STA 1013 : Statistics through Examples

Lecture 28: Statistical Hypothesis 2

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Warm up and review

Identify H_0 and H_a

1. The RDA (Recommended Daily Allowance) for iron intake for women under the age of 51 is 18 mg. We would like to see if the mean iron intake μ is below the RDA. What are the hypotheses?
2. The American Hospital Association claims that the mean cost to community hospitals per patient per day is $1,033. We would like to see if the mean actually exceeds this. What are the hypotheses?
3. A dog food manufacturer wants to know if the proper amount of dog food is being placed in 25 pound bags. They would like to see if the mean weight of bags is not 25 pounds. What are the hypotheses?

Rejection Region approach

- **Test Statistic under H_0 (when σ known):**

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

- We will reject H_0 if our test statistic (z_0) is **too "extreme."**

- "Too extreme" means the test statistic is located in the **rejection region**

Rejection Region approach

Rejection Regions

1. Left-Tailed test :

$$\textbf{Rejection Region} < -z_\alpha$$

2. Right-Tailed test :

$$\textbf{Rejection Region} > z_\alpha$$

3. Two-Tailed test :

$$\textbf{Rejection Region} > z_{\alpha/2} \text{ or } \textbf{Rejection Region} < -z_{\alpha/2}$$

Exercises

Alternative Hypothesis supported? ($\alpha = 0.05$)

1. $H_a : \mu < 75, n = 100, \bar{X} = 70, \sigma = 15$
2. $H_a : \mu < 75, n = 36, \bar{X} = 72, \sigma = 15$
3. $H_a : \mu > 12, n = 64, \bar{X} = 14, \sigma = 2$
4. $H_a : \mu > 1007, n = 225, \bar{X} = 1021, \sigma = 35$
5. $H_a : \mu \neq 2.55, n = 100, \bar{X} = 2.58, \sigma = 0.29$
6. $H_a : \mu \neq 156.2, n = 225, \bar{X} = 155.5, , \sigma = 29$

P-value approach

P-value

- Using a rejection region to determine the result of a test **just tells us whether or not to reject** H_0.
- A **P-value** additionally measures the strength of evidence against H_0
- We can use P-values to be more precise about the significance of a result.
- The definition of the P-value is basically :
  - If the null hypothesis is true, then how likely are the results we got?
  - the p-value is the probability of obtaining the observed results of a test, assuming that the null hypothesis is correct.

Motivation

Suppose you are gambling with a suspicious-looking person (hypothetically) and the rules are simple: if a coin flip is heads, you win \$1, if it is tails, he wins \$1.

1. After 10 flips he has won 8 times. If the coin is fair, the probability of winning at least 8 times is 0.0547. Is this enough to make you think that he is cheating?
2. Suppose he won 9 times. If the coin is fair, the probability of him winning at least 9 times is 0.0107. Do you think he is cheating?
3. Suppose he won all 10 times. If the coin is fair, the probability of this is less than 0.001. Do you think he is cheating?

P-value

- P-value $=$ P(Test statistic quantity is as extreme as what we observed)
- The "extreme" direction is determined by H_a
- It measures the **strength of the disagreementbetween the sample and** H_0
- The smaller the P-value, the stronger the evidence is against H_0

Hypothesis Test decisions Based on P-Values

- **Small P-value leads to reject H_0**

  : Under the assumption H_0 was true, the results we got would be rare.

- **Large P-value leads to can not reject H_0**

  : Under the assumption H_0 was true, the results we got would not be rare enough

The criteria for “small” : P-value $\leqslant \alpha$

- Reject H_0 when : $P \leqslant \alpha$
- Do not Reject H_0 when $P > \alpha$

YOU MUST CHOOSE α BEFORE THE TEST!!!!

P-value

1. Left-Tailed test

$$\textbf{P-value} = P(Z \leqslant z_0)$$

2. Right-Tailed test

$$\textbf{P-value} = P(Z \geqslant z_0)$$

3. Two-Tailed test

$$\begin{aligned}\textbf{P-value} &= P(Z \leqslant -|z_0| \text{ or } Z \geqslant |z_0|) \\ &= 2 * P(Z \geqslant |z_0|)\end{aligned}$$

Example : Starting salary

Starting salary

Columbia College advertises that the mean starting salary of its graduates is \$39,000. The Committee for Truth in Advertising, an independent organization, suspects that this claim is exaggerated and decides to conduct a hypothesis test to seek evidence to support its suspicion. Having formed the hypotheses, the Committee for Truth in Advertising selects a random sample of 100 recent graduates from the college. The mean salary of the graduates in the sample turns out to be \$37,000. And $\sigma = \$6,150$ can be assumed.

