# 树形穷举

# 树和图的遍历

Youtube频道: hwdong

博客: hwdong-net.github.io

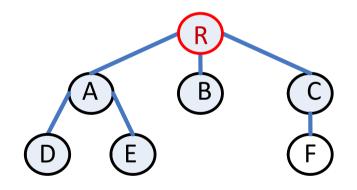
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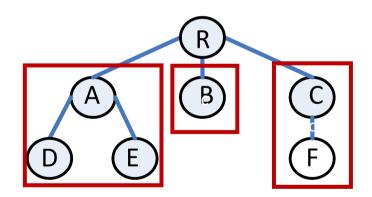
# 树

树是一种数据结构,它由节点和边组成,每个节点可以有零个或多个子节点,其中只有一个节点没有父节点,称为根节点。



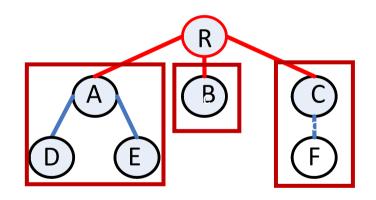


• 每个节点可以有零个或多子树。



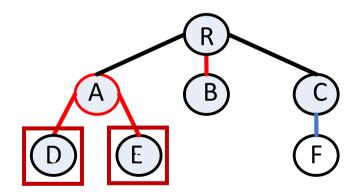
# 树

每个节点可以有零个或多子树。子树的根节点是这个 节点的孩子



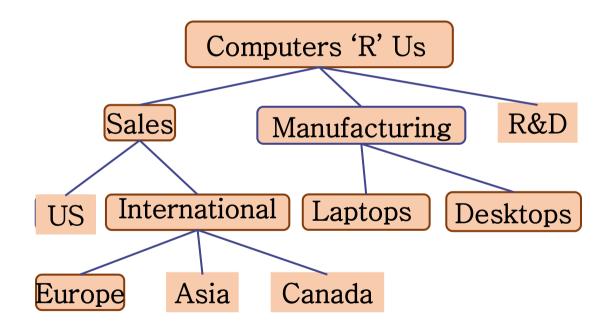
# 树

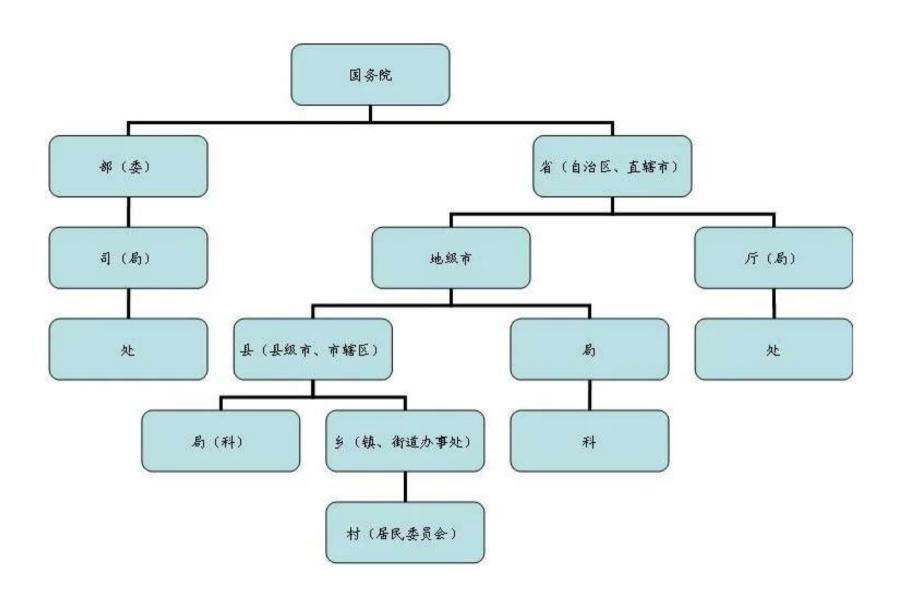
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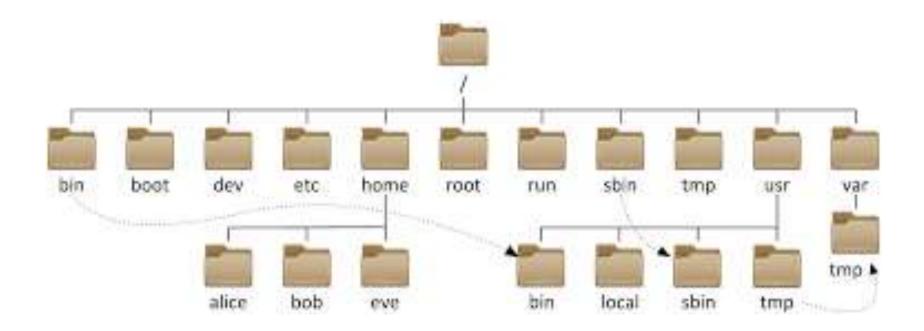
# 树的例子

• 组织结构

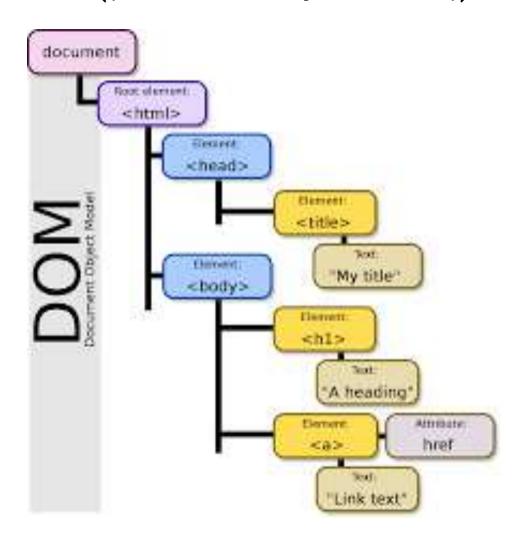




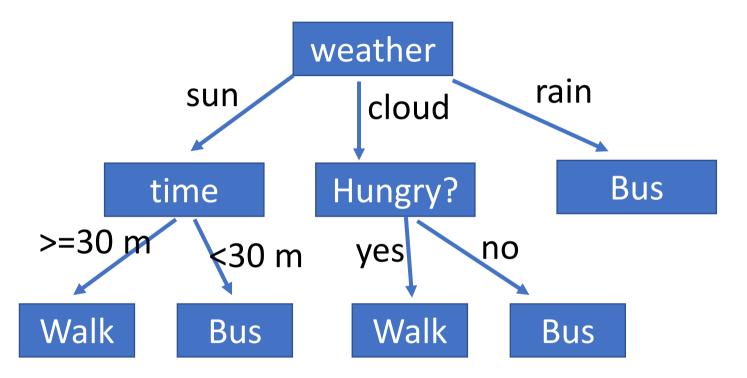
#### • 文件系统

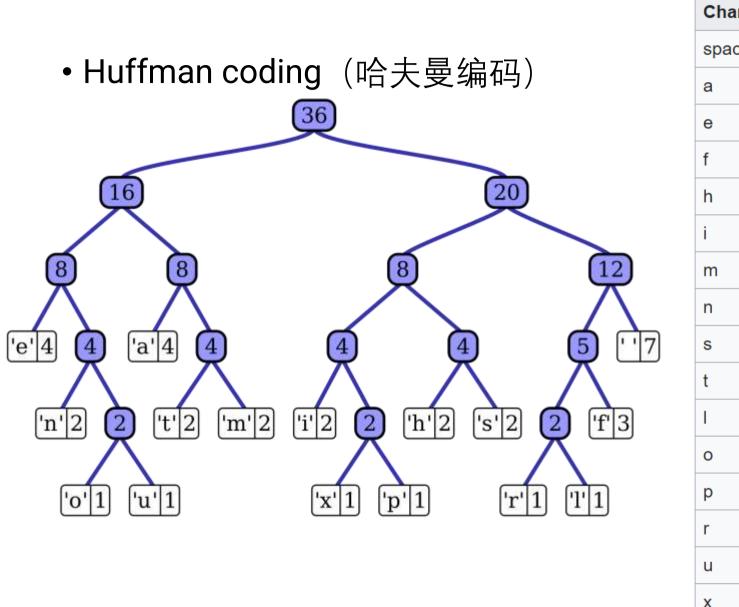


• HTML DOM ((Document Object Model))



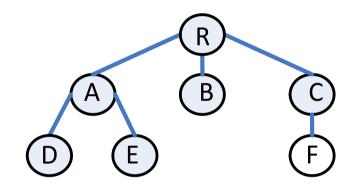
• Decision tree (决策树)



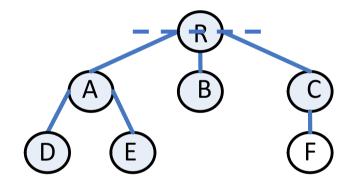


Char ♦	Freq +	Code +	
space	7	111	
а	4	010	
е	4	000	
f	3	1101	
h	2	1010	
i	2	1000	
m	2	0111	
n	2	0010	
s	2	1011	
t	2	0110	
1	1	11001	
O	1	00110	
р	1	10011	
r	1	11000	
u	1	00111	
x	1	10010	

• 一层一层地访问每个节点。

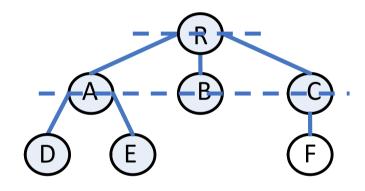


• 一层一层地访问每个节点。



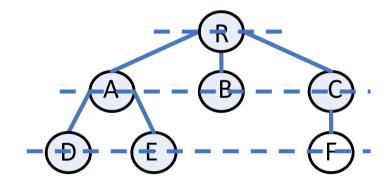
R

• 一层一层地访问每个节点。



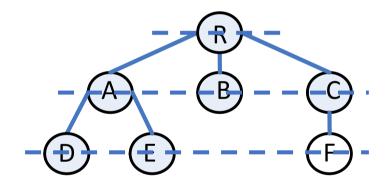
RABC

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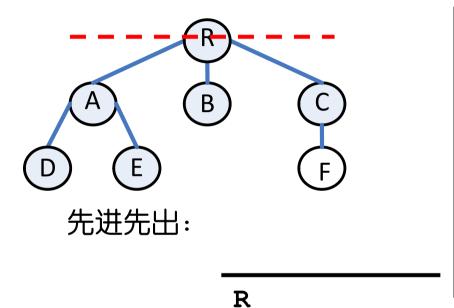


