Question 1 : (30 total points) Image data analysis with PCA

In this question we employ PCA to analyse image data

1.1 (3 points) Once you have applied the normalisation from Step 1 to Step 4 above, report the values of the first 4 elements for the first training sample in `Xtrn_nm`, i.e. `Xtrn_nm[0,:]` and the last training sample, i.e. `Xtrn_nm[-1,:]`.

```
First 4 elements of the first training sample in Xtrn_nm:
[-3.13725490e-06 -2.26797386e-05 -1.17973856e-04 -4.07058824e-04]

First 4 elements of the last training sample in Xtrn_nm:
[-3.13725490e-06 -2.26797386e-05 -1.17973856e-04 -4.07058824e-04]
```

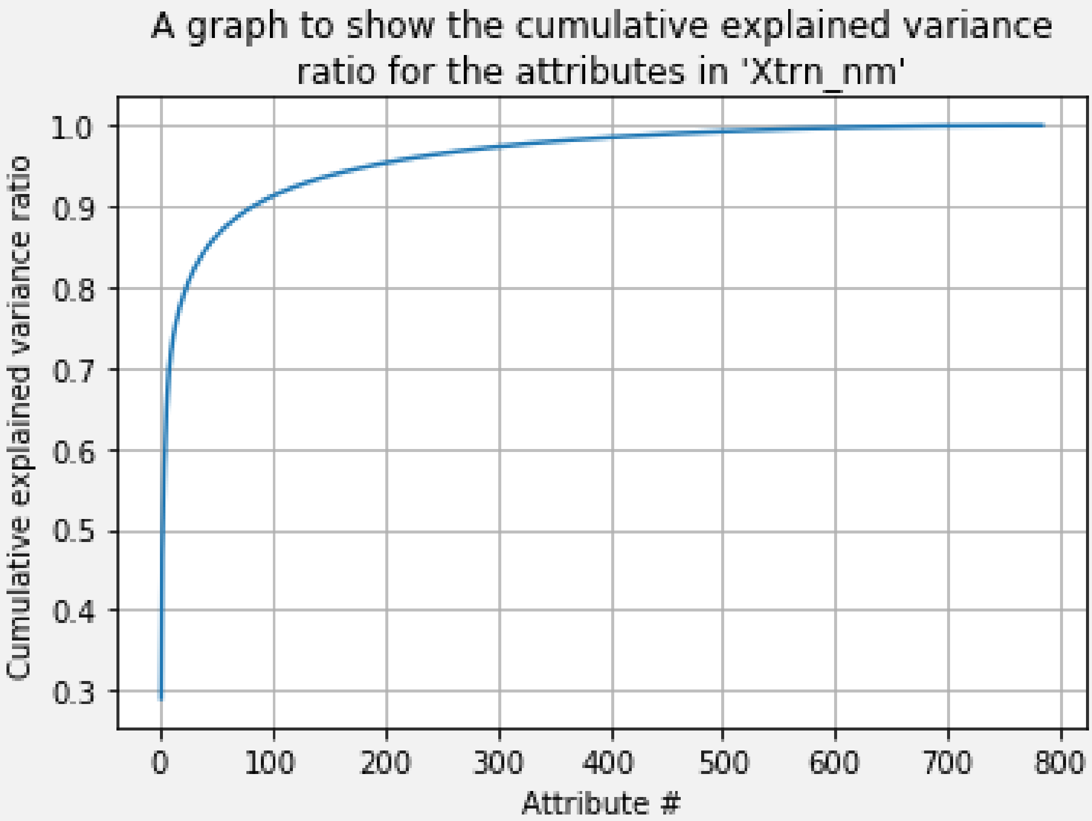
Question 1 continued on next page. . .

1.2 (4 points) Using `Xtrn` and Euclidean distance measure, for each class, find the two closest samples and two furthest samples of that class to the mean vector of the class.

