STA 1013 : Statistics through Examples

Lecture 27: Statistical Hypothesis 1

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Motivation

Hypothesis Testing

Everyone makes claims :

- Advertisers make claims about their products.
- Governments claim their programs are effective.
- Medical diagnoses are claims about the presence or absence of disease.
- Pharmaceutical companies make claims about the effectiveness of their drugs.

Q. **But how do we know whether any of these claims are true ?**

A. Statistics offers a way to test many claims, through a powerful set of techniques called **hypothesis testing.**

Gender Choice

A company called ProCare Industries, Ltd. once claimed that its product, called Gender Choice, could increase a woman's chance of giving birth to a baby girl. The company claimed that the chance of a baby girl could be increased "up to 80%,".

How could we test whether the Gender Choice claim is true?

One way would be to study a random sample of, say, 100 babies born to women who used the Gender Choice product.

- If the product **does not work**, we would expect about half of these babies to be girls.
- If it **does work**, we would expect **significantly more** than half of the babies to be girls.

Hypothesis Testing

The key question, then, is what constitutes **"significantly more."**

- If there were 97 girls among the 100 births in the sample, we would all agree that this constituted significantly more than half and that the product is probably effective.
- If there were only 52 girls among the 100 births, we'd probably agree that 52 was so close to half that we had no reason to think the product had any effect.

Q. But would we consider, say, 64 girls in the sample of 100 babies to be "significantly more" than half, and therefore think that the product might really work?

In statistics, we answer such questions through hypothesis testing.

Hypothesis Testing (procedure)

We would use to draw a conclusion about Gender Choice based on a sample in which there are 64 girls among 100 babies born to women who used the product.

Step 1. We begin by **assuming that Gender Choice do<u>es not work</u>**;

- That is, it does not increase the percentage of girls.
- If this is true, then we should expect about 50% girls among the population of all births to women using the product.

Step 2. Use our sample (with 64 girls among 100 births) to test the above assumption.

- We conduct this test by calculating the likelihood of drawing a random sample of 100 births in which 64% (or more) of the babies are girls **from a population in which the overall proportion of girls is only 50%.**

Hypothesis Testing (procedure)

Step 3. Conclusion

- If we find that a random sample of births is **fairly likely** to have 64% (or more) girls, then we do not have evidence that Gender Choice works.

- However, if we find that a random sample of births is **unlikely** to have at least 64% girls, then we conclude that the result in the Gender Choice sample is probably due to something other than chance—meaning that the product may be effective.

Hypothesis Testing (remaining issues)

The remaining issues in test of the effectiveness of Gender Choice

1. **How to calculate the likelihood ?**

2. **How to define "fairly likely" or "unlikely" ?**

Terminology

Hypothesis

Definitions

- A **hypothesis** is a claim about a population parameter (such as a population proportion π or population mean μ) or some other characteristic of a population.
- A **hypothesis test** is a standard procedure for testing a claim about a population parameter or some characteristic of a population.

Note : There are always **two hypotheses** in any statistical hypothesis test

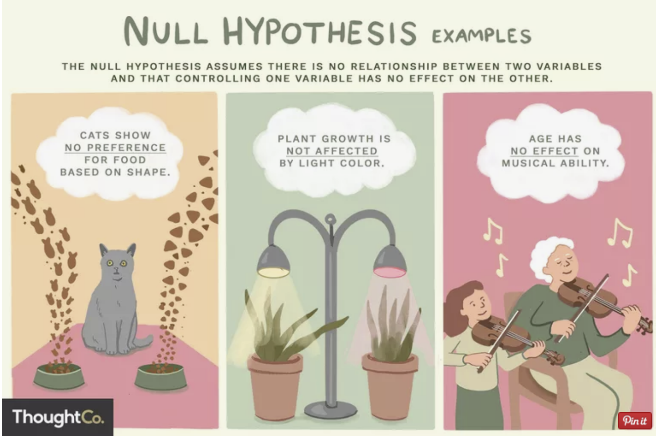
Null Hypothesis

The null hypothesis : H_0

- The null hypothesis is a general statement or default position that there is nothing new happening
- **Starting assumption** for a hypothesis test
- Assumes that there is **no meaningful relationship between two variables**
- The null hypothesis always claims a specific value for a population parameter and therefore takes the form of an **equality (=)**:

$$H_0 : \text{Population parameter} = \text{Claimed value}$$

Null Hypothesis



Null Hypothesis

To write a null hypothesis, first start by asking a question.

