STA 1013 : Statistics through Examples

Lecture 16: Probabilities for the standard normal distribution

Hwiyoung Lee

October 9, 2019

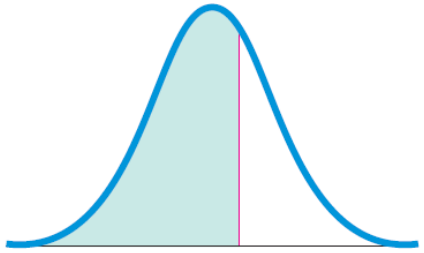
Department of Statistics, Florida State University

Warm up problems

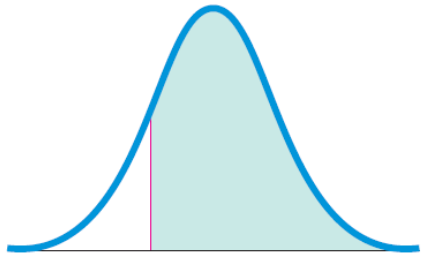
Find the Probability from the given z score

Find the area of the shaded region. The graph depicts the standard normal distribution N(0,1)

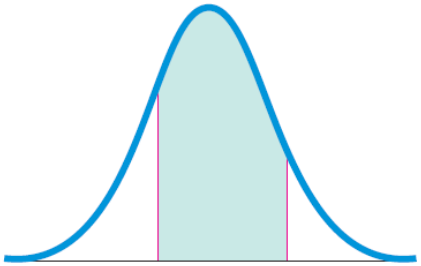
9.



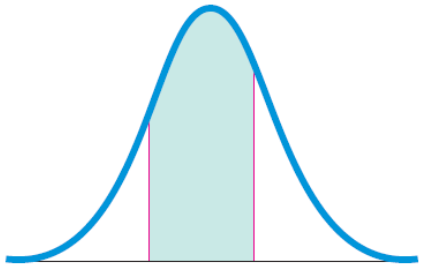
10.



11.



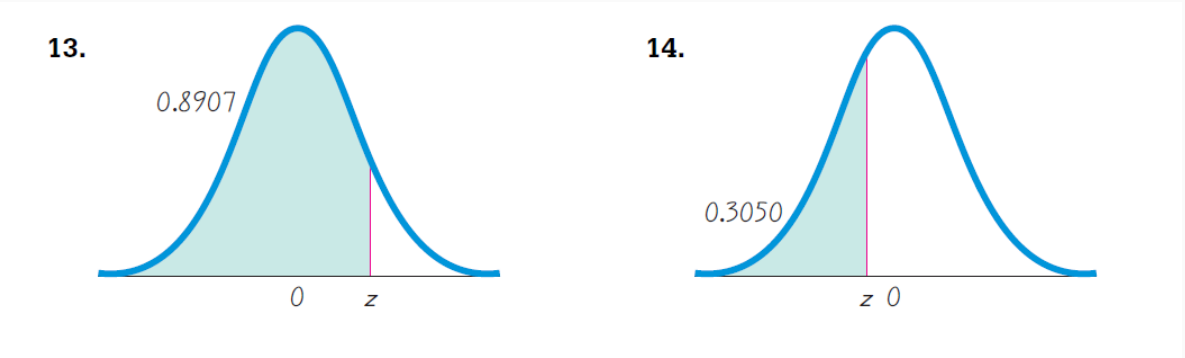
12.



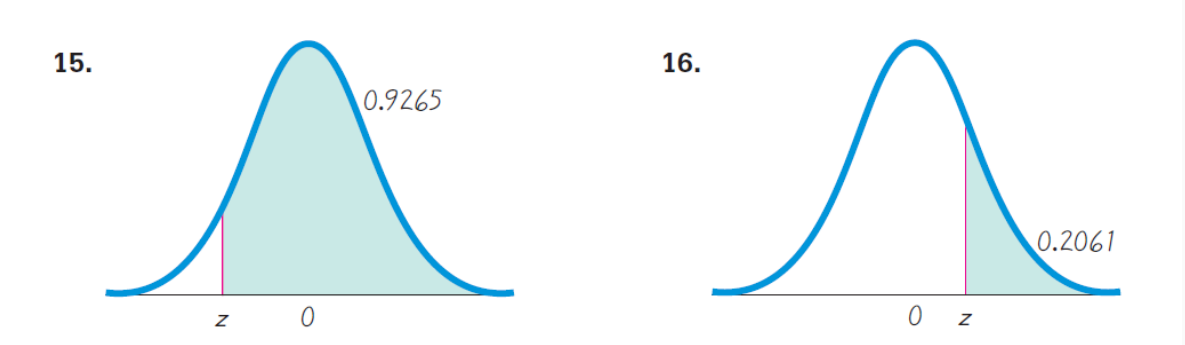
Find the z score from the given Probability

