# FOLD-RM: A Scalable and Efficient Inductive Learning Algorithm for Multi-Category Classification of Mixed Data

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## **Abstract**

FOLD-RM is an automated inductive learning algorithm for learning default rules for mixed (numerical and categorical) data. It generates an (explainable) answer set programming (ASP) rule set for *multi-category classification* tasks while maintaining efficiency and scalability. The FOLD-RM algorithm is competitive in performance with the widely-used XGBoost algorithm, however, unlike XGBoost, the FOLD-RM algorithm produces an explainable model. FOLD-RM outperforms XGBoost on some datasets, particularly large ones. FOLD-RM also provides human-friendly explanations for predictions.

KEYWORDS: Explainable AI, Data Mining, Inductive Logic Programming, Machine Learning

## 1 Introduction

Dramatic success of machine learning has led to an avalanche of applications of Artificial Intelligence (AI). However, the effectiveness of these systems is limited by the machines' current inability to explain their decisions to human users. That is mainly because statistical machine learning methods produce models that are complex algebraic solutions to optimization problems such as risk minimization or geometric margin maximization. Lack of intuitive descriptions makes it hard for users to understand and verify the underlying rules that govern the model. Also, these methods cannot produce a justification for a prediction they arrive at for a new data sample. The problem of explaining (or justifying) a model's decision to its human user is referred to as the model interpretability problem. The sub-field is referred to as Explainable AI (XAI).

The ILP learning problem is the problem of searching for a set of logic programming clauses from which the training examples can be deduced. ILP provides an excellent solution for XAI. ILP is a thriving field and a large number of such clause search algorithms have been devised (Muggleton et al. 2012; Cropper and Dumančić 2021). The search in these ILP algorithms is performed either top down or bottomup. A bottom-up approach builds most-specific clauses from the training examples and searches the hypothesis space by using generalization. This approach is not applicable to large-scale datasets, nor it can incorporate *negation-as-failure* (Baral 2003) into the hypotheses. A survey of bottom-up ILP systems and their shortcomings can be found elsewhere (Sakama 2005). In contrast, top-down approach starts with the most general clauses and then specializes them. A top-down algorithm guided by heuristics is better suited for large-scale and/or noisy datasets (Zeng et al. 2014).

The FOIL algorithm by Quinlan is a popular top-down inductive logic programming algorithm that learns a logic program. The FOLD algorithm by Shakerin et al is a novel top-down algorithm that learns default rules along with exception(s) that closely model human thinking (Shakerin et al. 2017). It first learns default predicates that cover positive examples while avoiding covering negative examples. Then it swaps the covered positive examples and negative examples and calls itself recursively to learn the exception to the default. It repeats this process to learn exceptions to exceptions, exceptions to exceptions, and so on. The FOLD-R++ algorithm designed by us is a new scalable ILP algorithm that builds upon

the FOLD algorithm to deal with the efficiency and scalability issues of the FOLD and FOIL algorithms (Wang and Gupta 2022). It introduces the prefix sum computation and other optimizations to speed up the learning process while providing human-friendly explanation for its prediction using the s(CASP) answer set programming system (Arias et al. 2018). However, all these algorithms focus on binary classification tasks, and cannot deal with multi-category classification tasks.

In this paper we propose a new ILP learning algorithm called FOLD-RM for multi-category classification that builds upon the FOLD-R++ algorithm. FOLD-RM also provides native explanations for prediction without external libraries or tools. Our experimental results indicates that the FOLD-RM algorithm is comparable in performance to traditional, popular machine learning algorithms such as XGBoost (Chen and Guestrin 2016) and Multi-Layer Perceptrons (MLP) (Aggarwal 2018). In most cases, FOLD-RM outperforms them in execution efficiency. Of course, neither XGBoost nor MLP are interpretable.

# 2 Background

### 2.1 Inductive Logic Programming

Inductive Logic Programming (ILP) (Muggleton 1991) is a subfield of machine learning that learns models in the form of logic programming rules comprehensible to humans. This problem is formally defined as: **Given** 

- A background theory B, in the form of an extended logic program, i.e., clauses of the form h ← l<sub>1</sub>,...,l<sub>m</sub>, not l<sub>m+1</sub>,..., not l<sub>n</sub>, where l<sub>1</sub>,...,l<sub>n</sub> are positive literals and not denotes negation-as-failure (NAF) (Baral 2003; Gelfond and Kahl 2014). We require that B has no loops through negation, i.e., it is stratified.
- 2. Two disjoint sets of ground target predicates  $E^+, E^-$  known as positive and negative examples, respectively
- 3. A hypothesis language of function free predicates L, and a refinement operator  $\rho$  under  $\theta$ -subsumption (Plotkin 1971) that would disallow loops over negation.

**Find** a set of clauses *H* such that:

- 1.  $\forall e \in E^+, B \cup H \models e$
- 2.  $\forall e \in E^-, B \cup H \not\models e$
- 3.  $B \wedge H$  is consistent.

