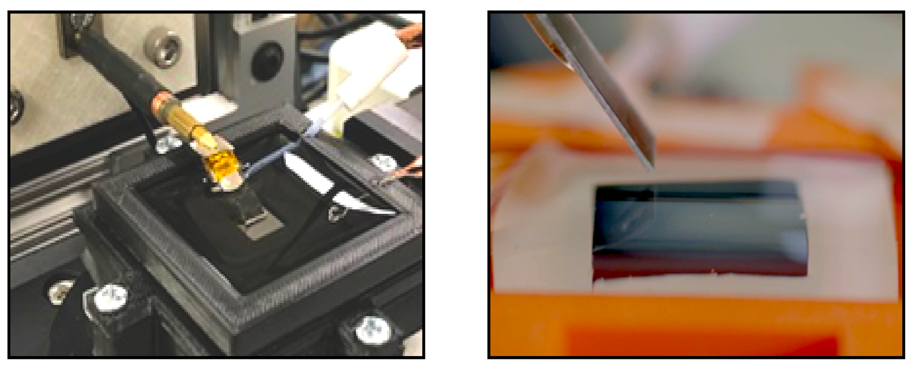
# Compressed Sensing Microscopy with Scanning Line Probes

Han-Wen (Henry) Kuo, Anna Dorfi, Daniel Esposito and John Wright

Columbia University

## Microscope with Line Probe

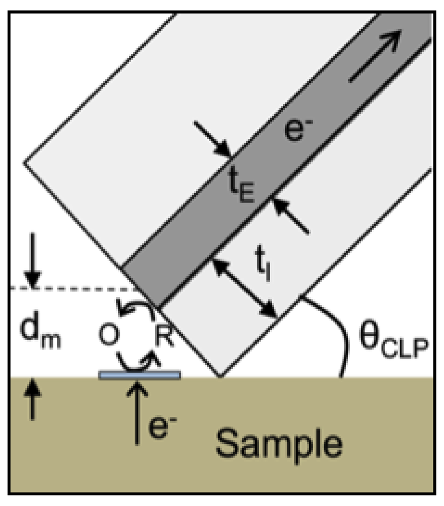
- Conventional scanning probe microscope takes *point measurements*; inefficient sampling.



**Line measurements** can improve efficiency by order of magnitude on structured signals.

## Electrochemical Line Probe

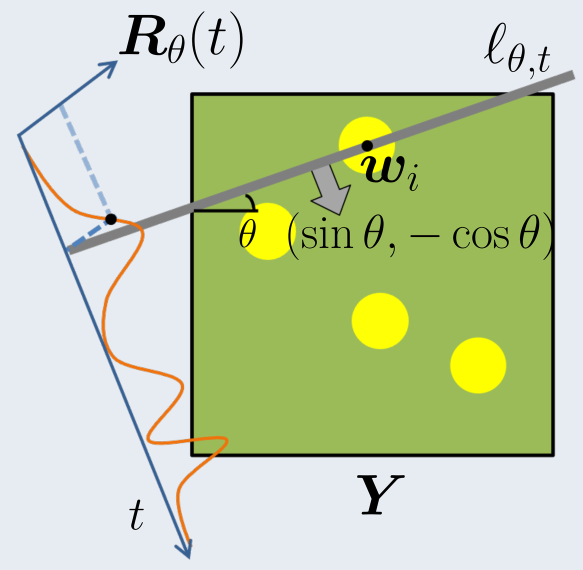
- Line probe measures redox reaction (O $\leftrightarrow$ R) electric current ($e^-$) induced by conducting layer and electroactive species on the sample.
- Insulating layers sandwich conducting layer w/one edge contacting sample at tilt angle $\theta_{\mathrm{CLP}}$. Distance of conducting layer to sample is $d_m$ [1].