RABCDEF

• 先访问的节点的孩子节点也先访问。A比C先访问,A的孩子D、E比C的孩子F先访问。

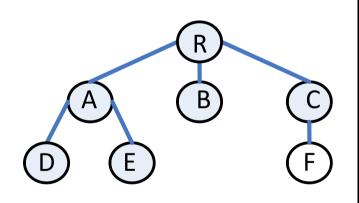


RABCDEF



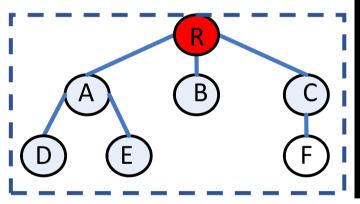
```
R A B C D E F
```

先序(根)深度优先遍历: 先访问当前节点, 再递归地 遍历它的所有子树。



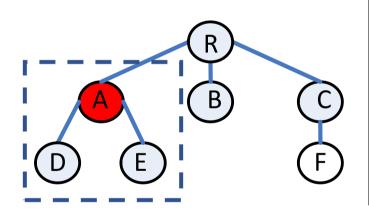
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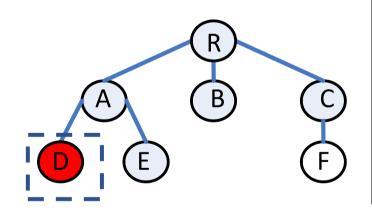
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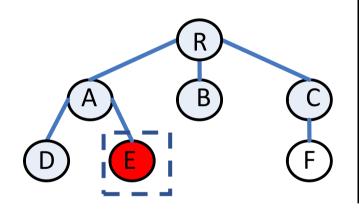
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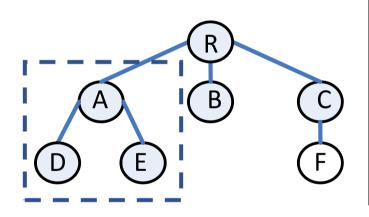
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```

RADF

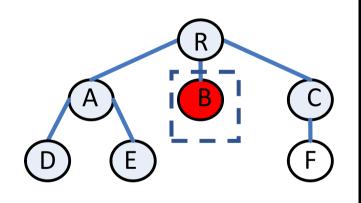
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```
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RADE

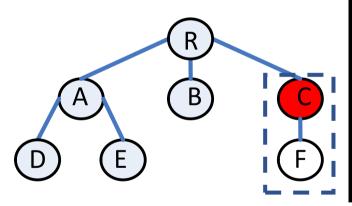
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```
preorder(node)
  if node is not null
    visit(node)
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```

RADEB

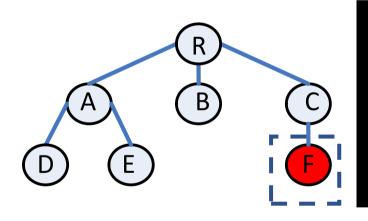
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```

RADEBC

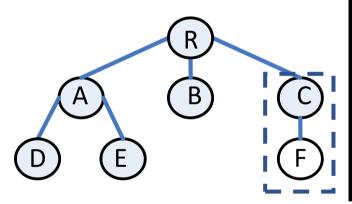
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RADEBCF

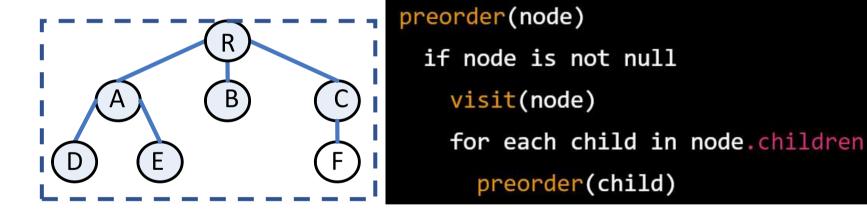
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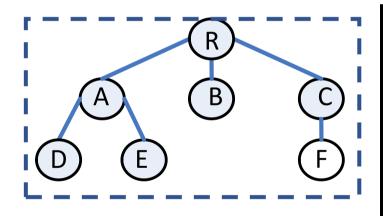
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RADEBCF

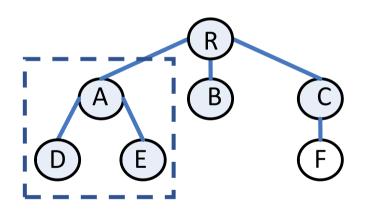
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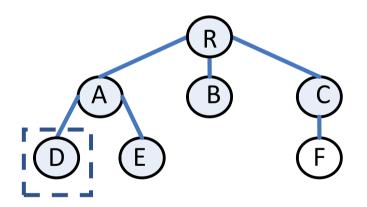
RADEBCF



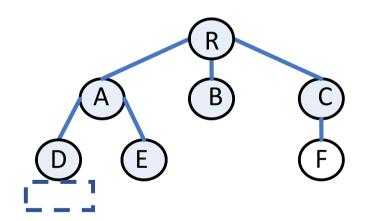
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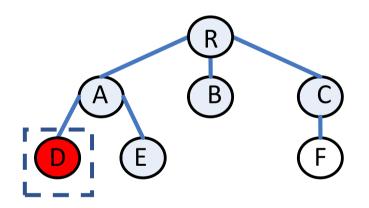
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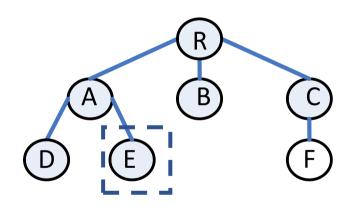
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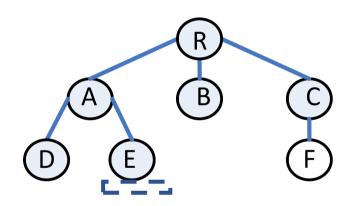
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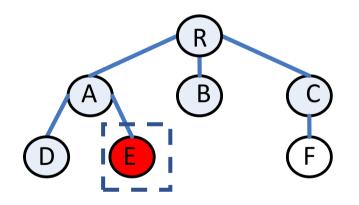
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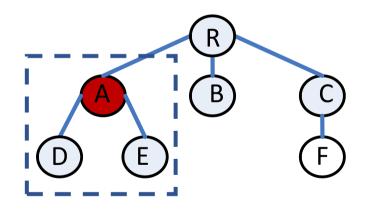


```
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    visit(node)
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postorder(node)
  if node is not null
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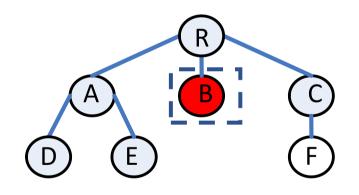
• 后序(根)深度优先遍历: 先递归地遍历当前节点的所有子树, 再访问当前节点。



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    for each child in node.children
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    visit(node)
```

DEA

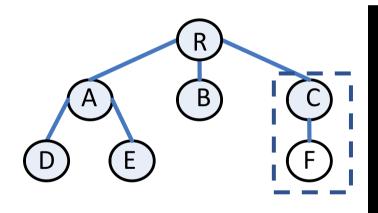
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```
postorder(node)
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```

DEAB

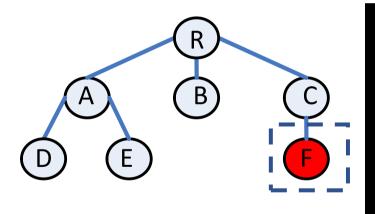
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DEAB

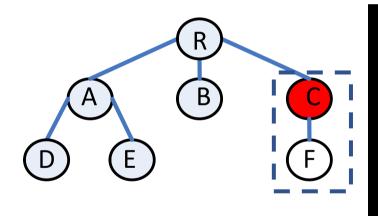
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DEABF

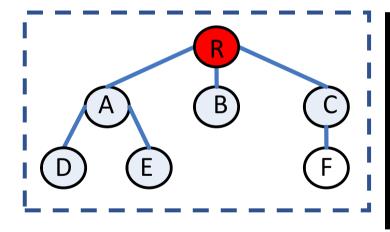
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DEABFC

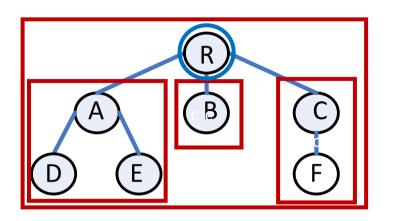
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postorder(node)
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  visit(node)
```

DEABFCR

### 树的(先序)深度优先遍历



