Aim of course

- to provide a framework for error analysis in research work
- to emphasise the importance of hypothesis testing

This will involve revisiting some topic you have met previously, but here the emphasis will be to understand the origin (and limitations) of various rules-of-thumb and techniques that are commonly encountered

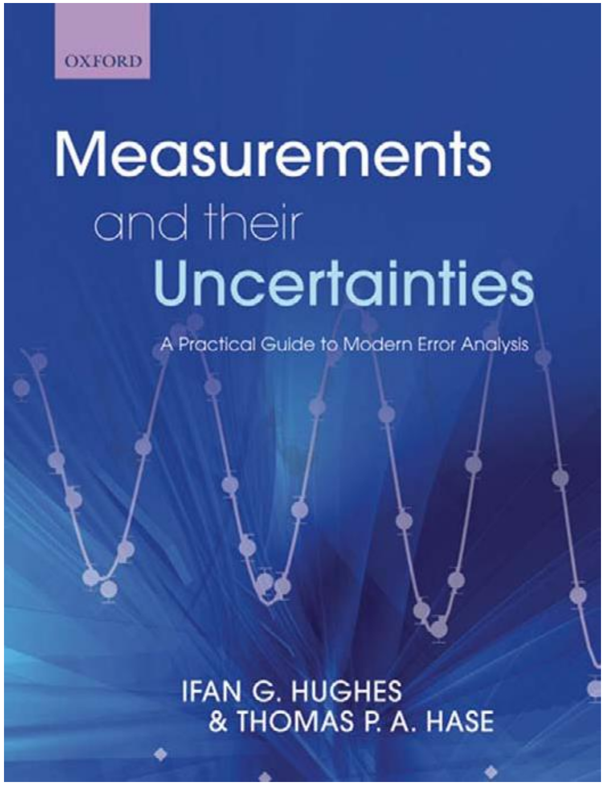
Course book

The course will be based on the book

Measurements and their Uncertainties

I G Hughes and T P A Hase

(OUP, 2010)



- the lecture slides are from the book
- Most homework questions are from the book

Session breakdown

The course will have four quarters:

I will not lecture for two hours. Sessions will also include time to work on problems, and to discuss previous week's work.

1. The first quarter is based on **Chapters 1–3**
2. The second quarter is based on **Chapters 4–5**
3. The third quarter is based on **Chapters 6–7**
4. The fourth quarter is based on **Chapters 8–9**

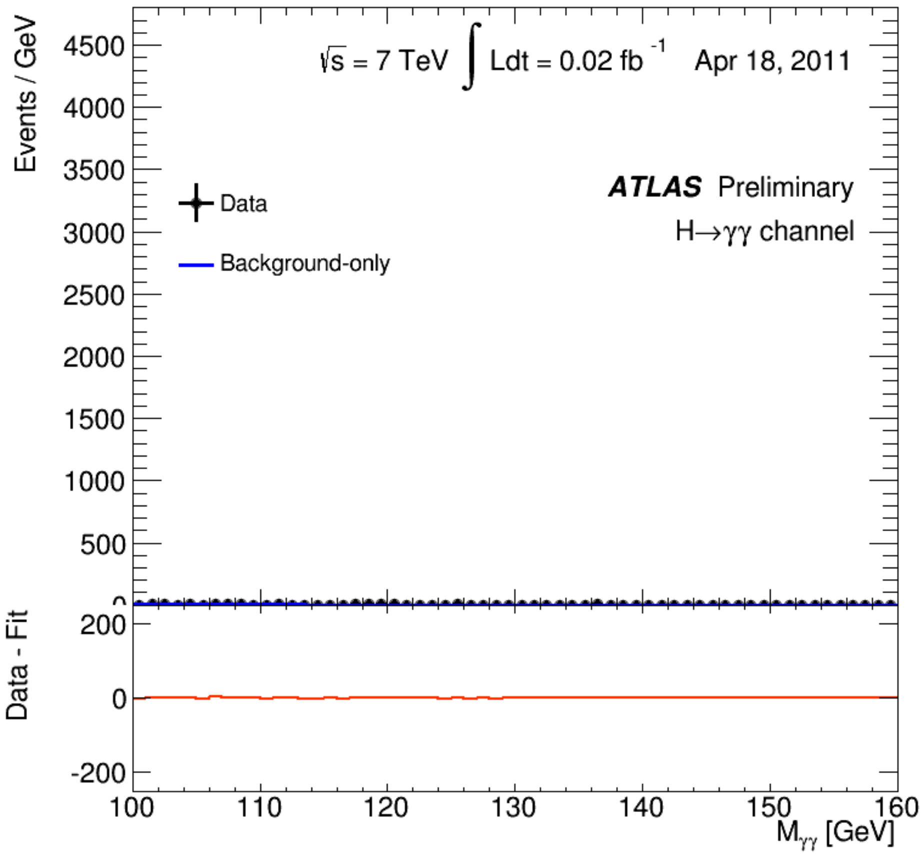
What is the role of experiments in the physical sciences?

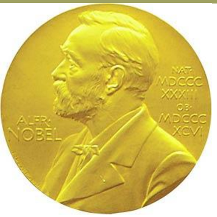
Nobel Laureate, ***Richard Feynman:***

"The principle of science, the definition, almost, is the following*: The test of all knowledge is experiment*. Experiment is the *sole judge* of scientific "truth"".*

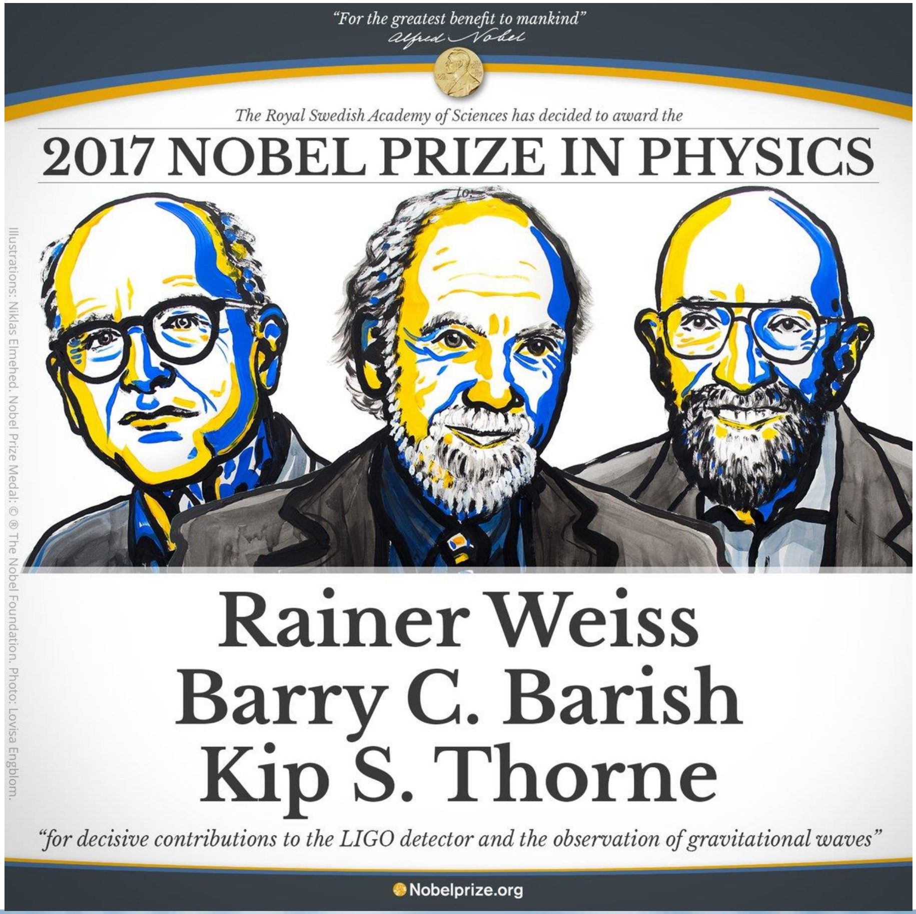
*Lectures on Physics, Vol. 1, Page 1 (1963)

2013 Nobel Prize in Physics





2017 Nobel Prize in Physics



2016 – Gravitational waves announced!