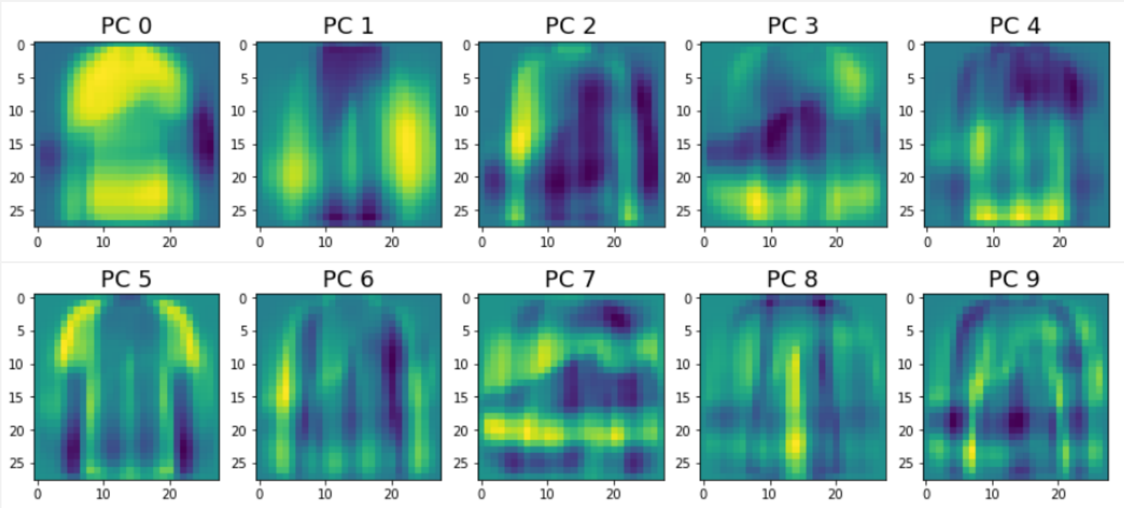
1.3 (3 points) Apply Principal Component Analysis (PCA) to the data of `Xtrn_nm` using sklearn.decomposition.PCA, and report the variances of projected data for the first five principal components in a table. Note that you should use `Xtrn_nm` instead of `Xtrn`.

Principal Component #	**Explained Variance**
1	19.81
2	12.112
3	4.106
4	3.382
5	2.625

1.4 (3 points) Plot a graph of the cumulative explained variance ratio as a function of the number of principal components, K, where $1 \leq K \leq 784$. Discuss the result briefly.



1.5 (4 points) Display the images of the first 10 principal components in a 2-by-5 grid, putting the image of 1st principal component on the top left corner, followed by the one of 2nd component to the right. Discuss your findings briefly.



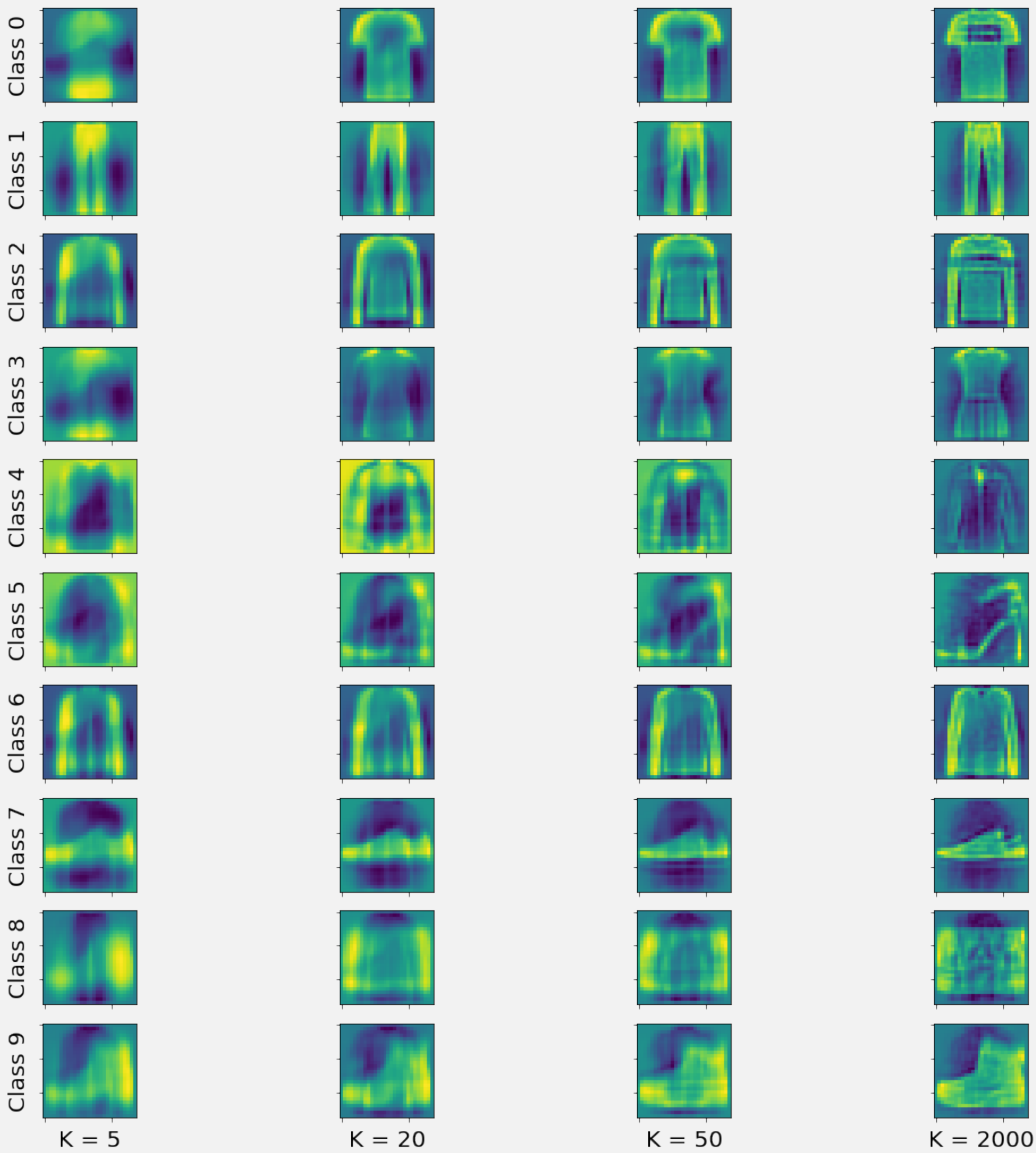
1.6 (5 points) Using `Xtrn_nm`, for each class and for each number of principal components $K = 5, 20, 50, 200$, apply dimensionality reduction with PCA to the first sample in the class, reconstruct the sample from the dimensionality-reduced sample, and report the Root Mean Square Error (RMSE) between the original sample in `Xtrn_nm` and reconstructed one.

