2002

$$x_i = (x_{i-37} + x_{i-100}) \bmod 2^{30}$$

(and discard the first 2000 numbers)

See also the **randtoolbox** vignette, Dutang and Würtz (2009)
A note on random number generation

Knuth-TAOCP-2002 – an R Implementation

```
randTAOCP <- function(seed = NULL) {
    local({
        R <- vector(mode = "numeric", length = 2000)
        R[1:100] <- qrandom::qrandomunif(n = 100, a = 0, b
        for (k in 101:2000)
            R[k] <- (R[k-37] + R[k-100]) %% 1
        k <- 2000; i <- 2000 - 37; j <- 2000 - 100
        frand <- function() {
            k <<- (k %% 2000) + 1
            i <<- (i %% 2000) + 1
            j <<- (j %% 2000) + 1
            z <- (R[i] + R[j]) %% 1
            R[k] <<- z
            return(z)
        }
        return(frand)
    })
}
```

Tests for RNGs

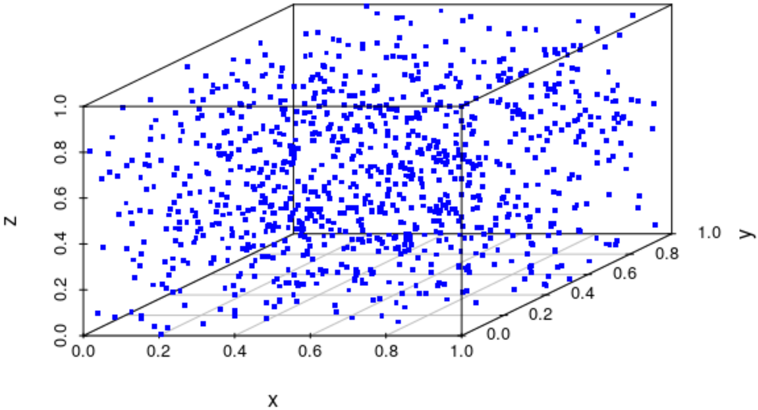
Testing Random Number Generators

- **RDieHarder**
  R Interface to the 'DieHarder' RNG Test Suite

  Not even 'Mersenne Twister' satisfies all these tests!
- Simple RNG tests, e.g.
  - Spectral test in d dimensions
  - Permutation rank distribution
  - Monte Carlo value for π
  - 'Greatest Common Divisor' test
  - Birthday spacing test
  - *Random Walk* tests

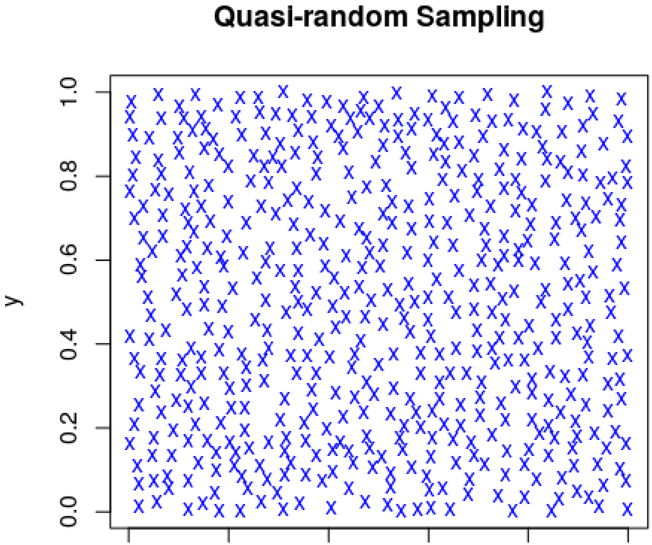
Example: 3D Spectral Test

Search for lattice structure (in all dimensions)



Example: Image Sampling

Mitchell's best-candidate algorithm for Poisson disk distribution



Example: Random Walks

“Irrfahrten und ihre Folgen”

