STA 1013 : Statistics through Examples

Lecture 25: Review for Quiz 3

Hwiyoung Lee

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Department of Statistics, Florida State University

Quiz 3

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Topics :

1. Standard Normal dist
2. Central Limit Theorem
3. Probability
4. Binomial dist

Notes :

- One piece of cheet sheet
- Calculator

1. Standard Normal distribution

Standard Normal distribution

Normal distribution with $\mu = 0, \sigma = 1$

- $Z \sim N(0, 1)$
- Z transformation (or Z score) :

$$X \sim N(\mu, \sigma) \;\rightarrow\; Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Example : $X \sim N(\mu = 5, \sigma = 4)$, and x is an obs of X

1. z score of $x = 3$

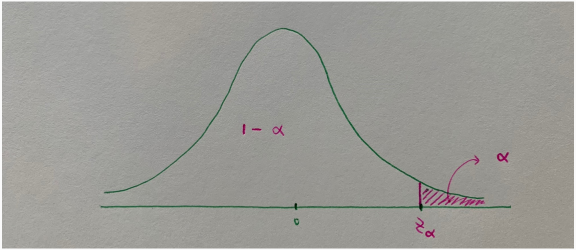
$$z = \frac{3 - 5}{4} = -0.5$$

2. z score of $x = 9$

$$z = \frac{9 - 5}{4} = 1$$

Critical value

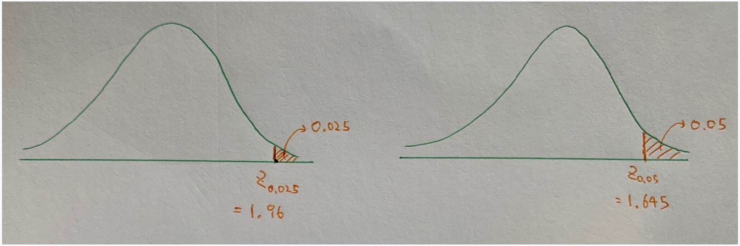
Critical value z_α : the z score with an area of α to its right.



- For the standard normal distribution, a critical value is a z score separating unlikely values from those that are likely to occur.
- α : small number (probability)

Some important critical values

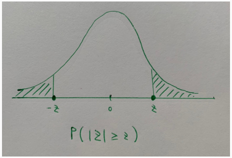
- $z_{0.025} = 1.96$ (invnorm(0.975.0,1))
- $z_{0.05} = 1.645$ (invnorm(0.95,0,1))



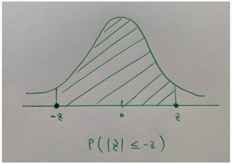
Probability absoulte value inequalities

$Z \sim N(0,1)$, and Z is symmetric about 0

1. $P(|Z| \geqslant z) = P(Z \leqslant -z \text{ or } Z \geqslant z) = P(Z \leqslant -z) + P(Z \geqslant z)$



2. $P(|Z| \leqslant z) = P(-z \leqslant Z \leqslant z)$



Note : $P(|Z| \geqslant z) = 1 - P(|Z| \leqslant z)$

Conversely, $P(|Z| \leqslant z) = 1 - P(|Z| \geqslant z)$

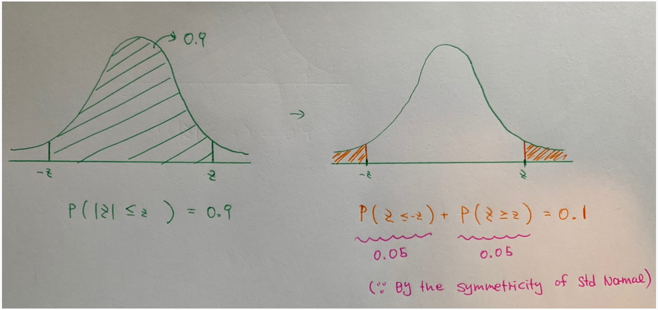
Solution for Practice Problem 2 (a)

$Z \sim N(0,1)$, Find $P(|Z| \leqslant z) = 0.90$

Step1. $P(-z \leqslant Z \leqslant z) = 0.9$

Step2. $P(Z \leqslant -z) + P(Z \geqslant z) = 0.1$

Step3. $P(Z \leqslant -z) = P(Z \geqslant z) = 0.05$ ($\because$ Symmetric about 0)