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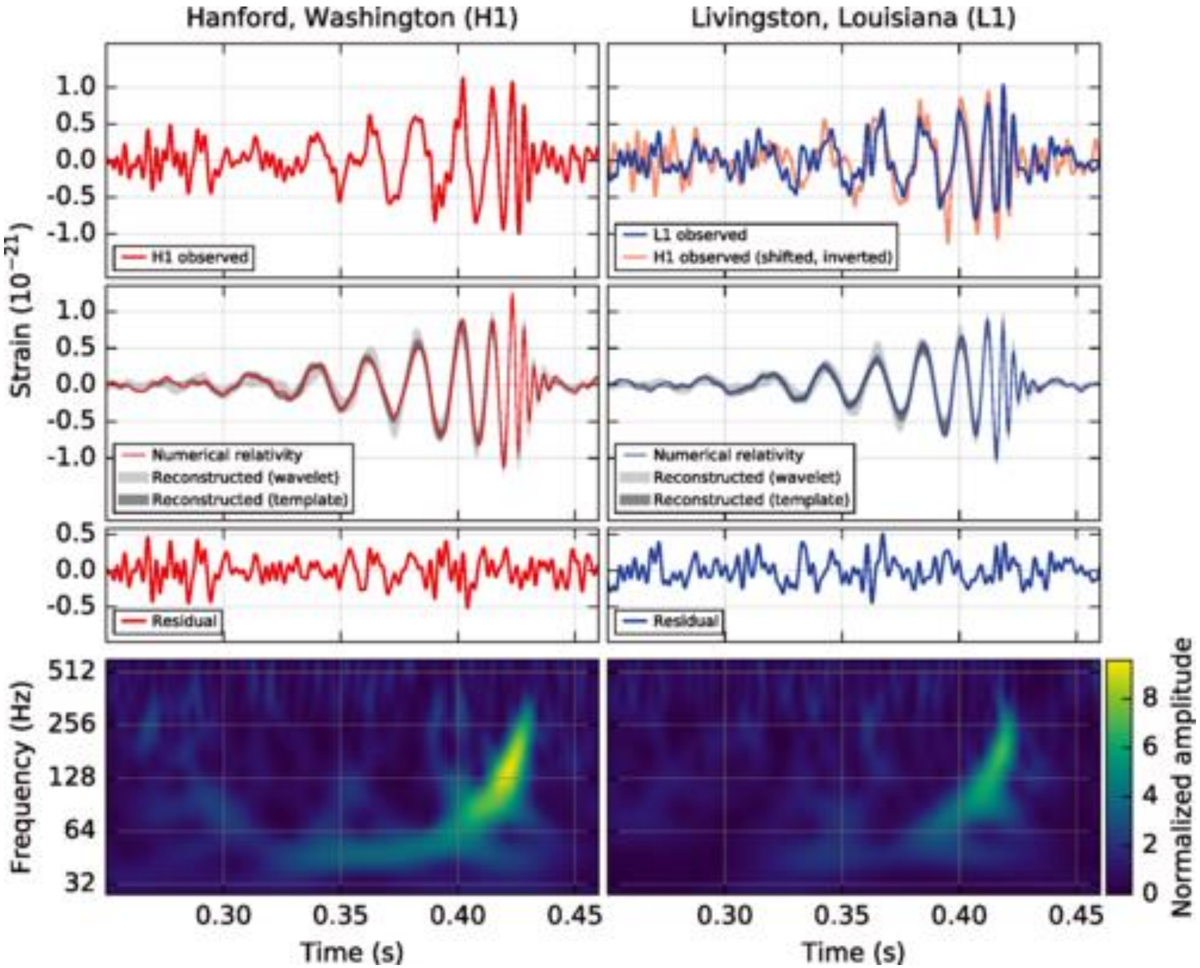


B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. **116**, 061102 (2016)

Gravitational waves – is it a signal?

The importance of error analysis

There are **two** important aspects to error analysis

An investigation is not complete until an analysis of the numbers to be reported has been conducted

