STA 1013 : Statistics through Examples

Lecture 8: Measure of Variation

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Overview

1. Measure of Variation
2. Range
3. Interquartile Range
4. Standard Deviation

Measure of Variation

Why variation matters ?

Imagine customers waiting in line for tellers at two different banks

- **Bank A** can enter any one of three different lines leading to three different tellers
- **Bank B** also has three tellers, but all customers wait in a single line and are called to the next available teller

Why variation matters ?

The following values are waiting times, in minutes, for 11 customers at each bank. The times are arranged in ascending order

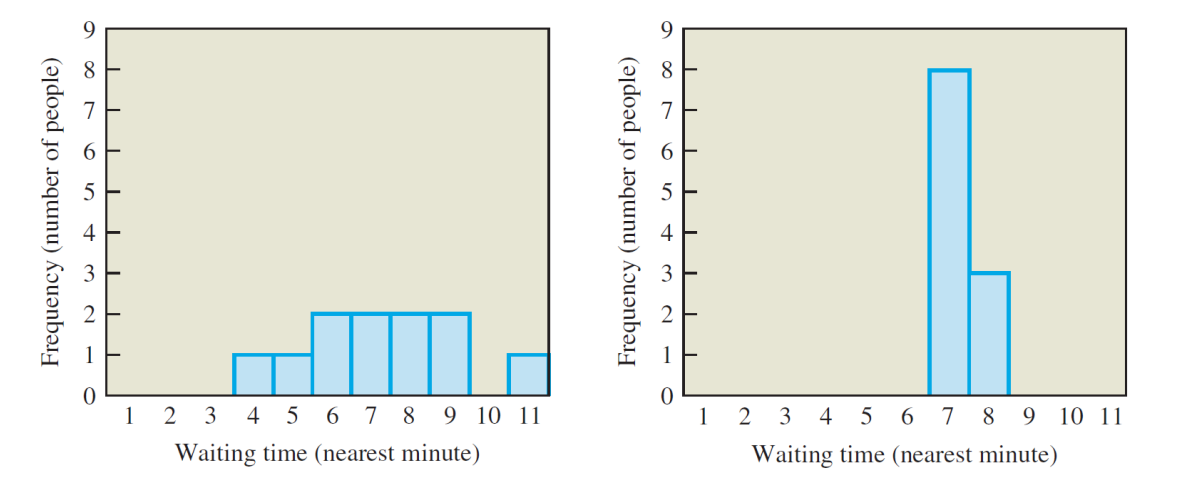
Bank A	4.1	5.2	5.6	6.2	6.7	7.2	7.7	7.7	8.5	9.3	11.0
Bank B	6.6	6.7	6.7	6.9	7.1	7.2	7.3	7.4	7.7	7.8	7.8

- Mean A : _____ , Median A : _____
- Mean B : _____ , Median B : _____

You'll probably find more unhappy customers at **Bank A** than at **Bank B**, but this is not because the average wait is any longer

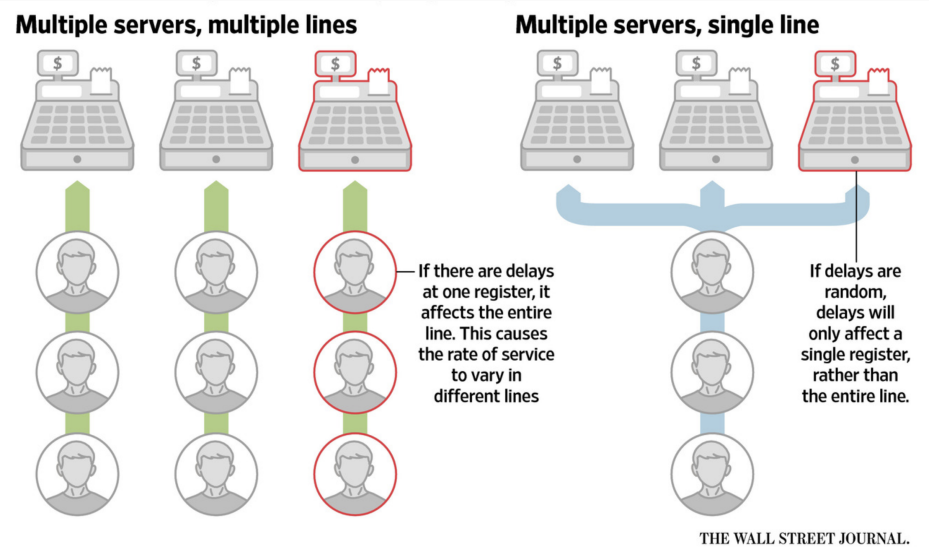
Why variation matters ?

- The difference in customer satisfaction comes from the variation at the two banks
- The waiting times at **Bank A** vary over a fairly wide range, so a few customers have long waits and are likely to become annoyed.
- In contrast, the variation of the waiting times at **Bank B** is small, so all customers feel they are being treated roughly equally.



Why variation matters ?