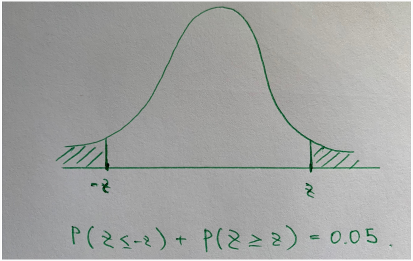
Find $z_{0.05} \rightarrow 1.645$ (invnorm(0.95,0,1))

Solution for Practice Problem 2 (b)

$Z \sim N(0,1)$, Find $P(|Z| \geqslant z) = 0.05$

Step1. $P(Z \leqslant -z) + P(Z \geqslant z) = 0.05$

Step2. $P(Z \leqslant -z) = P(Z \geqslant z) = 0.025$



Find $z_{0.025} \rightarrow 1.96$ (invnorm(0.975.0,1))

2. Central Limit Theorem

Central Limit Theorem

Central Limit Theorem (CLT)

Suppose the population distribution (**Not necessarily a normal distribution**) has population mean μ, and population standard deviation σ , then the distribution of sample means (Sampling distribution of $\bar{X}$) converges to $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$

- The distribution of sample means will be approximately a normal distribution for large sample sizes ($n \geqslant 30$).
- As $n \uparrow \rightarrow \frac{\sigma}{\sqrt{n}} \downarrow$

Example (Sample Size)

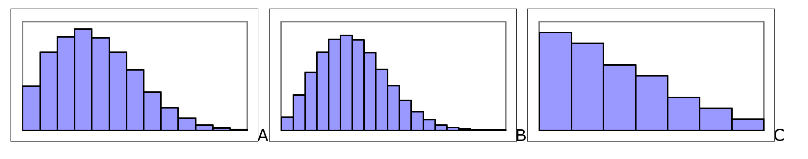
Two sampling distributions of $\bar{X}$ are described below. Which sampling distribution will have the lower standard deviation?

1. SRSs of 4 subjects each are selected and the mean is calculated for each sample.

2. SRSs of 16 subjects each are selected and the mean is calculated for each sample.

Example (Sample Size)

Match the sampling distributions of $\bar{X}$ shown below to these sample sizes: n=1, n=5, n=10.



When n=1, the sampling distribution of $\bar{X}$ is the same as the distribution of the individual values X, so it is the same as the original population distribution.

Binomial distribution and CLT

- X follows binomial distribution with the number of trials is 100, success probability $p = 0.6$. Suppose we draw random samples with size 36. Then what is the sampling distribution of $\bar{X}$?

- X follows binomial distribution with the number of trials is $1,000$, success probability $p = 0.5$. Suppose we draw random samples with size 36. Then what is the sampling distribution of $\bar{X}$?

3. Probability

Theoretical Methods for Equally Likely Outcomes

Step 1. Count the total number of possible outcomes

Step 2. Among all the possible outcomes, count the number of ways the event of interest, E, can occur

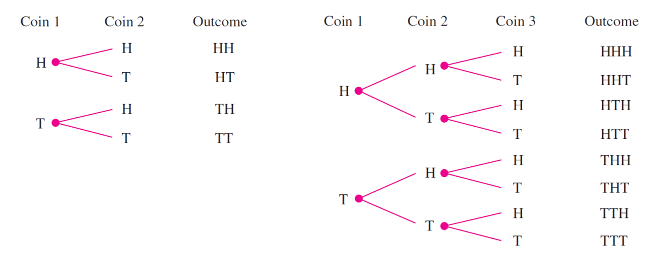
Step 3. Determine the probability, $P(E)$, from

$$\boxed{P(E) = \frac{\text{Number of ways E can occur}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)}}$$

Counting outcomes

- Independent : The simple multiplication
- Dependent : No general rule

Counting outcomes : Independent case



- Tree diagrams show the outcomes of tossing 2 and 3 coins
- Tossing 2 coins : 2×2
- Tossing 3 coins : $2 \times 2 \times 2$

$$\vdots$$

- Tossing k coins : 2^k

Counting outcomes : Dependent case

A computer program requires the user to enter a 7-digit registration code made up of the digits 1, 2, 4, 5, 6, 7, and 9. Each number has to be used, and **no number can be used more than once.**

- How many different registration codes are possible ?
- If the first three digits of the code have to be even numbers, how many different registration codes are possible ?
- If the first two digits of the code have to be odd numbers, how many different registration codes are possible ?