An understanding of the **dominant error** is useful when planning a strategy

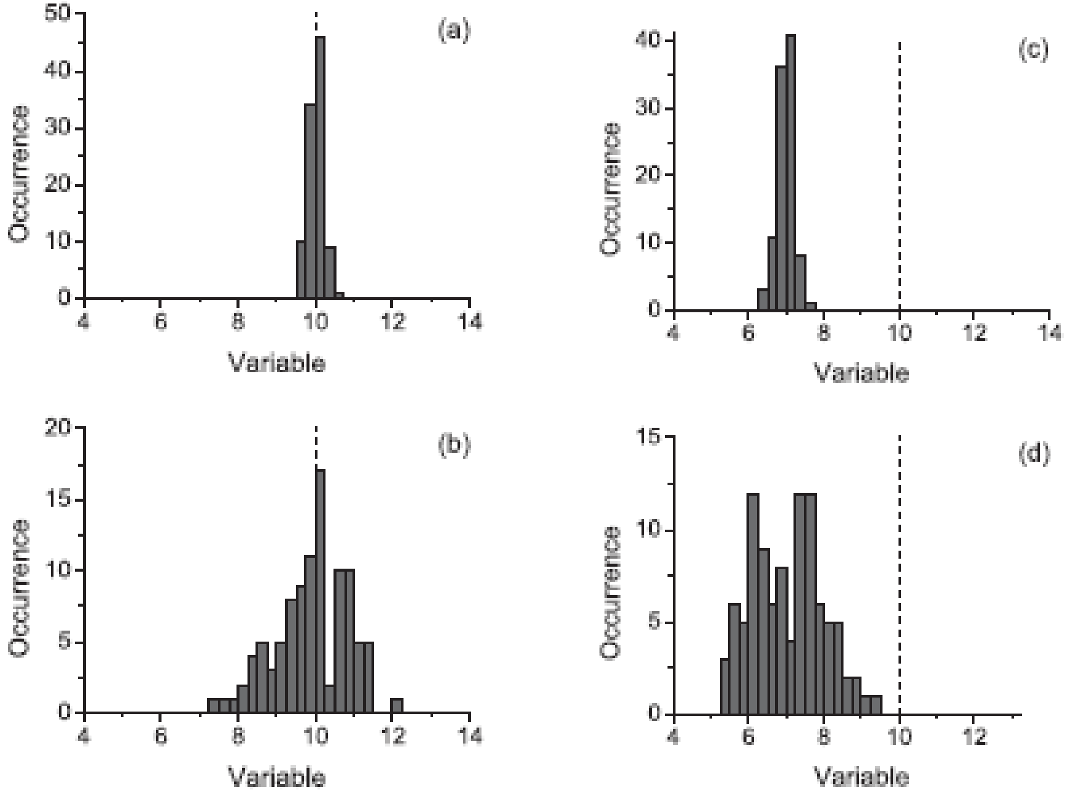
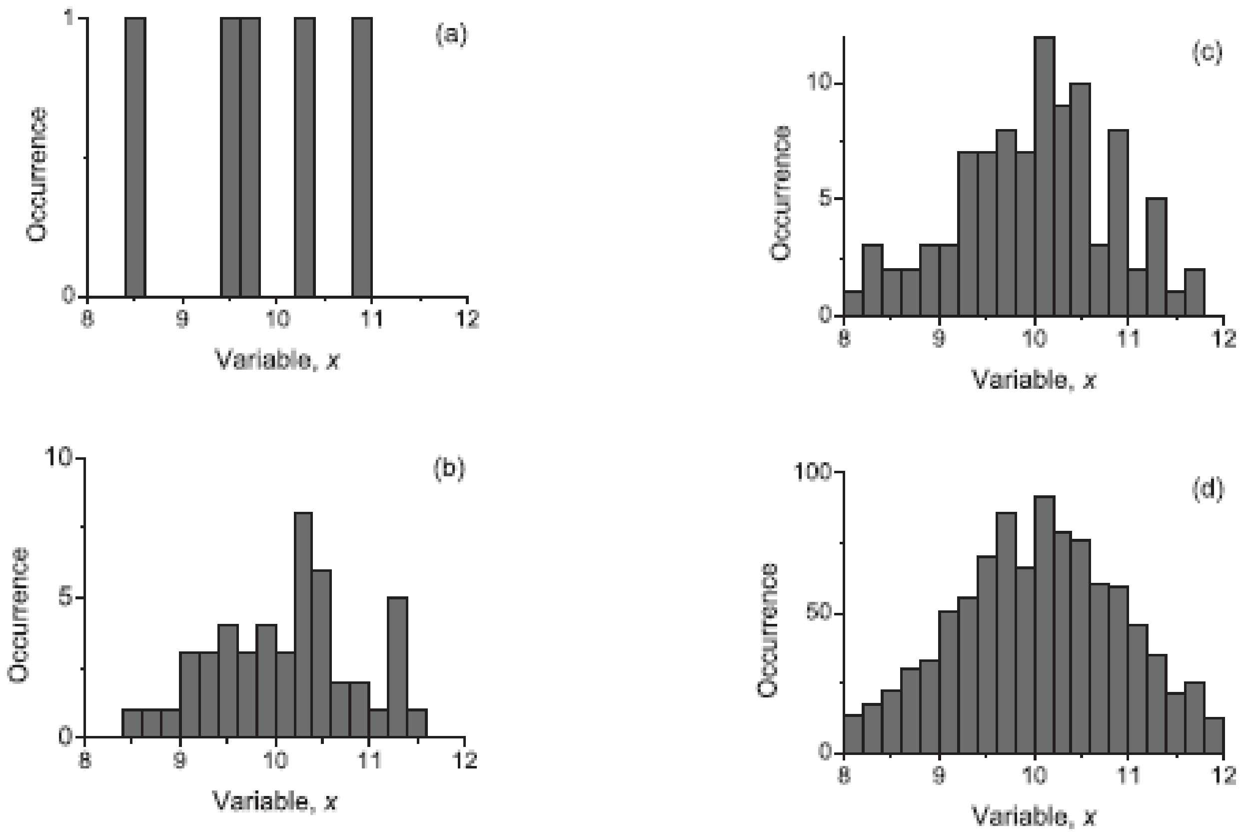


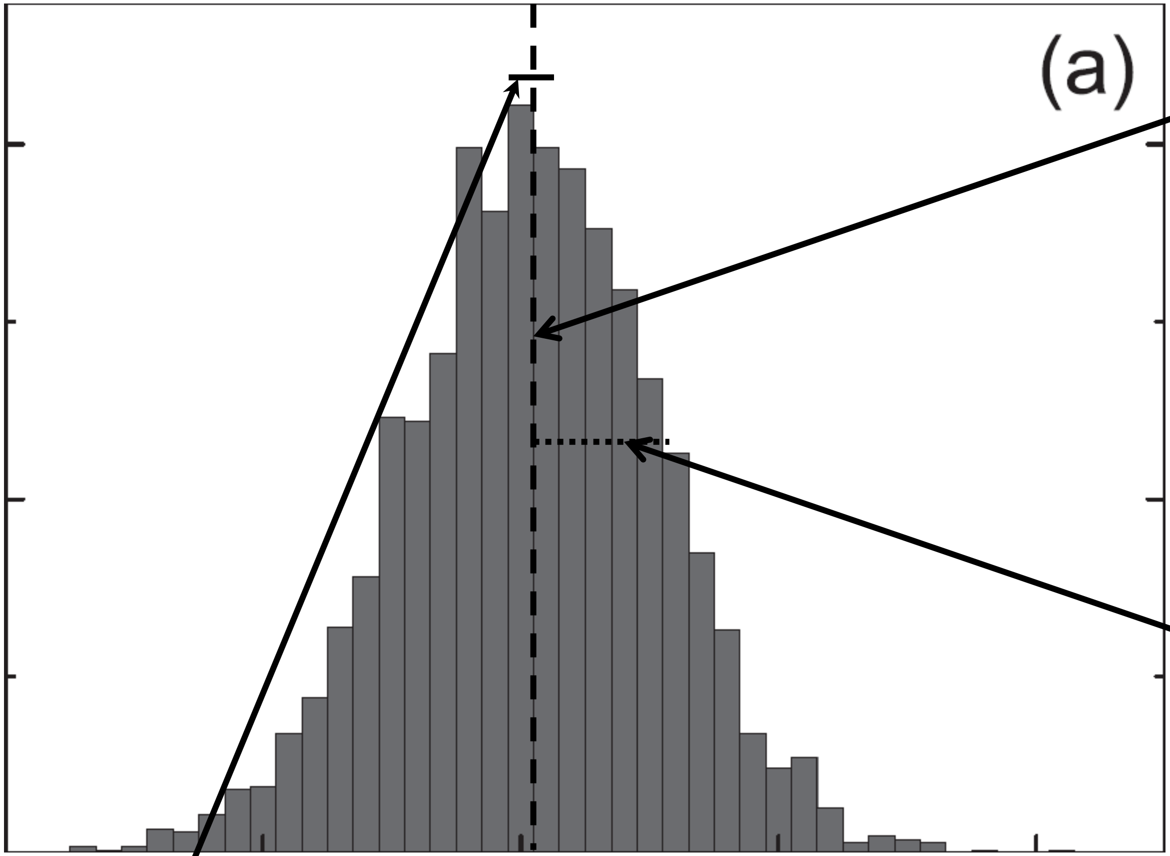
Fig. 1.2 Terminology used in error analysis. Simulations of 100 measurements are shown in histograms of constant bin-width. The extent of the scatter of the data (the width of the histogram) is a measure of the precision, and the position of the centre of the histogram relative to the dashed line represents the accuracy. The histograms show (a) precise and accurate, (b) imprecise and accurate, (c) precise and inaccurate and (d) imprecise and inaccurate sets of measurements.