A table to show the RMSE between the original and the reconstructed version of the first sample for every class with varying numbers of PCA components (K)

*Each class sample is reconstructed by reducing the sample to K dimensions and then is transformed back to the original number of dimensions, this is all done via the sklearn PCA implementation.

RMSE	K = 5	K = 20	K = 50	K = 200
Class = 0	0.256	0.15	0.128	0.062
Class = 1	0.198	0.14	0.095	0.037
Class = 2	0.199	0.146	0.123	0.08
Class = 3	0.146	0.107	0.084	0.056
Class = 4	0.118	0.103	0.088	0.046
Class = 5	0.181	0.159	0.142	0.091
Class = 6	0.129	0.096	0.072	0.046
Class = 7	0.166	0.128	0.106	0.062
Class = 8	0.223	0.145	0.123	0.093
Class = 9	0.184	0.151	0.122	0.072

1.7 (4 points) Display the image for each of the reconstructed samples in a 10-by-4 grid, where each row corresponds to a class and each row column corresponds to a value of $K = 5,\ 20,\ 50,\ 200$.



1.8 (4 points) Plot all the test samples (`Xtrn_nm`) on the two-dimensional PCA plane you obtained in Question 1.3, where each sample is represented as a small point with a colour specific to the class of the sample. Use the 'coolwarm' colormap for plotting.

Your Answer Here

Question 2 : (25 total points) Logistic regression and SVM

In this question we will explore classification of image data with logistic regression and support vector machines (SVM) and visualisation of decision regions.

2.1 (3 points) Carry out a classification experiment with multinomial logistic regression, and report the classification accuracy and confusion matrix (in numbers rather than in graphical representation such as heatmap) for the test set.