## 2.2 Default Rules

Default Logic (Reiter 1980; Gelfond and Kahl 2014) is a non-monotonic logic to formalize commonsense reasoning. A default *D* is an expression of the form

$$\frac{A:\mathbf{M}B}{\Gamma}$$

which states that the conclusion  $\Gamma$  can be inferred if pre-requisite A holds and B is justified.  $\mathbf{M}B$  stands for "it is consistent to believe B" (Gelfond and Kahl 2014). Normal logic programs can encode a default quite elegantly. A default of the form:

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n : \mathbf{M} \neg \beta_1, \mathbf{M} \neg \beta_2 \dots \mathbf{M} \neg \beta_m}{\gamma}$$

can be formalized as the following normal logic program rule:

$$\gamma : -\alpha_1, \alpha_2, \ldots, \alpha_n, \text{not } \beta_1, \text{not } \beta_2, \ldots, \text{not } \beta_m.$$

where  $\alpha$ 's and  $\beta$ 's are positive predicates and not represents negation as failure (under the stable model semantics (Baral 2003; Gelfond and Kahl 2014), we assume here). We call such rules default rules. Thus, the default  $\frac{bird(X):M\neg penguin(X)}{fly(X)}$  will be represented as the following ASP-coded default rule:

```
fly(X) :- bird(X), not penguin(X).
```

We call bird (X), the condition that allows us to jump to the default conclusion that X can fly, as the *default part* of the rule, and not penguin (X) as the *exception part* of the rule.

Default rules closely represent the human thought process (commonsense reasoning). FOLD-R and FOLD-R++ learn default rules represented as answer set programs. Note that the programs currently generated are stratified normal logic programs, however, we eventually hope to learn non-stratified programs too (Shakerin and Gupta 2018; Shakerin 2020) that are interpreted under the stable model semantics (Gelfond and Kahl 2014). Hence, we continue to use the term answer set program for a normal logic program in this paper. An advantage of learning default rules is that we can distinguish between exceptions and noise (Shakerin et al. 2017; Shakerin 2020).

### 3 The FOLD-R++ Algorithm

The FOLD-R++ algorithm (Wang and Gupta 2022) is a new ILP algorithm for binary classification devised by us that is built upon the FOLD algorithm (Shakerin et al. 2017; Shakerin 2020). The FOLD-RM algorithm builds upon the FOLD-R++ algorithm. FOLD-R++ increases the efficiency and scalability of the FOLD algorithm. The FOLD-R++ algorithms divides features into two categories: categorical features and numerical features. For a categorical feature, all the values in the feature would be considered as categorical values even though some of them are numbers. For categorical features, the FOLD-R++ algorithm only generates equality or inequality literals. For numerical features, the FOLD-R++ algorithm would try to read all the values as numbers, converting them to categorical values if conversion to numbers fails. FOLD-R++ additionally generates numerical comparison ( $\leq$  and >) literals for numerical values. For a mixed type feature that contains both categorical values and numerical values, the FOLD-R++ algorithm treats them as numerical features.

The FOLD-R++ algorithm employs information gain heuristic to guide literal selection during the learning process. It uses a simplified calculation process for information gain by using the number of true positive, false positive, true negative, and false negative examples that a literal can imply. The information gain for a given literal is calculated as shown in Algorithm 1.

## Algorithm 1 FOLD-R++ Algorithm: Information Gain function

```
1: function F(a,b)
        if a = 0 then
 2:
            return 0
 3:
 4:
        end if
        return a \cdot \log_2(\frac{a}{a+b})
 5:
 6: end function
 7: function IG(tp, fn, tn, fp)
 8:
        if fp + fn > tp + tn then
            return -\infty
 9:
        end if
10:
        return \frac{1}{tp+fp+tn+fn} \cdot (F(tp,fp) + F(fp,tp) + F(tn,fn) + F(fn,tn))
12: end function
```

The comparison between two numerical values or two categorical values in FOLD-R++ is straightforward, as commonsense would dictate, i.e., two numerical (*resp.* categorical) values are equal if they are identical, else they are unequal. However, a different assumption is made to compare a numerical value and a categorical value in FOLD-R++. The equality between a numerical value and a categorical value is always false, and the inequality between a numerical value and a categorical value is always true. Additionally, numerical comparison between a numerical value and a categorical value is always false. An example is shown in Table 1.

The FOLD-R++ algorithm starts with the clause p(...) := true., where p(...) is the target

comparison	5 = `k'	5 ≠ 'k'	5 ≤ 'k'	5 > 'k'
evaluation	False	True	False	False

Table 1. Comparisons between a numerical value and a categorical value in FOLD-R++

		i <sup>th</sup> feature values							
$\mathbf{E}^{+}$	1	2	2	4	5	X	X	У	
$\mathbf{E}^{-}$	1	3	4	У	У	У	Z		

value	1	2	3	4	5	X	у	Z
pos	1	2	0	1	1	2	1	0
psum <sup>+</sup>	1	3	3	4	5	N/A	N/A	N/A
neg	1	0	1	1	0	0	3	1
psum <sup>-</sup>	1	1	2	3	3	N/A	N/A	N/A

Table 2. Left: Examples and values on i<sup>th</sup> feature. Right: positive/negative count and prefix sum on each value

predicate to learn. It specializes this clause by adding literals to its body during the inductive learning process. It selects a literal to add that maximizes information gain (IG). The literal selection process is summarized in Algorithm 2. In line 2, pos & neg are dictionaries that hold, respectively, the numbers of positive & negative examples for each unique value. In line 3, xs & cs are lists that hold, respectively, the unique numerical and categorical values. In line 4, xp & xn are the total number of, respectively, positive & negative examples with numerical values; cp & cn are the same for categorical values. In line 11, the information gain of literal  $(i, \leq, x)$  is calculated by taking the parameters pos[x] as the number of true positive examples, xp - pos[x] + cp as the number of false negative examples, xn - neg[x] + cp as the number of true negative examples, and neg[x] as the number of false negative examples. After computing the prefix sum in line 6, pos[x] holds the total number of positive examples that has a value less than or equal to x. Therefore, xp - pos[x] represents the total number of positive examples that have a value greater than x. cp, the total number of positive examples that have a categorical value, is added to the number of false negative examples because of the assumption that numerical comparison between a numerical value and a categorical value is always false. The negative examples that have a value greater than x or a categorical value would be evaluated as false by literal $(i, \leq, x)$ , so xn - neg[x] is added as true negative parameter. And, cn, the total number of negative examples that has a categorical value, is added to true negative parameter. neg[x] means the number of negative examples that have the value less than or equal to x. neg[x]is added as false positive parameter because the evaluations of these examples by  $literal(i, \leq, x)$  are true. The information gain calculation processes of other literals also follows the comparison assumption mentioned above. Finally, the best\_info\_gain function returns the best score on information gain and the corresponding literal except the literals that have been used in current rule-learning process. For each feature, we compute the best literal, then the find\_best\_literal function returns the best literal among this set of best literals.