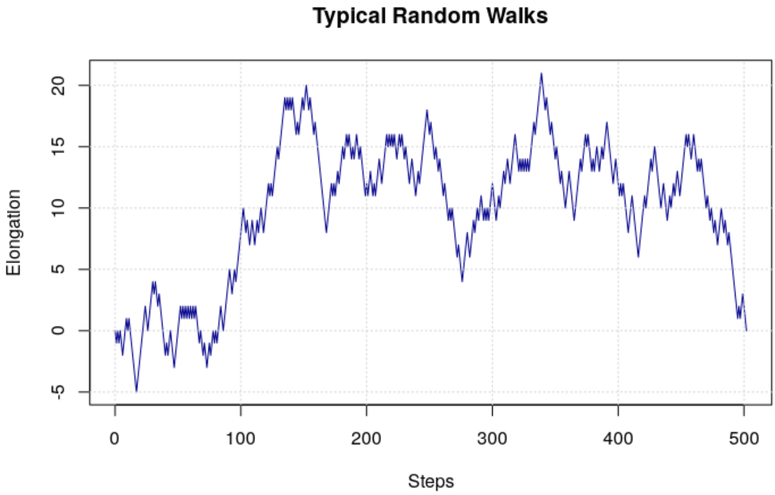
Definition (Pearson 1905)
A **random walk** consists of a succession of random steps on some discrete grid. An elementary example is the **symmetric** random walk on the integers that starts at 0 and at each step moves $+1$ or 1 with equal probability.

Theorem (Polya 1921).
A symmetric random walk in one or two dimensions will return to its starting point almost certainly *(i.e., with probability 1)*.

Applications in
Queing models, Brownian motion, stock markets, animal behavior, risk analysis, diffusion processes, game theory, ...

Random walks are fundamental for Markov processes.

Visualization of Random Walks



Original Project Idea

Goal

- Generate a million or so example curves, starting and ending in 0, by smoothing enough random walks (splines, etc.)
- Store these curves in appropriate databases
- Apply **Functional Data Analysis** (FDA) methods to classify, compare by similarity, and retrieve similar curves

Problem

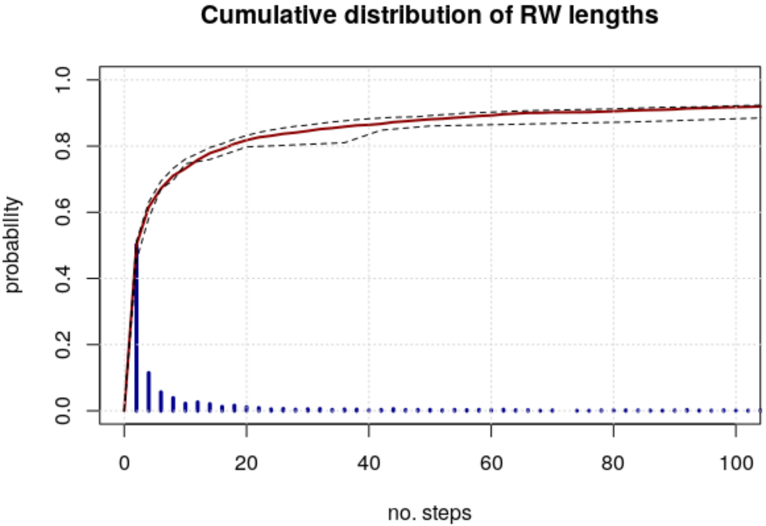
- Find enough nontrivial random walks returning to 0
- **Or**: What is the probability that a random walk returns to 0 after at most n steps?

Random Walks Step-by-Step

```
rwalk <- function(N, M) {
    result <- rep(0, N)
    for (i in 1:N) {
        steps <- 2
        a <- if (dqrunif(1) >= 0.5) 1 else -1
        a <- a + if (dqrunif(1) >= 0.5) 1 else -1
        while (a != 0) {
            steps <- steps + 2
            a <- a + if (dqrunif(1) >= 0.5) 1 else -1
            a <- a + if (dqrunif(1) >= 0.5) 1 else -1
            if (steps >= M) break
        }
        result[i] <- steps
    }
    result
}
```

Discussion on other, more compact approaches ?

Probability Distribution of RWs



Derive Minimum No. of Steps

```r
N <- 10000; M = 2048
result <- numeric(100)
for (i in 1:100) {                      # 100 simulation runs
    no_steps <- rwalk(N, M)             # vector of step lengths
    r <- rle(sort(no_steps))            # 'run length encoding'
    x <- r$values                       # steps
    y <- cumsum(r$lengths)/N            # probability
    ind <- which(y > 0.975)[1]          # where is p > 0.975
    result[i] <- x[ind]                 # store no. of steps
}

summary(result)
##  ...
```