Find the indicated z score. The graph depicts the standard normal distribution N(0,1)

Left tail



Right tail



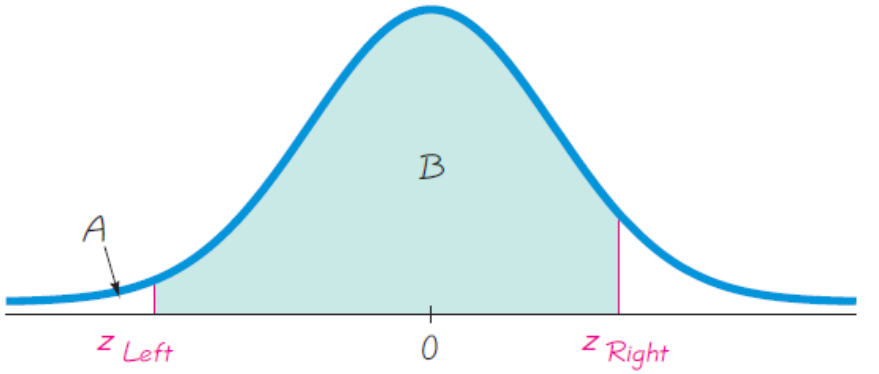
Probabilities of Normal distribution

Probabilities of Normal distribution

- $P(Z > z) = 1 - P(Z \leqslant z)$
- $P(Z = z) = 0$. Therefore
  - $P(Z \leqslant z) = P(Z < z)$
  - $P(Z \geqslant z) = P(Z > z)$
- $P(a < Z < b) = P(Z < b) - P(Z < a)$

Probabilities of Normal distribution

$$P(a < Z < b) = P(Z < b) - P(Z < a)$$



Shaded area $\mathcal{B}$ = (areas A and $\mathcal{B}$ combined) – (area A)

