CS446: Machine Learning	**Spring 2017**

Problem Set 2

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1. (a) If choose `Holiday` as the splitting attribute:

$$Gain(H) = Entropy(S) - \sum \frac{|S_v|}{|S|} Entropy(S_v) =$$

$$Entropy(S) - \{\frac{21}{50}[-\frac{20}{21}\log_{10}(\frac{20}{21}) - \frac{1}{21}\log_{10}(\frac{1}{21})] + \frac{29}{50}[-\frac{15}{29}\log_{10}(\frac{15}{29}) - \frac{14}{29}\log_{10}(\frac{14}{29})]\}$$
$$= Entropy(S) - 0.209$$

If choose `Exam Tomorrow` as the splitting attribute:

$$Gain(H) = Entropy(S) - \sum \frac{|S_v|}{|S|} Entropy(S_v) =$$

$$Entropy(S) - \{\frac{15}{50}[-\frac{10}{15}\log_{10}(\frac{10}{15}) - \frac{5}{15}\log_{10}(\frac{5}{15})] + \frac{35}{50}[-\frac{25}{35}\log_{10}(\frac{25}{35}) - \frac{10}{35}\log_{10}(\frac{10}{35})]\}$$
$$= Entropy(S) - 0.265$$

Thus, I will choose `Holiday` to get the highest information gain.

(b) Replace *Entropy* with *MajorityError*. For the first layer :
Choosing `Color`:

$$Gain = MajorityError(S) - \sum \frac{|S_v|}{|S|} MajorityError(S_v) =$$

$$\frac{7}{16} - (\frac{8}{16} \times \frac{3}{8} + \frac{8}{16} \times \frac{2}{8}) = \frac{7}{16} - \frac{5}{16}$$

Choosing `Size`:

$$Gain = \frac{7}{16} - (\frac{8}{16} \times \frac{3}{8} + \frac{8}{16} \times \frac{2}{8}) = \frac{7}{16} - \frac{5}{16}$$

Choosing `Act`:

$$Gain = \frac{7}{16} - (\frac{8}{16} \times \frac{2}{8} + \frac{8}{16} \times \frac{3}{8}) = \frac{7}{16} - \frac{5}{16}$$

Choosing `Age`:

$$Gain = \frac{7}{16} - (\frac{8}{16} \times \frac{2}{8} + \frac{8}{16} \times \frac{3}{8}) = \frac{7}{16} - \frac{5}{16}$$

All the gains are the same. Choose `Color` as the splitting attribute.
For the second layer at `Color` = Blue:
Choosing `Size`:

$$Gain = \frac{3}{8} - \frac{1}{8}$$

Choosing `Act`:

$$Gain = \frac{3}{8} - \frac{3}{8}$$

Choosing `Age`:

$$Gain = \frac{3}{8} - \frac{3}{8}$$

Choose `Size` as the splitting attribute.
For the second layer at `Color` = Red:
Choosing `Size`:

$$Gain = \frac{2}{8} - \frac{2}{8}$$

Choosing `Act`:

$$Gain = \frac{2}{8} - \frac{2}{8}$$

Choosing `Age`:

$$Gain = \frac{2}{8} - \frac{2}{8}$$

All the gains are the same. Choose `Size` as the splitting attribute.
For the third layer at `Color` = Blue, `Size` = Large:
Choosing `Act`:

$$Gain = \frac{1}{4} - \frac{1}{4}$$

Choosing `Age`:

$$Gain = \frac{1}{4} - \frac{1}{4}$$

All the gains are the same. Choose `Act` as the splitting attribute.
For the third layer at `Color` = Blue, `Size` = small, there is no need to split. The label of the node is F.

