

Question 1 : (30 total points) Image data analysis with PCA

In this question we employ PCA to analyse image data

1.1 (3 points) Once you have applied the normalisation from Step 1 to Step 4 above, report the values of the first 4 elements for the first training sample in `Xtrn_nm`, i.e. `Xtrn_nm[0,:]` and the last training sample, i.e. `Xtrn_nm[-1,:]`.

```
First 4 elements of the first training sample in Xtrn_nm:  
[-3.13725490e-06 -2.26797386e-05 -1.17973856e-04 -4.07058824e-04]  
  
First 4 elements of the last training sample in Xtrn_nm:  
[-3.13725490e-06 -2.26797386e-05 -1.17973856e-04 -4.07058824e-04]
```

1.2 (4 points) Using **Xtrn** and Euclidean distance measure, for each class, find the two closest samples and two furthest samples of that class to the mean vector of the class.

Class 0		Sample 59933 	Sample 58923 	Sample 33011 	Sample 51163 
Class 1		Sample 13767 	Sample 18720 	Sample 46375 	Sample 56855 
Class 2		Sample 3518 	Sample 53758 	Sample 53579 	Sample 18913 
Class 3		Sample 28687 	Sample 36680 	Sample 53509 	Sample 14842 
Class 4		Sample 30335 	Sample 43937 	Sample 17267 	Sample 5346 
Class 5		Sample 16895 	Sample 44193 	Sample 20982 	Sample 18906 
Class 6		Sample 344 	Sample 40687 	Sample 41019 	Sample 55023 
Class 7		Sample 51327 	Sample 58102 	Sample 47527 	Sample 51601 
Class 8		Sample 28998 	Sample 43370 	Sample 56147 	Sample 29088 
Class 9		Sample 32622 	Sample 28192 	Sample 26636 	Sample 33141 
	Mean sample	Closest sample to mean	2nd closest sample to mean	2nd furthest sample from mean	Furthest sample from mean

1.3 (3 points) Apply Principal Component Analysis (PCA) to the data of `Xtrn_nm` using `sklearn.decomposition.PCA`, and report the variances of projected data for the first five principal components in a table. Note that you should use `Xtrn_nm` instead of `Xtrn`.

Calculating the variances of the projected data in `Xtrn_nm` for the first five principal components

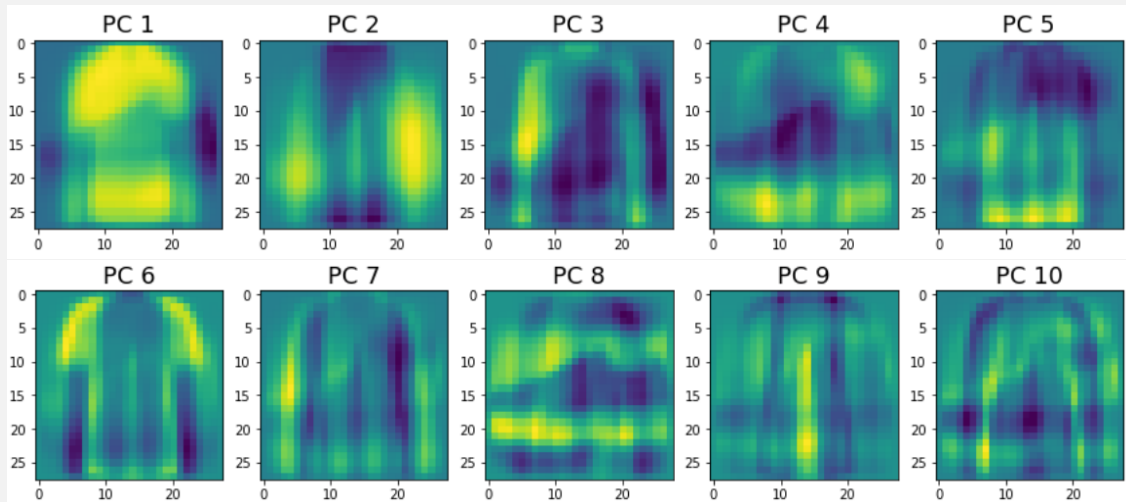
Principal Component #	Explained Variance
1	19.81
2	12.112
3	4.106
4	3.382
5	2.625

1.4 (3 points) Plot a graph of the cumulative explained variance ratio as a function of the number of principal components, K , where $1 \leq K \leq 784$. Discuss the result briefly.