Examples

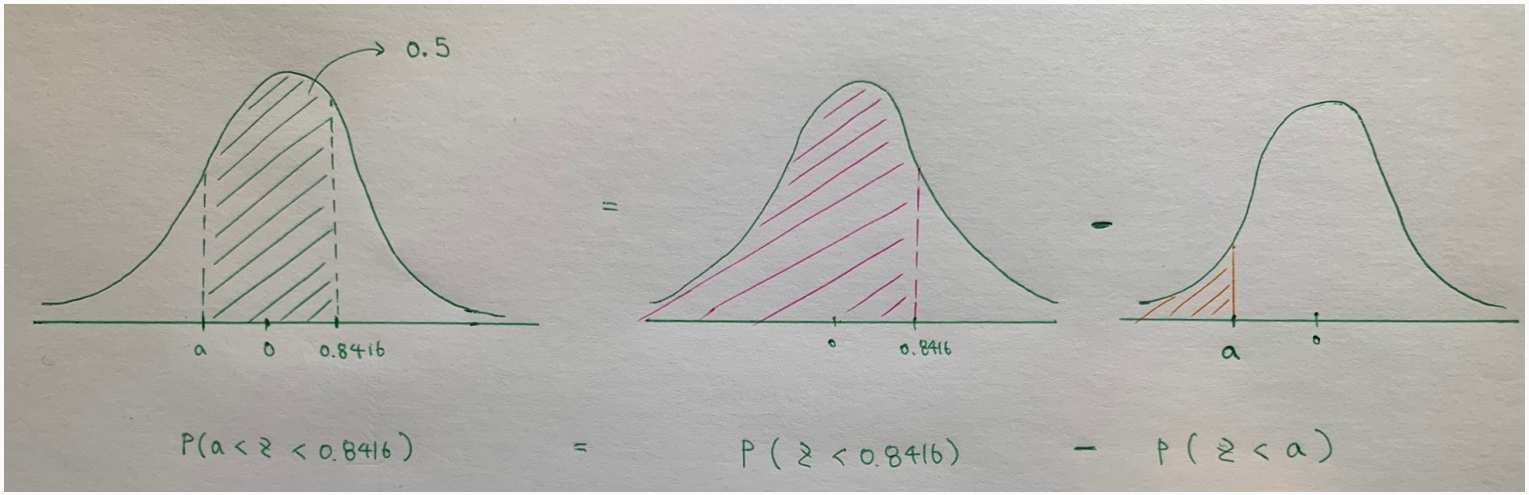
$Z \sim N(0, 1)$

Find

- $P(0.5 < Z < 1.5)$
- $P(a < Z < 0.8416) = 0.5$
- $P(-0.5244 < Z < b) = 0.3$

Probabilities of Normal distribution

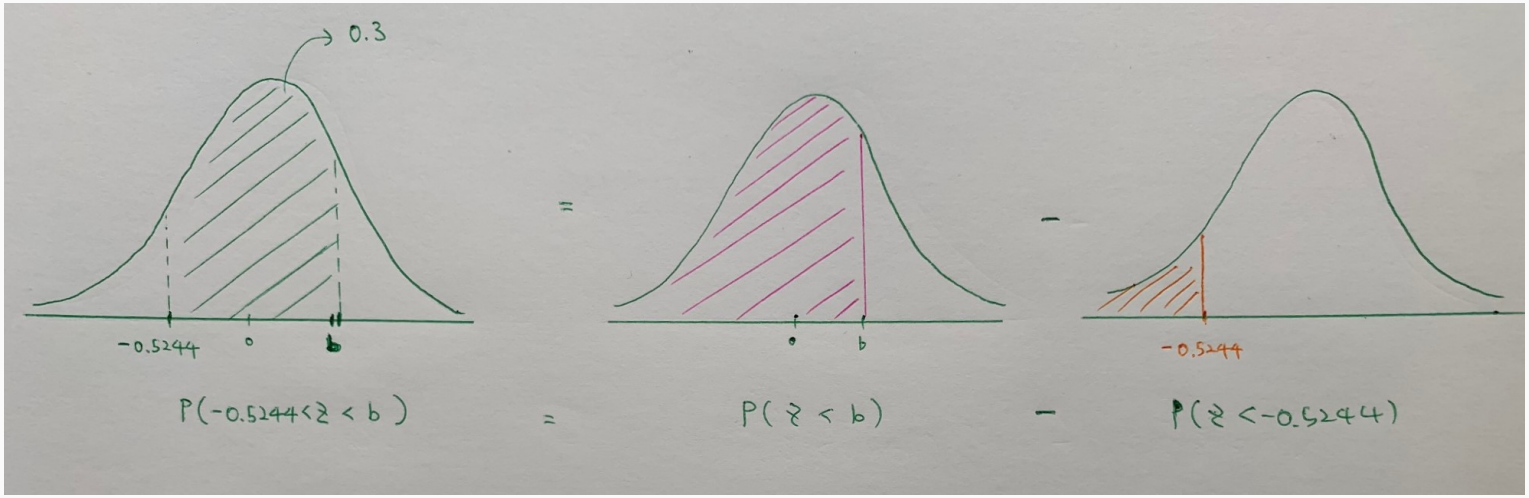
$Z \sim N(0,1)$, find $P(a < Z < 0.8416) = 0.5$



1. fine $P(Z < 0.8416) =$ _____ (use normalcdf)
2. find $\mathbf{P(Z < a)} = P(Z < 0.8416) - P(a < Z < 0.8416) =$
3. find a such that $P(Z < a) =$