When analysing statistical distributions of data the following 3 concepts are very useful:

- the mean
- the stan
- the stan



Mean, standard deviation, standard error



The **mean** tells us where the measurements are centred

The **standard deviation** gives us the width of the distribution (independent of *N*)

The **standard error** is the uncertainty in the location of the centre (improves with higher *N*)

Measurements and their Uncertainties, Chpt 2

PRL 117, 157202 (2016) PHYSICAL REVIEW LETTERS week ending 7 OCTOBER 2016

Anisotropic Polar Magneto-Optic Kerr Effect of Ultrathin Fe/GaAs(001) Layers due to Interfacial Spin-Orbit Interaction

M. Buchner,[1] P. Högl,[2] S. Putz,[2] M. Gmitra,[2] S. Günther,[3] M. A. W. Schoen,[1] M. Kronseder,[1] D. Schuh,[1] D. Bougeard,[1] J. Fabian,[2] and C. H. Back[1]

[1]*Institute of Experimental and Applied Physics, University of Regensburg, Regensburg 93040, Germany*
[2]*Institute of Theoretical Physics, University of Regensburg, Regensburg 93040, Germany*
[3]*Department of Materials, ETH Zürich, Zürich 8093, Switzerland*

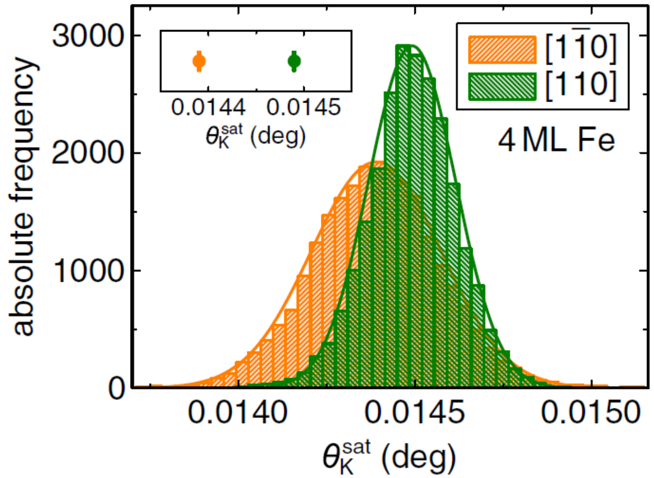


FIG. 2. Histograms of the Kerr rotation angles θ_K^{sat} for the laser being polarized along the [110] and [1$\bar{1}$0] crystallographic directions for 4 ML Fe/GaAs. The inset shows the mean values for both polarization directions with the corresponding error bars.

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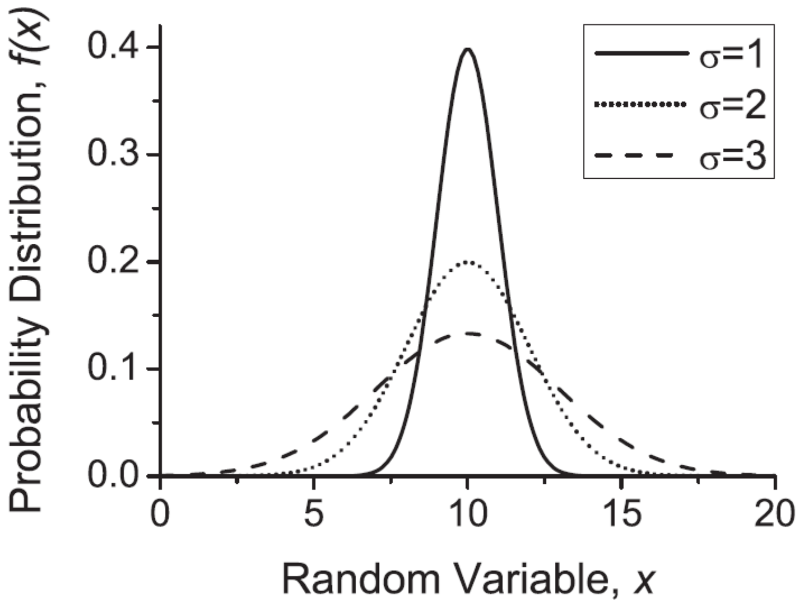
The continuous distribution which arises most often is the **Gaussian** or **normal distribution**



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right],$$

This is a **2** parameter distribution

- The mean $\bar{x}$
- The standard deviation **σ**

As we collect more data the **signal-to-noise** ratio improves

The precision with which we can ascertain the centre of the distribution improves.

We can imagine repeating measurements, and calculating the **standard deviation of the mean** (**SDOM**).

This quantity is also known as the **standard error**. It is $\sqrt{N}$ smaller than the standard deviation

$$\alpha = \frac{\sigma_{N-1}}{\sqrt{N}}$$

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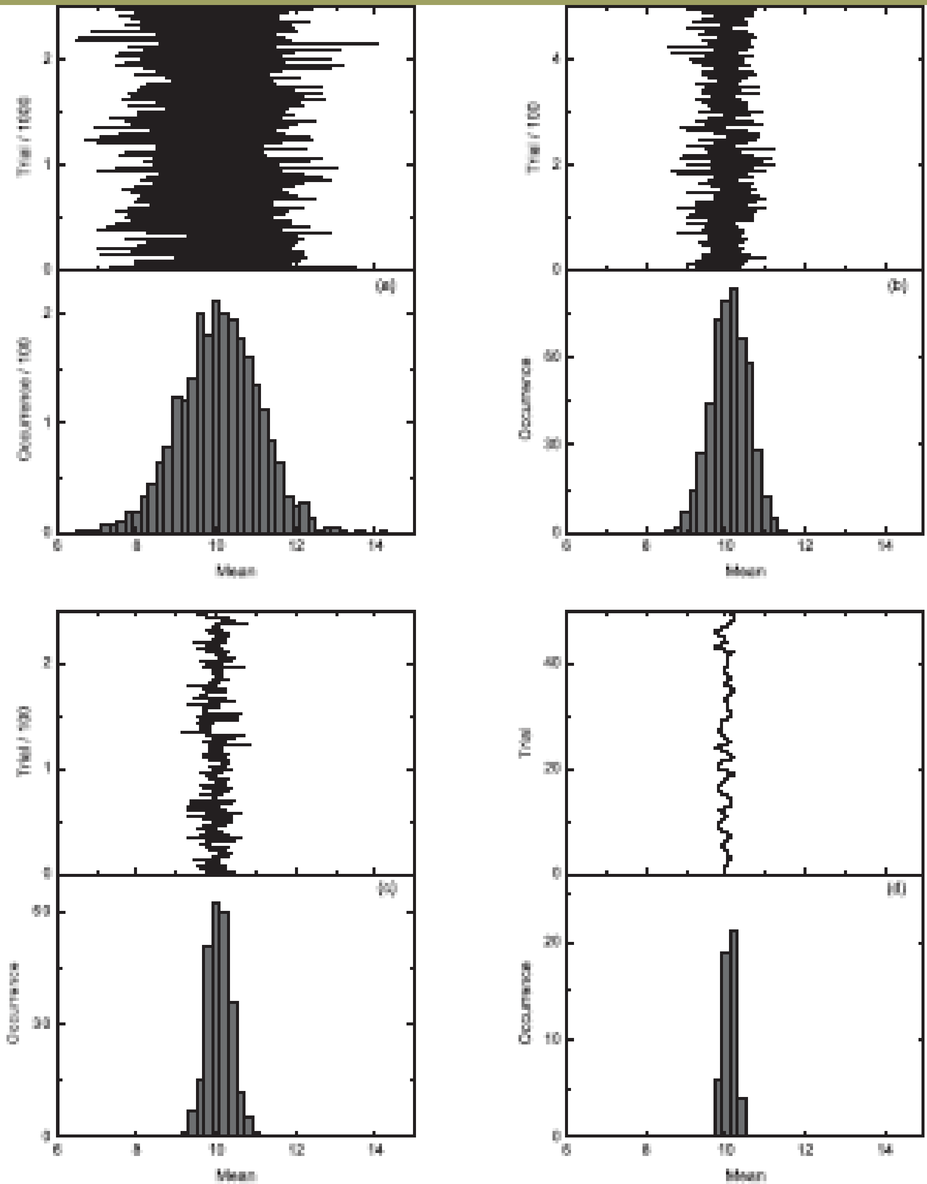