For the third layer at `Color` = Red, `Size` = Large:
Choosing `Act`:

$$Gain = \frac{1}{4} - \frac{1}{4}$$

Choosing `Age`:

$$Gain = \frac{1}{4} - \frac{1}{4}$$

All the gains are the same. Choose `Act` as the splitting attribute.
For the third layer at `Color` = Red, `Size` = Small:
Choosing `Act`:

$$Gain = \frac{1}{4} - \frac{1}{4}$$

Choosing `Age`:

$$Gain = \frac{1}{4} - \frac{1}{4}$$

All the gains are the same. Choose `Act` as the splitting attribute. For the forth layer at `Color` = Blue, `Size` = Large, `Act` = Dip, all the labels are Ts. Thus, the label of this node is set as T. For the forth layer at `Color` = Blue, `Size` = Large, `Act` = Stretch. Choose the only attribute, `Age`, as the splitting attribute. The label of the node `Age` = Adult is F and the other is T. For the forth layer at `Color` = Red, `Size` = Large, `Act` = Dip, all the labels are Ts. Thus, the label of this node is set as T. For the forth layer at `Color` = Red, `Size` = Large, `Act` = Stretch. Choose the only attribute, `Age`, as the splitting attribute. The label of the node `Age` = Adult is F and the other is T. For the forth layer at `Color` = Red, `Size` = Small, `Act` = Dip, all the labels are Ts. Thus, the label of this node is set as T. For the forth layer at `Color` = Red, `Size` = Small, `Act` = Stretch. Choose the only attribute, `Age`, as the splitting attribute. The label of the node `Age` = Adult is F and the other is T. Thus, The decision tree:

```
if Color = Blue :
  if Size = Large :
    if Act = Dip :
      class = T
    if Act != Dip :
      if Age = Adult :
        class = F
      if Age != Adult :
        class = T
  if Size != Large :
    class = F
if Color != Blue :
  if Size = Large :
    if Act = Dip :
      class = T
    if Act != Dip :
      if Age = Adult :
        class = F
      if Age != Adult :
        class = T
  if Size != Large :
    if Act = Dip :
      class = T
    if Act != Dip :
      if Age = Adult :
        class = F
      if Age != Adult :
        class = T
```

(c) No. Take 1.(b) as an example. when growing the tree, the algorithm sometimes will need to deal with a tie in information gains and generate some redundant nodes without knowing like the node with the splitting attribute `Size` in the subtree of `Color` = Red.

2. (a) I modified the source code, `FeatureGenerator.java`.

1. Modify the feature list.

```
    features = new String[] { "firstName0", "firstName1" , "firstName2" , "firstName3" , "firstName4","lastName0"
        , "lastName1" , "lastName2" , "lastName3" , "lastName4" }; // add features
```

2. Add theta as a feature.

```
    ff.add("theta=1"); // add theta features
    features = ff.toArray(new String[ff.size()]);
```

3. Change the capacity of the attributes

```
    FastVector attributes = new FastVector(12);             // mod capacity to 12
```

4. Extract extra information for new features.

```
    String firstName = parts[1].toLowerCase();
    String lastName = parts[2].toLowerCase();                   // Add for lastName

    Instance instance = new Instance(features.length + 1);      // add 1 theta
    instance.setDataset(instances);

    Set<String> feats = new HashSet<String>();
    /////////  add for new features  /////////
    for(int times = 0; times < 5; times++){
        if(firstName.length() > times ){
            feats.add("firstName"+times+"=" + firstName.charAt(times));
        }else
            feats.add("firstName"+times+"=" + " ");
    }
    for(int times = 0; times < 5; times++){
        if(lastName.length() > times ){
            feats.add("lastName"+times+"=" + lastName.charAt(times));
        }else
            feats.add("lastName"+times+"=" + " ");
    }
    feats.add("theta=1");
    /////////  add for new features  /////////
```

(b) **i. Stochastic gradient descent:**

We get features from the first 5 alphabets of first name and last name. As mentioned, '1' is added for each feature to represent θ. To fit line, $\vec{w}\vec{x_i} + \theta = y_i$, to the dataset, we use gradient descent method. Assuming the error function $Q(w)$, we could derive the learning method.

$$Q(w) = \frac{1}{2}\sum(\vec{w}\vec{x_i} + \theta - y_i)^2$$

According to the function, $Q(w)$, we modify the weight vector to minimize error function.

$$\vec{w'} = \vec{w} - \eta \times \vec{x_i}(\vec{w}\vec{x_i} + \theta - y_i)$$

Keep adjusting the weight vector until error rate of all training set achieving the threshold we set. Then, we sweep different values for threshold and learning rate. The result:

		Learning Rate			
		0.05	0.01	0.001	0.0001
Error Threshold	0.1	0.617	0.671	0.666	0.662
	0.01	0.619	0.645	0.640	0.639
	0.001	0.604	0.651	0.656	0.642

Coarse sweep of parameters and average accuracy

		Learning Rate		
		0.025	0.01	0.005
Error Threshold	0.25	0.624	0.613	0.617
	0.1	0.676	0.671	0.675
	0.05	0.680	0.672	0.683

Fine sweep of parameters and average accuracy

According to the table above, we choose learning rate = 0.005 and threshold = 0.05.