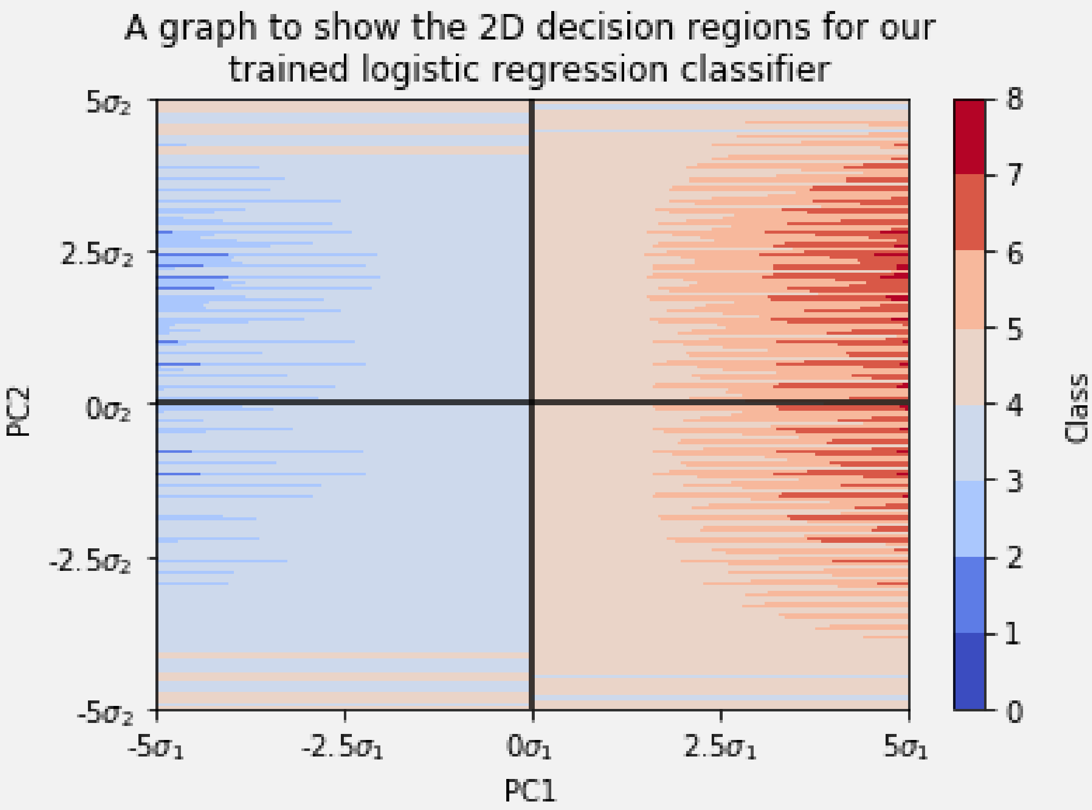
```
FREQUENCY CONFUSION MATRIX:

Predicted    0    1    2    3    4    5    6    7    8    9
Actual
0          819    5   27   31    0    2  147    0    7    0
1            3  953    4   15    3    0    3    0    1    0
2           15    4  731   14  115    0  128    0    6    0
3           50   27   11  866   38    1   46    0   11    1
4            7    5  133   33  760    0  108    0    3    0
5            4    0    0    0    2  911    0   32    7   15
6           89    3   82   37   72    0  539    0   15    1
7            1    1    2    0    0   56    0  936    5   42
8           12    2    9    4   10   10   28    1  945    0
9            0    0    1    0    0   20    1   31    0  941


PERCENTAGE CONFUSION MATRIX:

Predicted     0     1     2     3     4     5     6     7     8     9
Actual
0          78.9   0.5   2.7   2.9   0.0   0.2  17.5   0.0   0.7   0.0
1           0.3  97.0   0.4   1.4   0.3   0.0   0.4   0.0   0.1   0.0
2           1.4   0.4  72.2   1.3  11.0   0.0  15.3   0.0   0.6   0.0
3           4.8   2.7   1.1  82.4   3.6   0.1   5.5   0.0   1.1   0.1
4           0.7   0.5  13.1   3.1  72.4   0.0  12.9   0.0   0.3   0.0
5           0.4   0.0   0.0   0.0   0.2  93.8   0.0   3.1   0.7   1.5
6           8.6   0.3   8.1   3.5   6.9   0.0  64.3   0.0   1.5   0.1
7           0.1   0.1   0.2   0.0   0.0   5.8   0.0  89.7   0.5   4.2
8           1.2   0.2   0.9   0.4   1.0   1.0   3.3   0.1  92.6   0.0
9           0.0   0.0   0.1   0.0   0.0   2.1   0.1   3.0   0.0  94.7


Total classification accuracy = 84.01%
```

Question 2 continued on next page. . .