### Example 1

Given positive and negative examples,  $E^+$ ,  $E^-$ , with mixed type of values on  $i^{th}$  feature, the target is to find the literal with the best information gain on the given feature. There are 8 positive examples, their values on  $i^{th}$  feature are [1,2,2,4,5,x,x,y]. And, the values on  $i^{th}$  feature of the 7 negative examples are [1,3,4,y,y,y,z].

With the given examples and specified feature, the number of positive examples and negative examples for each unique value are counted first, which are shown as pos, neg on right side of Table 2. Then, the prefix sum arrays are calculated for computing heuristic as psum<sup>+</sup>, psum<sup>-</sup>. Table 3 shows the information gain for each literal, the literal(i, =, x) has been selected with the highest score.

## 4 The FOLD-RM Algorithm

The FOLD-R++ algorithm performs binary classification. We generalize the FOLD-R++ algorithm to perform multi-category classification. The generalized algorithm is called FOLD-RM. The FOLD-R++ algorithm is summarized in Algorithm 3. The FOLD-R++ algorithm generates an answer set programming rule set, in which all the rules have the same rule head. An example covered by any rule in the set would imply

## Algorithm 2 FOLD-R++ Algorithm, Find Best Literal function

```
Input: E^+, E^-, L_{used}
Output: best_lit
 1: function BEST_INFO_GAIN(E^+, E^-, i, L_{used})
 2:
          pos, neg \leftarrow count\_classification(E^+, E^-, i)
         xs, cs \leftarrow collect\_unique\_values(E^+, E^-, i)
 3:
         xp, xn, cp, cn \leftarrow count\_total(E^+, E^-, i)
 4:
         xs \leftarrow couting\_sort(xs)
 5:
         for j \leftarrow 1 to size(xs) do
 6:
                                                                                        ⊳ compute the prefix sum
              pos[xs_i] \leftarrow pos[xs_i] + pos[xs_{i-1}]
 7:
              neg[xs_i] \leftarrow neg[xs_i] + neg[xs_{i-1}]
 8:
         end for
 9:
         for x \in xs do
10:
              lit\_dict[literal(i, \leq, x)] \leftarrow IG(pos[x], xp - pos[x] + cp, xn - neg[x] + cn, neg[x])
11:
              lit\_dict[literal(i, >, x)] \leftarrow IG(xp - pos[x], pos[x] + cp, neg[x] + cn, xn - neg[x])
12:
         end for
13:
         for c \in cs do
14:
              lit\_dict[literal(i,=,x)] \leftarrow \text{IG}(pos[c], cp - pos[c] + xp, cn - neg[c] + xn, neg[c])
15:
              lit\_dict[literal(i, \neq, x)] \leftarrow IG(cp - pos[c] + xp, pos[c], neg[c], cn - neg[c] + xn)
16:
17:
         end for
         best, l \leftarrow best\_pair(lit\_dict, L_{used})
18:
19:
          return best, l
20: end function
    function FIND_BEST_LITERAL(E^+, E^-, L_{used})
21:
         best\_ig, best\_lit \leftarrow -\infty, invalid
22:
         for i \leftarrow 1 to N do
                                                                                   \triangleright N is the number of features
23:
              ig, lit \leftarrow \text{BEST\_INFO\_GAIN}(E^+, E^-, i, L_{used})
24:
              if best\_ig < ig then
25:
                   best\_ig, best\_lit \leftarrow ig, lit
26:
              end if
27:
28:
         end for
          return best_lit
29.
30: end function
```

the rule head is true. The FOLD-R++ algorithm generates a model by learning one rule at a time. Ruling out the already covered example in line 8 after learning a rule would help select better literal for remaining examples. In the rule learning process, the best literal would be selected according to the useful information it can provide for current training examples (line 15) till the literal selection fails. If the ratio of false positive examples to true positive examples drops below the threshold *ratio* in line 23, it would next learn exceptions by swapping residual positive and negative examples and calling itself recursively (line 29). Any examples that cannot be covered by the selected literals would be ruled out in line 17, 18. The *ratio* in line 23 represents the upper bound on the number of true positive examples to the number of false positive examples implied by the default part of a rule. It helps speed up the training process and reduces the number of rules learned.

Generally, avoiding covering negative examples by adding literals to the default part of a rule will reduce the number of positive examples the rule can imply. Explicitly activating the exception learning procedure

		Info Gain						
value	1	2	3	4	5	X	У	Z
$\leq$ value	-∞	-0.655	-0.686	-0.688	-0.672	N/A	N/A	N/A
> value	-0.667	-∞	-0.682	-0.647	-∞	N/A	N/A	N/A
= value	N/A	N/A	N/A	N/A	N/A	-0.598	-∞	-∞
$\neq$ value	N/A	N/A	N/A	N/A	N/A	-∞	-0.631	-0.637

Table 3. The info gain on  $i^{th}$  feature with given examples

(line 29) could increase the number of positive example a rule can cover while reducing the total number of rules generated. As a result, the interpretability is increased due to fewer rules being generated.