	Test set				
	fold1	fold2	fold3	fold4	fold5
Correct	45	39	32	50	35
Wrong	20	18	14	16	25
Accuracy	0.692	0.684	0.696	0.758	0.583

Result of cross validation on SGD

We obtain information from the table$\Rightarrow \bar{x} = 0.683, std = 0.0627$. Two tails with 99% confidence interval and degree of freedom = 4, T value should be 4.604.

$$-4.604 \leq \frac{\bar{x} - \mu}{\frac{std}{\sqrt{n}}} \leq 4.604$$

$$\Rightarrow -4.604 \leq \frac{0.683 - \mu}{\frac{0.0627}{\sqrt{5}}} \leq 4.604$$

$$\Rightarrow 0.8118 \geq \mu \geq 0.5534$$

ii. Decision tree:

	Test set				
	fold1	fold2	fold3	fold4	fold5
Correct	47	40	35	49	41
Wrong	18	17	11	17	19
Accuracy	0.723	0.702	0.761	0.742	0.683

Result of cross validation on decision tree

We obtain information from the table$\Rightarrow \bar{x} = 0.7222, std = 0.031$. Two tails with 99% confidence interval and degree of freedom = 4, T value should be 4.604.

$$-4.604 \leq \frac{\bar{x} - \mu}{\frac{std}{\sqrt{n}}} \leq 4.604$$

$$\Rightarrow -4.604 \leq \frac{0.7222 - \mu}{\frac{0.031}{\sqrt{5}}} \leq 4.604$$

$$\Rightarrow -0.064 \leq 0.7222 - \mu \leq 0.064$$

$$\Rightarrow 0.7862 \geq \mu \geq 0.6582$$

iii. Decision tree of depth 4:

	Test set				
	fold1	fold2	fold3	fold4	fold5
Correct	39	39	27	48	41
Wrong	26	18	19	18	19
Accuracy	0.600	0.684	0.587	0.727	0.683

Result of cross validation on decision tree of depth 4

We obtain information from the table$\Rightarrow \bar{x} = 0.6564, std = 0.0601$. Two tails with 99% confidence interval and degree of freedom = 4, T value should be 4.604.

$$-4.604 \leq \frac{\bar{x} - \mu}{\frac{std}{\sqrt{n}}} \leq 4.604$$

$$\Rightarrow -4.604 \leq \frac{0.6564 - \mu}{\frac{0.0601}{\sqrt{5}}} \leq 4.604$$

$$\Rightarrow -0.124 \leq 0.6564 - \mu \leq 0.124$$

$$\Rightarrow 0.7801 \geq \mu \geq 0.5318$$

iv. Decision tree of depth 8:

	Test set					
	fold1	fold2	fold3	fold4	fold5	Sum & Avg
Correct	48	41	29	47	41	206
Wrong	17	16	17	19	19	88
Accuracy	0.738	0.719	0.630	0.712	0.683	0.701

Result of cross validation on decision tree of depth 8

We obtain information from the table$\Rightarrow \bar{x} = 0.6967, std = 0.0421$. Two tails with 99% confidence interval and degree of freedom = 4, T value should be 4.604.

$$-4.604 \leq \frac{\bar{x} - \mu}{\frac{std}{\sqrt{n}}} \leq 4.604$$

$$\Rightarrow -4.604 \leq \frac{0.6967 - \mu}{\frac{0.0421}{\sqrt{5}}} \leq 4.604$$

$$\Rightarrow -0.0867 \leq 0.6967 - \mu \leq 0.0867$$

$$\Rightarrow 0.7834 \geq \mu \geq 0.6100$$

v. Decision stumps as features & Stochastic gradient descent:

		Test set				
		fold1	fold2	fold3	fold4	fold5
TD = 4	Wrong	25	20	19	16	16
	Correct	40	37	27	50	44
	Accuracy	0.615	0.649	0.587	0.758	0.733
TD = 8	Wrong	22	16	17	15	19
	Correct	43	41	29	51	41
	Accuracy	0.662	0.719	0.630	0.773	0.683
TD = -1	Wrong	21	17	21	19	20
	Correct	44	40	25	47	40
	Accuracy	0.677	0.702	0.543	0.712	0.667

Result of cross validation on 100 Decision stumps + SGD with different tree depth

When max tree depth = 4, $\Rightarrow \bar{x} = 0.668, std = 0.074$. We get:

$$0.821 \geq \mu \geq 0.516$$

When max tree depth = 8, $\Rightarrow \bar{x} = 0.693, std = 0.055$. We get:

$$0.806 \geq \mu \geq 0.580$$

When no limitation for max tree depth, $\Rightarrow \bar{x} = 0.660, std = 0.068$. We get:

$$0.800 \geq \mu \geq 0.521$$

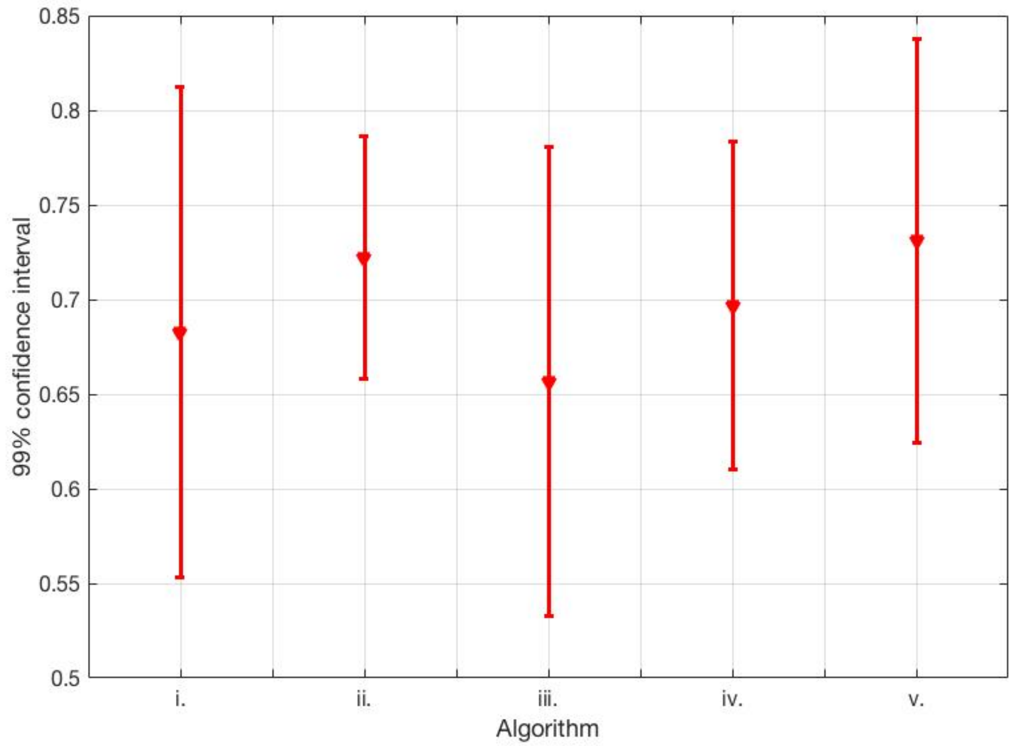
After tuning all possible combination parameters, I found that increasing samples when creating decision stumps will largely increase the accuracy. Here, depth of tree = 8, number of trees = 100, we sweep the ratio of removing train data set.