Probabilities of Normal distribution

$Z \sim N(0,1)$, find $P(-0.5244 < Z < b) = 0.3$

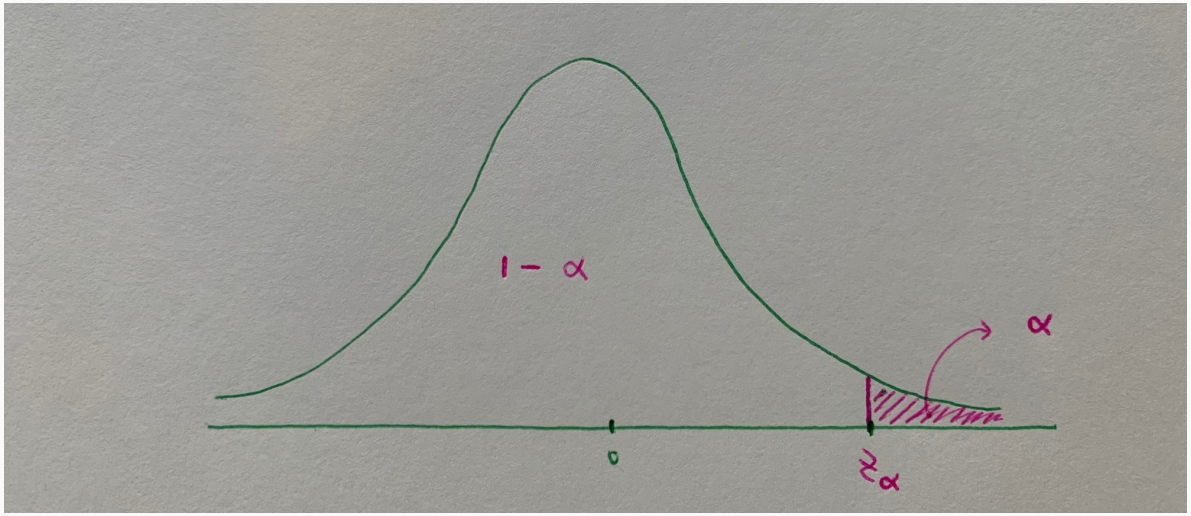


1. fine $P(Z < -0.5244) =$ _____ (use normalcdf)
2. find $\mathbf{P(Z < b)} = P(Z < -0.5244) + P(-0.5244 < Z < b) =$
3. find b such that $P(Z < b) =$

Critical value

Critical value

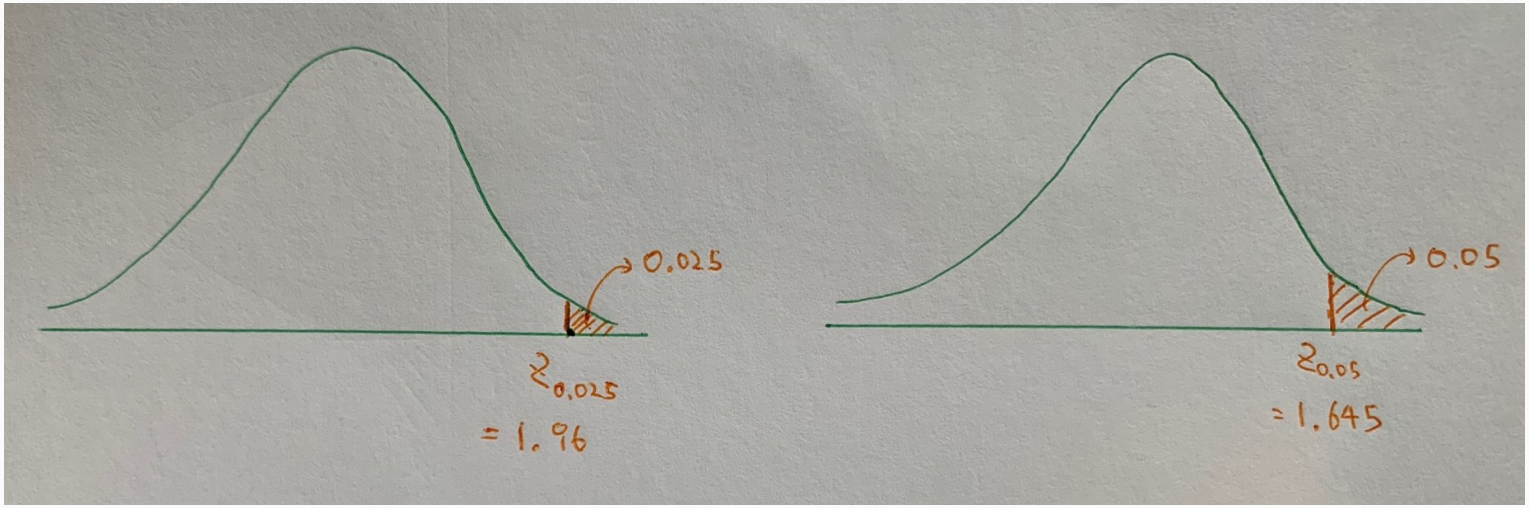
Critical value z_α : the z score with an area of α to its right.