Data drawn from a parent distribution with mean = 10 standard deviation = 1

(a) 2500 data points

(b) 500 means of average of 5

(c) 250 means of average of 10

(d) 50 means of average of 50

	mean	stdev
(a)	10.0	1.0
(b)	10.0	0.5
(c)	10.0	0.3
(d)	10.00	0.14

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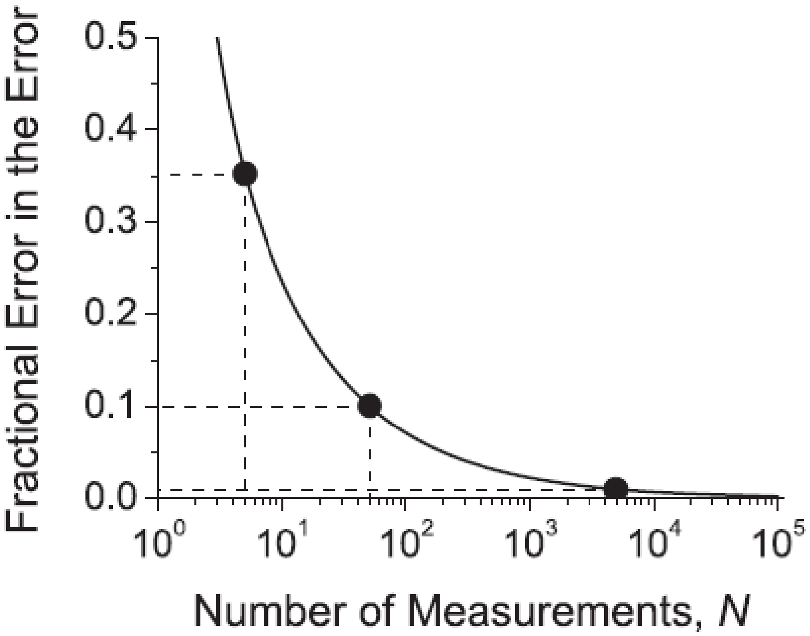
As experimental data have statistical fluctuations there is an

error in the error

The fractional error in the error decreases very slowly with increasing number of data *N*

$$\text{error in the error} = \frac{1}{\sqrt{2N-2}}$$



- If the error starts with a 1 quote the error to two sig. fig.
- else quote the error to one sig.fig.

The five golden rules

(1) The best estimate of a parameter is the mean.
(2) The error is the standard error in the mean.
(3) Round up the error to the appropriate number of significant figures.
(4) Match the number of decimal places in the mean to the standard error.
(5) Include units.

Quoting numbers: superfluous precision

Confidence limits and error bars

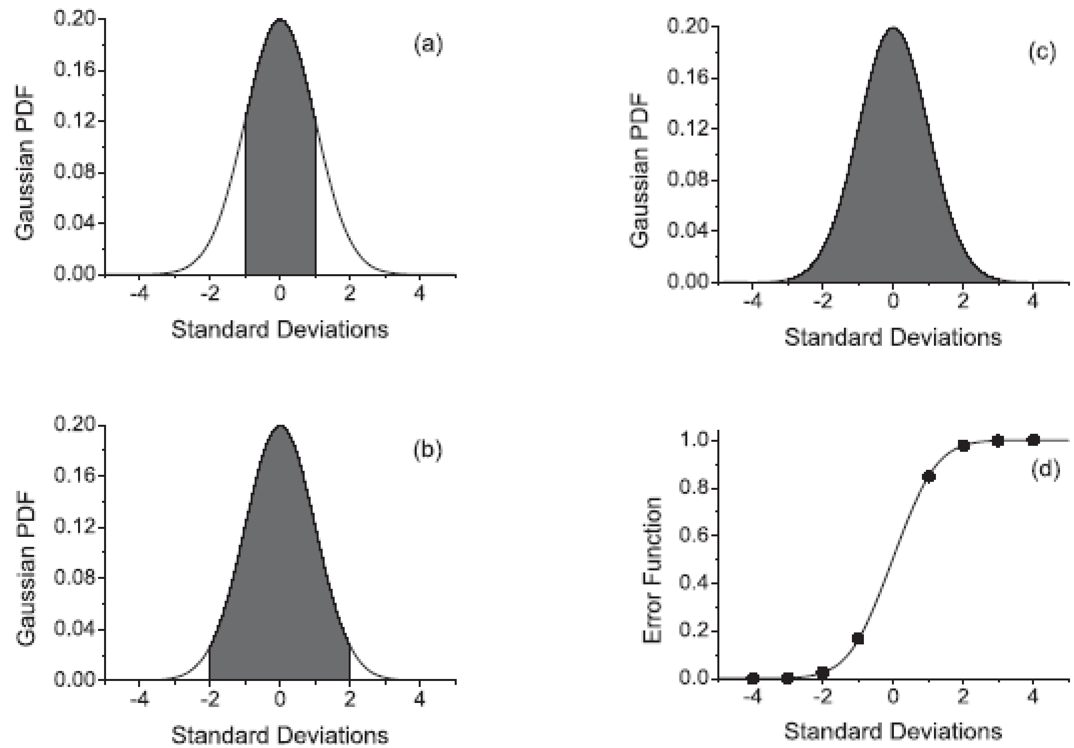


Fig. 3.3 The shaded areas of the Gaussian curves show the fraction of data within (a) one standard deviation of the mean, (b) two standard deviations, and (c) three standard deviations. The corresponding points on the error function, along with those for four standard deviations, are indicated in (d).

Table 3.1 The fraction of the data which lies within different ranges of a Gaussian probability distribution function.