The FOLD-RM algorithm performs multi-category classification. It generates rules that it can learn for each category. If an example cannot be implied by any rule in the learned rule set, it means the model fails to classify this example. The FOLD-RM algorithm, summarized in Algorithm 4, first finds a target literal that represents the category with most examples among the current training set (line 3). It next splits the training set into positive and negative examples based on the target literal (line 4). Then, it learns a rule to cover the target category (line 5) by calling the learn\_rule function of the FOLD-R++ Algorithm. The already covered examples would be ruled out from the training set in line 10, and the rule head would be changed to the target literal in line 11. However, there's a difference between the outputs of FOLD-RM and FOLD-R++. Unlike FOLD-R++, the output of FOLD-RM is a textually ordered answer set program, which means a rule is checked only if all the rules before it did not apply. The FOLD-RM system is publicly available at https://github.com/hwd404/FOLD-RM. Next, we illustrate FOLD-RM with a simple example.

## Example 2

The target is to learn rules for habitat using the FOLD-RM algorithm. B, E are background knowledge and training examples, respectively.

```
B: mammal(kitty). cat(kitty).
mammal(john). whale(john).
mammal(smoky). bear(smoky).
mammal(charlie). dog(charlie).
fish(nemo). clownfish(nemo).
E: habitat(charlie,land). habitat(john,water).
habitat(kitty,land).
```

For the first rule, the target predicate  $\{\text{habitat}(X, \text{land}): - \text{true}\}$  is specified at line 3 in Algorithm 4 because 'land' is the majority label. The find\_best\_literal function selects literal mammal (X) as result and adds it to the clause  $r = \{\text{habitat}(X, \text{land}): - \text{mammal}(X)\}$  at line 16 in Algorithms 3 because it provides the most useful information among literals  $\{\text{cat}, \text{whale}, \text{bear}, \text{dog}, \text{fish}\}$ . Then the training set rules out covered examples at line 17-18 in Algorithm 3,  $E^+ = \{\emptyset\}$ ,  $E^- = \{\text{john}, \text{nemo}\}$ . The default learning is finished at this point because the candidate literal cannot provide any further useful information. Therefore, the FOLD\_RPP function is called recursively with swapped positive and negative examples,  $E^+ = \{\text{john}, \text{nemo}\}$ ,  $E^- = \{\emptyset\}$ , to learn exceptions. In this case, an abnormal predicate  $\{\text{ab1}(X): -\text{whale}(X)\}$  is learned and added to the previously generated clause as  $r = \{\text{habitat}(X, \text{land}): -\text{mammal}(X)$ , not ab1(X)}. And the exception rule  $\{\text{ab1}(X): -\text{whale}(X)\}$  is added to the answer set program. FOLD-RM next learns rules for target predicate  $\{\text{habitat}(X, \text{water}): -\text{true}\}$  and two rules are generated as  $\{\text{habitat}(X, \text{water}): -\text{fish}(X)\}$  and  $\{\text{habitat}(X, \text{water}): -\text{whale}(X)\}$ . The generated final answer set program is:

```
habitat(X,land):- mammal(X), not ab1(X).
habitat(X,water):- fish(X).
habitat(X,water):- whale(X).
ab1(X):- whale(X).
```

The program above is a logic program, which means rules are not mutually exclusive. For correctness, a

## Algorithm 3 FOLD-R++ Algorithm

```
Input: target, B, E^+, E^-, ratio, L_{used}
                                                                                     ⊳ ratio is the exception ratio
Output: R = \{r_1, ..., r_n\}
                                                                                                       ⊳ R is rule set
 1: function FOLD_RPP(E^+, E^-, L_{used})
          while |E^+| > 0 do
 2:
              r \leftarrow \text{LEARN\_RULE}(E^+, E^-, L_{used})
 3:
              E_{TP} \leftarrow covers(r, E^+, true)
 4:
              if |E_{TP}| = 0 then
 5:
                   break
 6:
 7:
              end if
              E^+ \leftarrow E^+ \setminus E_{TP}
 8:
              R \leftarrow R \cup \{r\}
 9:
         end while
10:
          return R
12: end function
     function LEARN_RULE(E^+, E^-, L_{used})
13:
          while true do
14:
              l \leftarrow \text{FIND\_BEST\_LITERAL}(E^+, E^-, L_{used})
15:
              r \leftarrow add\_default(r, l)
16:
              E^+ \leftarrow covers(r, E^+, true)
17:
              E^- \leftarrow E^- \setminus covers(r, E^-, false)
18:
              if l is invalid then
19:
                   r \leftarrow remove\_default(r, l)
20:
                   break
21:
              end if
22:
              if |E^-| \leq |E^+| * ratio then
23:
                   flag \leftarrow true
24:
                   break
25:
              end if
26:
         end while
27:
28:
         if flag then
              AB \leftarrow \text{FOLD\_RPP}(E^-, E^+, L_{used} + L)
29.
              r \leftarrow add\_exception(r,AB)
30:
31:
          end if
          return r
32:
33: end function
```

rule should be checked only if all the earlier rules result in failure. FOLD-RM generates further rules to make the learned rules mutually exclusive. The program above is transformed as shown below.

```
\label{eq:habitat} \begin{array}{lll} \text{habitat}(X), \text{land}) :- & \text{habitat}(X). \\ \text{habitat}(X, \text{water}) :- & \text{habitat}(X), & \text{not habitat}(X). \\ \text{habitat}(X, \text{water}) :- & \text{habitat}(X), & \text{not habitat}(X), & \text{not habitat}(X), & \text{not habitat}(X). \\ \text{habitat}(X) :- & \text{mammal}(X), & \text{not abl}(X). \\ \text{habitat}(X) :- & \text{fish}(X). \\ \text{habitat}(X) :- & \text{whale}(X). \\ \text{abl}(X) :- & \text{whale}(X). \\ \end{array}
```