2.2 (3 points) Carry out a classification experiment with SVM classifiers, and report the mean accuracy and confusion matrix (in numbers) for the test set.

```
FREQUENCY CONFUSION MATRIX:

Predicted    0    1    2    3    4    5    6    7    8    9
Actual
0          845    4   15   32    1    0  185    0    3    0
1            2  951    2    6    0    0    1    0    1    0
2            8    7  748   12   98    0  122    0    8    0
3           51   31   11  881   36    1   39    0    5    0
4            4    5  137   26  775    0   95    0    2    0
5            4    0    0    0    0  914    0   34    4   22
6           72    1   79   40   86    0  533    0   13    0
7            0    0    0    0    0   57    0  925    4   47
8           14    1    8    3    4    2   25    0  959    1
9            0    0    0    0    0   26    0   41    1  930


PERCENTAGE CONFUSION MATRIX:

Predicted     0     1     2     3     4     5     6     7     8     9
Actual
0          77.9   0.4   1.5   3.0   0.1   0.0  22.5   0.0   0.3   0.0
1           0.2  98.8   0.2   0.6   0.0   0.0   0.1   0.0   0.1   0.0
2           0.7   0.7  74.6   1.1   9.4   0.0  14.8   0.0   0.8   0.0
3           4.7   3.2   1.1  83.5   3.4   0.1   4.7   0.0   0.5   0.0
4           0.4   0.5  13.7   2.5  74.2   0.0  11.5   0.0   0.2   0.0
5           0.4   0.0   0.0   0.0   0.0  93.5   0.0   3.3   0.4   2.2
6           6.6   0.1   7.9   3.8   8.2   0.0  64.7   0.0   1.3   0.0
7           0.0   0.0   0.0   0.0   0.0   5.8   0.0  89.5   0.4   4.7
8           1.3   0.1   0.8   0.3   0.4   0.2   3.0   0.0  94.3   0.1
9           0.0   0.0   0.0   0.0   0.0   2.7   0.0   4.0   0.1  93.2


Mean classification accuracy = 84.412%
```

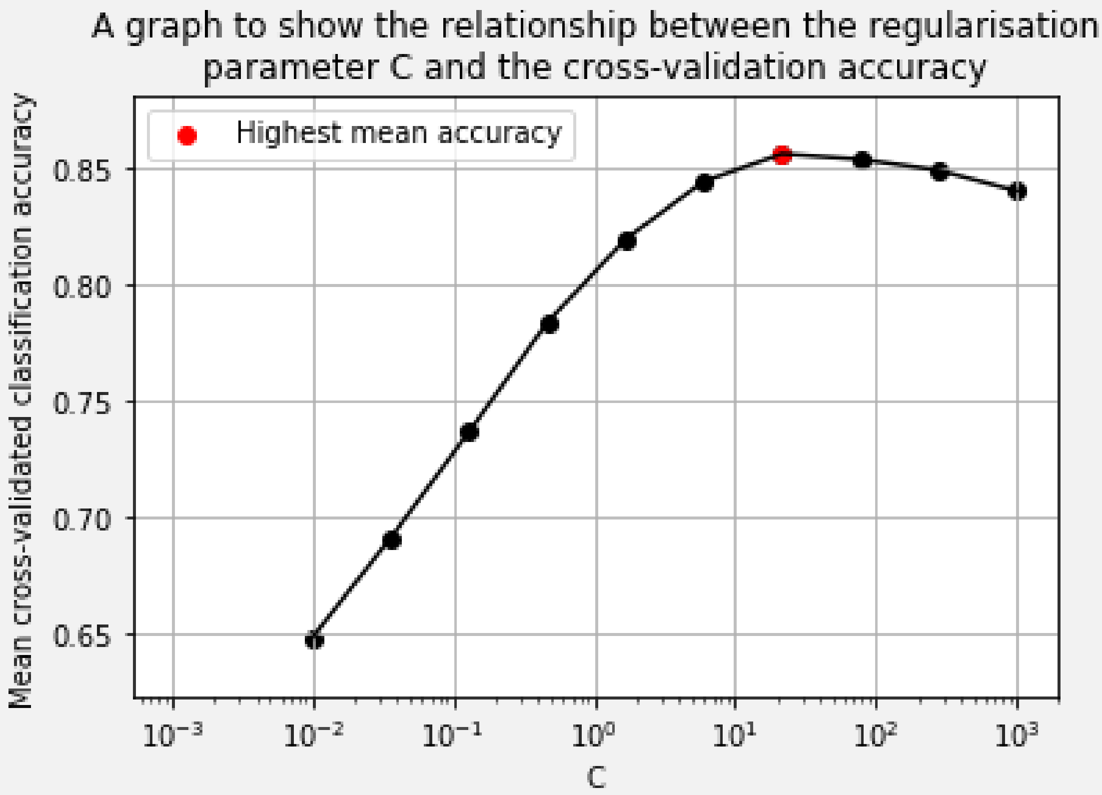
2.3 (6 points) We now want to visualise the decision regions for the logistic regression classifier we trained in Question 2.1.