A graph to show the relationship between the cumulative explained variance ratio and the number of principal components (K) for 'Xtrn_nm'



1.5 (4 points) Display the images of the first 10 principal components in a 2-by-5 grid, putting the image of 1st principal component on the top left corner, followed by the one of 2nd component to the right. Discuss your findings briefly.



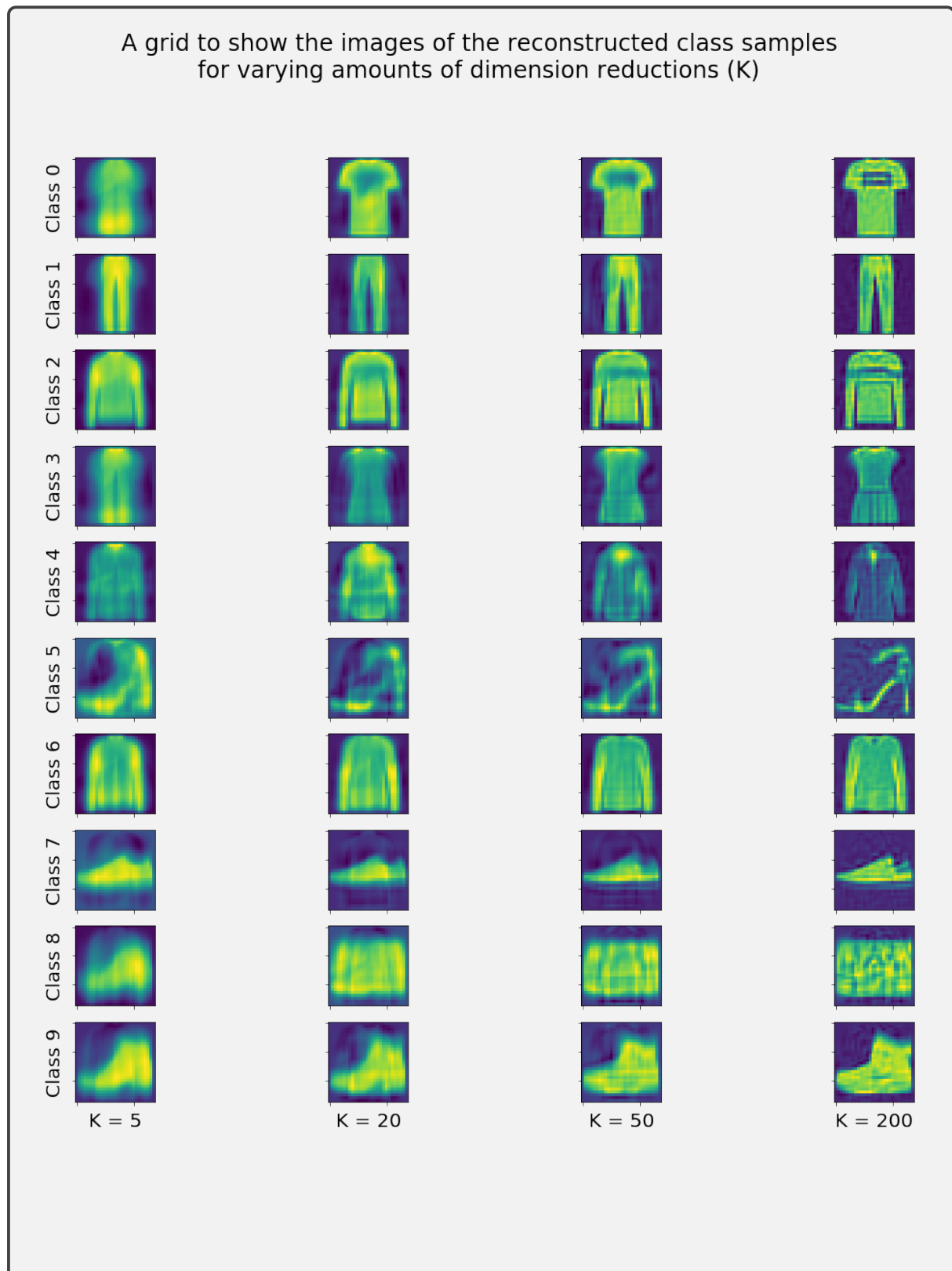
1.6 (5 points) Using `Xtrn_nm`, for each class and for each number of principal components $K = 5, 20, 50, 200$, apply dimensionality reduction with PCA to the first sample in the class, reconstruct the sample from the dimensionality-reduced sample, and report the Root Mean Square Error (RMSE) between the original sample in `Xtrn_nm` and reconstructed one.