- Multiple lines, Multiple tellers (Bank A) → Large variation
  - Picked the fastest line → short waiting time
  - Picked the slowest line → Long waiting time
- Single line, Multiple tellers (Bank B) → Small variation
  - May not be in the slowest one, also are not in the fastest



Range

Range

Range

The range of a set of data values is the difference between its highest and lowest data values:

$$\textbf{Range} = \textbf{max} - \textbf{min}$$

Bank data

Bank A	4.1	5.2	5.6	6.2	6.7	7.2	7.7	7.7	8.5	9.3	11.0
Bank B	6.6	6.7	6.7	6.9	7.1	7.2	7.3	7.4	7.7	7.8	7.8

- Range of Bank A : _____
- Range of Bank B : _____

Range

Consider the following two sets of quiz scores for nine students.

Quiz 1	1	10	10	10	10	10	10	10	10
Quiz 2	2	3	4	5	6	7	8	9	10

Which set has the greater range?

- Range of Quiz 1 : _____
- Range of Quiz 2 : _____

Would you also say that this set has the greater variation?

Range

- Aside from a single low score (1 : outlier), Quiz 1 has no variation at all because every other student got a 10.
- In contrast, no two students got the same score on Quiz 2, and the scores are spread throughout the list of possible scores.
- Quiz 2 therefore has greater variation even though Quiz 1 has greater range.

Interquartile Range

Quartile

- A better way to describe variation is to consider a few intermediate data values in addition to the high and low values.
- A common way involves looking at the quartiles, or values that divide the data distribution into quarters.

Quartile

- The first quartile (or lower quartile) :

$$\mathbf{Q_1} = \frac{n+1}{4} \text{ th observation}$$

- The second quartile (or middle quartile) is the **median** :

$$\mathbf{Q_2} = 2 \times \frac{n+1}{4} \text{ th observation}$$

- The third quartile (or upper quartile) :

$$\mathbf{Q_3} = 3 \times \frac{n+1}{4} \text{ th observation}$$

- Interquartile Range :

$$\mathbf{IQR} = \mathbf{Q_3} - \mathbf{Q_1}$$

Interquartile Range

Quiz data

Quiz 1	1	10	10	10	10	10	10	10	10
Quiz 2	2	3	4	5	6	7	8	9	10

- IQR of Quiz 1 : _____

  Q_1 : _____ , Q_2 : _____ , Q_3 : _____

- IQR of Quiz 2 : _____

  Q_1 : _____ , Q_2 : _____ , Q_3 : _____

Problem of IQR

Obs	1	2	3	4	5	6	7	8	9	10	11
Data 1	-100	-10	-5	-3	-1	0	1	3	5	10	100
Data 2	-5.2	-5.1	-5	-3	-1	0	1	3	5	5.1	5.2

- Mean of Data 1 = Mean of Data 2
- Median of Data 1 = Median of Data 2
- IQR of the Data 1 :
- IQR of the Data 2 :

Would you say that data set 1 and set 2 have the same variation?

Problem of IQR

- This is basically the range of the central 50% of the observations in the distribution
- **Problem** : The interquartile range does not take into account the variability of the total data (only the central 50%).
- We are "throwing out" half of the data.

Another measure which takes into account the whole data is needed. $\rightarrow$ **Standard deviation, Variance**

Five-Number Summary

The five-number summary for a data distribution consists of the following five numbers:

low value, **lower quartile**, **median**, **upper quartile**, **high value**

Eample : Five-Number Summary

Bank A	4.1	5.2	5.6	6.2	6.7	7.2	7.7	7.7	8.5	9.3	11.0
Bank B	6.6	6.7	6.7	6.9	7.1	7.2	7.3	7.4	7.7	7.8	7.8

Bank A

- min
- Q_1
- Q_2
- Q_3
- max

Bank B

- min
- Q_1
- Q_2
- Q_3
- max

Exercise

Data (n=16): 1, 1, 2, 2, 2, 2, 3, 3, 4, 4, 5, 5, 6, 7, 8, 10

- $\min$:
- Q_1 :
- Q_2 :
- Q_3 :
- $\max$:
- IQR :
- Range :

Outliers

- Identifying Outliers in a Sample of Real Data
- First calculate the lower and upper fences:

$$\text{LF} : Q_1 - 1.5 \times \text{IQR}$$
$$\text{UF} : Q_3 + 1.5 \times \text{IQR}$$

- Outliers are values that lie outside of the fences
  - Low outliers are values that are less than the lower fence
  - High outliers are values that are greater than the upper fence

Example

Example(n=15)

0, 0, 2, 3, 4, 7, 9, 12, 17, 18, 20, 22, 45, 56, 98

- LF : _____
- UF : _____
- Outlier : _____

Exercise : Identifying Outliers

- **Space Shuttle Flights** : Listed below are the durations (in hours) of a sample of all flights of NASA's Space Transport System (space shuttle):

  0 73 95 165 191 192 221 235
  235 244 259 262 331 376 381

  Answer : _____

- **blood Alcohol** : Blood alcohol concentrations of drivers involved in fatal crashes:

  0.01 0.12 0.13 0.14 0.16 0.16
  0.17 0.24 0.27 0.29 0.46

  Answer : _____

Standard Deviation

Standard Deviation

- The five-number summary characterizes variation well
- Statisticians prefer to describe variation with a single number
- The single number most commonly used to describe variation is called the **standard deviation**
- The standard deviation is a measure of **how widely data values are spread around the "mean" of a data set**
- To calculate a standard deviation,
  - First find the mean
  - Calculate how much each data value "deviates" from the mean

Deviation ?