2.4 (4 points) Using the same method as the one above, plot the decision regions for the SVM classifier you trained in Question 2.2. Comparing the result with that you obtained in Question 2.3, discuss your findings briefly.

Your Answer Here

2.5 (6 points) We used default parameters for the SVM in Question 2.2. We now want to tune the parameters by using cross-validation. To reduce the time for experiments, you pick up the first 1000 training samples from each class to create `Xsmall`, so that `Xsmall` contains 10,000 samples in total. Accordingly, you create labels, `Ysmall`.



Optimal accuracy achieved and it's respective value of C:

$Accuracy \approx 85.65\%$ where $C \approx 10^{1.33}$

2.6 (3 points) Train the SVM classifier on the whole training set by using the optimal value of C you found in Question 2.5.

Classification accuracy of our trained SVM using $\mathbf{C = 10^{\frac{4}{3}}}$

Training accuracy $\approx 86.768\%$
Testing accuracy $\approx 85.02\%$

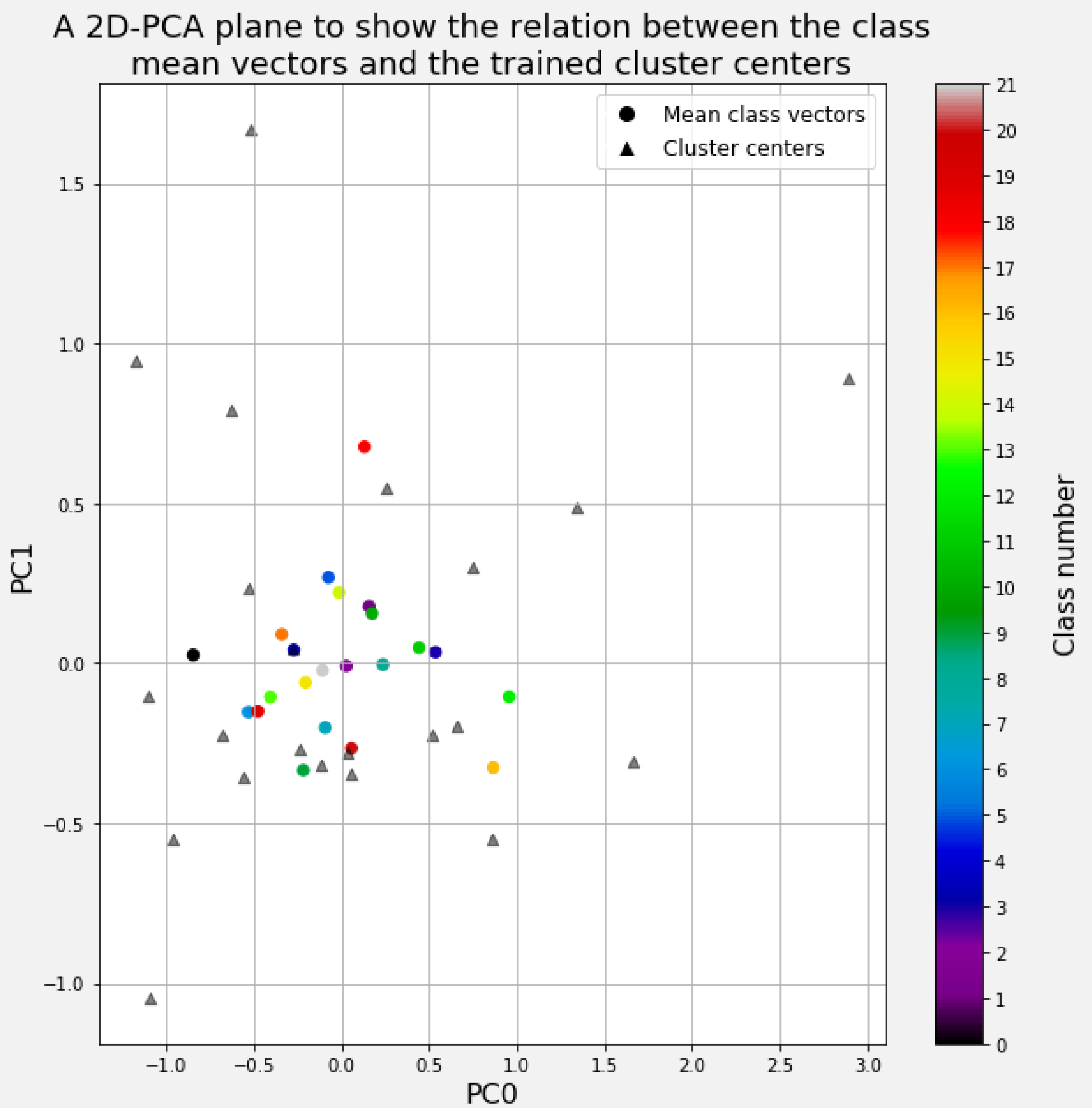
Question 3 : (20 total points) Clustering and Gaussian Mixture Models