Repeat this for different uniform RNGs in `rwalk()`

Simulation Results

Simulate 100 times and compute the 97.5% level:
10000 random walks – stopping at length 2048

```
# with `runif()`
> summary(result)
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    684     959    1018    1038    1120    1476

# with `dqrunif()`
> summary(result)
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    752     941    1013    1025    1108    1320

# with `randTAoCP()`
> summary(result)
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    806     944    1003    1026    1098    1302
```

Theory of Random Walks

The probability for returning to zero for the first time after exactly 2n steps is:

$$P(W = 2n) = \binom{2(n-1)}{n-1} \frac{1}{2^{2(n-1)}} \frac{1}{2n}$$

```
n <- 1:512
a <- choose(2*(n-1), n-1)/2^(2*(n-1))/(2*n)
w <- c(0, cumsum(a))
cbind(2*c(510:512), w[510:512])
```

```
##      [,1]      [,2]
## [1,] 1020 0.9749989
## [2,] 1022 0.9750234
## [3,] 1024 0.9750478
```

Remark about the $P = 0.99$ Case

`choose()` does not work for bigger numbers.
We need to aply the 'arbitrary-precision' package **gmp**.

```
n <- 1:3185
b2 <- as.bigz(2)
A <- chooseZ(b2*(n-1), n-1)/(b2^(b2*(n-1))*(b2*n))
W <- c(0, cumsum(as.numeric(A)))
cbind(2*c(3182:3185), W[3182:3185])
## [1,] 6364 0.9899971
## [2,] 6366 0.9899987
## [3,] 6368 0.9900002
## [4,] 6370 0.9900018
```

Appendices

JavaScript and R

Package **V8** provides an embedded JavaScript engine
(On Linux, the user needs to install `libv8-dev`)

Since version 2.0 (2019-02-07) it supports ECMAScript 6
i.e., version 6 that implements, e.g., ‘collections’

```
library(V8); js <- v8()
js$console()
js$eval("<JS code>")
js$source("<file.js>")
js$assign("var_name", <R object>)
js$get("var_name")
js$call("<JS function>", <args...>)
```

Objects will be exchanged using the JSON format.

Random Walks with JavaScript

```
function rwalk(N, M) {
    var result = new Array(N)
    var a = 0, steps
    for (var i = 0; i < N; i++) {
        steps = 2
        if (Math.random() >= 0.5) {a = 1} else {a = -1}
        if (Math.random() >= 0.5) {++a} else {--a}
        while (a != 0) {
            steps += 2
            if (Math.random() >= 0.5) {++a} else {--a}
            if (Math.random() >= 0.5) {++a} else {--a}
            if (steps >= M) break
        }
        result[i] = steps
    }
    return result
}
```

Results with Javascript

Find probabilities with 1 million random walks:

```
library(V8)
js <- v8()
# js$eval("function rwalk(N, M) { ... }")
js$source("rwalk.js")              #   user  system elaps
system.time(                       #  1.845   0.101   1.93
    js$eval("var noStepsJS
             noStepsJS = rwalk(10^6, 10^4)
             undefined") )
noStepsR <- js$get("noStepsJS")
...

## No. of steps for p >= 0.975: 1020
## No. of steps for p >= 0.990: 6380
```

Julia and R

Package **JuliaCall** provides an R interface to Julia,
a high-performance language for numerical computing.

Stable version 1.0 (2018-08-08) is backward-compatible.

```
library(JuliaCall); julia_setup()
julia_console()
julia_source("<file.jl>")
julia_command("<Julia code>")
julia_eval("var_name")
julia_assign("<var_name>", <R object>)
julia_call("<Julia function>", <args...>)
```

Objects will be exchanged using R6 and the JSON format.

Random Walks with Julia

```
rwalk = function(N, M)
    result = zeros(Int, N)
    for i in 1:N
        steps = 2
        rand() >= 0.5 ?  a = 1 : a = -1
        rand() >= 0.5 ? a += 1 : a -= 1
        while a != 0
            steps += 2
            rand() >= 0.5 ? a += 1 : a -= 1
            rand() >= 0.5 ? a += 1 : a -= 1
            if steps >= M; break; end
        end
        result[i] = steps
    end
    return result
end
```

Results with Julia

```
library(JuliaCall)
julia_setup()

js$source("rwalk.jl")
julia_command("rw = rwalk(10, 10);")  # compile functi
system.time(
    no_steps <- julia_eval("rwalk(1000000, 10000)") )
##  user  system elapsed
## 0.330   0.008   0.338

...

## No. of steps for p >= 0.975: 1020
## No. of steps for p >= 0.990: 6348
```