		Test set				
		fold1	fold2	fold3	fold4	fold5
Use 50% data	Wrong	26	21	20	21	21
	Correct	39	36	26	45	39
	Accuracy	0.6	0.632	0.565	0.682	0.65
Use 60% data	Wrong	20	11	20	14	22
	Correct	45	46	26	52	38
	Accuracy	0.692	0.807	0.565	0.788	0.633
Use 70% data	Wrong	23	18	16	10	16
	Correct	42	39	30	56	44
	Accuracy	0.646	0.684	0.652	0.848	0.733
Use 80% data	Wrong	15	15	17	14	18
	Correct	50	42	29	52	42
	Accuracy	0.769	0.737	0.660	0.788	0.7
Use 90% data	Wrong	20	18	17	18	16
	Correct	45	39	29	48	44
	Accuracy	0.692	0.684	0.630	0.727	0.733

Result of cross validation on 100 Decision stumps of depth 4 + SGD with different sampling ratio

The best case is $\bar{x} = 0.731, std = 0.052$ when randomly sampling 80% data.
99% confidence interval $\Rightarrow 0.837 \geq \mu \geq 0.624$

Comparison:



According to the 99% confidence interval we get from t-distribution, the performance of each method is not significantly different from the others. Moreover, the ranking of the first 4 methods is $ii. > iv. > i. > iii.$. However, $v.$ is really hard to compare with others because of the randomization process in it. We could find an average accuracy $= 0.731$ after adjusting the sampling ratio of training data set to 80% which is the best of all methods introduced. Thus, roughly speaking, the ranking might be $v. > ii. > iv. > i. > iii.$

Trees display for ***ii. iii. iv.*****:**
ii. decision tree:

```
if lastName0=m = 1:
  if firstName2=e = 1:
    class = +
  if firstName2=e = 0:
    if lastName1=o = 1:
      class = +
    if lastName1=o = 0:
      if firstName0=p = 1:
        class = -
      if firstName0=p = 0:
        if firstName0=r = 1:
          class = -
        if firstName0=r = 0:
          if firstName0=y = 1:
            class = -
          if firstName0=y = 0:
            if firstName1=u = 1:
              class = -
            if firstName1=u = 0:
              if lastName3=a = 1:
                if firstName3=r = 1:
                  class = +
                if firstName3=r = 0:
                  class = -
              if lastName3=a = 0:
                class = +
if lastName0=m = 0:
  if lastName1=l = 1:
    if firstName0=d = 1:
      class = -
    if firstName0=d = 0:
      class = +
  if lastName1=l = 0:
    if lastName2=l = 1:
      if firstName2=r = 1:
        class = -
      if firstName2=r = 0:
        if lastName4=n = 1:
          class = -
        if lastName4=n = 0:
          if firstName2=h = 1:
            class = -
```

```
      if firstName2=h = 0:
        class = +
if lastName2=l = 0:
  if lastName2=o = 1:
    if firstName0=b = 1:
      class = -
    if firstName0=b = 0:
      class = +
  if lastName2=o = 0:
    if firstName3=f = 1:
      class = +
    if firstName3=f = 0:
      if lastName4=l = 1:
        if firstName2=h = 1:
          class = -
        if firstName2=h = 0:
          if lastName0=l = 1:
            class = -
          if lastName0=l = 0:
            if lastName0=q = 1:
              class = -
            if lastName0=q = 0:
              class = +
      if lastName4=l = 0:
        if firstName1=o = 1:
          if lastName0=f = 1:
            class = -
          if lastName0=f = 0:
            if firstName2=e = 1:
              class = -
            if firstName2=e = 0:
              if firstName3=a = 1:
                if lastName0=h = 1:
                  class = +
                if lastName0=h = 0:
                  class = -
              if firstName3=a = 0:
                if firstName2=n = 1:
                  class = +
                if firstName2=n = 0:
                  if lastName2=n = 1:
                    class = +
                  if lastName2=n = 0:
                    if lastName1=a = 1:
                      class = -
```

```
            if lastName1=a = 0:
              if firstName3=g = 1:
                class = -
              if firstName3=g = 0:
                class = +
if firstName1=o = 0:
  if lastName0=l = 1:
    if firstName1=a = 1:
      if firstName0=d = 1:
        class = +
      if firstName0=d = 0:
        class = -
    if firstName1=a = 0:
      class = +
  if lastName0=l = 0:
    if lastName3=m = 1:
      if firstName2=r = 1:
        class = -
      if firstName2=r = 0:
        class = +
    if lastName3=m = 0:
      if firstName1=e = 1:
        if firstName2=n = 1:
          class = +
        if firstName2=n = 0:
          if firstName2=o = 1:
            if lastName0=b = 1:
              class = -
            if lastName0=b = 0:
              class = +
          if firstName2=o = 0:
            if lastName2=r = 1:
              if firstName0=m = 1:
                class = -
              if firstName0=m = 0:
                class = +
            if lastName2=r = 0:
              class = -
      if firstName1=e = 0:
        if firstName0=t = 1:
          if lastName4=e = 1:
            class = -
          if lastName4=e = 0:
            if lastName0=s = 1:
              class = -
```

```
    if lastName0=s = 0:
      class = +
if firstName0=t = 0:
  if firstName3=o = 1:
    if firstName0=a = 1:
      class = +
    if firstName0=a = 0:
      if firstName0=m = 1:
        class = +
      if firstName0=m = 0:
        class = -
  if firstName3=o = 0:
    if firstName4=o = 1:
      if firstName0=s = 1:
        class = -
      if firstName0=s = 0:
        class = +
    if firstName4=o = 0:
      if lastName0=s = 1:
        if firstName0=d = 1:
          class = +
        if firstName0=d = 0:
          if lastName4=h = 1:
            if firstName0=s = 1:
              class = -
            if firstName0=s = 0:
              class = +
          if lastName4=h = 0:
            class = -
      if lastName0=s = 0:
        if lastName3=l = 1:
          if firstName0=d = 1:
            class = -
          if firstName0=d = 0:
            class = +
        if lastName3=l = 0:
          class = -
```