Perform a statistical hypothesis test with significance level $\alpha = 0.05$

Example (Solution)

1. **State the Hypothesis**

$$H_0 : \mu = \$39,000$$
$$H_a : \mu < \$39,000$$

2. **Calculate the test Statistic**

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{37,000 - 39,000}{6,150/\sqrt{100}} = -3.25$$

3. **p-value**

P-value : $P(Z < z_0) = P(Z < -3.25) = 5.77E - 4$

4. **Conclusion :** P-value $< \alpha$ reject H_0

Example : Rental car

Rental car

In the United States, the average car is driven about 12,000 miles each year. The owner of a large rental car company suspects that for his fleet, the mean distance is greater than 12,000 miles each year. He selects a random sample of n $=$ 225 cars from his fleet and finds that the mean annual mileage for this sample is $\bar{X} = 12,375$ miles. population standard deviation $\sigma = 2,415$ can be assumed.

Based on these data, describe the process of conducting a hypothesis test (with $\alpha = 0.05$) and drawing a conclusion.

Example (Solution)

1. **State the Hypothesis**

$$H_0 : \mu = 12,000 \text{ miles}$$

$$H_a : \mu > 12,000 \text{ miles}$$

2. **Calculate the test Statistic**

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{12,375 - 12,000}{2,415/\sqrt{225}} = 2.33$$

3. **p-value**

$$\text{P-value} : P(Z > z_0) = P(Z > 2.33) = 0.0099$$

4. **Conclusion :** P-value $< \alpha$ reject H_0

Example : Body Temperature

Body Temperature

Consider the study in which University of Maryland researchers measured body temperatures in a sample of n = 106 healthy adults, finding a sample mean body temperature of $\bar{X} = 98.20F^{\circ}$, and assume that the population standard deviation is $\sigma = 0.62F^{\circ}$.

Determine whether this sample provides enough evidence for rejecting the common belief that mean human body temperature is $\mu = 98.6F^{\circ}$. Use significance level $\alpha = 0.05$

Example (Solution)

1. **State the Hypothesis**

$$H_0 : \mu = 98.6F^\circ \text{ vs } H_a : \mu \neq 98.6F^\circ$$

2. **Calculate the test Statistic**

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{98.20 - 98.6}{0.62/\sqrt{106}} = -6.64$$

3. **p-value**

P-value : $2P(Z > |z_0|) = 2P(Z > 6.64) = 3.1535E - 11$

4. **Conclusion :** P-value $< \alpha$ reject H_0

Find P-values (Hint : Use the **normalcdf** function)

1. $z_0 = -0.4$ for $H_a : \mu < 727$
2. $z_0 = -2.4$ for $H_a : \mu > 727$
3. $z_0 = -2.4$ for $H_a : \mu \neq 727$
4. $z_0 = 1.4$ for $H_a : \mu \neq 727$

Calculator (Z-Test)

Z-Test from statistics

1. Press the $\boxed{\text{stat}}$ and highlight **TESTS**
2. Scroll down to 1: **Z-Test**
3. Inpt : Highlight **Stats**
4. Enter values for
   - μ_0 : Claimed value in the null hypothesis (H_0)
   - σ : Population standard deviation
   - $\bar{X}$: Sample mean
   - n : sample size
   - μ : Select the test type ($\underbrace{\neq \mu_0}_{Two-Tailed}$, $\underbrace{< \mu_0}_{Left-Tail}$, $\underbrace{> \mu_0}_{Right-Tail}$)

Example 1. (Left-Tailed Test)

Mean time of ownership

A Roper poll used a sample of 100 randomly selected car owners. Within the sample, the mean time of ownership for a single car was 7.01 years. And the population standard deviation $\sigma = 3.74$ years can be assumed. Test the claim by the owner of a large dealership that the mean time of ownership for all cars is less than 7.5 years. $(\alpha = 0.05)$

1. **State the Hypothesis**
2. **Find the P-value**
3. **Conclusion**

Example 2. (Right-Tailed Test)

Hospital Time

According to a study by the Centers for Disease Control, the national mean hospital stay after childbirth is 2.0 days, and a population standard deviation of 1.2 days. Reviewing records at her own hospital, a hospital administrator calculates that the mean hospital stay for a sample of 81 women after childbirth is 2.2 days. Assumingthat the patients represent a random sample of the population, test the claim that this hospital keeps new mothers longer than the national average. ($\alpha = 0.1$)

1. **State the Hypothesis**

2. **Find the P-value**

3. **Conclusion**

Example 3. (Two-Tailed Test)

Cola

A random sample of 36 cans of regular Coke is obtained and the contents are measured. The sample mean is 12.19 oz. And the population standard deviation $\sigma = 0.11oz$ can be assumed. Test the claim that the contents of all such cans have a mean different from 12.00 oz, as indicated by the label. $(\alpha = 0.01)$

1. **State the Hypothesis**

2. **Find the P-value**

3. **Conclusion**