In this question we will explore K-means clustering, hierarchical clustering, and GMMs.

3.1 (3 points) Apply k-means clustering on `Xtrn` for $k = 22$, where we use sklearn.cluster.KMeans with the parameters `n_clusters=22` and `random_state=1`. Report the sum of squared distances of samples to their closest cluster centre, and the number of samples for each cluster.

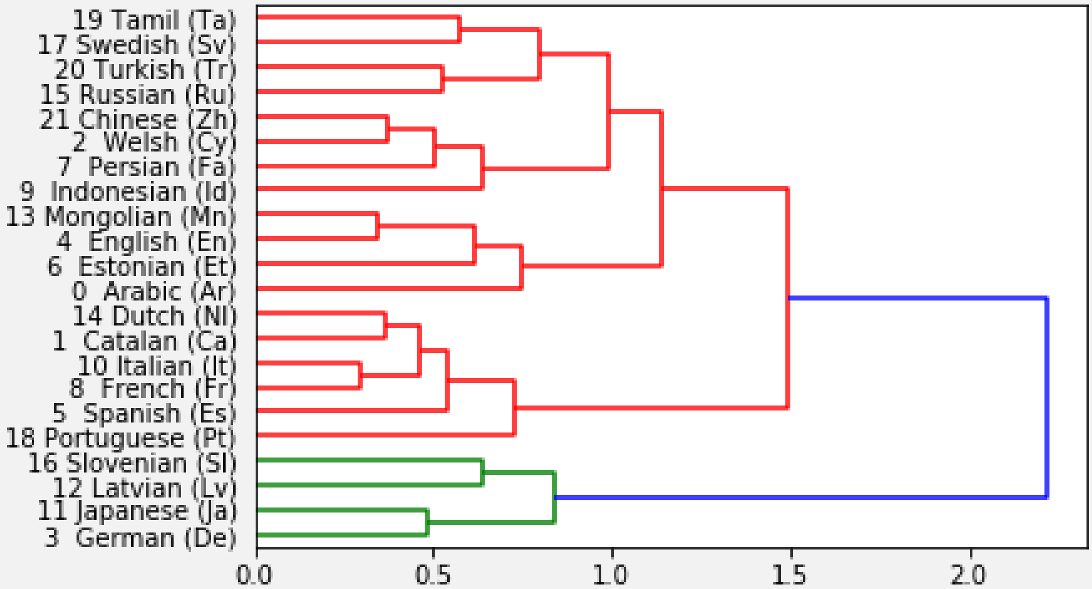
```
Sum of squared distances (Euclidean) of samples to their closest cluster center:
38185.81698349466

Number of samples for each cluster:
Cluster 1 = 1018
Cluster 2 = 1125
Cluster 3 = 1191
Cluster 4 = 890
Cluster 5 = 1162
Cluster 6 = 1332
Cluster 7 = 839
Cluster 8 = 623
Cluster 9 = 1400
Cluster 10 = 838
Cluster 11 = 659
Cluster 12 = 1276
Cluster 13 = 121
Cluster 14 = 152
Cluster 15 = 950
Cluster 16 = 1971
Cluster 17 = 1251
Cluster 18 = 845
Cluster 19 = 896
Cluster 20 = 930
Cluster 21 = 1065
Cluster 22 = 1466
```

3.2 (3 points) Using the training set only, calculate the mean vector for each language, and plot the mean vectors of all the 22 languages on a 2D-PCA plane, where you apply PCA on the set of 22 mean vectors without applying standardisation. On the same figure, plot the cluster centres obtained in Question 3.1.



