WCSP HW2 陳翰威 311513053

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1、

$$\underset{(1\times PK)}{z^H}\ \underset{PK\times(L+k)}{H_c} = \underset{1\times(L+k)}{e_d^T}$$

⇒ 若 $(L+k) < PK$，則 z 有無限多解 ⇒ 求出一組 minimum norm solution 讓 SINR 最大

$$\begin{cases} \text{minimize} & \|z\|_2^2 \\ \text{s.t.} & H_c^H z = e_d \end{cases}$$

$$L(z, v) = z^H z + v^T (H_c^H z - e_d)$$

① $\nabla_z L(z, v) = 2z + H_c v = 0 \Rightarrow z = -\frac{1}{2} H_c v$

② $\nabla_v L(z, v) = H_c^H z - e_d = 0 \Rightarrow H_c^H(-\frac{1}{2} H_c v) - e_d = 0 \Rightarrow v = -2(H_c^H H_c)^{-1} e_d$

根據①：$z = -\frac{1}{2} H_c v$

$= -\frac{1}{2} H_c (-2(H_c^H H_c)^{-1} e_d)$ 將②代入 v

$= H_c (H_c^H H_c)^{-1} e_d$ #

2、

$$z_{MS} = \arg\min_{z_{MS}} E\{|s(k-d) - z_{MS}^H x_c(k)|^2\}$$

$E\{|s(k-d) - z_{MS}^H x_c(k)|^2\}$

$= E\{[s(k-d) - z_{MS}^H x_c(k)][s(k-d) - z_{MS}^H x_c(k)]^T\} = E\{s(k-d)s(k-d)\} + z_{MS}^H E\{x_c(k) x_c(k)^H\} z_{MS} - E\{s(k-d) x_c^H(k)\} z_{MS}$

$- z_{MS}^H E\{x_c(k) s^H(k-d)\}$ …… ①

①對 z_{MS} 微分 ⇒ $2 z_{MS}^H E\{x_c(k) x_c(k)^H\} - E\{x_c(k) s(k-d)^H\} - E[x_c(k) s^H(k-d)]$

$= 2E[x_c(k) x_c(k)^H] z_{MS} - 2E[x_c(k) s(k-d)^H] = 0$

$\Rightarrow z_{MS} = E[x_c(k) x_c(k)^H]^{-1} E[x_c(k) s(k-d)^H]$

let $E[x_c(k) x_c(k)^H] = R_{c,x}$ So: $\underline{z_{MS} = R_{c,x}^{-1} \cdot h_{c,pr}}$ #

$E[x_c(k) s^H(k-d)] = h_{c,pr}$

3.

$$g_f, g_b = \arg\min E\left[\left|s(k-d) - g_f^H x_c(k) + g_b^H \hat{s}_b(k,d)\right|^2\right]$$

let

$$f(g_f, g_b) = E\left[\left|s(k-d) - g_f^H x_c(k) + g_b^H \hat{s}_b(k,d)\right|^2\right]$$

$$\nabla_{g_b} f(g_f, g_b) = E\left[2 \cdot \hat{s}_b(k,d) \cdot \left[s(k-d)^H - x_c(k)^H g_f + \hat{s}_b(k,d)^H g_b\right]\right]$$

$$= 2 \cdot E\left[\hat{s}_b(k,d) s(k-d)^H\right] - 2 E\left[\hat{s}_b(k,d) x_c(k)^H g_f\right]$$

$$+ 2 E\left[\hat{s}_b(k,d) \hat{s}_b(k,d)^H g_b\right]$$

$$= 0$$

$$\Rightarrow E\left[\hat{s}_b(k,d) s(k-d)^H\right] - E\left[\hat{s}_b(k,d) x_c(k)^H\right] g_f + E\left[\hat{s}_b(k,d) \hat{s}_b(k,d)^H\right] g_b = 0$$

$\because$ $s(k)$ is i.i.d with zero mean and unit variance

$$\therefore \begin{cases} E\left[\hat{s}_b(k,d) s(k-d)^H\right] = 0 \\ E\left[\hat{s}_b(k,d) \hat{s}_b(k,d)^H\right] = 1 \end{cases}$$

$$\Rightarrow g_b = E\left[\hat{s}_b(k,d) x_c^H(k)\right] g_f$$

$$= E\left[\hat{s}_b(k,d) \left(H_{c,f} S_f(k,d) + H_{c,pa} S_b(k,d)\right)^H\right] g_f$$

$$= E\left[\hat{S}_b(k,d) S_f(k,d)\right] H_{c,f}\, \xi_f + E\left[\hat{S}_b(k,d) S_b^H(k,d)\right] H_{c,pa}\, \xi_f$$

$$= H_{c,pa} \cdot \xi_f$$

$$\downarrow \quad \xi_f^H \cdot H_c = e^T \quad \Rightarrow \xi_f = H_c (H_c^H H_c)^{-1} \cdot e$$

$$\xi_b = H_{c,pa} \cdot H_c (H_c^H H_c)^{-1} e \quad \#$$

4、

(a)

① Least square method

② 雖然 Sample averge method 的複雜度較低，但他對 noise 更加敏感，所以在現實中，使用 least square method 是對克服干擾更好的方法

(b)

$$\hat{H} = \arg\min_{H} \sum_{k=1}^{N_s} \| x(k) - H s(k) \|^2$$

$$f(H) = \| x(k) - H s(k) \|^2$$ 要求 least square solution

$$= [x(k)^H - s(k)^H H^H][x(k) - H s(k)]$$

$$= x(k)^H x(k) - x(k)^H H s(k) - s(k)^H H^H x(k) + s(k)^H H^H H s(k)$$

$$\nabla_H f(H) = -x(k) s(k)^H - x(k) s(k)^H + 2 H s(k) s(k)^H = 0$$

$$\Rightarrow H s(k) s(k)^H = x(k) s(k)^H$$

$$\Rightarrow H = x(k) s(k)^H [s(k) s(k)^H]^{-1} \quad \#$$