- Q. Are teens better at math than adults?
- H_0. Age has no effect on mathematical ability.

- Q. Does taking aspirin every day reduce the chance of having a heart attack?
- H_0. Taking aspirin daily does not affect heart attack risk.

Alternative Hypothesis

The alternative hypothesis : H_a or H_1

- The alternative hypothesis is a position that states **something is happening, a new theory is true** instead of an old one (null hypothesis)
- A claim that the population parameter has a value that **differs from the value claimed in the null hypothesis**
- It may take one of the following forms:

(Left-tailed)	H_a : Population parameter $<$ Claimed value
(Right-tailed)	H_a : Population parameter $>$ Claimed value
(Two-tailed)	H_a : Population parameter $\neq$ Claimed value

Null & Alternative hypothesis (Gender choice example)

- **The null hypothsis :**
  - The null hypothesis is the claim that Gender Choice **does not work**
  - in which case the population proportion of girls born to women who use the product should be 50%, or 0.50.

$$H_0 : p = 0.5$$

- **The alternative hypothsis :**
  - The alternative hypothesis is the claim that Gender Choice **does work**
  - in which case the population proportion of girls born to women who use the product should be greater than 0.50.

$$H_0 : p > 0.5$$

Identifying Hypotheses

1. Nissan claims that the Leaf, an electric car, has a mean range of 110 miles between charges. A consumer group claims that the mean range is less than 110 miles.

2. The Ohio Department of Health claims that the average stay in Ohio hospitals after childbirth is greater than the national mean of 2.0 days.

3. wildlife biologist working in the African savanna claims that the actual proportion of female zebras in the region is different from the accepted proportion of 50%.

Possible outcomes of a Hypothesis Test

A hypothesis test always begins with the assumption that the null hypothesis is true.

Two Possible outcomes of a Hypothesis Test

There are two possible outcomes to a hypothesis test:

1. **Reject the null hypothesis (H_0)**, in which case we have evidence in support of the alternative hypothesis.

2. **Not reject the null hypothesis (H_0)**, in which case we do not have enough evidence to support the alternative hypothesis.

Possible outcomes of a Hypothesis Test

Note : "Accepting the null hypothesis" is **Not** a possible outcome

- Because the null hypothesis is always the starting assumption.
- The hypothesis test may not give us reason to reject this starting assumption.
- **But it cannot by itself give us reason to conclude that the starting assumption is true.**

Not reject the null hypothesis $\neq$ **Accepting the null hypothesis**

Hypothesis Test Outcomes (Example)

1. Nissan claims that the Leaf, an electric car, has a mean range of 110 miles between charges. A consumer group claims that the mean range is less than 110 miles.

$$H_0 : \mu = 110$$

$$H_a : \mu < 110$$

   - **Reject the null hypothesis** of $\mu = 110$ miles, in which case we have evidence in support of the consumer group's claim that the range is less than advertised
   - **Do not reject the null hypothesis**, in which case we lack evidence to support the consumer group's claim. Note, however, that this option does not imply that the advertised claim is true.

Hypothesis Test Outcomes (Example)

2. The Ohio Department of Health claims that the average stay in Ohio hospitals after childbirth is greater than the national mean of 2.0 days.

$$H_0 : \mu = 2.0$$
$$H_a : \mu > 2.0$$

- **Reject the null hypothesis** of $\mu = 2.0$ days, in which case we have evidence in support of the Health Department's claim that the mean stay in Ohio is greater than the national average.
- **Do not reject the null hypothesis**, in which case we lack evidence to support the Health Department's claim. Note, however, that this option does not imply that the Ohio average stay is actually equal to the national average stay of 2.0 days.