## Algorithm 4 FOLD-RM Algorithm

```
Input: target, B, E, ratio
                                                                                     ⊳ ratio is the exception ratio
Output: R = \{r_1, ..., r_n\}
                                                                                                       ⊳ R is rule set
 1: function FOLD_RM(E)
 2:
          while |E| > 0 do
              l \leftarrow \text{MOST}(E)
 3:
              E^+, E^- \leftarrow \text{SPLIT\_BY\_LITERAL}(E, l)
 4:
              r \leftarrow \text{LEARN\_RULE}(E^+, E^-, \emptyset)
 5:
              E_{TP} \leftarrow covers(r, E^+, true)
 6:
 7:
              if |E_{TP}| = 0 then
                   break
 8:
              end if
 9:
              E \leftarrow E^+ \cup E^- \setminus E_{TP}
10:
              r \leftarrow add\_head(r, l)
11:
              R \leftarrow R \cup \{r\}
12:
         end while
13:
          return R
14:
15: end function
16: function MOST(E)
          for e \in E do
17:
              count[label_e] \leftarrow count[label_e] + 1
18:
19:
20:
         label_{most} \leftarrow FIND\_MOST(count)
          return literal(index_{label}, =, label_{most})
21:
    end function
22:
    function SPLIT_BY_LITERAL(E, l)
23.
          for e \in E do
24.
              if EVALUATE(e, l) is true then
25:
                   E^+ \leftarrow E^+ \cup \{e\}
26:
              else
27:
                   E^- \leftarrow E^- \cup \{e\}
28:
29:
              end if
         end for
30:
         return E^+, E^-
31:
32: end function
```

# 5 Experimental Results

In this section, we present our experiments on standard UCI benchmarks. The XGBoost Classifier is a well-known classification model and used as a baseline model in our experiments. The settings used for XGBoost Classifier is kept simple without limiting its performance. Multi-Layer Perceptron (MLP) is another widely-used classification model that can deal with generic classification tasks. However, both XGBoost model and MLP cannot take mixed type (numerical and categorical values in a row or a column) as training data without pre-processing. For mixed type data, one-hot encoding (Aggarwal 2018) has been used for data preparation. For binary classification, we use accuracy, precision, recall, and F<sub>1</sub> score as evaluation metrics. For the multi-category classification tasks, following convention, we use accuracy, weighted average of

			XGB	oost.	Classi	ifier	FOLD-R++					
Data Set	Shape	Acc	Prec	Rec	F1	T(ms)	Acc	Prec	Rec	F1	T(ms)	#Rules
acute	120,7	1	1	1	1	35	0.99	1	0.99	0.99	2.5	2.6
autism	704,18	0.97	0.98	0.98	0.97	76	0.95	0.96	0.97	0.97	47	24.3
breast-w	699,10	0.95	0.97	0.96	0.96	78	0.96	0.97	0.96	0.97	28	10.2
cars	1728,7	1	1	1	1	77	0.98	1	0.97	0.98	48	12.2
credit-a	690,16	0.85	0.83	0.83	0.83	368	0.84	0.92	0.79	0.84	100	10.0
ecoli	336,9	0.76	0.76	0.62	0.68	165	0.96	0.95	0.94	0.95	28	11.4
heart	270,14	0.80	0.81	0.83	0.81	112	0.79	0.79	0.83	0.81	44	11.7
ionosphere	351,35	0.88	0.86	0.96	0.90	1,126	0.92	0.93	0.94	0.93	392	12.0
kidney	400,25	0.98	0.98	0.98	0.98	126	0.99	1	0.98	0.99	27	5.0
kr vs. kp	3196,37	0.99	0.99	0.99	0.99	210	0.99	0.99	0.99	0.99	361	18.4
mushroom	8124,23	1	1	1	1	378	1	1	1	1	476	8.0
sonar	208,61	0.53	0.54	0.84	0.65	1,178	0.78	0.81	0.75	0.78	419	11.6
voting	435,17	0.95	0.94	0.95	0.94	49	0.95	0.94	0.94	0.94	16	10.5
adult	32561,15	0.86	0.88	0.94	0.91	274,655	0.84	0.86	0.95	0.90	10,069	16.7
rain in aus	145460,24	0.83	0.84	0.95	0.89	285,307	0.78	0.87	0.84	0.85	279,320	40.5

Table 4. Comparison of XGBoost and FOLD-R++ on various Datasets

Macro precision, weighted average of Macro recall, and weighted average of Macro  $F_1$  score to compare models (Grandini et al. 2020).

Both FOLD-R++ and FOLD-RM algorithms *do not* need any encoding for training. After specifying the numerical features, they can deal with mixed type data directly, i.e., no one-hot encoding is needed. Even missing values are handled and do not need to be provided. We implemented both algorithms with Python. The hyper-parameter *ratio* is simply set as 0.5 for all the experiments. And all the learning processes have been run on a desktop with Intel i5-10400 CPU @ 2.9 GHz and 32 GB RAM. To have good performance test, we performed 10-fold cross-validation test on each dataset and average classification metrics and execution time are shown. The best performer is highlighted in boldface.