## Line Scan Math Model

- *Line projection*: integrate current over line $\ell_{\theta,t}$

$$\mathcal{L}_\theta[\boldsymbol{Y}](t) := \int_{\ell_{\theta,t}} \boldsymbol{Y}(\boldsymbol{w})\, d\boldsymbol{w}$$
$$:= \int_s \boldsymbol{Y}\left(s \cdot \boldsymbol{u}_\theta + t \cdot \boldsymbol{u}_\theta^\perp\right) ds.$$

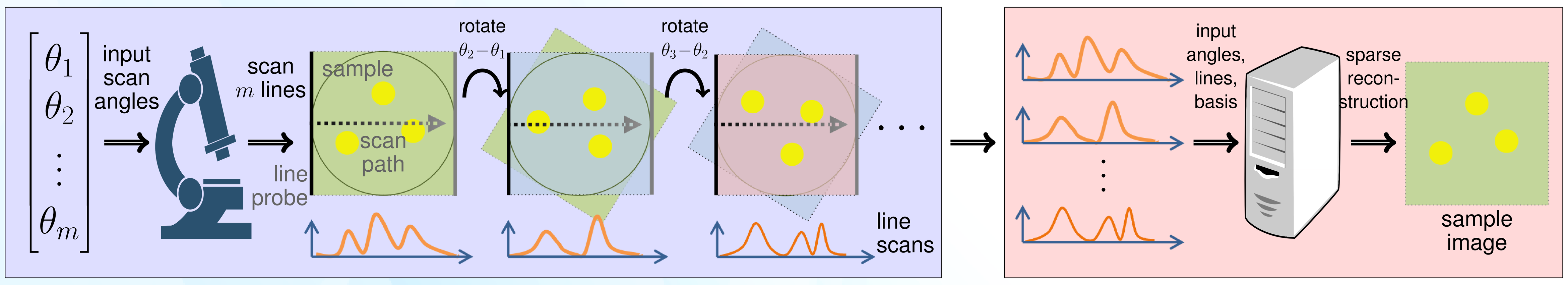



- *Line scans*: sweep line probe $m$ times at different angle with PSF $\boldsymbol{\psi}$ along scan path

$$\widetilde{\boldsymbol{R}} = \tfrac{1}{\sqrt{m}}[\boldsymbol{\psi} * \mathcal{L}_{\theta_1}[\boldsymbol{Y}], \ldots, \boldsymbol{\psi} * \mathcal{L}_{\theta_m}[\boldsymbol{Y}]]$$
$$:= \boldsymbol{\psi} * \mathcal{L}_\Theta[\boldsymbol{Y}]$$

take equispaced discrete samples $\boldsymbol{R} = \mathcal{S}[\widetilde{\boldsymbol{R}}]$.

## Schematic of Microscopic Line Scans




## Compressed Sensing with Line Scans

Line scan is **delocalized**, more efficient when sample **spatially sparse** image.

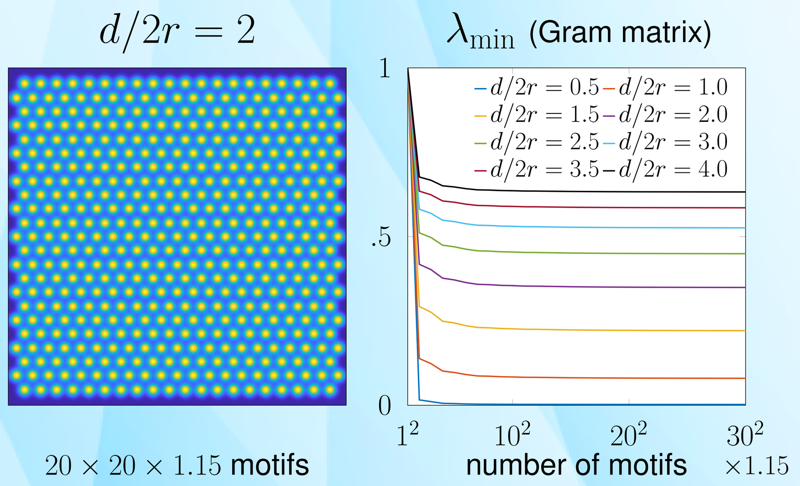
We study using the line probe to measure image consists of multiple electroactive discs with small known radius (spatially sparse), it is more efficient than point probe with raster scan.

