STA 1013 : Statistics through Examples

Lecture 20: Basic Probability

Hwiyoung Lee

October 23, 2019

Department of Statistics, Florida State University

Probability

Terminology

- **Outcomes** : Possible results of observations or experiments
- **Event** : A collection of one or more outcomes that share a property of interest
- **Sample space** : A collection of all possible outcomes

Examples of Outcomes and Sample space

Tossing a coin

- Head, Tail

Rolling a single die

- $1, 2, 3, 4, 5, 6$

Choosing a card from a deck of cards

- All 52 possible cards : (Ace of Hears, Two of Hearts, $\cdots$)

Examples of Events

An event can be just one outcome :

- Getting a tail when tossing a coin
- Getting a "5" when rolling a single die

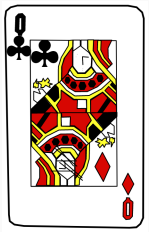
An event can include more than one outcome :

- Choosing a "King" from a deck of cards
- Getting an even number when rolling a single die

Probability

- Probability is a measure quantifying the likelihood that events will occur
- Probability quantifies as a number between 0 and 1
- 0 indicates impossibility :

  Example : club and diamond on one card draw



- 1 indicates certainty :

  Example : Probability that sun will rise tomorrow

Theoretical Methods for Equally Likely Outcomes

Step 1. Count the total number of possible outcomes

Step 2. Among all the possible outcomes, count the number of ways the event of interest, E, can occur

Step 3. Determine the probability, $P(E)$, from

$$P(E) = \frac{\text{Number of ways E can occur}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)}$$

Examples

Guessing Birthdays

You select a person at random from a large group at a conference. What is the probability that the person selected has a birthday in July? Assume 365 days in a year.

Letter

Each of the letters of the word MISSISSIPPI are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is an I?

Examples

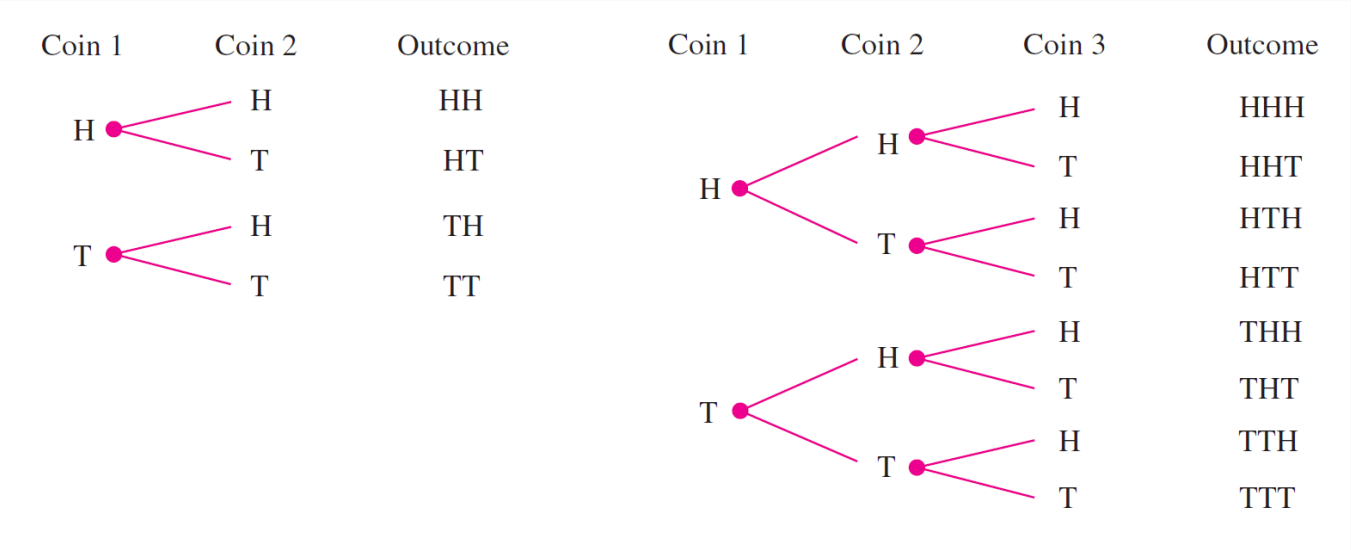
A card is chosen at random from a deck of 52 playing cards. What is the probability the card chosen is a Queen or a King?