The XGBoost Classifier utilizes decision tree ensemble method to build model and provides good performance. Performance comparison of FOLD-R++ and XGBoost is shown in Table 4. The FOLD-R++ algorithm is comparable to XGBoost classifier for classification, but it's more efficient in terms of execution time, especially on datasets with many unique feature values.

For multi-category classification experiments, we collected 15 datasets for comparison. The drug consumption dataset has many output attributes, we perform training on heroin, crack, and semer attributes. The shape and label distribution of the datasets used is shown in Table 5. We first compare the performance of FOLD-RM and XGBoost in Table 6. XGBoost performs much better on datasets avila and yeast, and FOLD-RM performs much better on datasets ecoli, dry-bean, eeg, and weight-lifting. After analyzing these dataset, FOLD-RM seems to perform better on more complicated datasets with mixed type values. XG-Boost seems to perform better on the datasets that have limited information. However, for those datasets for which FOLD-RM has similar performance with XGBoost, FOLD-RM is more efficient in terms of execution speed. In addition, FOLD-RM is explainable/interpretable, and XGBoost is not.

The comparison with MLP is presented in Table 7. For most datasets, FOLD-RM can achieve equivalent scores, similar to the comparison with XGBoost, FOLD-RM performs much better on datasets ecoli, drybean, eeg, and weight-lifting, while MLP performs much better on datasets avila and yeast. MLP takes much more time for training than XGBoost because of its algorithmic complexity. Like the XGBoost classifier, for complex datasets with mixed values, MLP also suffers from pre-processing complications such as having to use one-hot encoding.

RIPPER algorithm (Cohen 1995) is a popular rule induction algorithm that generates conjunctive normal form (CNF) formulas. 7 datasets have been used for comparison between RIPPER and FOLD-RM. We did not find the RIPPER algorithm implementation with multi class classification. Therefore, we have collected the accuracy data from (Asadi and Shahrabi 2016) and performed the same experiment with the same datasets with the FOLD-RM algorithm. Two-thirds of the dataset was used for training in (Asadi and Shahrabi 2016) and the remaining one-third used as the test set. We follow the same convention. For each dataset, this pro-

dataset	shape	distribution
anneal	898,39	'3': 684, 'U': 40, '1': 8, '5': 67, '2': 99
avila	20867,11	'A':8572, 'F':3923, 'H':1039, 'E':2190, 'I':1663, 'Y':533
		'D':705,'X':1044,'G':893,'W':89,'C':206,'B':10
ecoli	336,9	'cp':143, 'im':77, 'imS':2, 'imL':2, 'imU':35, 'om':20, 'omL':5, 'pp':52
drug-heroin	1885,13	'CL0':1605, 'CL1':68, 'CL2':94, 'CL3':65, 'CL5':16, 'CL6':13, 'CL4':24
drug-crack	1885,13	'CL0':1627,'CL1':67,'CL2':112,'CL3':59,'CL5':9,'CL4':9,'CL6':2
drug-semer	1885,13	'CL0': 1877, 'CL2': 3, 'CL3': 2, 'CL4': 1, 'CL1':2
dry-bean	13611,17	'SEKER': 2027, 'BARBUNYA': 1322, 'BOMBAY': 522
		'CALI': 1630, 'HOROZ': 1928, 'SIRA': 2636, 'DERMASON': 3546
eeg	14980,15	'0': 8257, '1': 6723
intention	12330,18	'FALSE': 10422, 'TRUE': 1908
nursery	12960,9	'recommend': 2, 'priority': 4266
		'not_recom': 4320, 'very_recom': 328, 'spec_prior': 4044
pageblocks	5473,11	'1': 4913, '2': 329, '4': 88, '5': 115, '3': 28
parkinson	756,754	'1': 564, '0': 192
pendigits	10992,17	'8': 1055, '2': 1144, '1': 1143, '4': 1144, '6': 1056
		'0': 1143, '5': 1055, '9': 1055, '7': 1142, '3': 1055
wine	178,14	'1': 59, '2': 71, '3': 48
weight-lift	4024,155	'E': 1370, 'A': 1365, 'D': 276, 'B': 901, 'C': 112
yeast	1484,10	'MIT': 244, 'NUC': 424, 'CYT': 463, 'ME1': 44, 'EXC': 35, 'ME2': 51
		'ME3': 163, 'VAC': 30, 'POX': 20, 'ERL': 5, '0.18': 2, '0.16': 2, '0.37': 1
wall-robot	5456,25	'Slight-Right-Turn': 826, 'Sharp-Right-Turn': 2097
		'Move-Forward': 2205, 'Slight-Left-Turn': 328
flags	194,10	'2': 36, '6': 15, '1': 60, '0': 40, '5': 27, '3': 8, '4': 4, '7': 4
glass	214,10	'1': 70, '2': 76, '3': 17, '5': 13, '6': 9, '7': 29
optidigits	3823,65	'0': 376, '7': 387, '4': 387, '6': 377, '2': 380
		'5': 376, '8': 380, '1': 389, '9': 382, '3': 389

