

STA 1013 : Statistics through Examples

Lecture 12: Normal Distribution (Probability)

Hwiyoung Lee

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Department of Statistics, Florida State University

1. Find the Probability from the given value
2. Find the Value from the given probability

Limitations of Empirical Rule

The empirical rule is an approximation (very rough calculation), and only valid when probabilities and numbers are related $\sigma, 2\sigma, 3\sigma$.

Example

IQ scores of adults, and those scores are normally distributed with a mean of 100 and a standard deviation of 15

- Find the probability that a randomly selected adult has an IQ between 90 and 110
- Find the probability that a randomly selected adult has an IQ less than 118
- Find P_{90} , which is the IQ score separating the bottom 90% from the top 10%
- Find the first quartile Q_1 , which is the IQ score separating the bottom 25% from the top 75%

Limitations of Empirical Rule

- Sometimes we need exact calculations
- Web calculator for Normal probability :

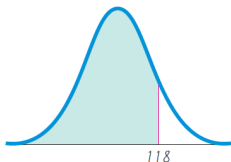
<https://istats.shinyapps.io/NormalDist/>

Find the Probability from the given value

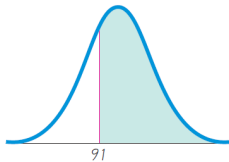
Find the Probability from the given number

Find the area (probability) of the shaded region. The graphs depict IQ scores of adults, and those scores are normally distributed with a mean of 100 and a standard deviation of 15

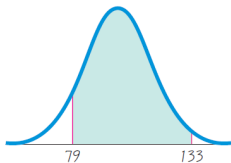
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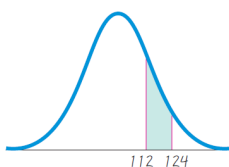
6.



7.



8.



Find the Probability from the given number

We randomly select 500 FSU students.

- How many students are expected to have IQ greater than 70
- How many students are expected to have IQ between 110 and 120 (referred to as bright normal)

Find the Probability from the given number

Pregnancy Lengths

Lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days.

- What is the percentage of pregnancies that last less than 250 days?
- What is the percentage of pregnancies that last more than 300 days?
- If a birth is considered premature if the pregnancy lasts less than 238 days, what is the percentage of premature births?

Find the Value from the given probability

Percentiles

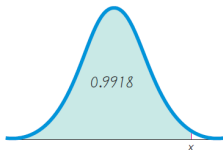
The α th percentile of a data set is the smallest value in the set with the property that $\alpha\%$ of the data values are less than or equal to it.

- α th percentile denoted by P_α
- P_α separates the bottom $\alpha\%$ from the top $1 - \alpha\%$
- Example :
 - Q_1 : the 25th percentile
 - Q_2 : the 50th percentile
 - Q_3 : the 75th percentile

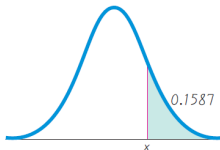
Find the Value from the given probability

Find the indicated IQ score, and round to the nearest whole number. The graphs depict IQ scores of adults, and those scores are normally distributed with a mean of 100 and a standard deviation of 15

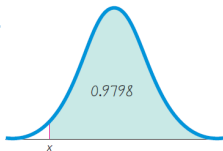
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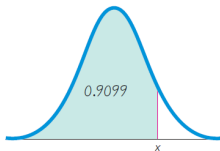
10.



11.



12.



Find the Value from the given probability

- Find P_{90} , which is the IQ score separating the bottom 90% from the top 10%
- Find the first quartile Q_1 , which is the IQ score separating the bottom 25% from the top 75%
- Find the third quartile Q_3 , which is the IQ score separating the top 25% from the others

Find the Value from the given probability

Mensa International

Mensa International calls itself “the international high IQ society,” and it has more than 100,000 members. Mensa states that “candidates for membership of Mensa must achieve a score at or above the 98th percentile on a standard test of intelligence (a score that is greater than that achieved by 98 percent of the general population taking the test).”

- Find the 98th percentile for the population of Wechsler IQ scores. This is the lowest score meeting the requirement for Mensa membership

Exercise

Navy pilots

Men's heights are normally distributed with mean 69.5 in. and standard deviation 2.4 in. Women's heights are normally distributed with mean 63.8 in. and standard deviation 2.6 in. The U.S. Navy requires that fighter pilots have heights between 62 in. and 78 in.

- Find the percentage of women meeting the height requirement. Are many women not qualified because they are too short or too tall?
- Find the percentage of men meeting the height requirement. Are many men not qualified because they are too short or too tall?

Exercise

- c. If the Navy changes the height requirements so that all women are eligible except the shortest 2% and the tallest 2%, what are the new height requirements for women?

- d. If the Navy changes the height requirements so that all men are eligible except the shortest 1% and the tallest 1%, what are the new height requirements for men?