```
DFS(V){
直接访问V;
for(V的每个孩子W)
DFS(W);
}
```

```
DFS(R) \begin{cases} R & A & D \\ DFS(D) & DFS(D) \end{cases}
DFS(E) \begin{cases} E \\ B \end{cases}
DFS(C) \begin{cases} C \\ DFS(F) \end{cases}
```



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## 图的基本概念

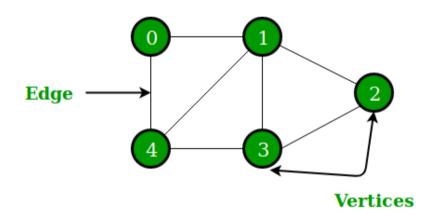
#### Graph

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博客: hwdong-net.github.io



• 图G(V,E)是一个顶点集合V和边集合E。



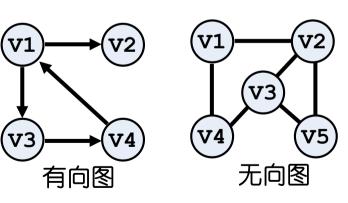
### 冬

•图G(V,E)是一个顶点集合V和边集合E。

• 无向图(undirected graph)中,边是一个无序顶点对:e=(u,v)

• 有向图(directed graph)中,边是一个有序顶点

对:e=<u,v>

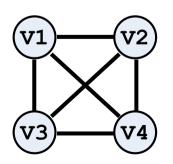


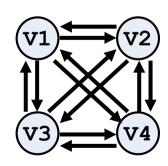
### 冬

- •图G(V,E)是一个顶点集合V和边集合E。
- 无向图(undirected graph)中,边是一个无序顶点对:e=(u,v)
- 有向图(directed graph)中,边是一个有序顶点对:e=<u,v>
- 多边图(a multigraph): 2个顶点之间可以有多个边。
- self-loop: 一个边的2个顶点是同一个顶点
- 简单图:没有self-loop边的图。

### 完全图

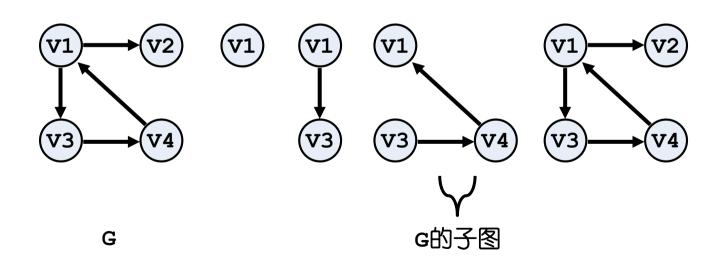
- 若无向图任意2个顶点之间都有一条边,则此图为完全 无向图。
- 若有向图任意2个顶点u,v都有2条反向的弧<u,v>和 <v,u>, 则此图为完全有向图。
- 完全图其实就是边/弧的数量达到最大值





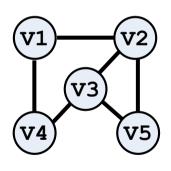
### 子图

- 有两个图G={V, E},G'={V',E'}
- 如果 ♥ V, € 'E, 则称 G'为 G 的 子 图



### 邻接点、度

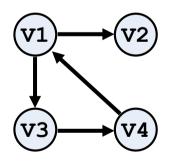
- u, v之间如果有一条边(u,v)或弧<u,v>,就称它们互为邻接点。
- 一个顶点的邻接点或边的个数, 称为该顶点的度。



- TD(V1) = 2
- TD(V2) = 3
- TD(V3) = 3
- TD(V4) = 2
- TD(V5) = 2

### 入度和出度

• 对有向图: 顶点的出度是以它为弧尾的弧的个数, 而入度是以它为弧头的弧的个数。度= 入度+出度



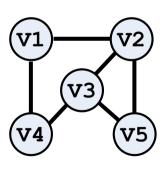
- ID (V1)=1
- OD (V1) = 2
- TD(V1) = ID(v1) + OD(v1) = 3

### 度和边数的关系

• 一个有n个顶点, e条边或弧的图满足:

$$e = \frac{1}{2} \sum_{i=1}^{n} TD(v_i)$$

• 即边(或弧)的总数 = 各个顶点的度的总数的一半



- TD(V1)=2 e=6
- TD(V2) = 3
- TD(V3) = 3
- TD(V4) = 2
- TD(V5) = 2

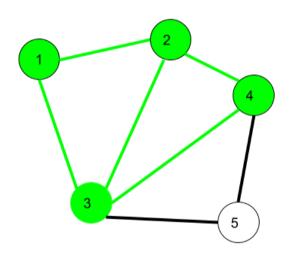
# 路径、简单路径、trail、回路,简单回路

- Walk (路径):从一个顶点经过一系列边到达另外一个顶点, 所经过的顶点和边的序列。
- Trail:没有重复边的路径
- Path (简单路径):没有重复顶点和边的路径。
- 回路(Circuit): 没有重复边(可能有重复顶点)、起点 终点相同的路径
- Cycle(简单回路):除起点终点相同、没有重复顶点和重复边的回路

**Walk (路径)**: 从一个顶点经过一系列边到达另外一个顶点, 所经过的顶点和边的序列。

Vertices may repeat. Edges may repeat (Closed or Open)

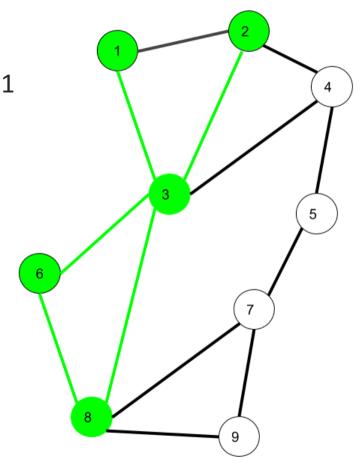
如: 1-2-3-4-2-1



**Trail**: Vertices may repeat. Edges cannot repeat



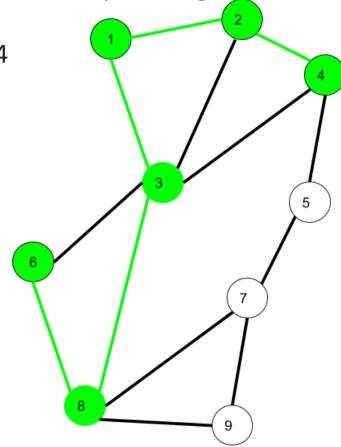
如: 1-3-8-6-3-2 1-3-8-6-3-2-1



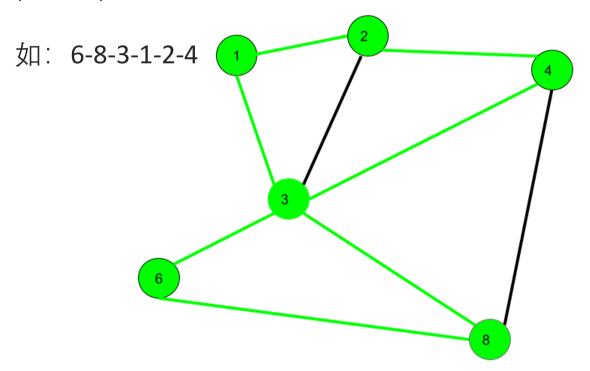
path: Vertices cannot repeat. Edges cannot repeat



如: 6-8-3-1-2-4



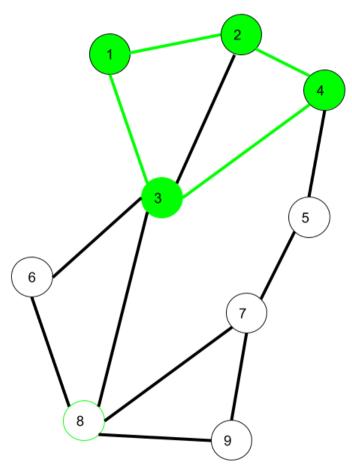
circuit : Vertices may repeat. Edges cannot repeat
(Closed)



cycle: Vertices cannot repeat. Edges cannot repeat

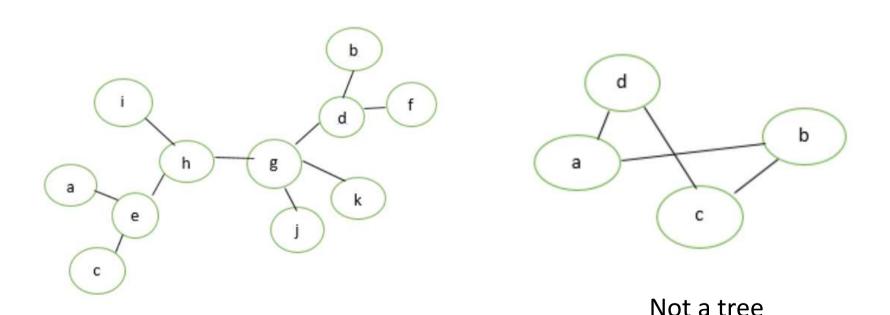
(Closed)

如: 1-2-4-3-1



### 树是一种特殊的图

•没有(简单)回路(cycle)的连通图称为树。



## 图的遍历

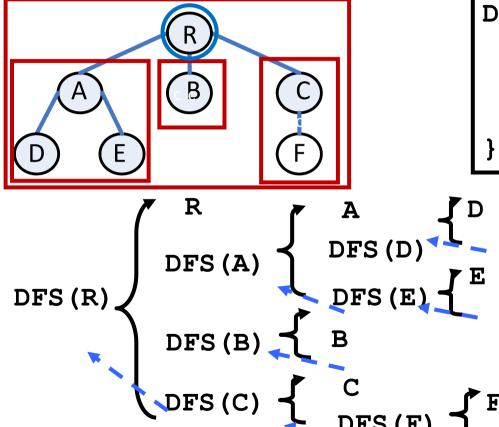
Youtube频道: hwdong

博客: hwdong-net.github.io

### 图的遍历

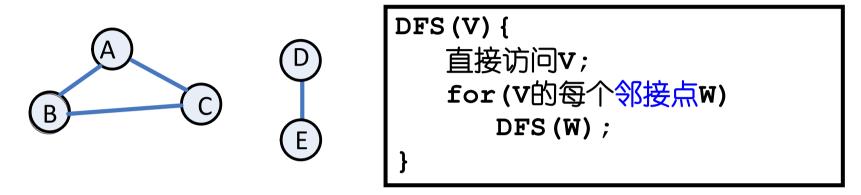
- 访问图中所有顶点, 并且每个顶点仅被访问一次
- 类似于树的先根遍历和层次遍历,有图的**深度优先遍历** 和**广度优先遍历**

### 回顾树的先根遍历



```
DFS(V){
直接访问V;
for(V的每个孩子W)
DFS(W);
}
```

## 树的遍历方法是否不修改就适用于图的遍历呢?



问题 1:由于图存在环路,所以会导致无限循环

解决方法:设置顶点访问标志

问题 2: 由于图不一定连通,从一个顶点出发的遍历只能访问其

所在的连通分量中的所有顶点

解决方法: 重复调用从一个顶点出发的DFS

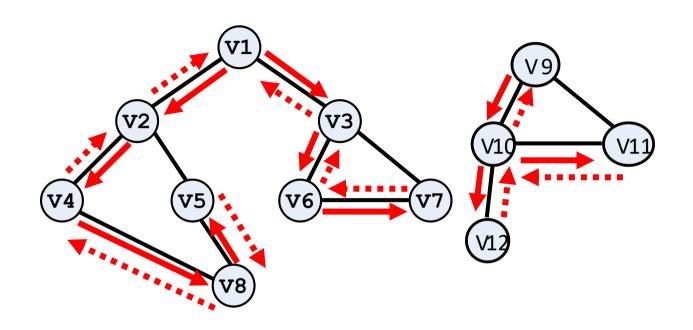
## 图的深度优先遍历