Table 5. The shape and label distribution of UCI datasets

			FOI	LD-RN	1		XGBoost					
Dataset	Acc	Prec	Rec	F1	rules	T(ms)	Acc	Prec	Rec	F1	T(ms)	
anneal	0.99	1	0.99	0.99	17	68	0.99	0.99	0.99	0.99	295	
avila	0.33	0.49	0.33	0.39	37	4,459	1	1	1	1	4,897	
ecoli	0.80	0.82	0.80	0.80	48	45	0.42	0.79	0.42	0.50	806	
drug-heroin	0.84	0.74	0.84	0.78	11	145	0.84	0.77	0.84	0.79	1,266	
drug-crack	0.85	0.75	0.85	0.80	21	160	0.85	0.76	0.85	0.81	1,116	
drug-semer	0.99	0.99	0.99	0.99	8	70	1	0.99	1	0.99	393	
dry-bean	0.91	0.91	0.91	0.91	195	13,885	0.29	0.87	0.29	0.37	3,458	
eeg	0.78	0.78	0.78	0.77	170	5,161	0.50	0.70	0.50	0.54	340	
intention	0.90	0.89	0.90	0.90	68	2,728	0.90	0.89	0.90	0.89	114,161	
nursery	0.97	0.97	0.97	0.96	62	1,150	0.88	0.93	0.88	0.89	24,100	
pageblocks	0.97	0.97	0.97	0.96	61	1,176	0.95	0.94	0.95	0.94	81,416	
parkinson	0.81	0.80	0.81	0.79	15	8,472	0.84	0.84	0.84	0.83	527	
pendigits	0.96	0.96	0.96	0.96	211	2,574	0.91	0.92	0.91	0.91	54,102	
wine	0.94	0.97	0.94	0.95	8	17	0.93	1	0.93	0.96	49	
weight-lift	1	1	1	1	15	1,940	0.51	0.81	0.51	0.57	224,140	
yeast	0.08	0.15	0.08	0.10	5	116	0.45	0.5	0.45	0.45	8,629	
wall-robot	0.99	0.99	0.99	0.99	31	2,559	0.99	0.99	0.99	0.99	403	

Table 6. Comparison of FOLD-RM and XGBoost on UCI Datasets

			FOI	LD-RN	Л				MLP					
Dataset	Acc	Prec	Rec	F1	rules	T(ms)	Acc	Prec	Rec	F1	T(ms)			
anneal	0.99	1	0.99	0.99	17	68	0.99	0.99	0.99	0.99	462			
avila	0.33	0.49	0.33	0.39	37	4,459	0.90	0.90	0.90	0.90	73,610			
ecoli	0.80	0.82	0.80	0.80	48	45	0.52	0.91	0.52	0.61	411			
drug-heroin	0.84	0.74	0.84	0.78	11	145	0.82	0.77	0.82	0.79	1,093			
drug-crack	0.85	0.75	0.85	0.80	21	160	0.84	0.77	0.84	0.80	1,061			
drug-semer	0.99	0.99	0.99	0.99	8	70	1	0.99	1	0.99	518			
dry-bean	0.91	0.91	0.91	0.91	195	13,885	0.57	0.92	0.57	0.66	11,292			
eeg	0.78	0.78	0.78	0.77	170	5,161	0.49	0.68	0.49	0.54	5,946			
intention	0.90	0.89	0.90	0.90	68	2,728	0.84	0.76	0.84	0.78	218,087			
nursery	0.97	0.97	0.97	0.96	62	1,150	0.91	0.94	0.91	0.91	943			
pageblocks	0.97	0.97	0.97	0.96	61	1,176	0.93	0.91	0.93	0.92	6,452			
parkinson	0.81	0.80	0.81	0.79	15	8,472	0.82	0.82	0.82	0.81	1,416			
pendigits	0.96	0.96	0.96	0.96	211	2,574	0.99	0.99	0.99	0.99	6,732			
wine	0.94	0.97	0.94	0.95	8	17	0.97	1	0.97	0.98	189			
weight-lift	1	1	1	1	15	1,940	0.54	0.89	0.54	0.58	52,643			
yeast	0.08	0.15	0.08	0.10	5	116	0.41	0.49	0.41	0.38	3,750			
wall-robot	0.99	0.99	0.99	0.99	31	2,559	0.88	0.88	0.88	0.88	8,141			

Table 7. Comparison of FOLD-RM and MLP on UCI Datasets

Dataset	RIPPER Acc	FOLD-RM Acc
ecoli	0.80	0.80
flags	0.61	0.58
glass	0.63	0.63
nursery	0.72	0.96
optidigits	0.90	0.90
pageblocks	0.97	0.96
pendigits	0.95	0.95

Table 8. Comparison of RIPPER and FOLD-RM on UCI Datasets

cess was repeated 50 times, the average of accuracy is shown in Table 8. Both algorithms have similar accuracy on most datasets, though FOLD-RM outperforms on nursery dataset. Ripper is explainable, as it outputs CNF formulas. However, the CNF formulae generated tend to have large number of literals. In contrast, FOLD-RM rules are succinct due to use of negation as failure and they have an operational semantics by virtue of being a logic program.

## 6 Prediction and Justification

The FOLD-RM algorithm generates rules that can be interpreted by the human user to understand the patterns and correlations that are implicit in the table data. These rules can also be used to make prediction given new data input. Thus FOLD-RM serves as a machine learning algorithm in its own right. However, making good predictions is not enough for critical tasks such as disease diagnosis and loan approval. FOLD-RM Comes with a built-in prediction and justification facility. We illustrate this justification facility via an example.