Counting outcomes : Dependent case

Each digit can be chosen from 0 to 9. How many 3-digit numbers can be formed with ?

1. Repetitions are allowed.
2. Repetiotion is Not allowed.
3. Last digit must be odd (repetitions are allowed).
4. No repeats and last digit must be odd.
5. The first digit is not 0 (repetiotions are allowed).
6. The first digit is not 0 (repetition is not allowed)

Probability Rules

1. Complement Event

$$P(E) = 1 - P(E^c) \ \text{ or } \ P(E^c) = 1 - P(E)$$

2. Joint Probability

$$P(A \cap B) = \frac{\text{No. of outcomes from A and B}}{\text{Total No. of possible outcomes}}$$

3. Compound Probability

$$\begin{aligned} P(A \cup B) &= \frac{\text{No. of outcomes from A onlt or B only or Both}}{\text{Total No. of possible outcomes}} \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

Probability Rules

4. Mutually Exclusive Event

$$P(A \cap B) = 0$$

Note : For mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

5. Independt Event

$$P(A \cap B) = P(A) \cdot P(B)$$

Conditional Probability

- Conditional probability is calculating the probability of an event given that another event has already occured.
- Probability of an event when we know a prior information
- Conditional probability formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example : Conditional Probability

Type	Red	Black	Total
Ace	2	2	4
Non-Ace	24	24	48
Total	26	26	52

Table 1: Contingency table; Color and Ace

- $P(\text{Ace}) = \frac{4}{52}$, $P(\text{Red}) = \frac{26}{52}$
- $P(\text{Ace} \mid \text{Red}) = \frac{2}{26}$, $P(\text{Red} \mid \text{Ace}) = \frac{2}{4}$

Nore : When we calculate the conditional probability, the denominator will be the **total for the row or column in the table that corresponds to the condition**

Example : Conditional Probability

- Mr. and Mrs. Jones have two children. What is the conditional probability that their children are both boys, given that they have at least one son?

- Consider a family that has two children. We are interested in the children's genders. Our sample space is S=(G,G),(G,B),(B,G),(B,B). Also assume that all four possible outcomes are equally likely. What is the probability that both children are girls given that the first child is a girl?

Expectation and Variance

1. Expectation $E(X)$

$$E(X) = \sum_x x \cdot P(X = x)$$

2. Variance $\operatorname{Var}(X)$

$$\begin{aligned}\operatorname{Var}(X) &= E(X^2) - \{E(X)\}^2 \\ &= \sum_x x^2 \cdot P(X = x) - \left(\sum_x x \cdot P(X = x)\right)^2\end{aligned}$$

4. Binomial distribution

Expectation, Variance, and probability

X follows binomial distribution with the number of trials is n, and success probability p.

1. $E(X) = np$
2. $\text{Var}(X) = np(1-p)$
3. $P(X = x|n, p) =$ binompdf(n,p,x)
4. $P(X \leqslant x|n, p =$ binomcdf(n,p,x)

Example Binomial distribution

An air conditioner made by a certain company is made of 50 distinct parts. Each part has a .004 probability of being defective.

1. What is the probability that a randomly selected air conditioner will not work perfectly? [**Note that it will not work perfectly if even one of these parts is defective**.]

2. What is the probability that a randomly selected air conditioner will have less than 10 defective parts ?

3. What is the probability that a randomly selected air conditioner will have at least 10 defective parts ?

Example Binomial distribution

The coins used at a casino are biased; the probability a coin lands on heads is 0.4. You play a game where **you must flip a coin ten times.** You win \$2 every time the coin lands on heads and you win nothing when the coin lands on tails.

1. What is the probability that you win \$10 ?
2. What is the probability that you win less than \$16 ?
3. What is the probability that you win at least \$6 ?
4. If you pay \$4 to play this game, should you play ?