Centred on mean	$\pm\sigma$	$\pm1.65\sigma$	$\pm2\sigma$	$\pm2.58\sigma$	$\pm3\sigma$
Measurements within range	68%	90%	95%	99.0%	99.7%
Measurements outside range	32%	10%	5%	1.0%	0.3%
	1 in 3	1 in 10	1 in 20	1 in 100	1 in 400

Range centered on Mean	$\pm\sigma$	$\pm 2\sigma$	$\pm 3\sigma$
Measurements within Range	68%	95%	99.7%
Measurements outside Range	32% 1 in 3	5% 1 in 20	0.3% 1 in 400

The error is a statement of probability. The standard deviation is used to define a **confidence level** on the data.

Measurements and their Uncertainties, p 26

When comparing your measurement of a quantity with a known or accepted value adopt the following terminology:

If your result and the accepted value differ by:
Up to 1 standard error it is in EXCELLENT AGREEMENT
Between 1 and 2 standard errors it is in REASONABLE AGREEEMENT
More than 3 standard errors it is in DISAGREEMENT
For a breakthrough discovery (like finding the Higgs boson) we need 5 standard errors

Measurements and their Uncertainties, p 28

5 standard errors for a "Discovery"

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21 June 2012 Last updated at 16:58

Sha

Excitement builds over Higgs data

By Paul Rincon
Science editor, BBC News website

Statistics of a 'discovery'

- Particle physics has an accepted definition for a "discovery": a five-sigma level of certainty
- The number of standard deviations, or sigmas, is a measure of how unlikely it is that an experimental result is simply down to chance, in the absence of a real effect
- Similarly, tossing a coin and getting a number of heads in a row may just be chance, rather than a sign of a "loaded" coin
- The "three sigma" level represents about the same likelihood of tossing nine heads in a row
- Five sigma, on the other hand, would correspond to tossing more than 21 in a row
- Unlikely results are more probable when several experiments are carried out at once - equivalent to several people flipping coins at the same time
- With independent confirmation by other experiments, five-sigma findings become accepted discoveries

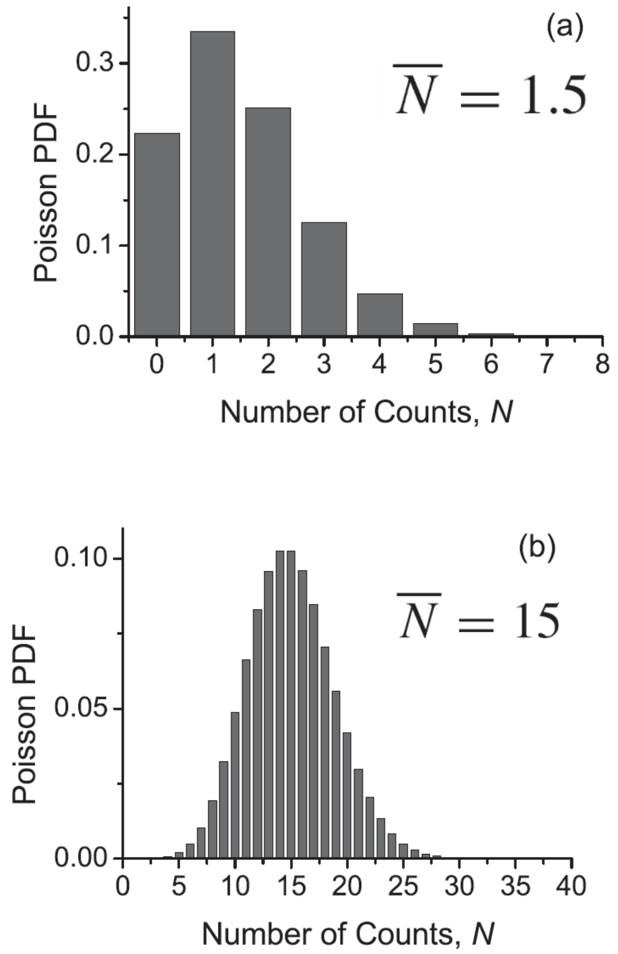
Poisson distribution

- **Counting rare events**
- **All events are independent**
- **Average rate does not change as a function of time**

$$P\left(N;\overline{N}\right)=\frac{\exp\left(-\overline{N}\right)\overline{N}^{N}}{N!}.$$

$\text{Mean} = \overline{N}$

$\text{Standard Deviation} = \sqrt{\overline{N}}$



Example of Poisson distribution

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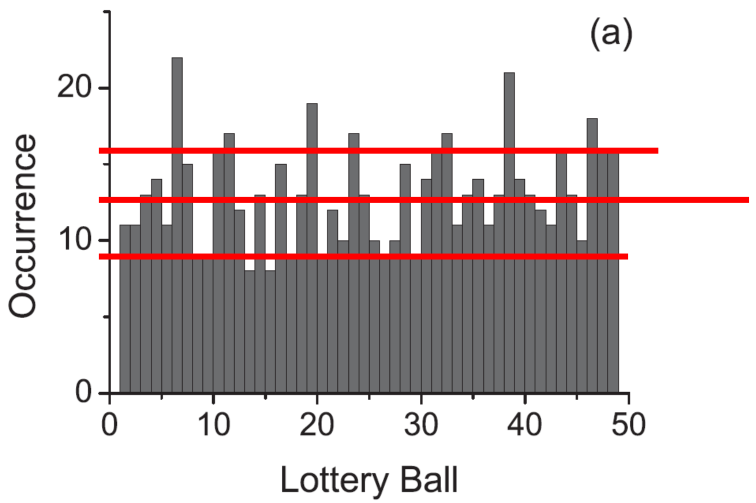
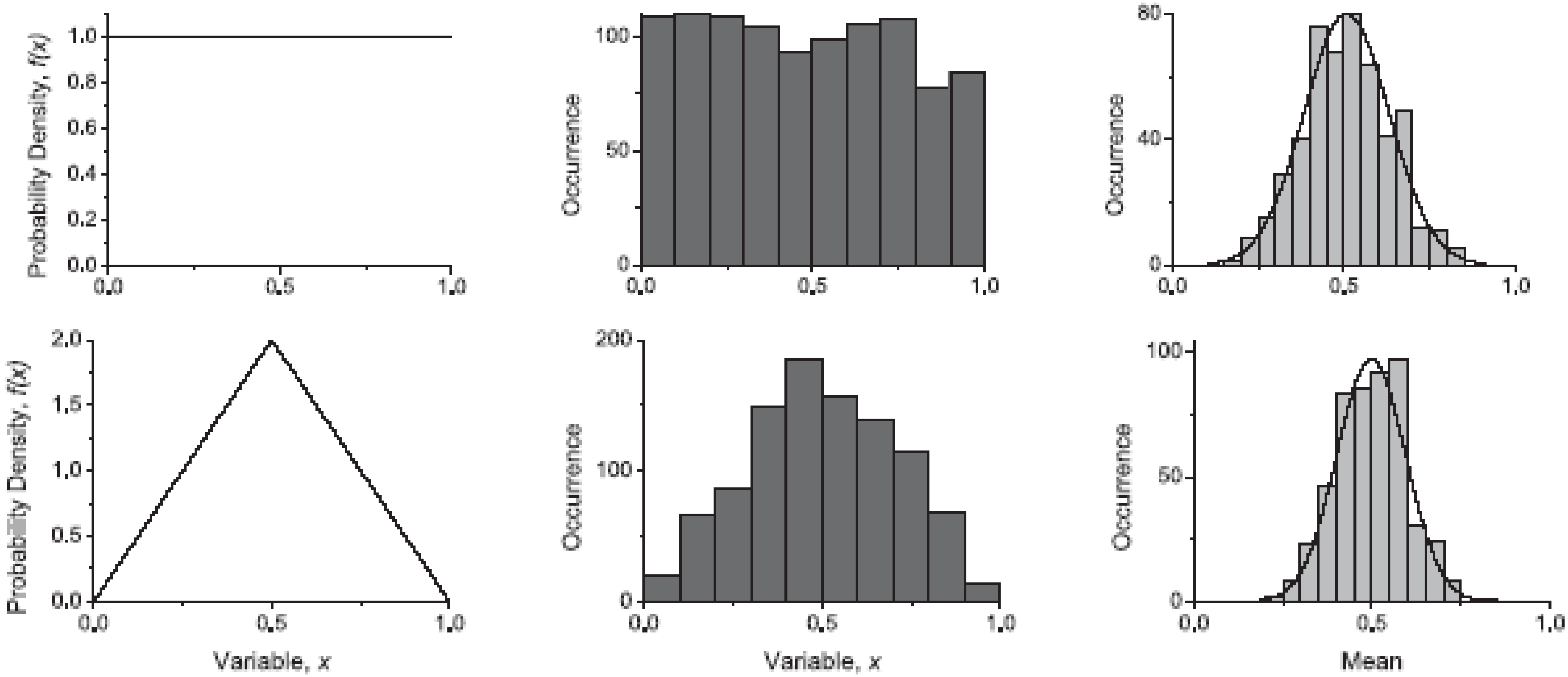
106 draws in a year. 6 balls drawn from 49.
We expect each ball to be seen 13 times.
The occurrence of each ball follows
Poissson statistics.