A table to show the RMSE between the original and the reconstructed version of the first sample for every class with varying numbers of PCA components (K)

*Each class sample is reconstructed by reducing the sample to K dimensions and then is transformed back to the original number of dimensions, this is all done via the sklearn PCA implementation.

RMSE	K = 5	K = 20	K = 50	K = 200
Class = 0	0.256	0.15	0.128	0.063
Class = 1	0.198	0.14	0.095	0.037
Class = 2	0.199	0.146	0.124	0.08
Class = 3	0.146	0.107	0.083	0.056
Class = 4	0.118	0.103	0.088	0.045
Class = 5	0.181	0.159	0.142	0.09
Class = 6	0.129	0.096	0.072	0.045
Class = 7	0.166	0.128	0.106	0.063
Class = 8	0.223	0.145	0.124	0.094
Class = 9	0.184	0.151	0.122	0.072

1.7 (4 points) Display the image for each of the reconstructed samples in a 10-by-4 grid, where each row corresponds to a class and each row column corresponds to a value of $K = 5, 20, 50, 200$.



1.8 (4 points) Plot all the test samples (`Xtrn_nm`) on the two-dimensional PCA plane you obtained in Question 1.3, where each sample is represented as a small point with a colour specific to the class of the sample. Use the 'coolwarm' colormap for plotting.

Your Answer Here

Question 2 : (25 total points) Logistic regression and SVM

In this question we will explore classification of image data with logistic regression and support vector machines (SVM) and visualisation of decision regions.

2.1 (3 points) Carry out a classification experiment with **multinomial logistic regression**, and report the classification accuracy and confusion matrix (in numbers rather than in graphical representation such as heatmap) for the test set.

FREQUENCY CONFUSION MATRIX:

Predicted Actual	0	1	2	3	4	5	6	7	8	9
0	819	5	27	31	0	2	147	0	7	0
1	3	953	4	15	3	0	3	0	1	0
2	15	4	731	14	115	0	128	0	6	0
3	50	27	11	866	38	1	46	0	11	1
4	7	5	133	33	760	0	108	0	3	0
5	4	0	0	0	2	911	0	32	7	15
6	89	3	82	37	72	0	539	0	15	1
7	1	1	2	0	0	56	0	936	5	42
8	12	2	9	4	10	10	28	1	945	0
9	0	0	1	0	0	20	1	31	0	941

PERCENTAGE CONFUSION MATRIX:

Predicted Actual	0	1	2	3	4	5	6	7	8	9
0	78.9	0.5	2.7	2.9	0.0	0.2	17.5	0.0	0.7	0.0
1	0.3	97.0	0.4	1.4	0.3	0.0	0.4	0.0	0.1	0.0
2	1.4	0.4	72.2	1.3	11.0	0.0	15.3	0.0	0.6	0.0
3	4.8	2.7	1.1	82.4	3.6	0.1	5.5	0.0	1.1	0.1
4	0.7	0.5	13.1	3.1	72.4	0.0	12.9	0.0	0.3	0.0
5	0.4	0.0	0.0	0.0	0.2	93.8	0.0	3.1	0.7	1.5
6	8.6	0.3	8.1	3.5	6.9	0.0	64.3	0.0	1.5	0.1
7	0.1	0.1	0.2	0.0	0.0	5.8	0.0	89.7	0.5	4.2
8	1.2	0.2	0.9	0.4	1.0	1.0	3.3	0.1	92.6	0.0
9	0.0	0.0	0.1	0.0	0.0	2.1	0.1	3.0	0.0	94.7

Total classification accuracy = 84.01%

2.2 (3 points) Carry out a classification experiment with **SVM classifiers**, and report the mean accuracy and confusion matrix (in numbers) for the test set.