### Case 1: Highly small and separated discs

- Let image has $k$ discs radius $r$ with centers separated by at least $\frac{2}{C}k^2 r$, then three iid uniform random line scans recover the image w.p. at least $1-C$.

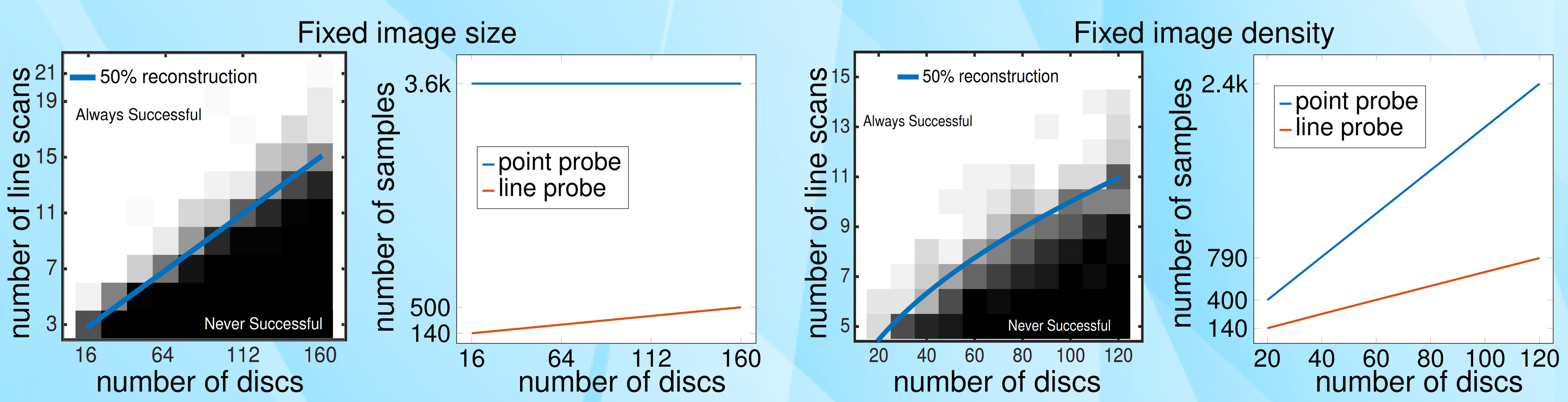
### Case 2: Stable injectivity with infinite line projections

- Infinitely many line projection is partially *coherent* with discs $\boldsymbol{D}$ (distance $d$, radius $r$):
$$\mathbb{E}_\Theta \left\langle \mathcal{L}_\Theta[\boldsymbol{D} * \boldsymbol{\delta}_{\boldsymbol{w}_i}], \mathcal{L}_\Theta[\boldsymbol{D} * \boldsymbol{\delta}_{\boldsymbol{w}_j}] \right\rangle \approx \tfrac{1}{\sqrt{1+d^2/4r^2}}.$$
not conventional ideal CS measurement.
- When the discs are **separated** ($d/2r > 1$), then $\mathbb{E}_\Theta \mathcal{L}_\Theta[\boldsymbol{D} * \cdot]$ is stably injective over the sparse support, regardless of support number.
- Infinite line projections $\mathbb{E}_\Theta \mathcal{L}_\Theta$ is *lowpass*, can recover discs with enough separation [2].




### Case 3: Sparse recovery with finite line projections

- When discs are $d > 2r$ **separated**, the number of line scans required for exact recovery is about **linear proportional** to the number of discs.