Counting outcomes

Counting outcomes

Suppose process A has **a** possible outcomes and process B has **b** possible outcomes. Assuming the outcomes of the processes do not affect each other, the number of different outcomes for the two processes combined is $\mathbf{a} \times \mathbf{b}$. This idea extends to any number of processes. For example, if a third process C has **c** possible outcomes, the number of all possible outcomes for the three processes combined is $\mathbf{a} \times \mathbf{b} \times \mathbf{c}$.

Counting outcomes (Tossing coins)



- Tree diagrams show the outcomes of tossing 2 and 3 coins
- Tossing 2 coins : 2×2
- Tossing 3 coins : $2 \times 2 \times 2$

$$\vdots$$

- Tossing k coins : 2^k

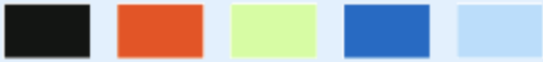
Counting outcomes (Cars)

Example: You are buying a new car.

There are **2** body styles:

sedan or hatchback

There are **5** colors available:



There are **3** models:

- GL (standard model),
- SS (sports model with bigger engine)
- SL (luxury model with leather seats)

How many total choices?

Counting outcomes (Counting Children)

What is the probability that a randomly selected family with three children has two girls and one boy? Assume that births of boys and girls are equally likely.

Solution

Step 1. There are two possible outcomes for each birth: boy (B) or girl (G). For a family with three children, the total number of possible outcomes (birth orders) is 2 * 2 * 2 = 8: BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG.

Step 2. Of these eight possible outcomes, three of them have two girls and one boy: BGG, GBG, and GGB.

Step 3. Therefore, the probability that a family with three children has two girls and one boy is

Counting outcomes (Depent case)

Independent or Dependent?

- The simple multiplication only works when all choices are independent of each other.
- If one choice affects another choice (i.e. depends on another choice), then a simple multiplication is not right.

Example : To use a locker in the Leach center, you must choose a four-digit PIN number. Each digit can be chosen from 0 to 9.

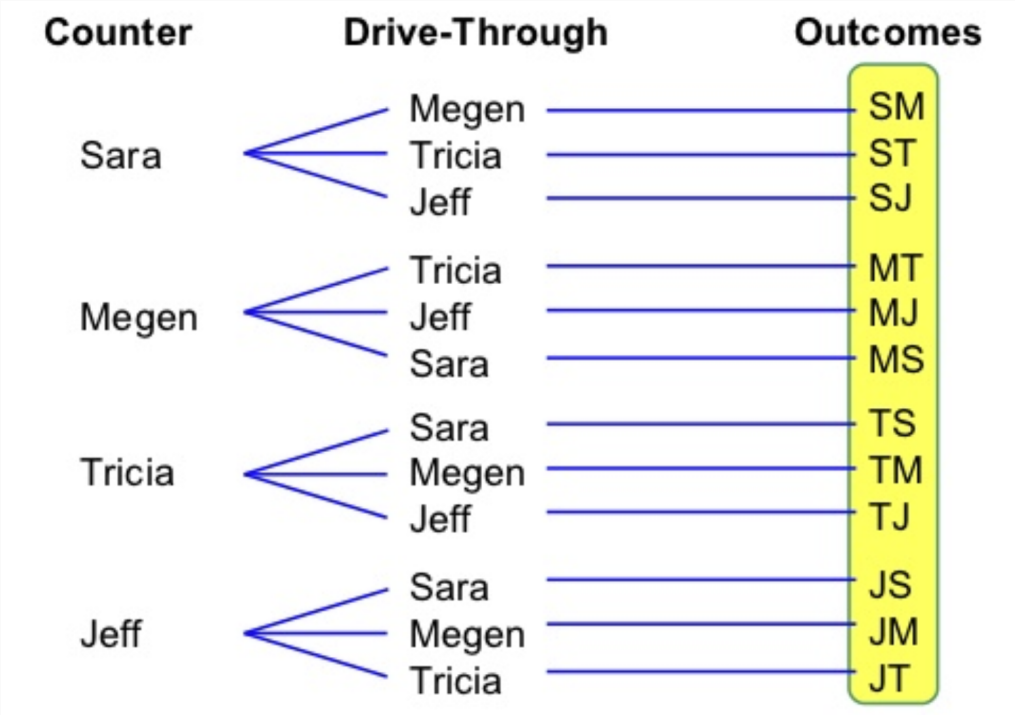
- How many possible PIN numbers can you choose ?
- If No number can be used more than once, how many possible PIN numbers can you choose ?