Hypothesis Test Outcomes (Example)

3. Wildlife biologist working in the African savanna claims that the actual proportion of female zebras in the region is different from the accepted proportion of 50%.

$$H_0 : \pi = 0.5$$
$$H_a : \pi \neq 0.5$$

- **Reject the null hypothesis** of $\pi = 0.5$, in which case we have evidence in support of the biologist's claim that the accepted value is wrong.
- **Do not reject the null hypothesis**, in which case we lack evidence to support the biologist's claim. <u>Note, however, that this does not imply that the accepted value is correct.</u>

Drawing Conclusion

Q. How do we decide whether the result from the sample should lead us to reject or not reject the null hypothesis?

A. Deciding whether the sample result was **likely** or **unlikely** to have occurred by chance if the null hypothesis is true.

1. **Fairly Likely** to occur : Not reject H_0
2. **Unlikely** to occur : Reject H_0

Note : Likelihood, Fairly Likely, Unlikely can be determined by

- Sifnificance level α
- Test Statistic

Hypothesis Testing for Population Means

Statistical Hypothesis for μ

- Two-tailed hypothesis test :

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

- Left-tailed hypothesis test :

$$H_0 : \mu = \mu_0$$
$$H_a : \mu < \mu_0$$

- Right-tailed hypothesis test :

$$H_0 : \mu = \mu_0$$
$$H_a : \mu > \mu_0$$

Two types of approach for testing Population mean

1. Critical Value Approach (Rejection Region Approach)
2. P-value Approach

1. Critical Value Approach (Rejection Region Approach)

Critical Value Approach to Hypothesis Testing

Steps:

1. State the hypotheses
2. Decide significance level α
3. Calculate test statistic
4. Determine the critical value, and rejection region
5. If test statistic falls in rejection region, reject H_0.
   Otherwise, do not reject H_0
6. Interpret the result in the context of the situation

Significance levels, Test Statistic

Significance level (α) :

- They are the same ones we used for confidence intervals
- common values : 0.1, 0.05, 0.01

Test Statistic under H_0 (when σ known):

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Note : We will reject H_0 if our test statistic is **too "extreme."**

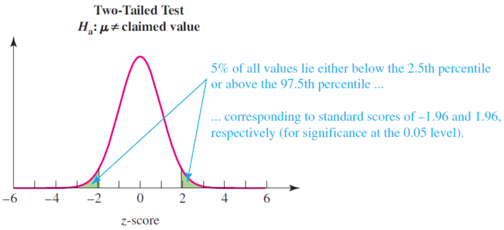
- "Too extreme" means the test statistic is located in the **rejection region**

Rejection Regions

1. Two-Tailed test:

- Here the null hypothesis will be rejected if the test statistic is too far from 0.
- This happens when the sample mean is too far from μ_0
- We will have a rejection region on the left and the right.

$$\textbf{Rejection Region} > z_{\alpha/2} \text{ or } \textbf{Rejection Region} < -z_{\alpha/2}$$



Rejection Regions

2. **Left-Tailed test:**

- Here the null hypothesis will be rejected if the test statistic is too much less than 0.
- This happens when our sample mean is much smaller than μ_0
- We will have a rejection region only on the left

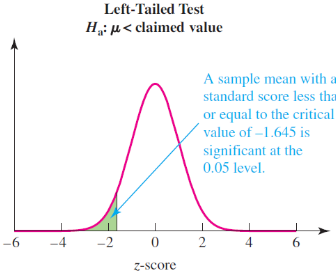
$$\textbf{Rejection Region} < -z_\alpha$$

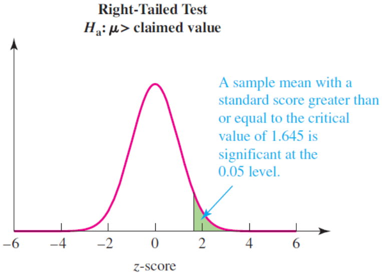
3. **Right-Tailed test:**

- Here the null hypothesis will be rejected if the test statistic is too much larger than 0.
- This means that the sample mean is much larger than μ_0.
- We will have a rejection region only on the right.