```
DFS(V){
    直接访问V;
    flag(v) = 1;
    for(V的每个邻接点W)
        if(!flag(w))
        DFS(W);
}
```

从一个顶点出发的深度优先遍历

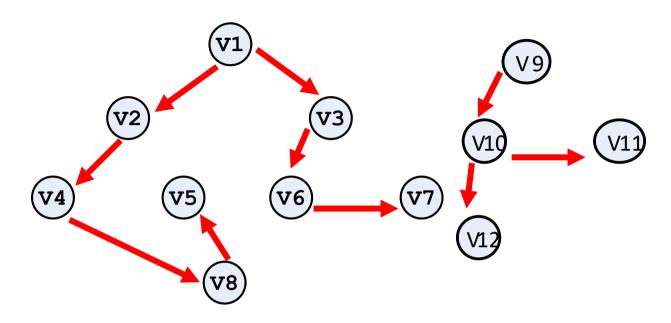
```
DFS(G) {
  for(每个v)
  flag[v]= 0;
  for(每个v)
  if(!flag(v))
  DFS(v);
}
```

整个图的深度优先遍历



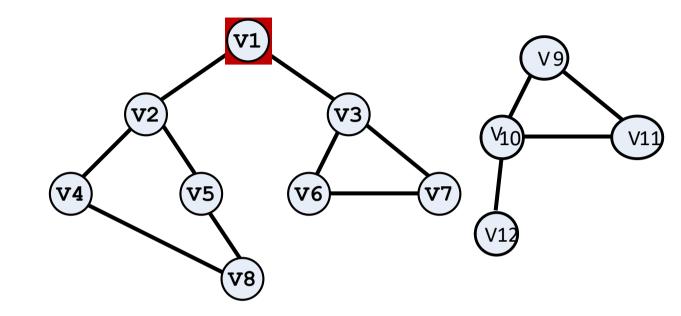
V1->V2->V4->V8->V5->V3->V6->V7 V9->V10->V12->V11

## 图的深度优先遍历是一个树形穷举过程

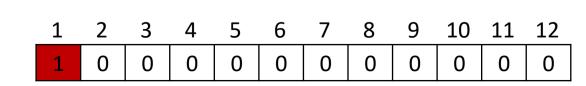


深度优先搜索树

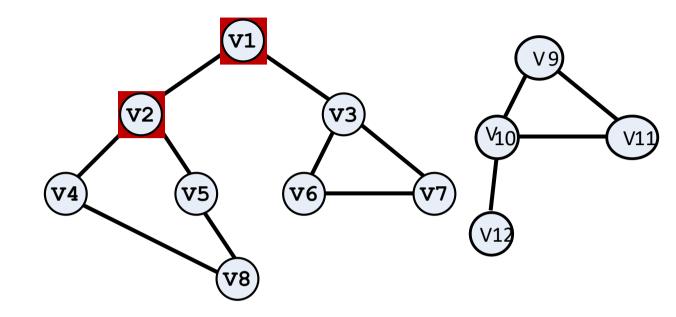
DFS(V1) 访问V1做标记

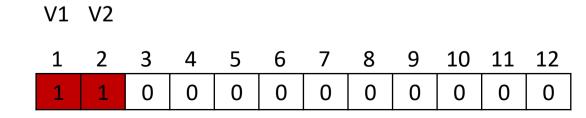


V1

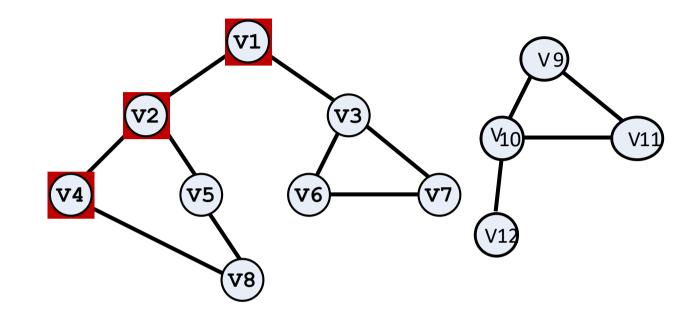


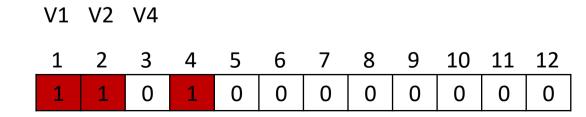
DFS(V1) 访问V1做标记 DFS(V2) 访问V2做标记

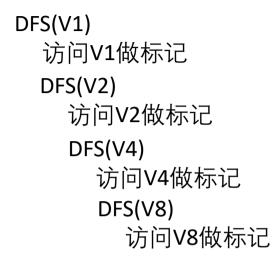


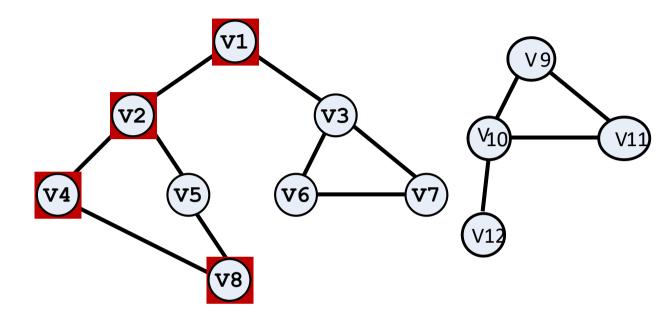


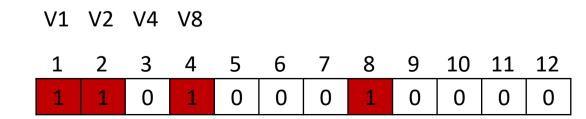
DFS(V1) 访问V1做标记 DFS(V2) 访问V2做标记 DFS(V4) 访问V4做标记

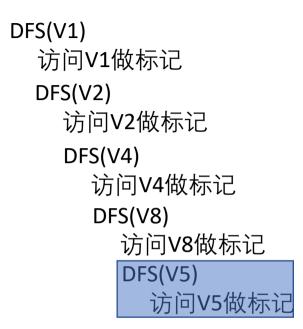


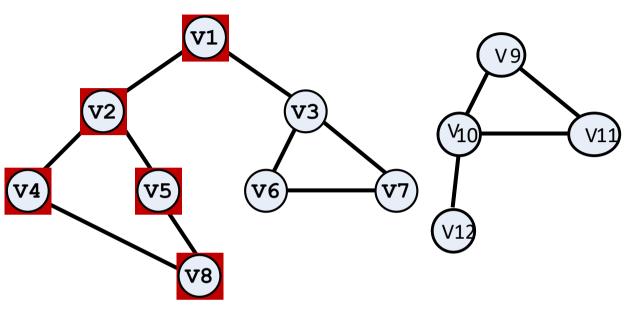


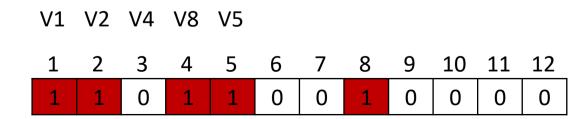


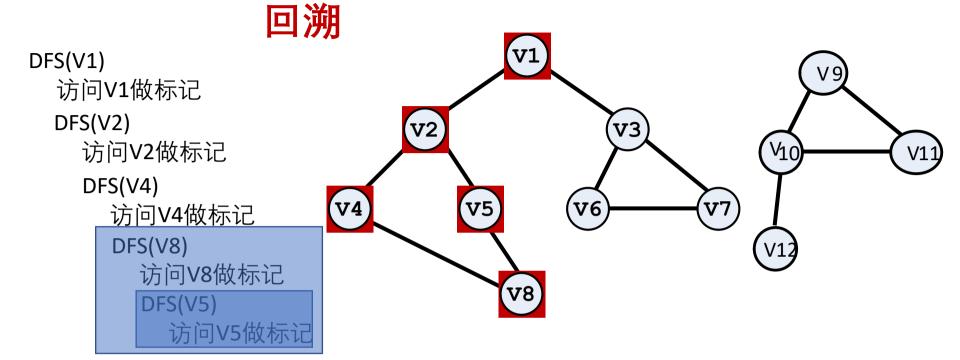


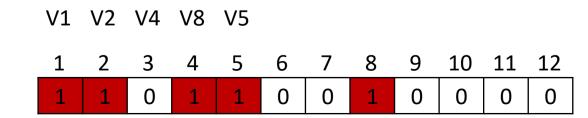


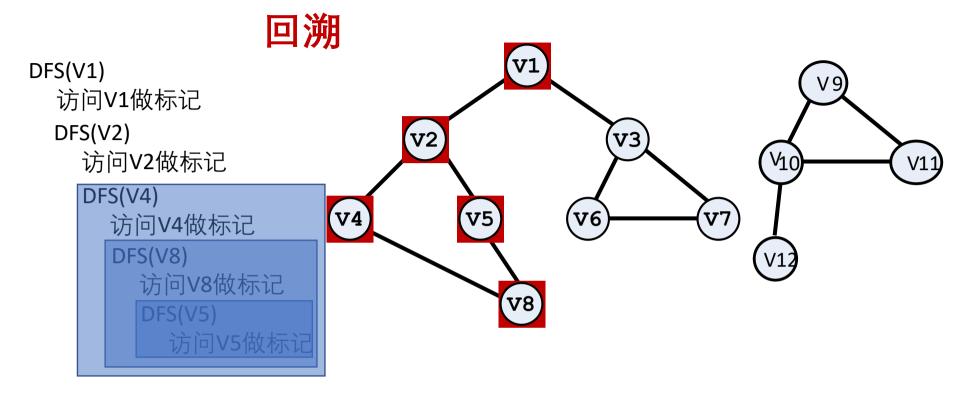


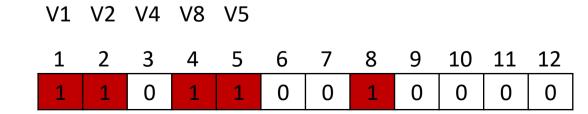


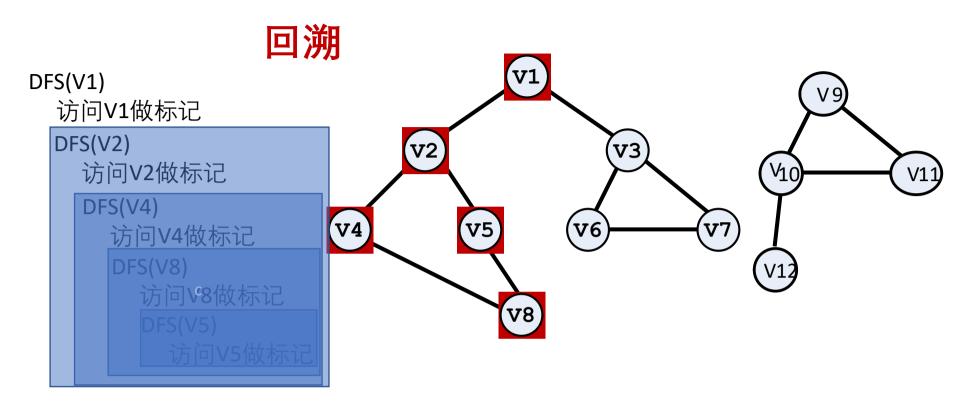


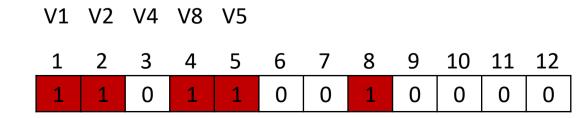


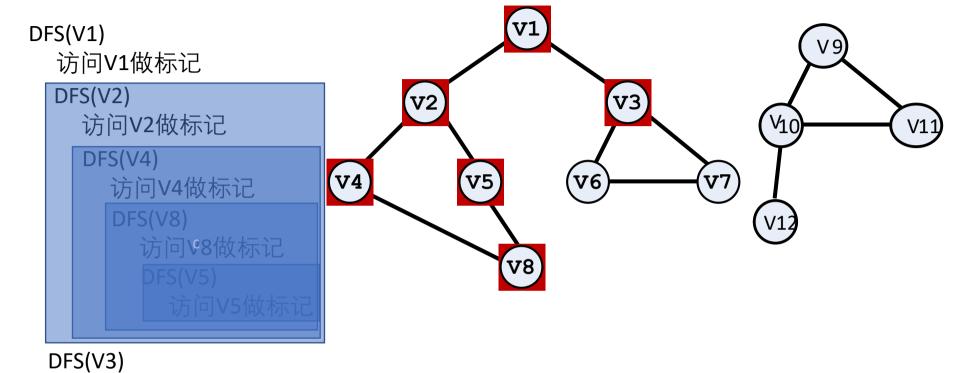




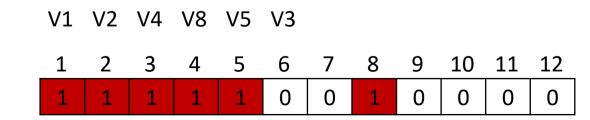








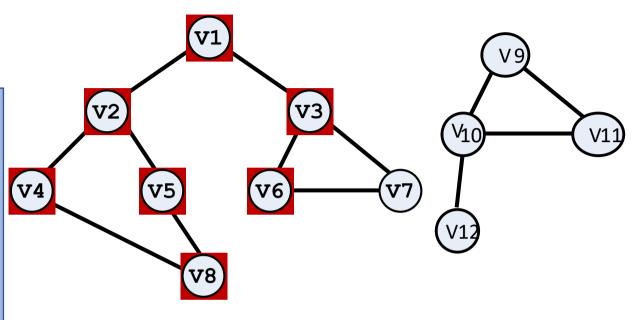
访问V3做标记



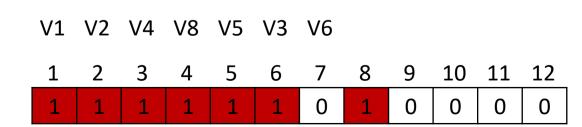