Example : Starbucks

Starbucks

The manager of Starbucks on W. Tennesse st. needs to hire two employees, one to work at the counter and one to work at the drive-through window. Sara, Megen, Tricia, and Jeff all applied for a job. How many possible ways are there for the manager to place the applicants ?

Solution


Counter
Drive-Through
Outcomes
Sara
Megen
Tricia
Jeff
Megen
Tricia
Jeff
Sara
Tricia
Sara
Megen
Jeff
Jeff
Sara
Megen
Tricia
SM
ST
SJ
MT
MJ
MS
TS
TM
TJ
JS
JM
JT

Examples

- A license plate begins with three letters. If the possible letters are A, B, C, D and E, how many different outcomes of these letters can be made if no letter is used more than once?

- A zip code contains 5 digits. How many different zip codes can be made with the digits 0–9 if no digit is used more than once and the first digit is not 0?

Examples

A computer program requires the user to enter a 7-digit registration code made up of the digits 1, 2, 4, 5, 6, 7, and 9. Each number has to be used, and no number can be used more than once.

- How many different registration codes are possible ?

- If the first three digits of the code have to be even numbers, how many different registration codes are possible ?

Examples

Each digit can be chosen from 0 to 9. How many 3-digit numbers can be formed with ?

1. Repetitions are allowed.
2. Repetiotion is Not allowed.
3. Last digit must be odd (repetitions are allowed).
4. No repeats and last digit must be odd.
5. The first digit is not 0 (repetiotions are allowed).
6. The first digit is not 0 (repetition is not allowed)

Probability rules

Complement of an event

Probability of an Event **Not** occurring

- Suppose we are interested in the probability that a particular event or outcome does not occur.
- For example, consider the probability of a wrong answer on a multiple-choice question with five possible answers.
- The probability of answering correctly with a random guess is $1/5$, so the probability of not answering correctly is $4/5$.
- Notice that the sum of the two probabilities must be 1, because the answer must be either right or wrong.

Complement of an event

A complement of an event E can be stated as that which does **NOT** contain the occurrence of E. (Not occuring E)

- $P(E)$: denote the probability of the event E will occur
- $P(E^c)$: denote the probability of the event E wil not occur
- Relation between $P(E)$, and $P(E^c)$

$$P(E^c) = 1 - P(E)$$

Examples

Week Days.

What is the probability of randomly selecting a day of the week and **not** getting Monday ?

Dice.

If a fair die is rolled, what is the probability of getting number greater than 2 ?

Examples

Card

A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a card that is **not a king ?**

Coin

A single coin is tossed 5 times. What is the probability of getting **at least one head ?**

Joint and Compound Probability

Joint Probability

- $P(A \cap B) = P(A \text{ and } B)$: The probability of the occurrence of both A and B at the same time.

$$P(A \cap B) = \frac{\text{No. of outcomes from A and B}}{\text{Total No. of possible outcomes}}$$