## Example 3

The "annealing" UCI dataset is a multi-category classification task which contains 798 training examples and 100 test examples and their classes based on features such as steel, carbon, hardness, condition, and strength, etc.. FOLD-RM generates the following answer set program with 20 rules for 5 classes, which is pretty concise and precise:

```
classes (X,'3'): - not surface_quality (X,'?'), not ab1(X),
                   not ab2(X), not ab3(X), not ab4(X).
classes (X,'2'): - condition (X,'s'), not ab5 (X).
classes (X,'3'): - not carbon (X,'00'), not ab6 (X).
classes (X,'5') := family(X,'tn').
classes (X,'u') := steel(X,'a'), not ab7(X).
classes (X,'2') := \text{thick}(X,N32), N32>0.8, not ab8(X),
\label{eq:continuous} \begin{array}{c} \text{not ab9}\,(X)\,, \text{ not ab10}\,(X)\,.\\ \text{classes}\,(X,\,'\,3') \ :- \ \text{not steel}\,(X,\,'\,s')\,, \text{ not ab11}\,(X)\,, \text{ not ab6}\,(X)\,. \end{array}
classes (X,'1') := family(X,'?').
classes (X,'1'):- family (X,'zs').
ab1(X) :- hardness(X,'85').
ab2(X) := strength(X,'600').
                                                      ab3(X) := carbon(X.'10').
ab4(X) := hardness(X,'80'), cbond(X,'?').
ab5(X) :- not steel(X,'r'), not enamelability(X,'2').
                                                      ab7(X) :- carbon(X,'03').
ab9(X) :- steel(X,'s').
ab6(X) := steel(X,'a').
ab8(X) := steel(X,'r').
ab10(X) :- not temper_rolling(X,'?').
                                                     ab11(X) :- not family(X,'?').
```

The above generated rule set achieves 0.99 accuracy, 0.99 weighted Macro precision, 0.99 weighted Macro recall, and 0.99 weighed Macro F1 score. The justification tree generated by the FOLD-RM system for the 8<sup>th</sup> test example is shown below:

```
Proof Tree for example number 8:
the value of classes is 2 DOES HOLD because
    the value of condition is 's' which should equal 's' (DOES HOLD)
    exception ab5 DOES NOT HOLD because
    the value of steel is 'r' which should not equal 'r' (DOES NOT HOLD)
    the value of enamelability is '?' which should not equal '2' (DOES HOLD)
{'condition: S', 'enamelability: ?', 'steel: R'}
```

This justification tree is also shown in another manner: by showing which rules were involved in the proof/justification. For each call in each rule that was invoked, FOLD-RM shows whether it is true ([T]) or false ([F]). The head of each applicable rule is similarly annotated. We illustrate this for the 8<sup>th</sup> test example:

```
 \begin{tabular}{ll} [F] ab5(X) := not & [T] steel(X,'r'), not & [F] enamelability(X,'2'). \\ [T] classes(X,'2') := & [T] condition(X,'s'), not & [F] ab5(X). \\ \{'condition: S', 'enamelability: ?', 'steel: R'\} \end{tabular}
```

### 7 Conclusions and Related Work

In this paper we presented FOLD-RM, an efficient and highly scalable algorithm for multi-category classification tasks. FOLD-RM can generate explainable answer set programs and human-friendly justification for predictions. Our algorithm does not need any encoding (such as one-hot encoding) for data preparation. Compared to the well-known classification models like XGBoost and MLP, our new algorithm has similar performance in terms of accuracy, weighted macro precision, weighted macro recall, and weighted macro  $F_1$  score. However, our new approach is much more efficient and interpretable than these other approaches. It is remarkable that an ILP system is comparable in accuracy to state-of-the-art traditional machine learning systems.

ALEPH (Srinivasan 2001) is a well-known ILP algorithm that employs bottom-up approach to induce rules for non-numerical data. Also, no automatic method is available for the specialization process. A tree-ensemble based rule extraction algorithm (Takemura and Inoue 2021) is proposed by Takemura and Inoue, its performance relies on trained tree-ensemble model. It may also suffer from scalability issue because its running time is exponential in the number of valid rules.

In practice, statistical Machine Learning models show good performance for classification. Extracting rules from statistical models is also a long-standing research topic. Rule extraction algorithms are of two kinds: 1) Pedagogical (learning rules from black box models without looking into internal structures), such as, TREPAN (Craven and Shavlik 1995), which learns decision trees from neural networks 2) Decompositional (learning rules by analysing the models inside out) such as, SVM+Prototypes (Nuñez et al. 2006), which employs clustering algorithm to extract rules from SVM classifiers by utilizing support vectors. RuleFit by Friedman and Popescu is another rule extraction algorithm that learns sparse linear models with

original feature decision rules from shallow tree ensemble model for both classification and regression tasks. However, its interpretability decreases when too many decision rules have been generated. Also, simpler approaches that are a combination of statistical method with ILP have been extensively explored. The kFOIL system (Landwehr et al. 2006) incrementally learns kernel for SVM FOIL style rule induction. nFOIL (Landwehr et al. 2005) is an integration of Naive Bayes model and FOIL. TILDE (Blockeel and De Raedt 1998) is another top-down rule induction algorithm based on C4.5 decision tree, it can achieve similar performance with Decision Tree. However, it would suffer from scalability issue when there are too many unique numerical values in the dataset. For most datasets we experimented with, the number of leaf nodes in the trained C4.5 decision tree is much more than the number of rules that FOLD-R++/FOLD-RM generate. The FOLD-RM algorithm outperforms the above methods in efficiency and scalability due to (i) its use of learning defaults, exceptions to defaults, exceptions, and so on (i) its top-down nature, and (iii) its use of improved method (prefix sum) for heuristic calculation.

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