$$\textbf{Rejection Region} > z_\alpha$$

Rejection Regions (One tailed)





Example : Starting salary

Starting salary

Columbia College advertises that the mean starting salary of its graduates is \$39,000. The Committee for Truth in Advertising, an independent organization, suspects that this claim is exaggerated and decides to conduct a hypothesis test to seek evidence to support its suspicion. Having formed the hypotheses, the Committee for Truth in Advertising selects a random sample of 100 recent graduates from the college. The mean salary of the graduates in the sample turns out to be \$37,000. And $\sigma = \$6,150$ can be assumed.

Perform a statistical hypothesis test with significance level $\alpha = 0.05$

Example : Salary

1. State the Hypothesis

2. Calculate the test Statistic

3. Determine the critical value, and rejection region

4. Conclusion

Example (Solution)

1. **State the Hypothesis**

$$H_0 : \mu = \$39,000$$

$$H_a : \mu < \$39,000$$

2. **Calculate the test Statistic**

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{37,000 - 39,000}{6,150/\sqrt{100}} = -3.25$$

3. **Determine the critical value, and rejection region**

$$\text{Rejection Region} < -z_{0.05} = -1.645$$

4. **Conclusion :** strong reason to reject the null hypothesis and conclude that Columbia College officials did indeed exaggerate the mean starting salary of its graduates.

Example : Rental car

Rental car

In the United States, the average car is driven about 12,000 miles each year. The owner of a large rental car company suspects that for his fleet, the mean distance is greater than 12,000 miles each year. He selects a random sample of n $=$ 225 cars from his fleet and finds that the mean annual mileage for this sample is $\bar{X} = 12,375$ miles. population standard deviation $\sigma = 2,415$ can be assumed.

Based on these data, describe the process of conducting a hypothesis test (with $\alpha = 0.05$) and drawing a conclusion.

Example : Rental car

1. State the Hypothesis

2. Calculate the test Statistic

3. Determine the critical value, and rejection region

4. Conclusion

Example (Solution)

1. **State the Hypothesis**

$$H_0 : \mu = 12,000 \text{ miles}$$

$$H_a : \mu > 12,000 \text{ miles}$$

2. **Calculate the test Statistic**

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{12,375 - 12,000}{2,415/\sqrt{225}} = 2.33$$

3. **Determine the critical value, and rejection region**

$$\text{Rejection Region} > z_{0.05} = 1.645$$

4. **Conclusion :** We therefore have strong reason to reject the null hypothesis and conclude that the rental car fleet mean really is greater than the national mean

Body Temperature

Consider the study in which University of Maryland researchers measured body temperatures in a sample of n = 106 healthy adults, finding a sample mean body temperature of $\bar{X} = 98.20F^{\circ}$, and assume that the population standard deviation is $\sigma = 0.62F^{\circ}$.

Determine whether this sample provides enough evidence for rejecting the common belief that mean human body temperature is $\mu = 98.6F^{\circ}$. Use significance level $\alpha = 0.05$

Example : Body Temperature

1. State the Hypothesis
2. Calculate the test Statistic
3. Determine the critical value, and rejection region
4. Conclusion

Example (Solution)

1. **State the Hypothesis**

$$H_0 : \mu = 98.6F^\circ \text{ vs } H_a : \mu \neq 98.6F^\circ$$

2. **Calculate the test Statistic**

$$z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{98.20 - 98.6}{0.62/\sqrt{106}} = -6.64$$

3. **Determine the critical value, and rejection region**

$$\text{Rejection Region} < -z_{0.05/2} = -1.96 \text{ or}$$

$$\text{Rejection Region} > z_{0.05/2} = 1.96$$

4. **Conclusion :** strong reason to reject the null hypothesis and conclude that conclude that human mean body temperature is not equal to $98.6F^\circ$.