Average of Deviation :

$$\frac{\text{Sum of (Data value - mean)}}{\text{The number of observations}} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n}$$

- The above is not a good measure
- Why can't we simply compute the average deviation about the mean ?
- The summation in the numerator is always 0

$$\sum_{i=1}^{n}(x_i - \bar{x}) = \sum_{i=1}^{n} x_i - n\bar{x} = \sum_{i=1}^{n} x_i - n\frac{\sum_{i=1}^{n} x_i}{n} = 0$$

- The average deviation will not work.

Example

Obs	1	2	3	4	5	6	7	8	9	10	11
Data 1	-100	-50	-10	-5	-1	**0**	1	5	10	50	100
Data 2	-5	-4	-3	-2	-1	**0**	1	2	3	4	5

- Data 1 has larger variation than Data 2
- Data 1 : $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$
- Data 2 : $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

Average of deviation $(x_i - \bar{x})$ shouldn't be used !!!

Example

The means of the Bank A and Bank B are **7.2**

Bank A		**Bank B**	
x_i	$x_i - \bar{x}$	x_i	$x_i - \bar{x}$
4.1	-3.1	6.6	-0.6
5.2	-2	6.7	-0.5
5.6	-1.6	6.7	-0.5
6.2	-1	6.9	-0.3
6.7	-0.5	7.1	-0.1
7.2	0	7.2	0
7.7	0.5	7.3	0.1
7.7	0.5	7.4	0.2
8.5	1.3	7.7	0.5
9.3	2.1	7.8	0.6
11.0	3.8	7.8	0.6
$\sum x_i - \bar{x}$	0	$\sum x_i - \bar{x}$	0

Standard Deviation

Standard Deviation

$$s = \sqrt{\frac{\text{Sum of (Data value - mean)}^2}{\text{The number of observations-1}}} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\mathbf{n-1}}}$$

- Use Squared Deviation $(x_i - \bar{x})^2$
- By squaring the deviation, we eliminate the problem of the deviations summing to zero
- Note we divide by $\mathbf{(n-1)}$, **not** $\mathbf{n}$
- $(n-1)$ is called **the degree of freedom (df)**

Variance

Variance

$$s^2 = \frac{\text{Sum of (Data value - mean)}^2}{\text{The number of observations-1}} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\mathbf{n-1}}$$

- The variance is the standard deviation squared
- Conversely **standard deviation** $= \sqrt{\textbf{variance}}$

Example

The means of the Bank A and Bank B are **7.2**

Bank A			**Bank B**		
x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
4.1	-3.1	9.61	6.6	-0.6	0.36
5.2	-2	4	6.7	-0.5	0.25
5.6	-1.6	2.56	6.7	-0.5	0.25
6.2	-1	1	6.9	-0.3	0.09
6.7	-0.5	0.25	7.1	-0.1	0.01
7.2	0	0	7.2	0	0
7.7	0.5	0.25	7.3	0.1	0.01
7.7	0.5	0.25	7.4	0.2	0.04
8.5	1.3	1.69	7.7	0.5	0.25
9.3	2.1	4.41	7.8	0.6	0.36
11.0	3.8	14.44	7.8	0.6	0.36
SS	$\sum(x_i - \bar{x})^2$	38.46	$\sum(x_i - \bar{x})^2$		1.98
Variance (s^2)	$\frac{\sum(x_i-\bar{x})^2}{n-1}$	3.846	Variance (s^2)	$\frac{\sum(x_i-\bar{x})^2}{n-1}$	0.198
S.D (s)	$\sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$	1.961	S.D (s)	$\sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$	0.444

Exercise

body Temperatures : Body temperatures (in degrees Fahrenheit) of randomly selected normal and healthy adults:

Data	Deviation $(x_i - \bar{x})$	Deviation^2 $\left((x_i - \bar{x})^2\right)$
98.6		
98.6		
98.0		
98.0		
99.0		
98.4		
98.4		
98.4		
98.4		
98.6		

- Standard Deviation :

Note : Population version

Population Standard Deviation

$$\sigma = \sqrt{\frac{\text{Sum of (Data value - population mean)}^2}{\text{Population size}}} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{\mathbf{N}}}$$

- Divide by N
- Rarely used (We usually don't know the whole population)
  - μ :usually unknown
  - N : usually unknown
- Population variance (σ^2) :

$$\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{\mathbf{N}}$$