FREQUENCY CONFUSION MATRIX:

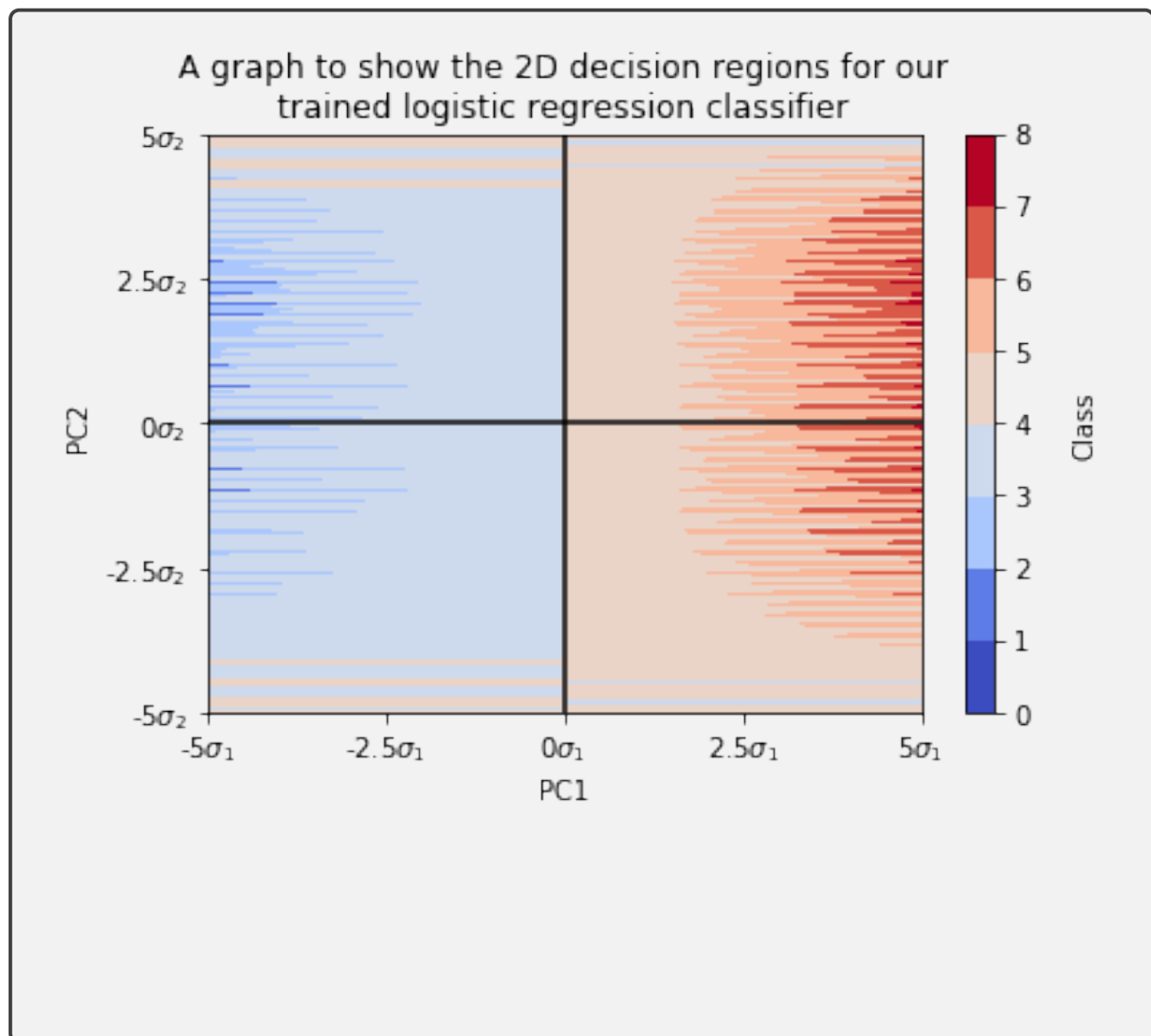
Predicted Actual	0	1	2	3	4	5	6	7	8	9
0	845	4	15	32	1	0	185	0	3	0
1	2	951	2	6	0	0	1	0	1	0
2	8	7	748	12	98	0	122	0	8	0
3	51	31	11	881	36	1	39	0	5	0
4	4	5	137	26	775	0	95	0	2	0
5	4	0	0	0	0	914	0	34	4	22
6	72	1	79	40	86	0	533	0	13	0
7	0	0	0	0	0	57	0	925	4	47
8	14	1	8	3	4	2	25	0	959	1
9	0	0	0	0	0	26	0	41	1	930