### Finding: Sample Complexity of Line Scans

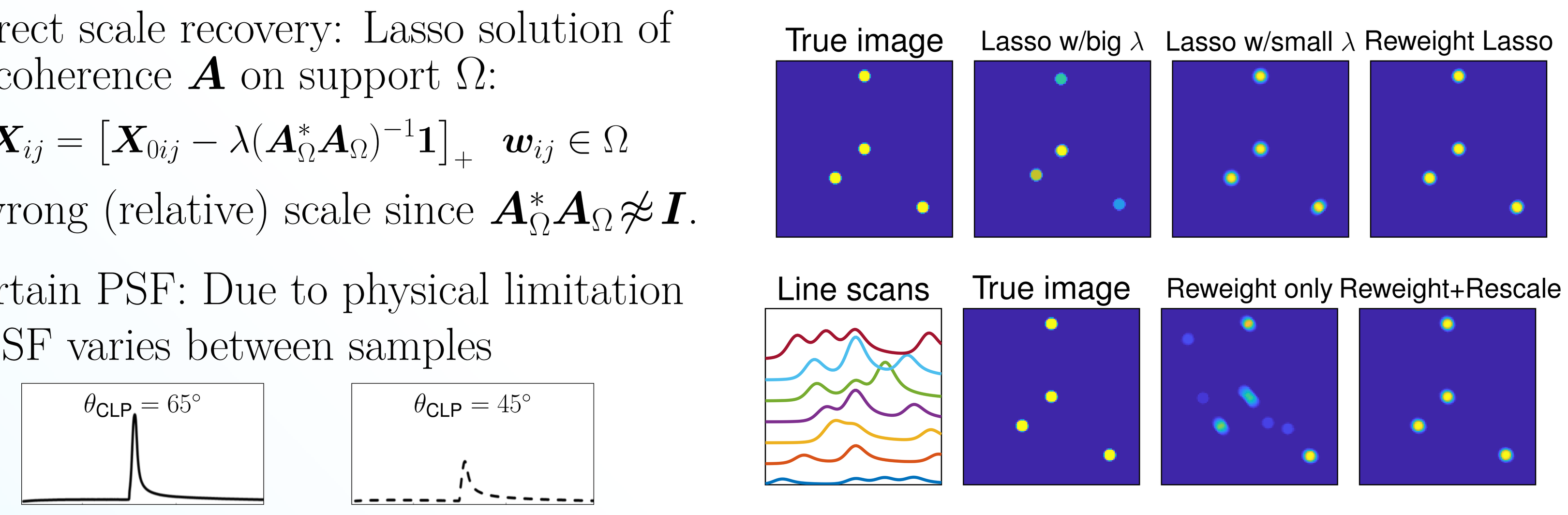
*When local features are well separated, the number of line scans required for exact recovery is about linear proportional to number of discs.*

## Image Reconstruction from a few Line Scans

Practical reconstruction from line scans poses additional difficulties: the real measurements are partially **coherent** and exhibit **nonidealities**.

### Problems of Vanilla Lasso for Reconstruction

- Incorrect scale recovery: Lasso solution of high coherence $\boldsymbol{A}$ on support $\Omega$:
$$\boldsymbol{X}_{ij} = \left[\boldsymbol{X}_{0ij} - \lambda(\boldsymbol{A}_\Omega^* \boldsymbol{A}_\Omega)^{-1} \boldsymbol{1}\right]_+ \quad \boldsymbol{w}_{ij} \in \Omega$$
has wrong (relative) scale since $\boldsymbol{A}_\Omega^* \boldsymbol{A}_\Omega \not\approx \boldsymbol{I}$.
- Uncertain PSF: Due to physical limitation the PSF varies between samples




