STA 1013 : Statistics through Examples

Lecture 21: Basic Probability 2

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October 28, 2019

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Independent

Independent Events

When multiple events occur, if the outcome of one event **Does Not** affect the outcome of the other events, they are called **independent events.**

Example : A die is rolled twice. The outcome of the first roll doesn't affect the second outcome. These two are independent events.

Independent Events

Two events are independent if the outcome of one event does not affect the probability of the other event. Consider two independent events A and B with probabilities P(A) and P(B). The probability that A and B occur together is

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

This principle can be extended to any number of independent events. For example, the probability of A, B, and a third independent event C is

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

Independent Events

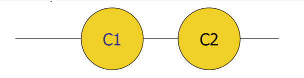
- A coin is tossed five times. What is the probability of getting five consecutive tails ?

- Suppose a pack contains 4 blue, 2 red and 3 black pens. If a pen is drawn at random from the pack, **Replaced** and the process repeated 2 more times, what is the probability of drawing 2 blue pens and 1 black pen?

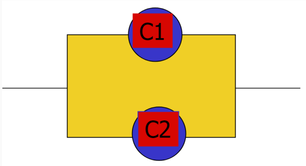
Example : Independent Events

The probability $P(C_1 \text{ fails}) = 0.1$, and $P(C_2 \text{ fails}) = 0.2$, and they are **independent**.

- What is the probability that the series system fails ?



- What is the probability that the parallel system fails ?



Dependent Events

When two events occur, if the outcome of one event affects the outcome of the other, they are called dependent events.

$$P(A \cap B) \neq P(A) \times P(B)$$

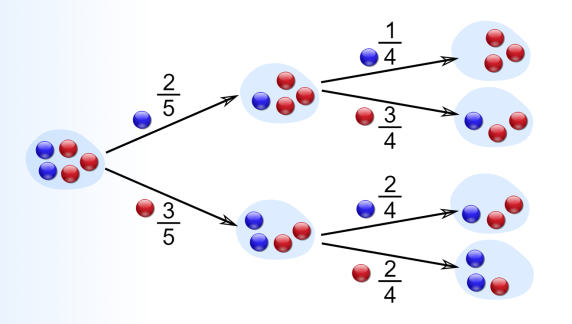
Example : The game of bingo involves drawing labeled buttons from a bin at random, **without replacement**. There are 75 buttons, 15 for each of the letters B, I, N, G, and O. What is the probability of drawing two B buttons in the first two selections?

Dependent Events

A bag contains 3 red marbles and 2 blue marbles. Two marbles are drawn at random without replacement.

- If the first marble drawn is red, what is the probability the second marble is blue?
- If the first marble drawn is red, what is the probability the second marble is red?
- If the first marble drawn is blue, what is the probability the second marble is blue?
- If the first marble drawn is blue, what is the probability the second marble is red?

Dependent Events



Dependent Events

Probability that dependent events A and B occur together is

$$P(A \cap B) = P(A) \times P(B \text{ given } A)$$

Probability of dependent events A, B, and C is

$$P(A \cap B \cap C) = P(A) \times P(B \text{ given } A) \times P(C \text{ given } A \text{ and } B)$$

Example : Dependent Events

You are playing the bingo card shown below. The caller has 50 numbers left to call. What is the probability that you will get bingo with the next 2 numbers called?

Example : Dependent Events

Need 7, and 44 for bingo with the next 2 numbers called.

- 1st call : $P(7 \text{ or } 44) = \frac{2}{50}$
- 2nd call : $P(\text{remaining number}) = \frac{1}{49}$

Thus probability that you will get bingo with the nex 2 numbers called? $\frac{2}{50} \times \frac{1}{49}$

Conditional Probability

Conditional Probability

- Conditional probability is calculating the probability of an event given that another event has already occured .
- Conditional Probability of B given A

$$P(B|A) = P(B \text{ given } A)$$

- In other words, event A has already happened, now what is the chance of event B ?

Example : Rollina two dice

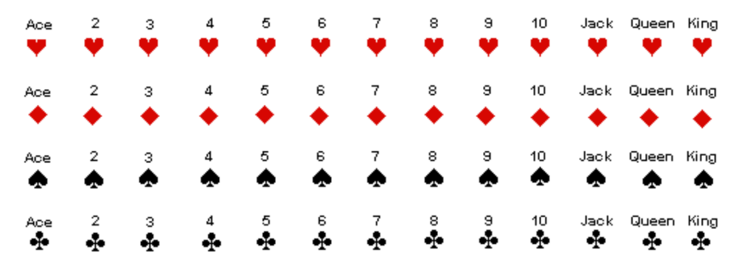
I roll a fair die twice and obtain two numbers Let

- X_1 : result of the first roll
- X_2 : result of the second roll

Given that I know $X_1 + X_2 = 7$, what is the probability that $X_1 = 4$ or $X_2 = 4$?

Example : Conditional Probability

A deck of cards



Conditional Probability using contingency table

Type	Red	Black	Total
Ace	2	2	4
Non-Ace	24	24	48
Total	26	26	52

Table 1: Contingency table; Color and Ace

- $P(\text{Ace})$
- $P(\text{Red})$
- $P(\text{Ace} \mid \text{Red}) =$ Probability of ace given that it is an red card (Note : reviesed sample space is 26)
- $P(\text{Red} \mid \text{Ace}) =$ Probability of red card given that it is an ace (Note : Revised sample space is 4)

Conditional Probability formula

- **The formula for conditional probability**

$$\mathbf{P(B|A) = \frac{P(A \cap B)}{P(A)}}$$

- $P(\text{Ace} \mid \text{Red}) = \frac{P(\text{Ace} \ \cap \ \text{Red})}{P(\text{Red})} = \frac{2/52}{26/52} = \frac{2}{26}$

- $P(\text{Red} \mid \text{Ace}) = \frac{P(\text{Red} \ \cap \ \text{Ace})}{P(\text{Ace})} = \frac{2/52}{4/52} = \frac{2}{4}$

Example

- Mr. and Mrs. Jones have two children. What is the conditional probability that their children are both boys, given that they have at least one son?

- Consider a family that has two children. We are interested in the children's genders. Our sample space is S=(G,G),(G,B),(B,G),(B,B). Also assume that all four possible outcomes are equally likely. What is the probability that both children are girls given that the first child is a girl?

General rule of the joint probability

From the conditional probability formula,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

we can generalize $P(A \cap B)$

$$\begin{aligned} P(A \cap B) &= P(A|B)P(B) \\ &= P(B|A)P(A) \end{aligned}$$

Conditoinal probability when two events are independent

Use the definition of the independence $P(A \cap B) = P(A)P(B)$

1. $P(A|B) = P(A)$

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} \\ &= P(A) \end{aligned}$$

2. $P(B|A) = P(B)$

$$\begin{aligned} P(B|A) &= \frac{P(B \cap A)}{P(A)} = \frac{P(B)P(A)}{P(A)} \\ &= P(B) \end{aligned}$$

Example : Conditional probability (Independence case)

I toss a fair coin twice and obtain head or tail Let

- X_1 : result of the first toss (H or T)
- X_2 : result of the second toss (H or T)

Given that I know $X_1 = T$, what is the probability that $X_2 = T$?

I roll a fair die twice and obtain two numbers Let

- X_1 : result of the first roll
- X_2 : result of the second roll

Given that I know $X_1 = 6$, what is the probability that $X_2 = 5$?