- For the standard normal distribution, a critical value is a z score separating unlikely values from those that are likely to occur.
- α : small number (probability)

Some important critical values

- $z_{0.025} = 1.96$
- $z_{0.05} = 1.645$



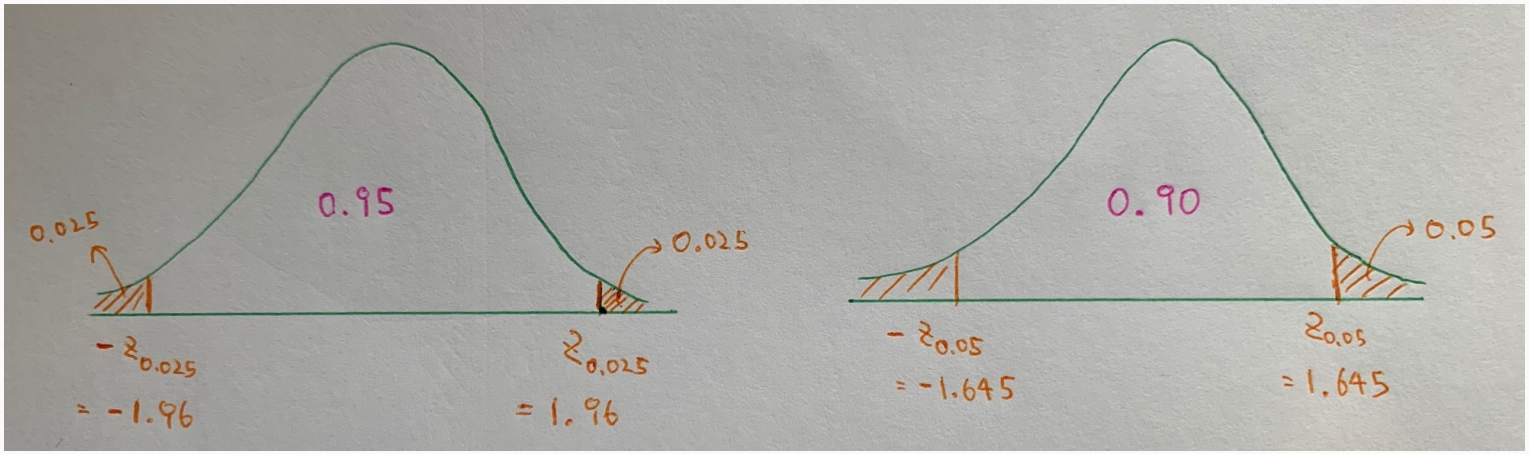
We can find critical values With our calculator

- $z_{0.025}$: invnorm(0.975.0,1)
- $z_{0.05}$: invnorm(0.95,0,1)

Some important critical values

- $z_{0.025} = 1.96$, $-z_{0.025} = -1.96$
- $z_{0.05} = 1.645$, $-z_{0.05} = -1.645$