#### DFS(V1)

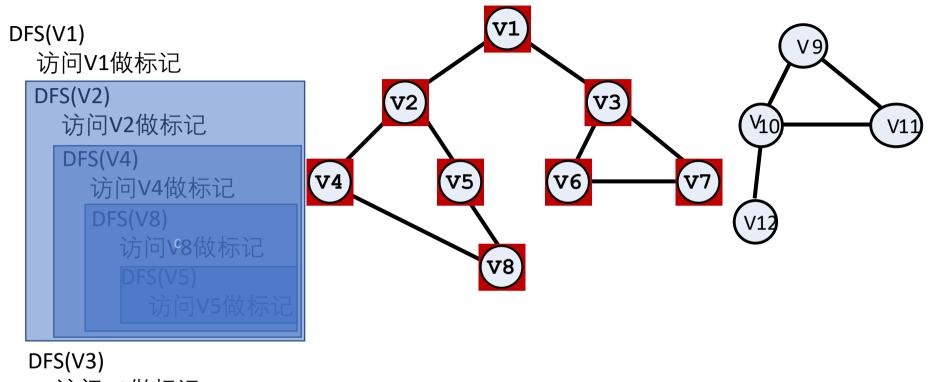
访问V1做标记

# DFS(V2) 访问V2做标记 DFS(V4) 访问V4做标记 DFS(V8) 访问V8做标记 DFS(V5) 访问V5做标记

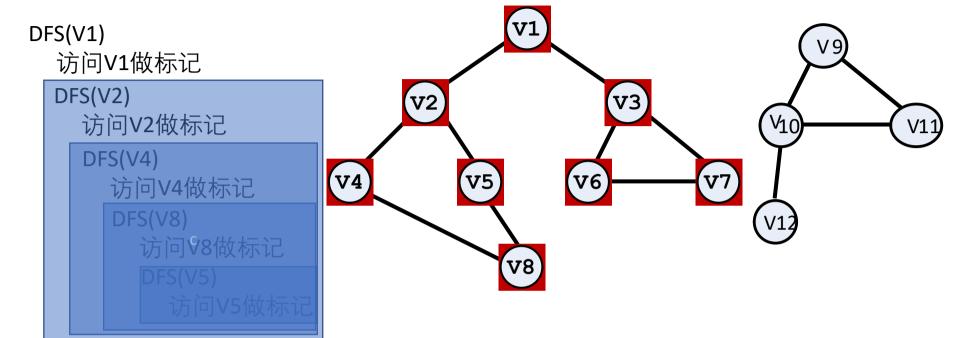


#### DFS(V3) 访问V3做标记 DFS(V6) 访问V6做标记





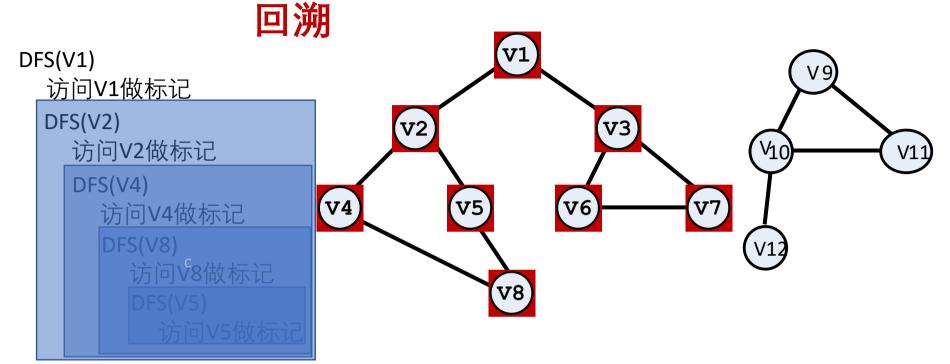
访问V3做标记 DFS(V6) 访问V6做标记	V1	V2	V4	V8	V5	V3	V6	V7				
DFS(V7)	1	2	3	4	5	6	7	8	9	10	11	12
访问V7做标记	1	1	1	1	1	1	1	1	0	0	0	0



DFS(V3) 访问V3做标记 DFS(V6) 访问V6做标记 DFS(V7)

)FS(V7) 访问V7做标记

V1	V2	V4	V8	V5	V3	V6	V7				
							8				
1	1	1	1	1	1	1	1	0	0	0	0



#### DFS(V3)

访问V3做标记

DFS(V6) 访问V6做标记 DFS(V7) 访问V7做标记

V1	V2	V4	V8	V5	V3	V6	V7				
							8				
1	1	1	1	1	1	1	1	0	0	0	0

## 回溯



访问V1做标记

#### DFS(V2)

访问V2做标记

#### DFS(V4)

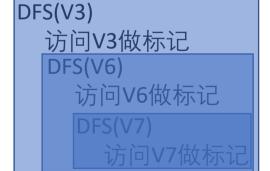
访问V4做标记

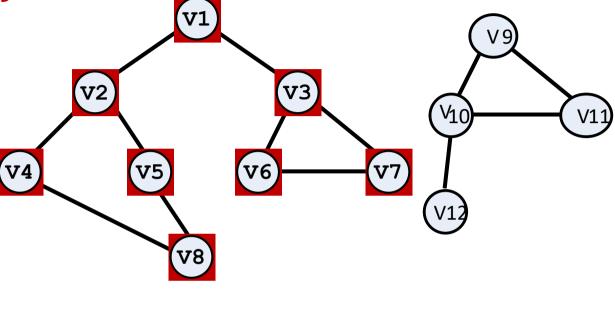
DFS(V8)

访问V8做标记

DFS(V5)

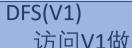
访问V5做标证





V1	V2	V4	V8	V5	V3	V6	V7				
							8				
1	1	1	1	1	1	1	1	0	0	0	0

## 回溯



访问V1做标记

DFS(V2)

访问V2做标记

DFS(V4)

访问V4做标记

DFS(V8)

访问 48 做标记

|DF3(V3) | 访问V5做标<del>:</del>

#### DFS(V3)

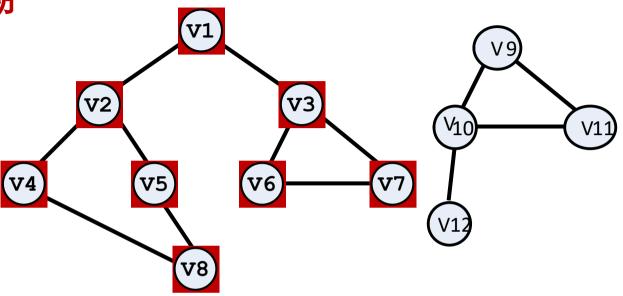
访问V3做标记

DFS(V6)

访问V6做标记

DFS(V7)

访问V7做标记



V1 V2 V4 V8 V5 V3 V6 V7

1	2	3	4	5	6	7	8	9	10	11	12

1	1	1	1	1	1	1	1	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---

## 深度优先遍历

回溯法

## 时间复杂度

- 1) 初始化访问标志 O(n)
- 2) 每个顶点的DFS,对每个顶点,要访问其邻接点:

对于邻接表,次数为顶点的度,而所有顶点的度之和为2e,加上顶点都访问一次,共(n+2e)

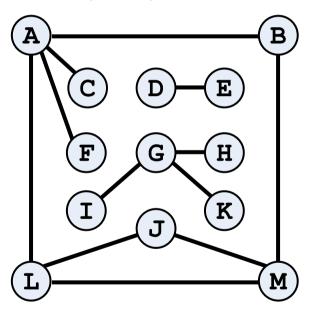
对于邻接矩阵,次数为n\*n