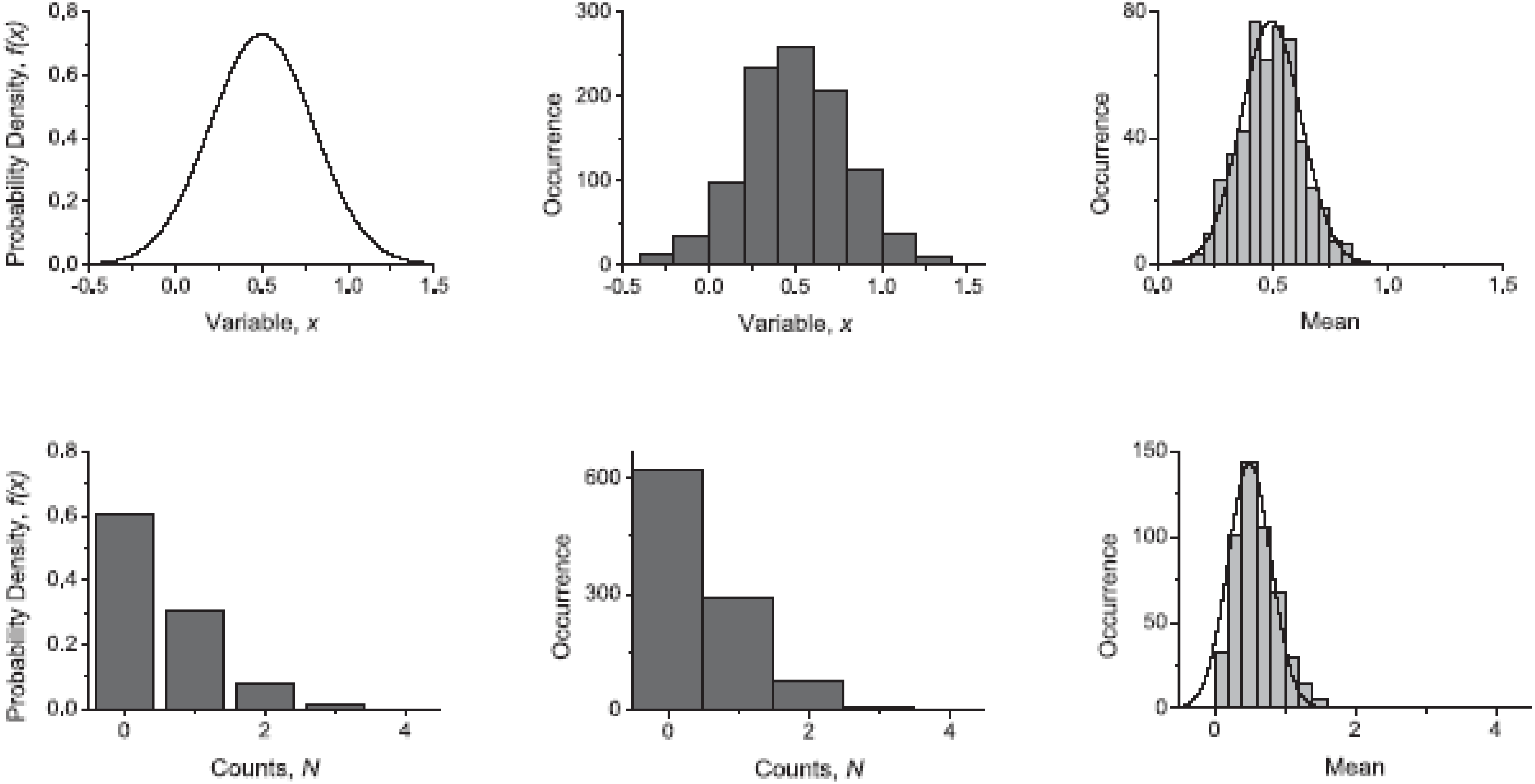
Therefore we expect 2/3rds of occurrences to be between

$$N \pm \sqrt{N}$$

The sum of a large number of independent random variables, each with finite mean and variance, will tend to be normally distributed, irrespective of the distribution function of the random variable.

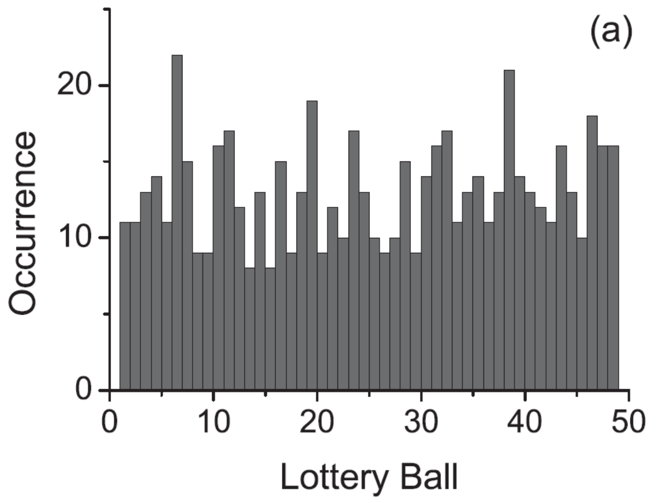


The Central Limit Theorem



Example of The Central Limit Theorem – the National Lottery

There are 106 draws in a year. 6 balls drawn from 49.
Therefore we expect each ball to be seen 13 times.
The distribution is expected to be uniform.