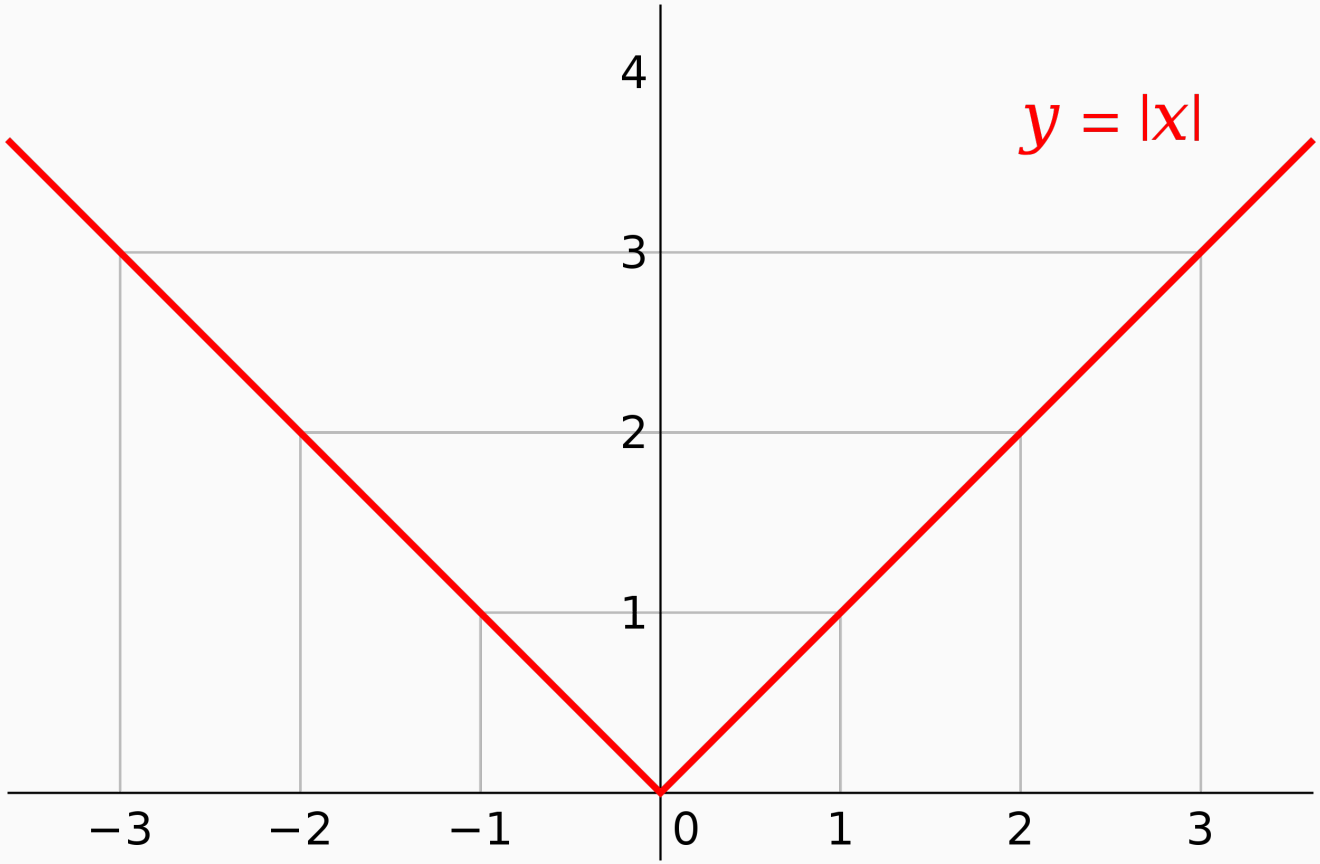
Since the standard normal distribution is symmetric about 0



- $P(-1.96 < Z < 1.96) = 0.95$
- $P(-1.645 < Z < 1.645) = 0.9$

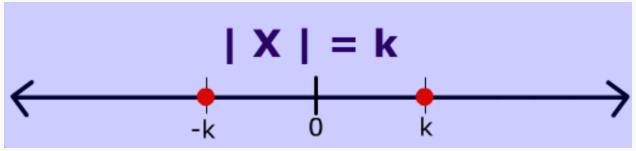
Absolute value

Graph of the absolute value function

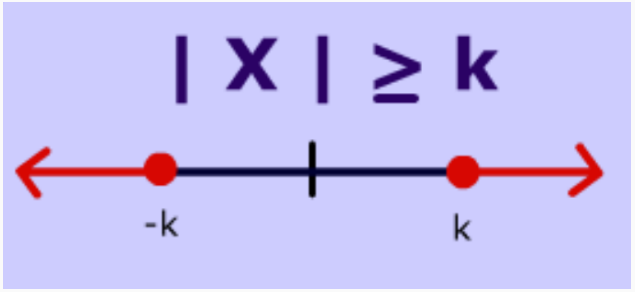


Basic absoulte value inequalities

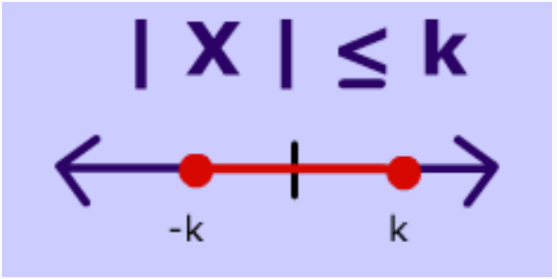
1. If $|X| = K$, then $X = K$ or $X = -K$