```
DFS(G) {
    for(每个v)
    flag = 0;
    for(每个v)
    if(!flag(v))
    DFS(v);
}
```

## 练习

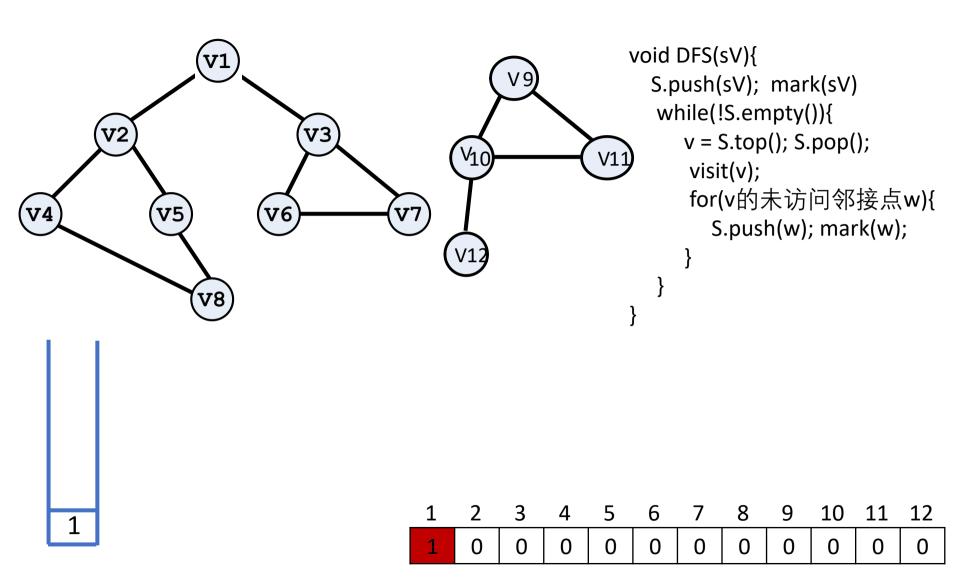
• 写出对下图进行深度优先遍历的结果

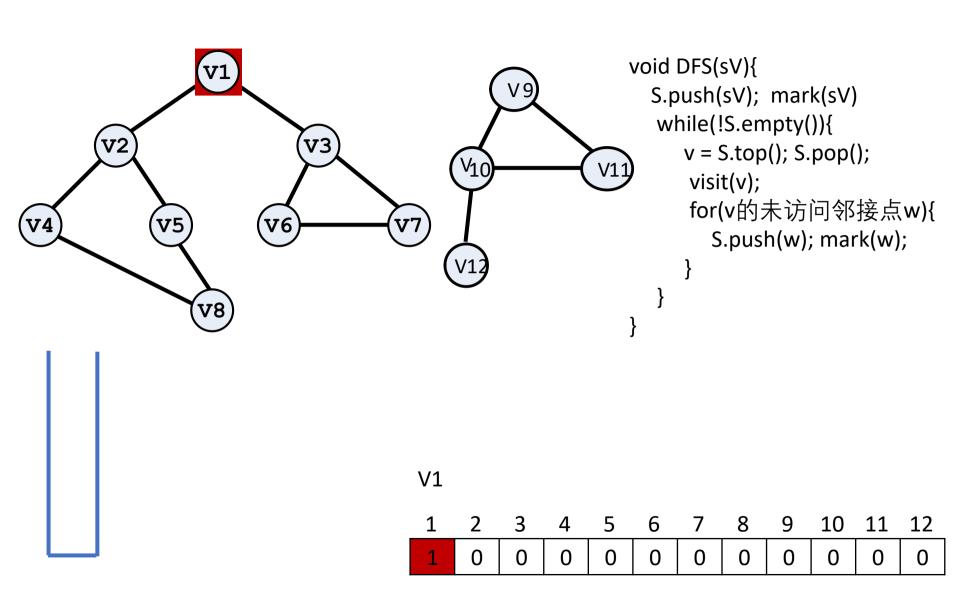
ABMJLCF DE GHIK

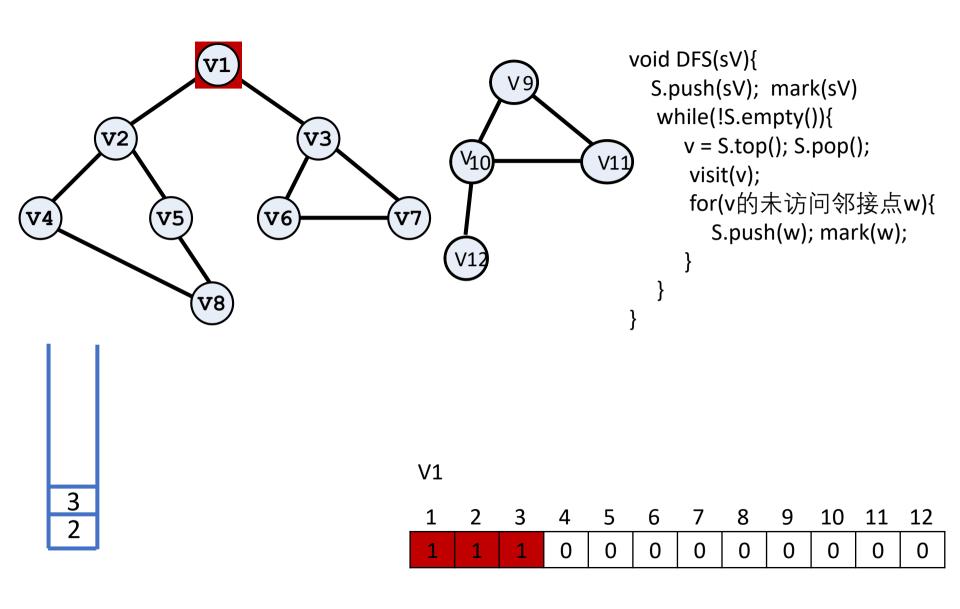


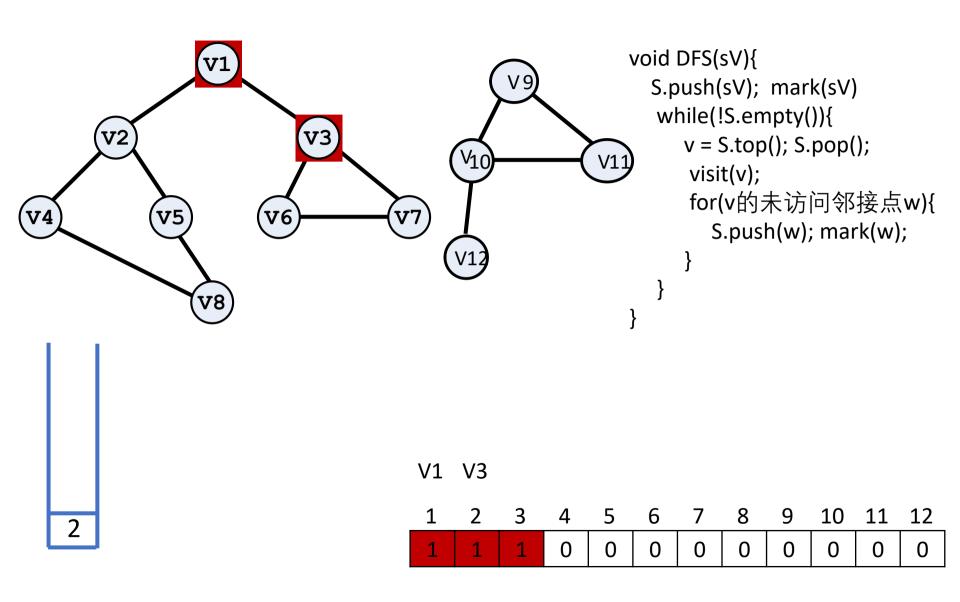
## 基于堆栈的深度优先遍历

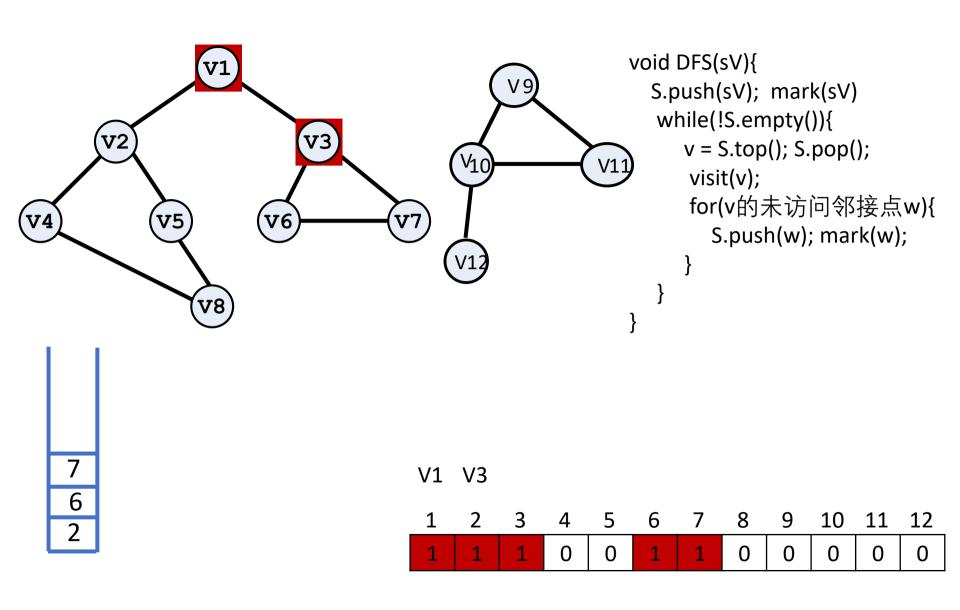
```
void DFS(sV){
 S.push(sV); mark(sV) //入栈、设标记
  while(!S.empty()){
    v = S.top(); S.pop();
     visit(v);
                         //访问它
     for(v的未访问邻接点w){
       S.push(w); mark(w); //入栈、设标记