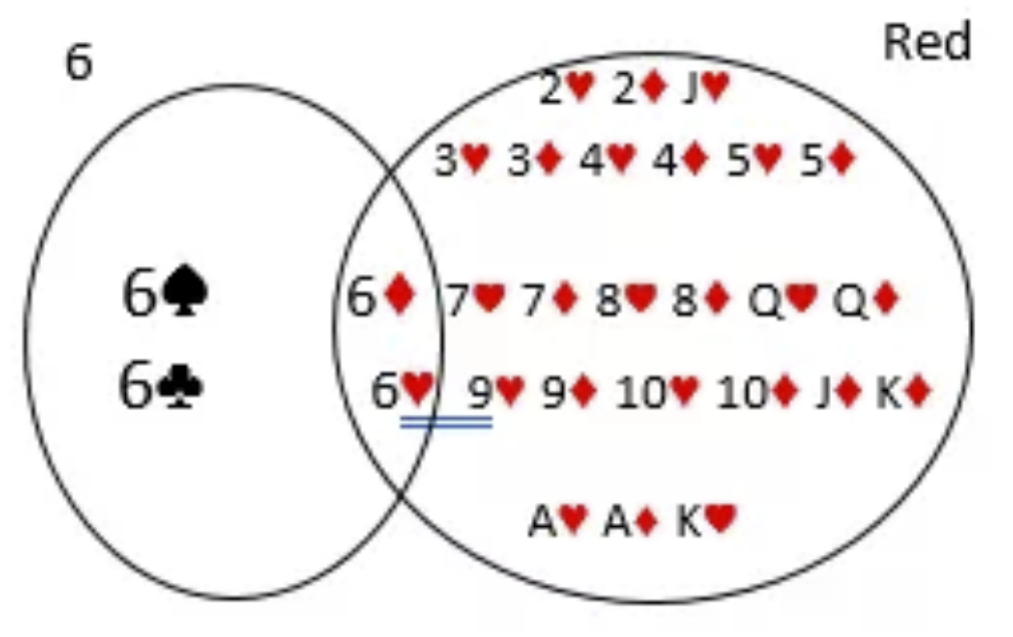
Compound Probability

- $P(A \cup B) = P(A \text{ or } B)$: The probability of the occurrence of at least one of the events.

$$P(A \cup B) = \frac{\text{No. of outcomes from A onlt or B only or Both}}{\text{Total No. of possible outcomes}}$$

Example : Joint Probability

What is the joint probability of picking up a card that is both red and 6 ?

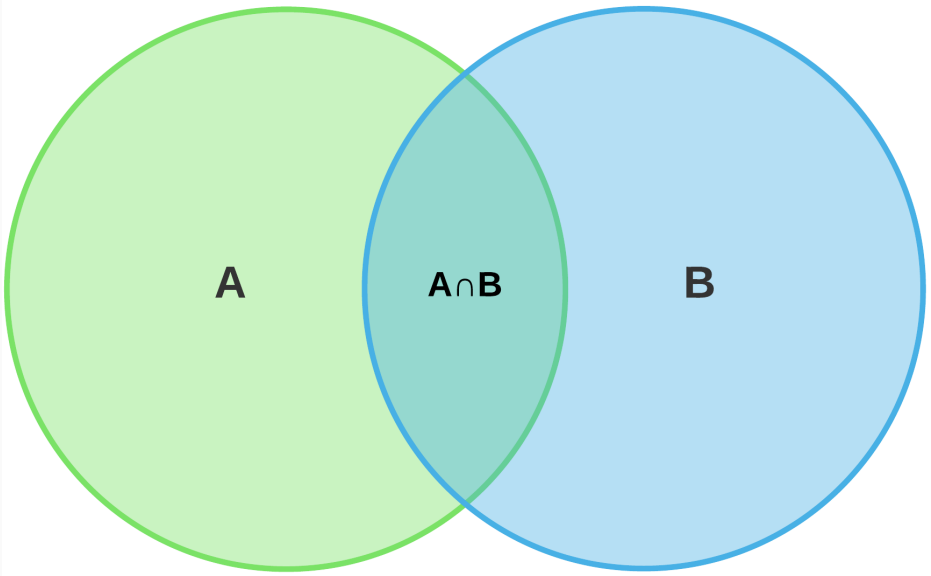


Compound Probability

Formula for compound probability :