### Algorithm: Reweighting Calibrated Lasso

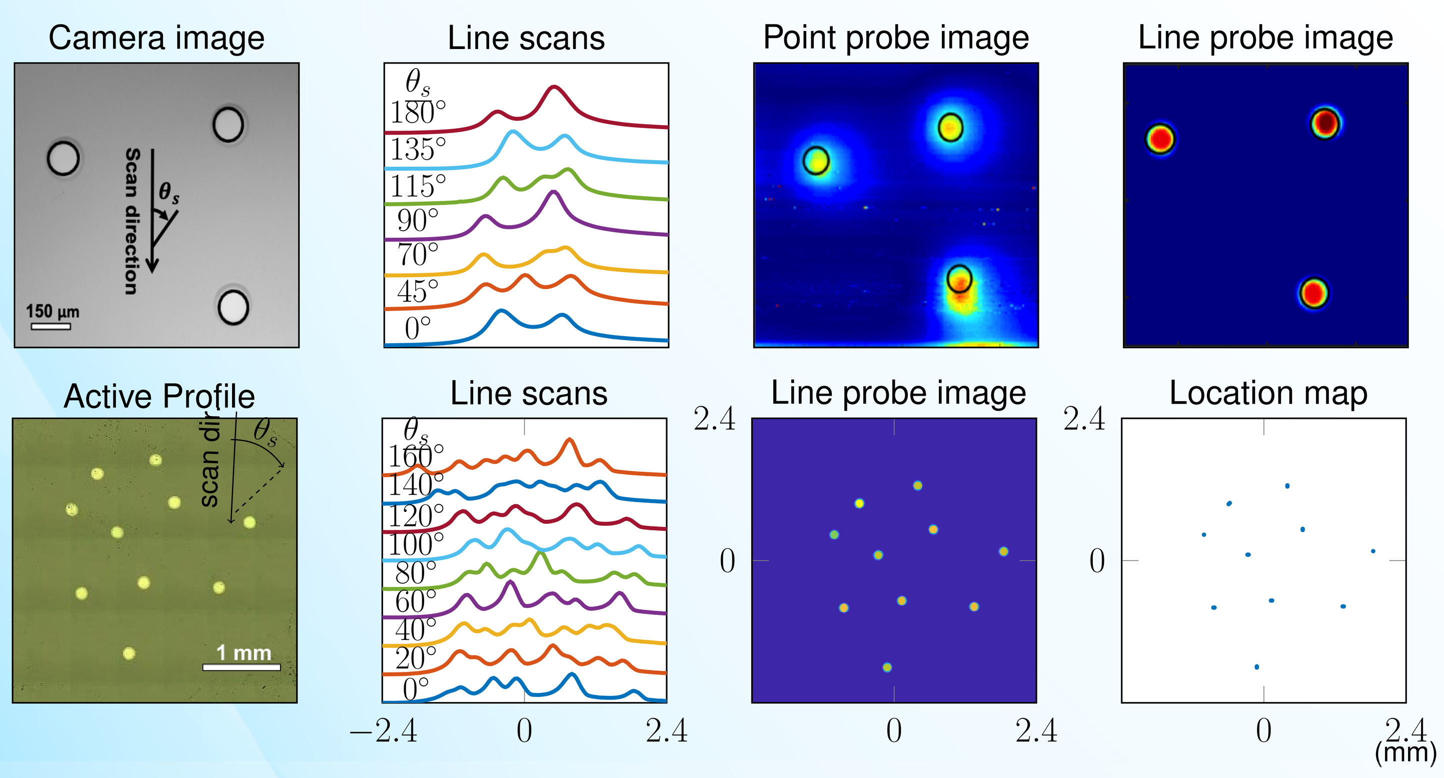
*We solve minimization for location map $\boldsymbol{X}$ and PSF parameters $\boldsymbol{p}$*

$$\min_{\boldsymbol{X}\geq 0, \boldsymbol{p}\in\mathcal{P}} \sum_{ij} \boldsymbol{\lambda}_{ij}^{(k)} \boldsymbol{X}_{ij} + \sum_{i=1}^m \tfrac{1}{2} \left\| \mathcal{S}\{\boldsymbol{\psi}(\boldsymbol{p}_i) * \mathcal{L}_{\theta_i}[\boldsymbol{D} * \boldsymbol{X}]\} - \boldsymbol{R}_i \right\|_2^2$$

*with reweighed sparse penalty* $\boldsymbol{\lambda}_{ij}^{(k)} = C/(\boldsymbol{X}_{ij} + \varepsilon)$.

### Real Data Experiments

- We demonstrate reconstruction from line scan on 3, 10 Pt discs samples.





### References

[1] O'Neil, G. D., Kuo H. W., Lomax, D. N., Wright, J. and Esposito, D. V., "Scanning Line Probe Microscopy: Beyond the Point Probe.", Analytical chemistry 90.9 (2018).

[2] Candès, E. J. and Fernandez-Granda, C., "Toward a mathematical theory of super-resolution", *Comm. on pure and applied Mathematics* 67.6 (2014).


Columbia University in the City of New York

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