```

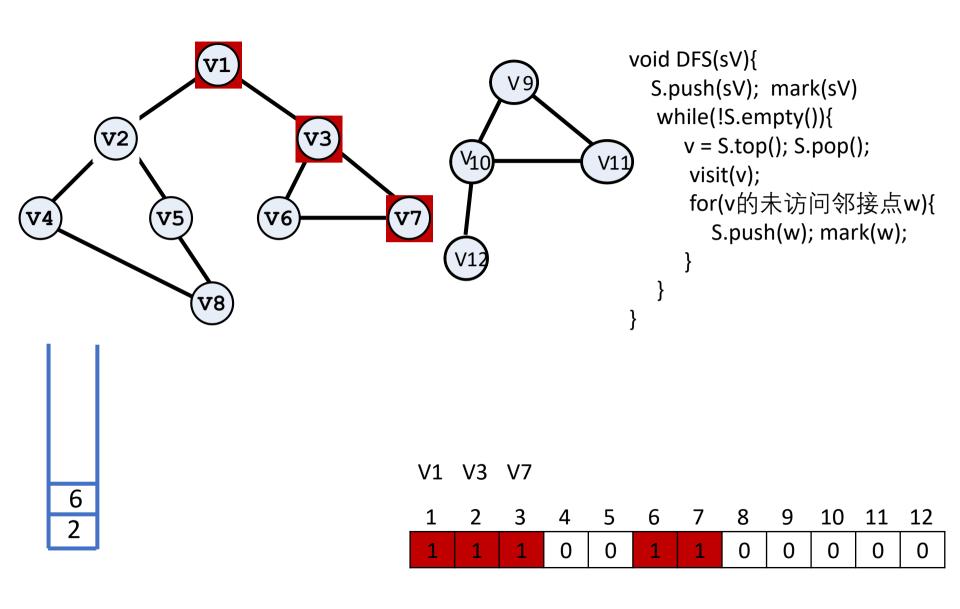


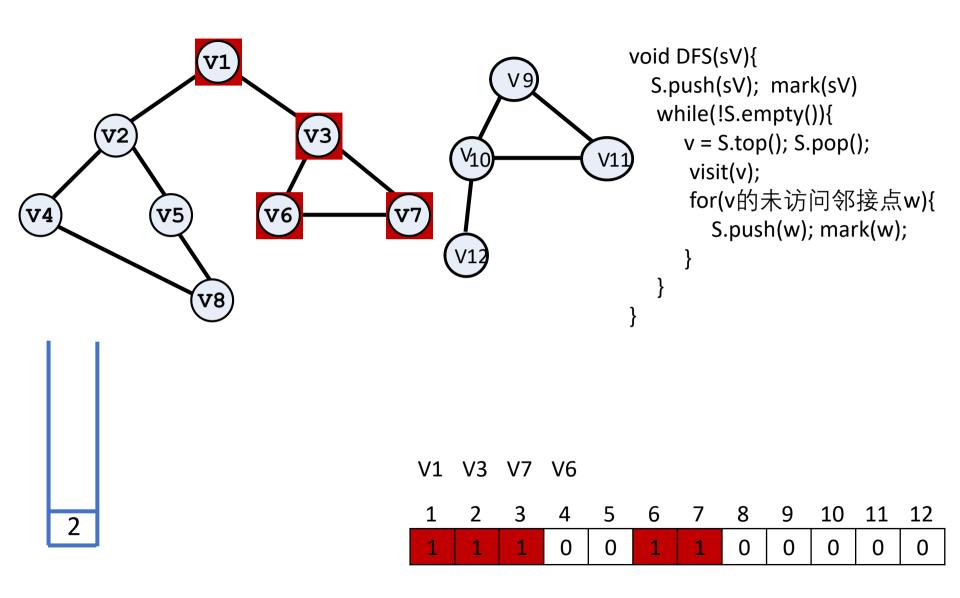


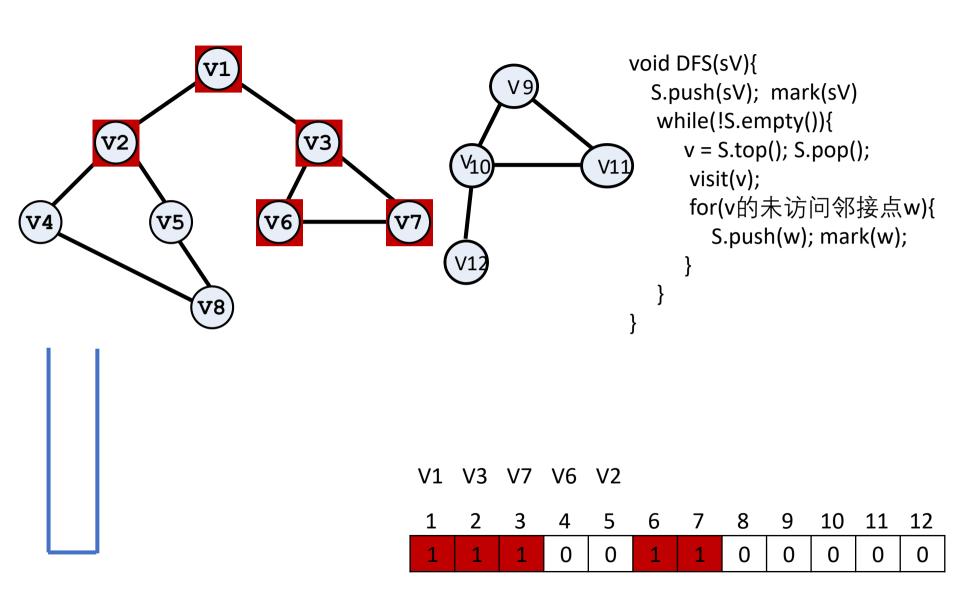


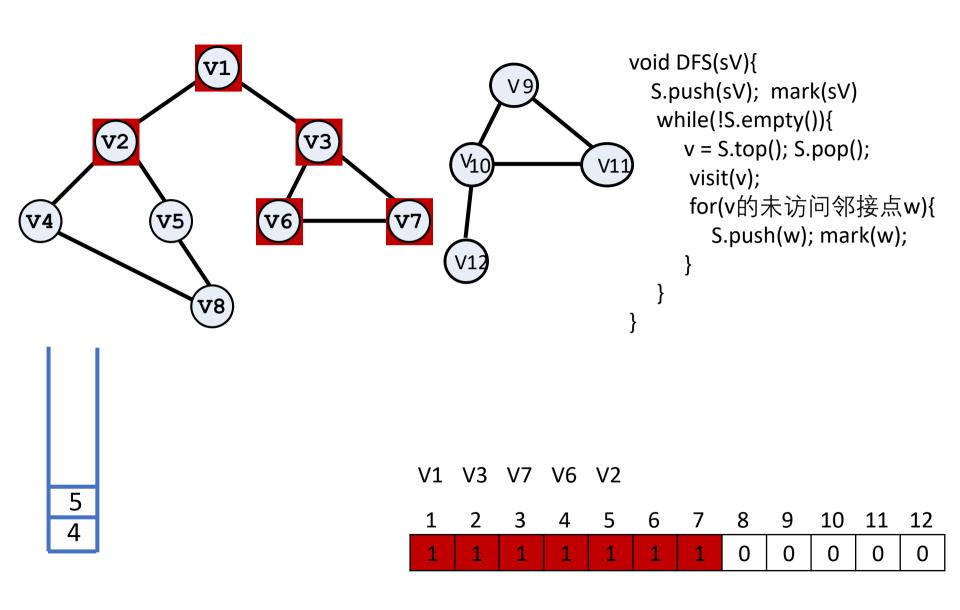


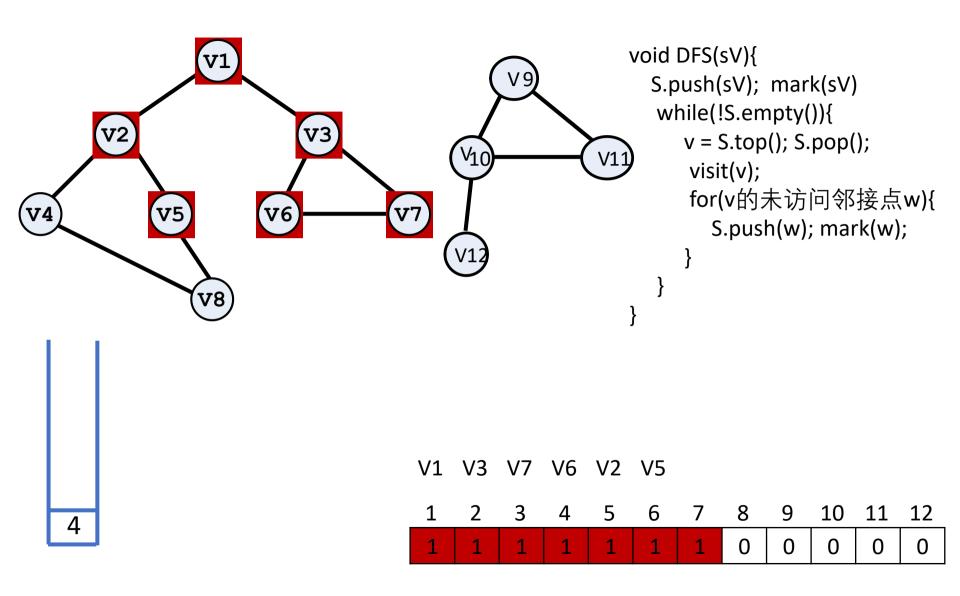


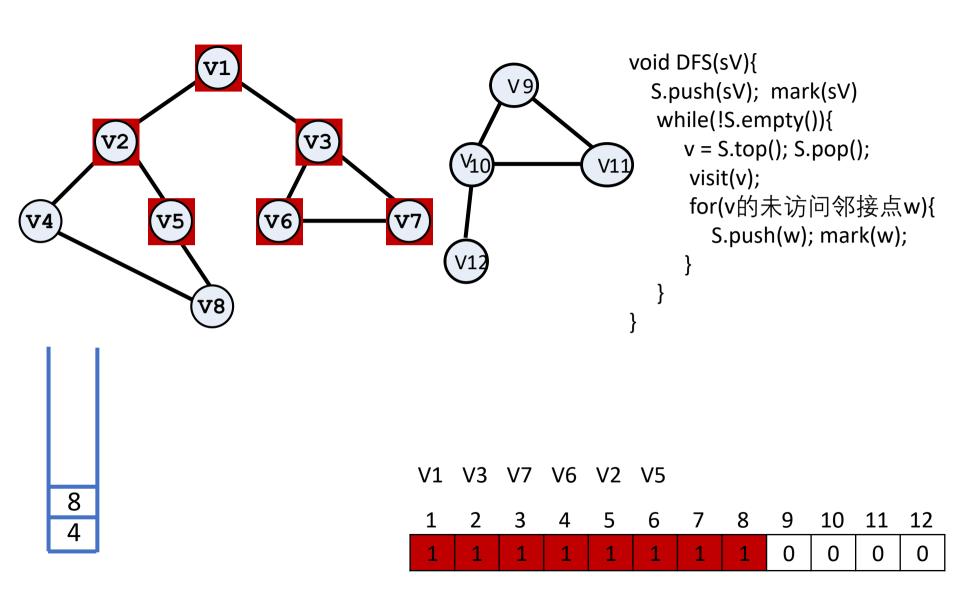


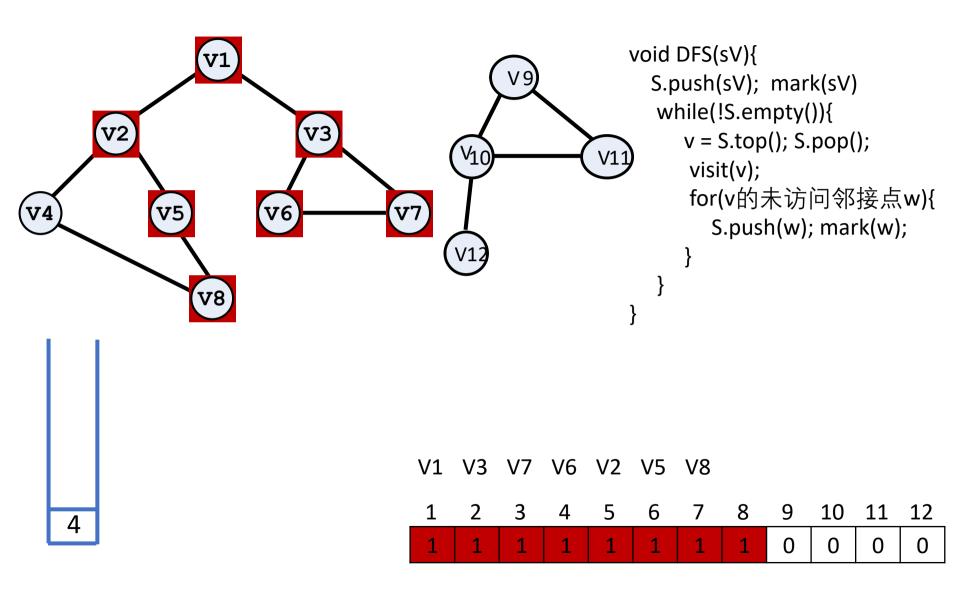


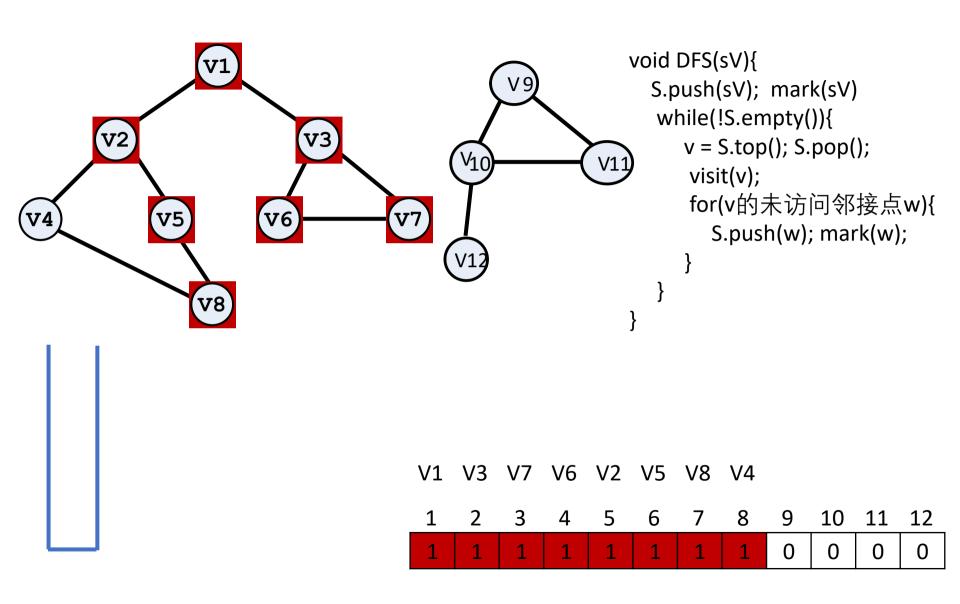








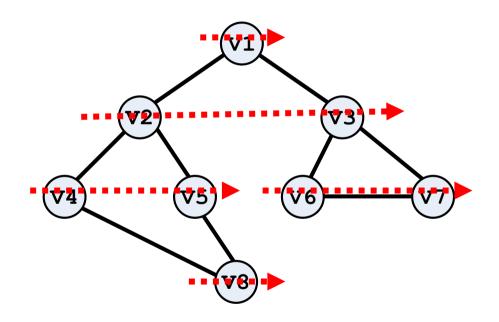




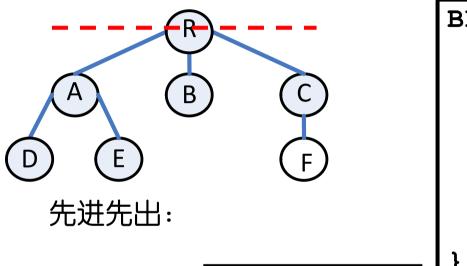
# 图的广度优先遍历

# 图的广度优先遍历

• 类似于树的广度优先(层次)遍历



V1->V2->V3->V4->V5->V6->V7->V8



R