2. If $|X| \geqslant K$, then $X \geqslant K$ or $X \leqslant -K$



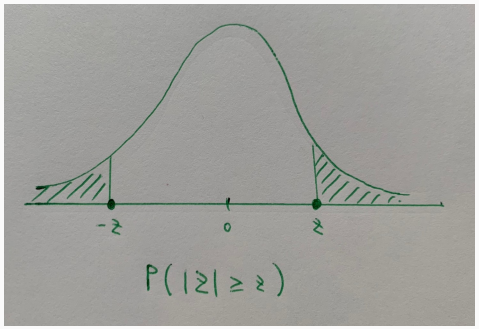
3. If $|X| \leqslant K$, then $-K \leqslant X \leqslant K$



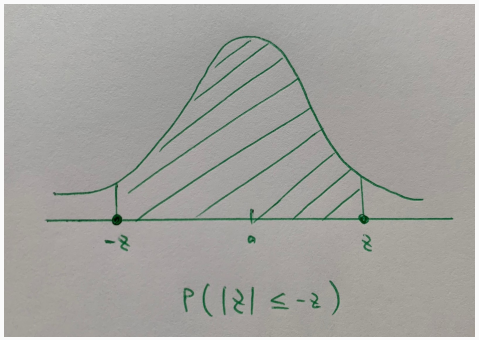
Probability absoulte value inequalities

$Z \sim N(0,1)$, and Z is symmetric about 0

1. $P(|Z| \geqslant z) = P(Z \leqslant -z \text{ or } Z \geqslant z) = P(Z \leqslant -z) + P(Z \geqslant z)$



2. $P(|Z| \leqslant z) = P(-z \leqslant Z \leqslant z)$



Note : $P(|Z| \geqslant z) = 1 - P(|Z| \leqslant z)$

Conversely, $P(|Z| \leqslant z) = 1 - P(|Z| \geqslant z)$

Probability absoulte value inequalities

$Z \sim N(0, 1)$

1. $P(|Z| \leqslant z) = 0.90$

2. $P(|Z| \geqslant z) = 0.05$

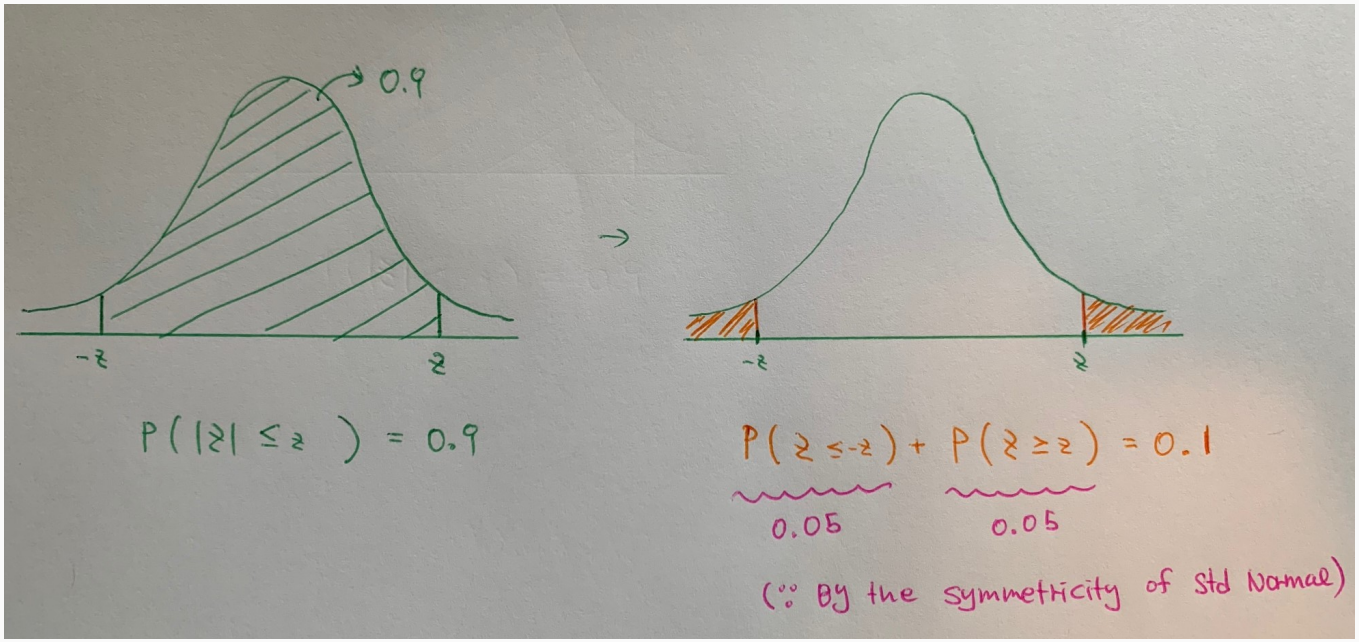
Probability absoulte value inequalities

$Z \sim N(0,1)$, Find $P(|Z| \leqslant z) = 0.90$

Step1. $P(-z \leqslant Z \leqslant z) = 0.9$

Step2. $P(Z \leqslant -z) + P(Z \geqslant z) = 0.1$

Step3. $P(Z \leqslant -z) = P(Z \geqslant z) = 0.05$ ($\because$ Symmetric about 0)