PERCENTAGE CONFUSION MATRIX:

Predicted Actual	0	1	2	3	4	5	6	7	8	9
0	77.9	0.4	1.5	3.0	0.1	0.0	22.5	0.0	0.3	0.0
1	0.2	98.8	0.2	0.6	0.0	0.0	0.1	0.0	0.1	0.0
2	0.7	0.7	74.6	1.1	9.4	0.0	14.8	0.0	0.8	0.0
3	4.7	3.2	1.1	83.5	3.4	0.1	4.7	0.0	0.5	0.0
4	0.4	0.5	13.7	2.5	74.2	0.0	11.5	0.0	0.2	0.0
5	0.4	0.0	0.0	0.0	0.0	93.5	0.0	3.3	0.4	2.2
6	6.6	0.1	7.9	3.8	8.2	0.0	64.7	0.0	1.3	0.0
7	0.0	0.0	0.0	0.0	0.0	5.8	0.0	89.5	0.4	4.7
8	1.3	0.1	0.8	0.3	0.4	0.2	3.0	0.0	94.3	0.1
9	0.0	0.0	0.0	0.0	0.0	2.7	0.0	4.0	0.1	93.2

Mean classification accuracy = 84.412%

2.3 (6 points) We now want to visualise the decision regions for the logistic regression classifier we trained in Question 2.1.



2.4 (4 points) Using the same method as the one above, plot the decision regions for the SVM classifier you trained in Question 2.2. Comparing the result with that you obtained in Question 2.3, discuss your findings briefly.

Your Answer Here

2.5 (6 points) We used default parameters for the SVM in Question 2.2. We now want to tune the parameters by using cross-validation. To reduce the time for experiments, you pick up the first 1000 training samples from each class to create `Xsmall`, so that `Xsmall` contains 10,000 samples in total. Accordingly, you create labels, `Ysmall`.



Optimal accuracy achieved and it's respective value of C :

Accuracy $\approx 85.65\%$ where $C \approx 10^{1.33}$

2.6 (3 points) Train the SVM classifier on the whole training set by using the optimal value of C you found in Question [2.5](#).

Classification accuracy of our trained SVM using $C = 10^{\frac{4}{3}}$

Training accuracy $\approx 86.768\%$

Testing accuracy $\approx 85.02\%$

Question 3 : (20 total points) Clustering and Gaussian Mixture Models

In this question we will explore K-means clustering, hierarchical clustering, and GMMs.