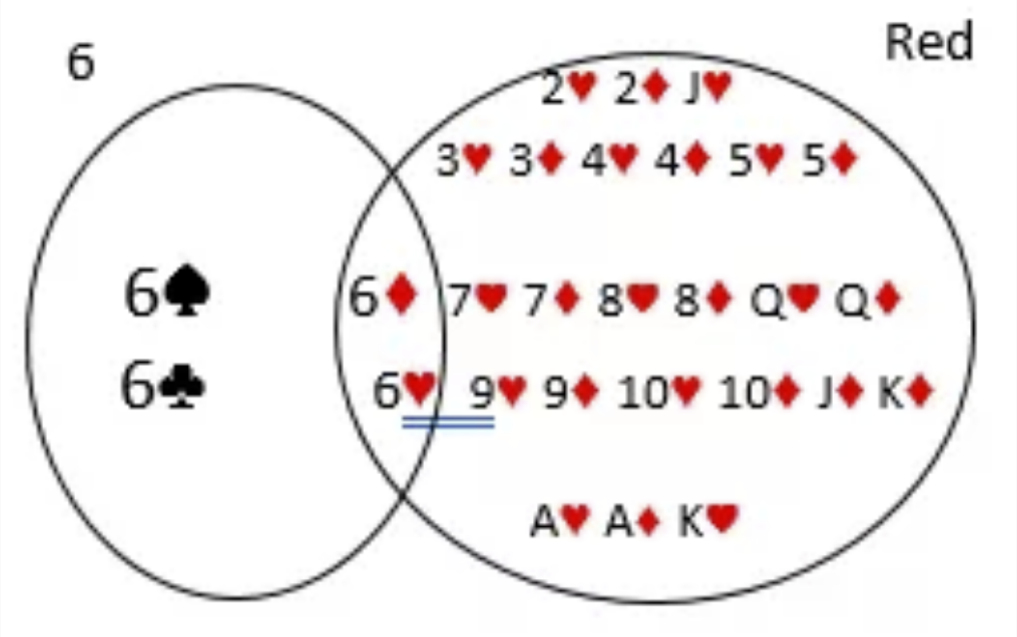
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Venn diagram

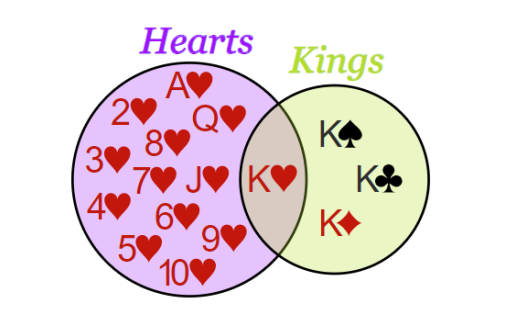


Compound Probability

Example : What is the joint probability of picking up a card that is red or 6 ?

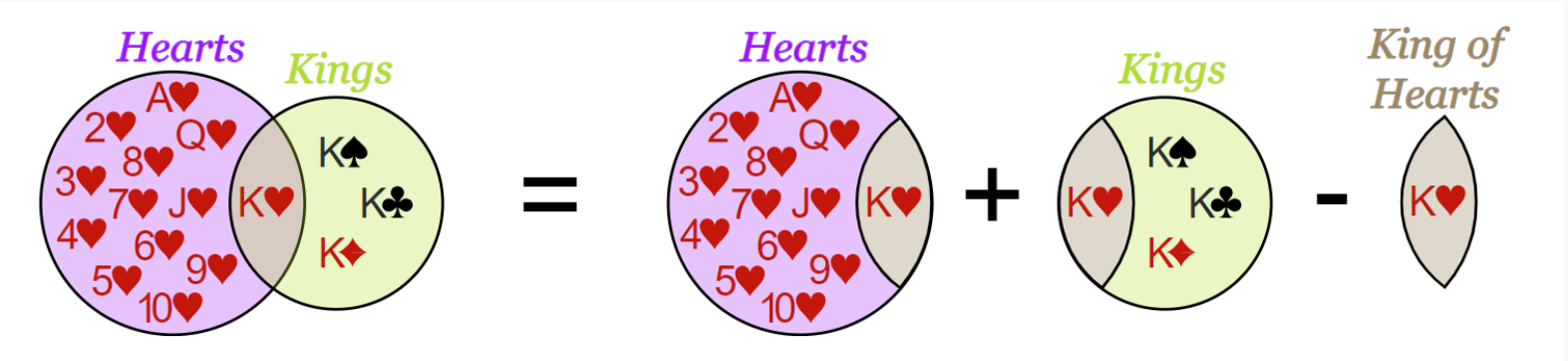


Example : Hearts, Kings



- Hearts **and** Kings together is only the King of Hearts:

- Hearts **or** Kings



Examples : Compound Probability

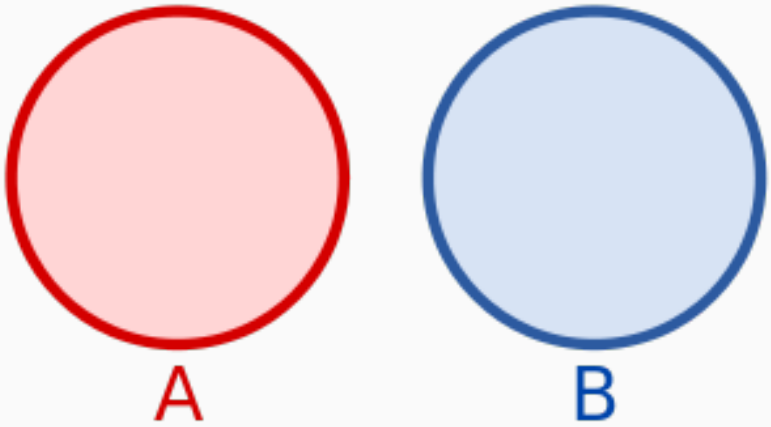
1. The probability that a student belongs to a club is 0.4. The probability that a student works part time is 0.5. The probability that a student belongs to a club AND works part time is 0.05. What is the probability that a student belongs to a club OR works part time ?

2. Let A = owns a car, B = has a pet. P(A) = 0.87, P(B) = 0.57, P(A or B) = 0.53. What is the probability that a student owns a car AND has a pet ?

Mutually exclusive events

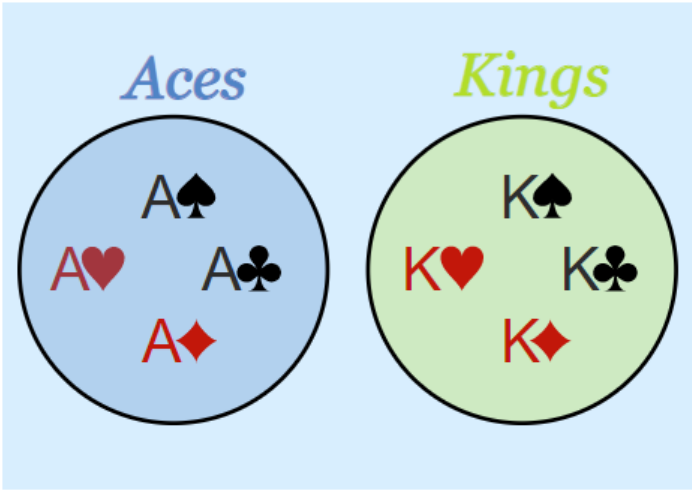
When two events cannot occur at the same time, they are considered **mutually exclusive.** (Non ovelapping)

- For a mutually exclusive event, $P(A \cap B) = P(A \text{ and } B) = 0$.

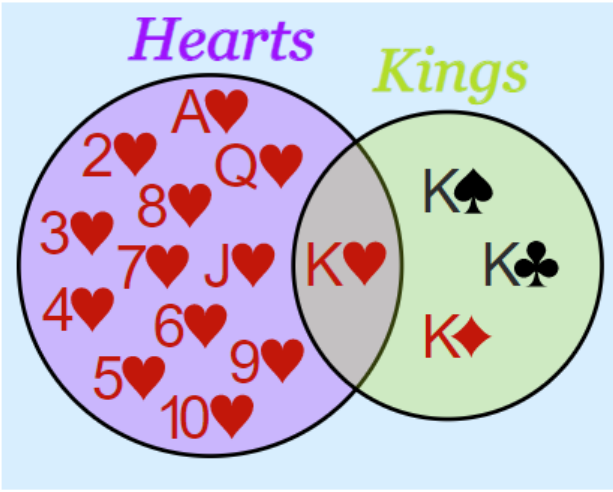


- Examples
  1. Rolling a die : A : Even, B : Odd
  2. Choosing a card : A : Red, B : Black

Example



Aces and Kings are
Mutually Exclusive
(can't be both)



Hearts and Kings are
not Mutually Exclusive
(can be both)

Compound Probability : Mutually exclusive events

Recall : General rule of addition (Compound Probability)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are **mutually exclusive**

$$P(A \cup B) = P(A) + P(B)$$