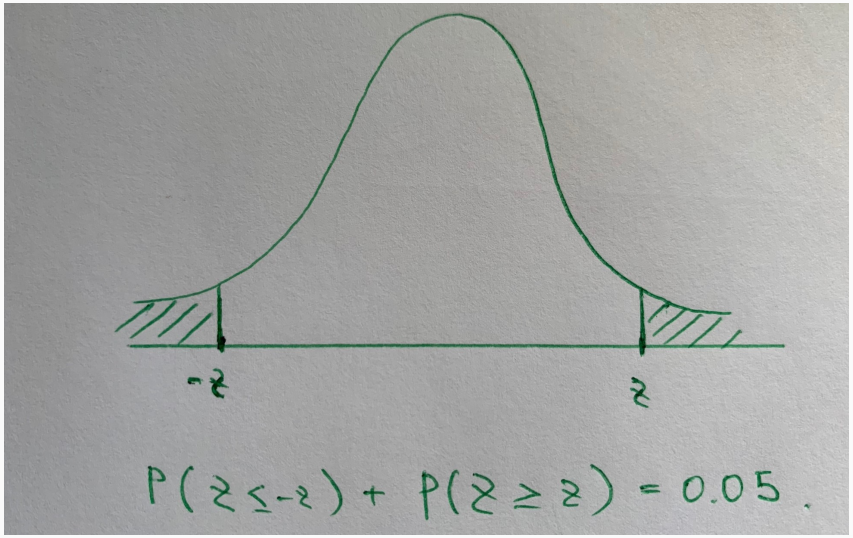
Find $z_{0.05}$

Probability absoulte value inequalities

$Z \sim N(0,1)$, Find $P(|Z| \geqslant z) = 0.05$

Step1. $P(Z \leqslant -z) + P(Z \geqslant z) = 0.05$

Step2. $P(Z \leqslant -z) = P(Z \geqslant z) = 0.025$



Find $z_{0.025}$

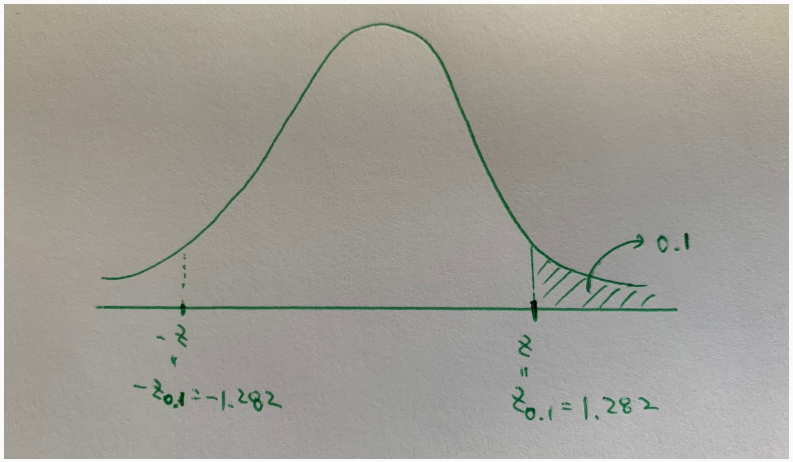
Mistakes in Exam

$X \sim N(0,1)$, Find $P(|X| \leqslant x) = 0.90$

- Common mistake : $x = 1.282$? (**Wrong !!**)
- Because,

$$P(|X| \leqslant 1.282) = P(-1.282 \leqslant X \leqslant 1.282) = 0.8!!$$

- My guess



LSQA : Liberal Studies Quantitative Assessment

- Topics :
  - Normal distribution
  - Central Limit theorem
- 6 Questions
- Test date
  - Option 1 : Oct 18 (Fri)
  - Option 2 : Oct 21 (Mon)