```
BFS(R) {
R入队;
while(|
队头V
V的孩
R.A.B.
```

```
日

日

日

上

大

送

大

出

R A B C

D E F
```

```
R A B C D E F
```

```
日
日
日
大
送
先
出
:
R A B C D E F
```

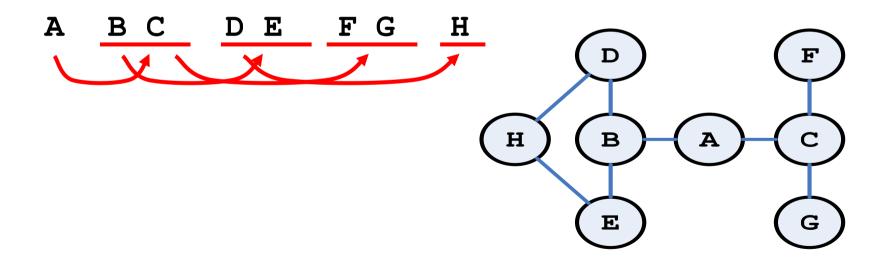
#### BFS(v)

```
queue<int> Q; //初始化队列
Q.push(v0); flag[v0] = 1; //入队并做标记
while( !Q.empty()){ //队列不空
  v = Q.front();Q.pop(); //出队一个顶点
                        //访问它
  visit(v);
  for(v的未被访问邻接点w){
      Q.push(w); flag[w] = 1; //入队并做标记
```

#### BFS(G)

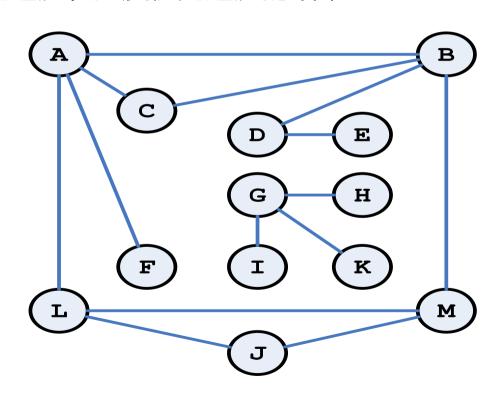
### 练习

写出对下图进行广度优先遍历的结果



#### 图的遍历: 广度优先遍历

写出对下图进行深度优先遍历和广度优先遍历的结果

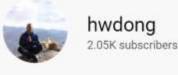


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