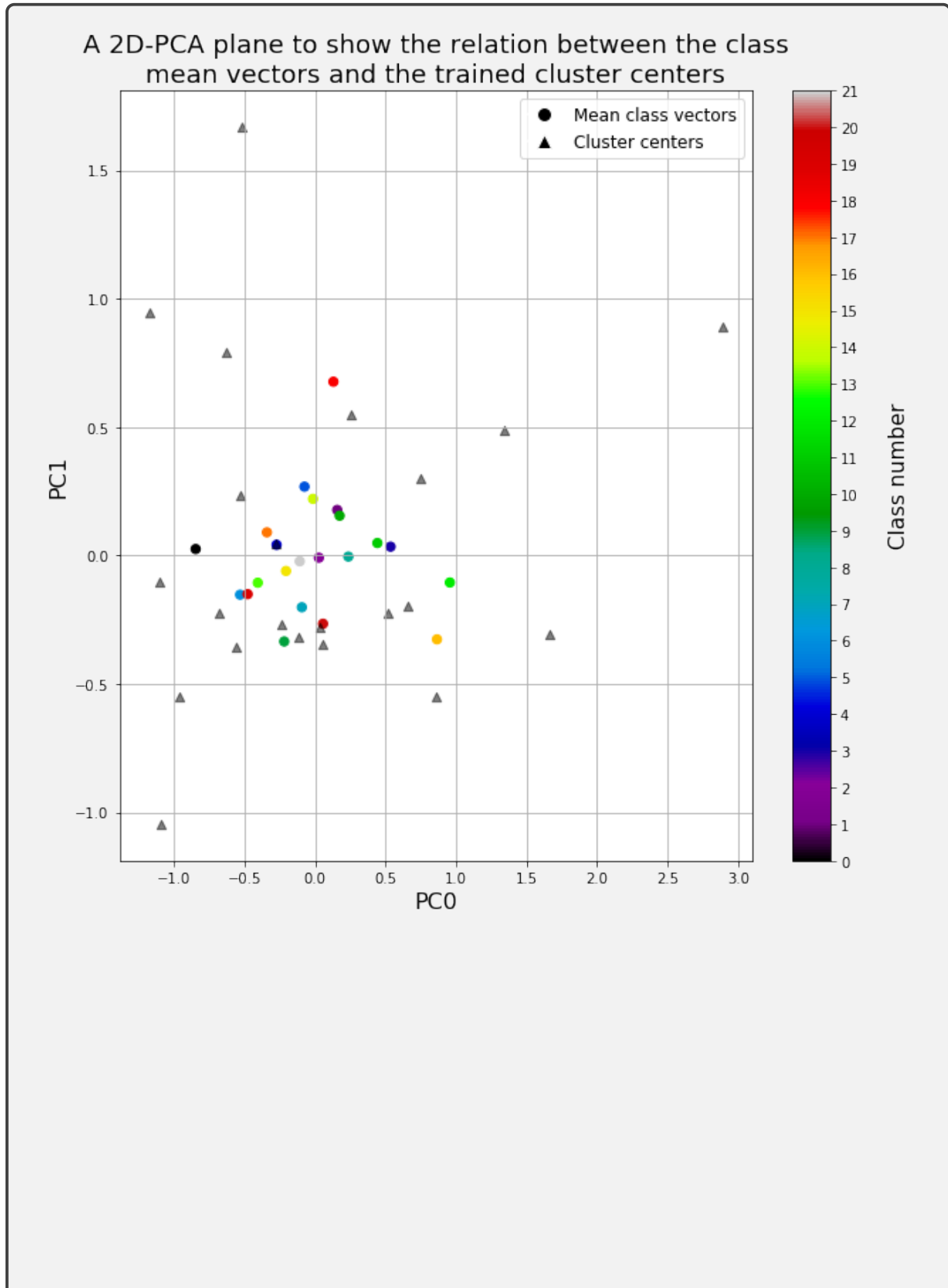
3.1 (3 points) Apply k-means clustering on `Xtrn` for $k = 22$, where we use `sklearn.cluster.KMeans` with the parameters `n_clusters=22` and `random_state=1`. Report the sum of squared distances of samples to their closest cluster centre, and the number of samples for each cluster.

```
Sum of squared distances (Euclidean) of samples to their closest cluster center:  
38185.81698349466
```

```
Number of samples for each cluster:
```