iii. decision tree of depth 4:

```
if lastName2=l = 1:
  if firstName2=r = 1:
    class = -
  if firstName2=r = 0:
    if firstName2=m = 1:
      class = -
    if firstName2=m = 0:
      class = +
if lastName2=l = 0:
  if lastName2=o = 1:
    if firstName0=d = 1:
      class = -
    if firstName0=d = 0:
      if firstName2=l = 1:
        class = -
      if firstName2=l = 0:
        class = +
  if lastName2=o = 0:
    if firstName3=f = 1:
      class = +
    if firstName3=f = 0:
      if lastName0=m = 1:
        if firstName0=n = 1:
          class = -
        if firstName0=n = 0:
          class = +
      if lastName0=m = 0:
        if lastName1=l = 1:
          class = +
        if lastName1=l = 0:
          class = -
```

iv. decision tree of depth 8:

```
if firstName3=f = 1:
  class = +
if firstName3=f = 0:
  if lastName0=c = 1:
    class = -
  if lastName0=c = 0:
    if lastName4=l = 1:
      if lastName0=q = 1:
        class = -
      if lastName0=q = 0:
```

```
        class = +
    if lastName4=l = 0:
      if firstName0=r = 1:
        if firstName1=o = 1:
          class = +
        if firstName1=o = 0:
          if firstName1=a = 1:
            class = +
          if firstName1=a = 0:
            if firstName1=e = 1:
              class = +
            if firstName1=e = 0:
              class = -
      if firstName0=r = 0:
        if lastName0=m = 1:
          if firstName2=n = 1:
            class = -
          if firstName2=n = 0:
            if firstName0=p = 1:
              class = -
            if firstName0=p = 0:
              if lastName2=t = 1:
                if firstName0=t = 1:
                  class = +
                if firstName0=t = 0:
                  class = -
              if lastName2=t = 0:
                class = +
        if lastName0=m = 0:
          if lastName0=l = 1:
            if firstName1=a = 1:
              if firstName0=d = 1:
                class = +
              if firstName0=d = 0:
                class = -
            if firstName1=a = 0:
              class = +
          if lastName0=l = 0:
            if lastName3=m = 1:
              if firstName2=r = 1:
                class = -
              if firstName2=r = 0:
                class = +
            if lastName3=m = 0:
              if lastName2=l = 1:
```

```
  if firstName2=r = 1:
    class = -
  if firstName2=r = 0:
    class = +
if lastName2=l = 0:
  if lastName3=l = 1:
    class = +
  if lastName3=l = 0:
    class = -
```