What is the AVERAGE value of the balls drawn?

Example of The Central Limit Theorem – the National Lottery

lotto
Plus5

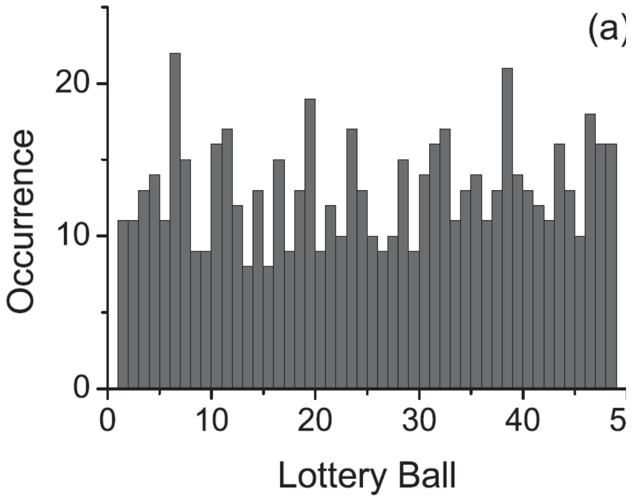
Draw history for Lotto and Plus 5

Download draw history for Lotto and Plus 5

Date	Game	Ball numbers
October		
Wed 09 Oct 13	**Lotto**	4 - 11 - 27 - 34 - 36 - 45
Sat 05 Oct 13	**Lotto**	5 - 15 - 17 - 19 - 20 - 47
Wed 02 Oct 13	**Lotto**	16 - 17 - 21 - 27 - 30 - 37

Calculate the AVERAGE value of the balls drawn for these 3 draws

The sum of a large number of independent random variables, each with finite mean and variance, will tend to be normally distributed, irrespective of the distribution function of the random variable.



Examples of the CLT

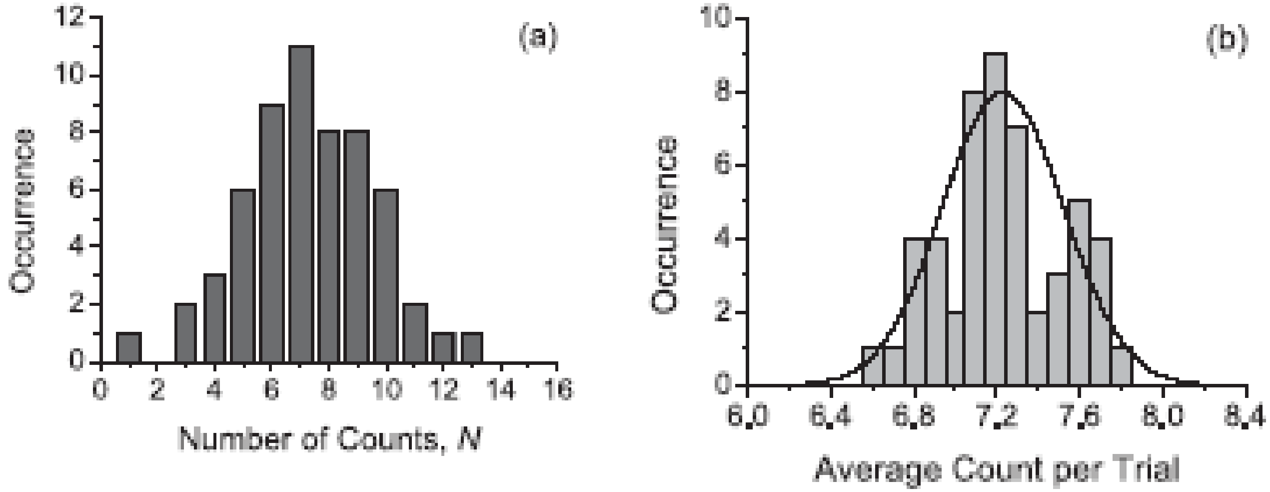


Fig. 3.8 Part (a) shows the result of a radioactive decay experiment. 423 counts were recorded in 58 seconds; the histogram shows the occurrences of the number of counts in one-second intervals about a mean of 7.3 counts per second. On repeating this experiment 51 times, it is possible to plot the distribution of the means, as is done in (b). Here it is seen that the distribution of means is (i) well described by a Gaussian, and (ii) significantly narrower than the sample Poisson distribution.

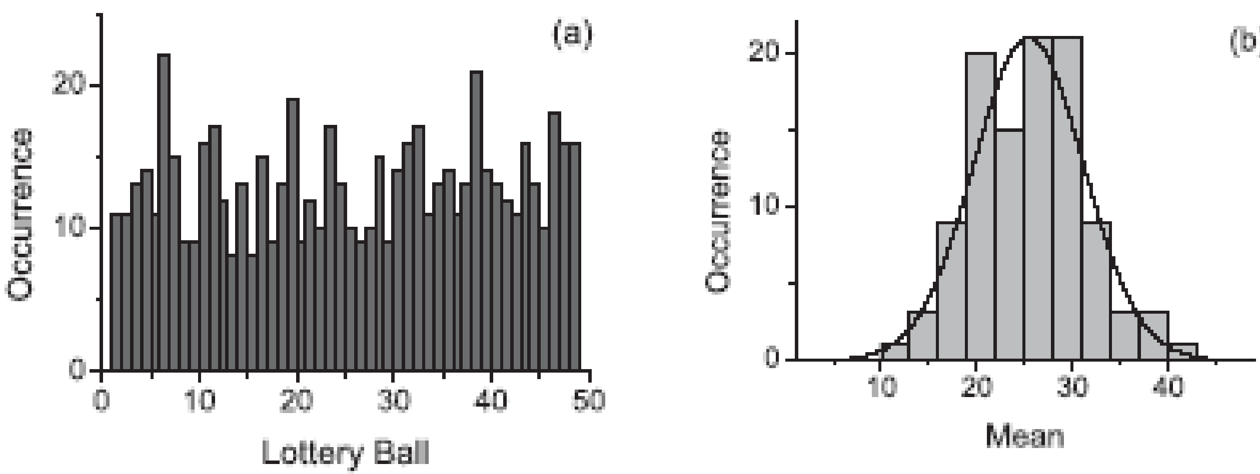


Fig. 3.9 Part (a) shows a histogram of the occurrences of the 49 balls in all 106 national lottery draws for the year 2000. If there was no bias a uniform distribution of 13 occurrences per ball would be expected; the experimental data are in good agreement with this within the statistical fluctuations. Six balls are chosen in each draw, the average number was calculated, and the histogram of the 106 results is plotted. As expected, a Gaussian distribution is obtained for the means, with a narrower standard deviation (by a factor of $\sqrt{6}$) compared with the parent distribution.

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"…error analysis is a participation, rather than a spectator, sport."

Please complete homework before 5pm next Monday (14th October).

For MiSCaDA students via https://notebooks.dmaitre.phyip3.dur.ac.uk/miscada-da/hub/login

For PhD students either the notebook server OR email me a document