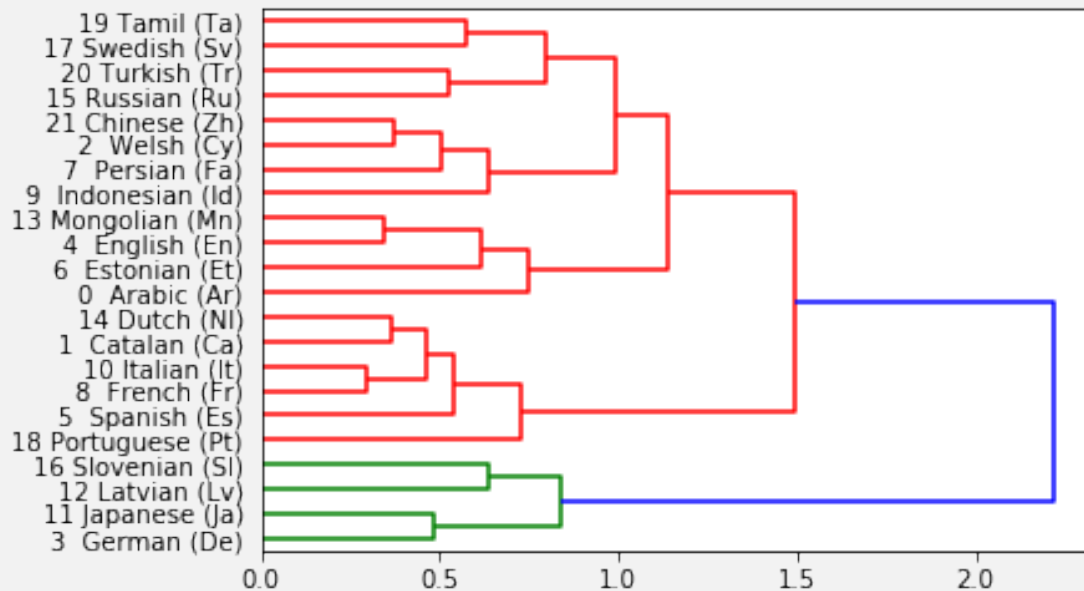
```
Cluster 1 = 1018  
Cluster 2 = 1125  
Cluster 3 = 1191  
Cluster 4 = 890  
Cluster 5 = 1162  
Cluster 6 = 1332  
Cluster 7 = 839  
Cluster 8 = 623  
Cluster 9 = 1400  
Cluster 10 = 838  
Cluster 11 = 659  
Cluster 12 = 1276  
Cluster 13 = 121  
Cluster 14 = 152  
Cluster 15 = 950  
Cluster 16 = 1971  
Cluster 17 = 1251  
Cluster 18 = 845  
Cluster 19 = 896  
Cluster 20 = 930  
Cluster 21 = 1065  
Cluster 22 = 1466
```


3.2 (3 points) Using the training set only, calculate the mean vector for each language, and plot the mean vectors of all the 22 languages on a 2D-PCA plane, where you apply PCA on the set of 22 mean vectors without applying standardisation. On the same figure, plot the cluster centres obtained in Question 3.1.



3.3 (3 points) We now apply hierarchical clustering on the training data set to see if there are any structures in the spoken languages.

A dendrogram to show the hierarchical clustering of the mean vectors for each language



This dendrogram illustrates how each cluster from our dataset is composed and thus ultimately represents the hierarchy of similarities between the classes in our data.

From this we can deduce that Italian and French have the most similar mean vectors and thereby are the most "similar" languages. In contrast we can see that Latvian has the most dissimilar mean vector from all other languages and thereby is the most "unique" language.

The figure consists of three vertically stacked plots, each showing a hierarchical clustering dendrogram and a corresponding bar chart for 100 samples. The x-axis for all plots represents the samples, labeled with two-letter codes (e.g., AA, AB, AC, etc.).

- Top Plot:** The y-axis ranges from 0 to 8. The dendrogram shows a hierarchical structure with a red vertical line at approximately sample 25. The bar chart shows the height of each sample's cluster at the red line threshold.
- Middle Plot:** The y-axis ranges from 0.0 to 2.5. The dendrogram shows a hierarchical structure with a red vertical line at approximately sample 25. The bar chart shows the height of each sample's cluster at the red line threshold.
- Bottom Plot:** The y-axis ranges from 0 to 5. The dendrogram shows a hierarchical structure with a red vertical line at approximately sample 25. The bar chart shows the height of each sample's cluster at the red line threshold.

3.5 (6 points) We now consider Gaussian mixture model (GMM), whose probability distribution function (pdf) is given as a linear combination of Gaussian or normal distributions, i.e.,

Your Answer Here