3.3 (3 points) We now apply hierarchical clustering on the training data set to see if there are any structures in the spoken languages.

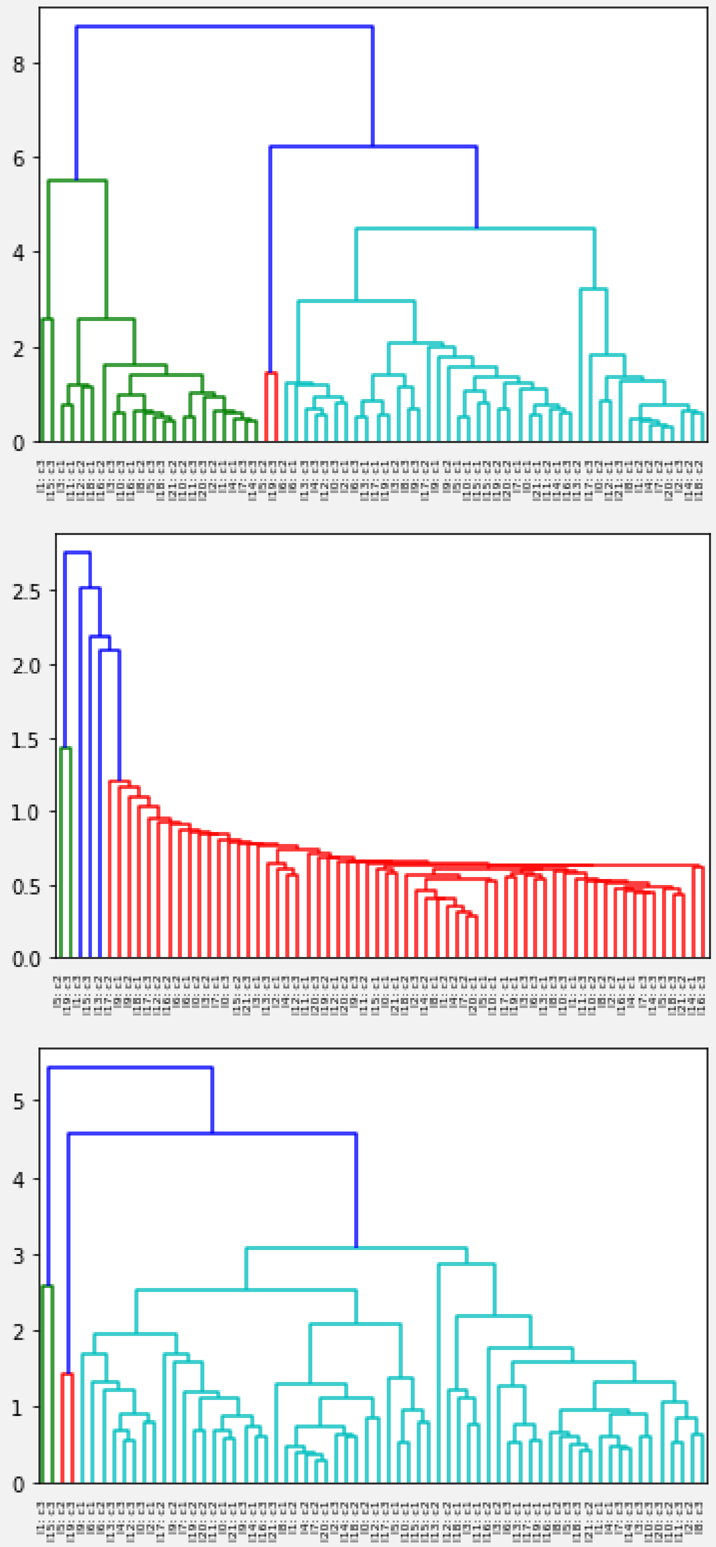
A dendogram to show the hierarchical clustering of the mean vectors for each language



This dendogram illustrates how each cluster from our dataset is composed and thus ultimately represents the hierarchy of similarities between the classes in our data.

From this we can deduce that Italian and French have the most similar mean vectors and thereby are the most "similar" languages. In contrast we can see that Latvian has the most dissimilar mean vector from all other languages and thereby is the most "unique" language.

3.4 (5 points) We here extend the hierarchical clustering done in Question 3.3 by using multiple samples from each language.



3.5 (6 points) We now consider Gaussian mixture model (GMM), whose probability distribution function (pdf) is given as a linear combination of Gaussian or normal distributions, i.e.,

Your Answer Here